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# INFORMATION REVELATION IN ALL-PAY CONTESTS WITH UNCERTAINTIES ABOUT THE NUMBER OF PARTICIPANTS

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Abstract. *In contests, the number of participants usually is private information of contest designers and whether and how to reveal this information to participants is one important decision for contest designers. Based on game theory, this paper addresses this problem in an all-pay contest setting. Two kinds of objectives in contests (i.e., maximizing the expected aggregate effort and maximizing the expected highest effort) are considered. First, the contest designer is assumed to have complete commitment ability. That is, he can ex-ante commit to his revelation policy before he learns the number of participants. The results show that if he wants to maximize the expected highest effort, then he should always commit to reveal his information. However, if he wants to maximize the expected aggregate effort, then his revelation policy depends on prior information about the number of participants. Further, the commitment ability of the contest designer is assumed to be limited and he decides strategically whether and how to reveal his information after he observes the number of participants. The results show that in this case the contest designer cannot conceal his information and the exact number of participants is always revealed.*

**Keywords:** All-pay contests, Information revelation, The number of participants, Game theory, Commitment

1. **Introduction.** Contests are widely used to describe economic, political and social activities with irreversible investments such as political lobbying, job promotions, sport competitions, patent races and R&D races (e.g., [1-4]). In these activities, participants exert efforts in order to win prizes and bear the costs of their efforts whether they win or not. Contest designers benefit from all participants' efforts or only benefit from the highest effort. For example, in a job promotion within a firm, the participant who makes the highest effort wins and yet the firm benefits from all participants' efforts. Thus, when the firm designs the contest, his goal is to maximize participants' aggregate efforts. While in a competition for construction contracts or in a patent race, the contest designer is only interested in the best achievement and what he wants is to maximize the highest effort. Surprisingly, most of the existing studies on contests have focused on the problem of maximizing aggregate effort (e.g., [5-12]) and only a few of studies have considered the problem of maximizing the highest effort (e.g., [13,14]).

Most of the literature on contests focuses on the setting in which the number of participants is fixed and known by the contest designer and participants (e.g., [15-19]). However, many real-life contests do not match with this setting. For example, the government invites some selected constructions firms to participate in the competition for a construction contract. Each firm's decision about whether to accept this invitation is his own secret. Therefore, the number of participants is ex-ante unknown to both the government and all constructions firms. However, as the designer of the contest, the government can know exactly the number of participants after he receives every firm's response to the invitation. However, constructions firms usually cannot have access to this information. An analogous situation also occurs in job promotion competitions and R&D races. This information asymmetry between the contest designer and participants provides the contest designer with the opportunity to manipulate participants' posterior beliefs about the number of competitors and then to affect their efforts by strategically concealing or revealing his private information. Then, questions naturally arise. Should the contest designer reveal the number of participants if he wants to maximize participants' aggregate efforts, and how should he do if he only wants to maximize the highest effort? This paper aims to address such these questions in an all-pay contest setting with private valuations.

In this paper, the number of participants is no longer confined to follow the binomial distribution or the Poisson distribution. And its realized number is privately known by the contest designer. Every participant only knows his own valuation for the prize, but he knows neither the number of competitors nor the valuations of competitors. Our equilibrium analysis result shows that the expected effort of each participant reaches its maximum level when he only knows that there is at least one competitor. Then following the classical literature on information revelation, we assume that the contest designer has complete commitment ability. That is, he can commit to a revelation policy before he learns the realized number of participants. After observing that realized number, he faithfully fulfills his pre-commitment. Under the assumption that participants' valuations for the prize are independently and uniformly distributed, we show that the contest designer should always commit to reveal fully his information if he wants to maximize the expected highest effort. However, this result no longer holds true if the contest designer wants to maximize the expected aggregate effort.

We further consider the case where the contest designer cannot make a pre-commitment and only can decide his revelation strategy after he learns the realized number of participants. For every realized number of participants, he makes a choice among fully concealing his information, fully revealing his information and partially revealing his information (i.e., revealing a range in which the number of participants lies). However, when he makes his decision in this case, he should take rational participants' responses to his revelation strategy into account, because rational participants can anticipate how he does and then can infer more precise information about the number of participants. For example, the contest designer will fully conceal his information if and only if there is only one participant. Therefore, the rational participant can infer that he is the only participant if the contest designer keeps silent. In other word, it is impossible and unnecessary for the contest designer to conceal this information. Our result shows this result always holds true no matter what the realized number of participants is and no matter what the goal of the contest designer is.

Although contests were extensively studied since Tullock's famous paper, only a handful of studies considered contests with a random number of participants. And most of them only investigated how uncertainties about the number of participants affected participants' equilibrium efforts and did not involve whether the contest designer should reveal the number of participants (e.g., [20-25]). Only a few of papers considered whether to reveal the number of participants in contests and auctions (e.g., [26-29]). For example, in a Tullock contest setting where participants had identical and commonly known valuations for the prize, Fu et al. [27], Feng and Lu [28], and Chen et al. [29] found that whether the contest designer should commit to reveal the number of participants depending on the concave-convex feature of a characteristic function. Without exception, the contest designer was assumed to maximize the aggregate effort in these papers. However, in this paper we consider both the goal of maximizing the aggregate effort and the goal of maximizing the highest effort. As far as we know, we are the first one to study revelation of the number of participants under the goal of maximizing the highest effort.

Moreover, most of the literature on information revelation assumed that the contest designer had commitment ability and then analyzed whether he should pre-commit to reveal his information (e.g., [30,31]), but the study on strategic revelation of the contest designer who has no commitment ability is rather rare. Using the same model as Fu et al. [27], Lim and Matros [32] concluded that the exact number of participants was always revealed when the contest designer decided whether to reveal his information after he learned the realized number of participants. In this paper, we find that this result also holds true, no matter the contest designer wants to maximize the aggregate effort or just wants to maximize the highest effort. In their analyses, Lim and Matros assumed that the number of participants followed the binomial distribution and the contest designer either fully concealed or fully revealed his information. However, in this paper, we allow the contest designer to reveal partly his information (i.e., revealing a range in which the number of participants lies).

The rest of this paper is organized as follows. Section 2 introduces a contest with private valuations and uncertainties about the number of participants. Section 3 solves the symmetric Bayesian equilibrium and analyzes how information about the number of participants affects the effort which is exerted by a participant. Under two different goals, Section 4 discusses the optimal pre-commitment of the contest designer and Section 5 studies his strategic revelation. Section 6 concludes this paper.

## 2. **The Model.**

Consider a contest in which there is one indivisible prize and $2 \leq m \leq +\infty$ potential participants. The number of actual participants is a random variable, which is denoted by $\xi$ and distributed according to a commonly known prior probability distribution $\Pr(\xi = n) = p_n$, $n = 0, 1, 2, \ldots, m$. That is, there are $n$ participants in the contest with a probability of $p_n$. Obviously, $0 \leq p_n < 1$ and $\sum_{n=0}^{m} p_n = 1$. Only the contest designer knows the realized number of participants. In order to maximize the aggregate effort of participants or the highest effort of participants, he has to decide whether and how to reveal this information to participants before they exert efforts. We consider two different cases. In one case, the contest designer commits to always reveal or conceal the number of participants before he learns its realized number. In the other case, the contest designer strategically decides whether and how to reveal the number of participants after he learns the realized number.

Participants' valuations for the prize are independently drawn from the probability distribution function $F(\cdot)$ with the density function $f(\cdot)$ and the support $[0, 1]$, which is common knowledge. However, each participant's valuation for the prize is his own private information.

All participants simultaneously exert efforts and the participant who exerts the highest effort certainly wins the prize. Ties are broken randomly. Efforts are costly and non-refundable. Therefore, all participants must bear the costs of their efforts whether they win or not. In other word, the contest takes the form of an all-pay auction. In this paper, the marginal cost of an effort is assumed to be the same for all participants and is normalized to 1. Therefore, the payoff of a participant who has a valuation $v$ and exerts an effort $b$ is $v - b$ if he wins. Otherwise, his payoff is $-b$.

After receiving information released by the contest designer, participants update their beliefs about the number of participants. For ease of narration, the following notations are used in the later analyses.

$\xi \in \Theta$: information about the number of participants. That is, the number of participants $\xi$ lies in the set $\Theta$, where $\Theta \subset \{0, 1, 2, \ldots, m\}$.

$\theta_{n,\Theta}$: the posterior probability of having $n \in \Theta$ actual participants in the contest.

$b(v, \Theta)$: the symmetric equilibrium effort function when participants have information $\xi \in \Theta$.

$b_n(v)$: the symmetric equilibrium effort function when the number of participants $n$ is fixed and commonly-known.

Therefore, according to Bayes rules, we have

$$\theta_{n,\Theta} = \Pr(\xi = n \,|\xi \in \Theta) = \frac{p_n}{\sum\limits_{n \in \Theta} p_n}, \quad \forall n \in \Theta. \tag{1}$$

Obviously, if $\Theta$ is a set with only one element, then participants know the exact number of participants.

## 3. **Participants' Equilibrium Efforts.**

Participants exert efforts to maximize their individual payoffs. A higher valuation for the prize impels a participant to exert more effort. Therefore, we assume that $\frac{db(v,\Theta)}{dv} > 0$.

**Theorem 3.1.** *When participants have information $\xi \in \Theta$, the symmetric equilibrium effort which is exerted by a participant with valuation $v$ is*

$$b(v, \Theta) = \sum_{n \in \Theta} \theta_{n,\Theta} \left[ vF^{n-1}(v) - \int_0^v F^{n-1}(y)dy \right], \tag{2}$$

*where $\theta_{n,\Theta}$ is defined as Equation (1).*

**Proof:** Suppose all other participants follow this symmetric equilibrium. One participant (participant 1) with valuation $v$ mimics the participant with valuation $x$ and exerts an effort $b(x, \Theta)$. By the total probability formula, participant 1 wins the prize with a probability of $\sum\limits_{n \in \Theta} \theta_{n,\Theta} F^{n-1}(x)$ and his expected payoff is $E(v, x) = v \sum\limits_{n \in \Theta} \theta_{n,\Theta} F^{n-1}(x) - b(x, \Theta)$.

The maximization of $E(v, x)$ with respect to $x$ leads to the following equation

$$\frac{\partial E(v, x)}{\partial x} = v \sum_{n \in \Theta} \theta_{n,\Theta}(n - 1)F^{n-2}(x)f(x) - \frac{db(x, \Theta)}{dx} = 0. \tag{3}$$

At the symmetric equilibrium $x = v$, therefore, Equation (3) yields

$$v \sum_{n \in \Theta} \theta_{n,\Theta}(n - 1)F^{n-2}(v)f(v) - \frac{db(v, \Theta)}{dv} = 0. \tag{4}$$

Combining the boundary condition $b(0, \Theta) = 0$ with Equation (4), we solve that $b(v, \Theta) = \sum\limits_{n \in \Theta} \theta_{n,\Theta} \left[vF^{n-1}(v) - \int_0^v F^{n-1}(y)dy\right]$. It is easy to verify $\frac{db(v,\Theta)}{dv} = \sum\limits_{n \in \Theta} \theta_{n,\Theta} v$ $F^{n-1}(v) > 0$. By Equation (3) and Equation (4), we have $\frac{\partial E(v,x)}{\partial x} = \sum\limits_{n \in \Theta} \theta_{n,\Theta}(n-1)(v-x)$ $F^{n-2}(x)f(x)$. If $x < v$, then $\frac{\partial E(v,x)}{\partial x} > 0$. If $x > v$, then $\frac{\partial E(v,x)}{\partial x} < 0$. Thus, maximization of $E(v, x)$ is indeed achieved at $x = v$ and $b(v, \Theta) = \sum\limits_{n \in \Theta} \theta_{n,\Theta} \left[vF^{n-1}(v) - \int_0^v F^{n-1}(y)dy\right]$ is the symmetric equilibrium. □

From a special case of Theorem 3.1, we easily get the expression of $b_n(v)$ and the following corollary.

**Corollary 3.1.** *If the number of participants $n$ is fixed and commonly-known, then the symmetric equilibrium effort which is exerted by a participant with valuation $v$ is*

$$b_n(v) = vF^{n-1}(v) - \int_0^v F^{n-1}(y)dy. \quad (5)$$

Note that $b_n(v)$ is a non-monotonic function of $n$. For example, if $v$ is distributed uniformly over the interval $[0, 1]$, then $b_n(v) = \frac{n-1}{n}v^n$ is a strictly increasing function of $n$ if $n < n^* = \left[\sqrt{\frac{1}{1-v}}\right] + 1$ and is a strictly decreasing function of $n$ if $n > n^*$. Here, $\left[\sqrt{\frac{1}{1-v}}\right]$ is the maximum integer which is no more than $\sqrt{\frac{1}{1-v}}$. Especially, if $0 < v < \frac{3}{4}$, then $n^* = 2$ and $b_n(v)$ decreases as $n \geq 2$ increases. If $v$ is large enough (very close to 1), then $n^* \to \infty$ and $b_n(v)$ always increases as $n$ increases. In other words, participants' responses to the increasing number of competitors depend on their valuations.

Using Equation (5), we can rewrite Equation (2) as follows

$$b(v, \Theta) = \sum_{n \in \Theta} \theta_{n,\Theta} b_n(v). \quad (6)$$

That is, the effort $b(v, \Theta)$ when a participant is ambiguous about the number of competitors is a weighted average of the effort $b_n(v)$ when he knows exactly that there are $n-1$ competitors and the weight is the probability of having $n-1$ competitors. Owing to the fact that $b_n(v)$ is a non-monotonic function of $n$, it is difficult to know intuitively that how information $\xi \in \Theta$ affects participants' efforts.

**Theorem 3.2.** *If $\Theta$ is an arbitrary subset of the set $\{2, 3, \ldots, m\}$, then $b(v, \Theta) \geq b(v, \Theta \cup \{1\})$. If $p_1 > 0$, then the inequality strictly holds.*

**Proof:** According to Equation (6) and $b_1(v) = 0$, we have $b(v, \Theta \cup \{1\}) = \sum_{n \in \Theta} \theta_{n,\Theta\cup\{1\}} b_n(v)$ and $b(v, \Theta) = \sum_{n \in \Theta} \theta_{n,\Theta} b_n(v)$. Thus, $b(v, \Theta\cup\{1\}) - b(v, \Theta) = \sum_{n \in \Theta} \left(\theta_{n,\Theta\cup\{1\}} - \theta_{n,\Theta}\right) b_n(v)$.

By Equation (1), we know that for $\forall n \in \Theta$, $\theta_{n,\Theta\cup\{1\}} = \frac{p_n}{p_1 + \sum_{n \in \Theta} p_n} \leq \frac{p_n}{\sum_{n \in \Theta} p_n} = \theta_{n,\Theta}$. Thus, $b(v, \Theta \cup \{1\}) - b(v, \Theta) \leq 0$. If $p_1 > 0$, then $\theta_{n,\Theta\cup\{1\}} < \theta_{n,\Theta}$ and $b(v, \Theta \cup \{1\}) - b(v, \Theta) < 0$. □

Theorem 3.2 implies that letting participants clearly realize competition (i.e., the existence of competitors) is beneficial to the contest designer. This conclusion relies on neither the prior distributions of the number of participants nor the distributions of participants' valuations. Then, does a participant exert higher effort when he knows the existence of more competitors? The following theorem indicates that the answer is negative.

**Theorem 3.3.** *If participants' valuations are distributed uniformly over the interval $[0, 1]$, then the expected effort of each participant when he knows the existence of at least $s - 1$ competitors is a monotone non-increasing function of $s \geq 2$.*

**Proof:** Let $\Theta_s = \{s, s+1, \ldots, m\}$, $s \geq 2$ and let $\Gamma(s)$ be the expected effort of each participant when he knows the existence of at least $s-1$ rivals (i.e., $\xi \in \Theta_s$). By Equation

(5) and Equation (6), we have

$$\Gamma(s) = E(b(v,\Theta_s) = E\left(\sum_{n\in\Theta_s} \theta_{n,\Theta_s}\frac{n-1}{n}v^n\right)$$
$$= \int_0^1 \sum_{n\in\Theta_s} \theta_{n,\Theta_s}\frac{n-1}{n}v^n dv = \sum_{n=s}^{m} \theta_{n,\Theta_s}\frac{n-1}{n(n+1)},$$
$$\Gamma(s+1) = E(b(v,\Theta_{s+1}) = E\left(\sum_{n\in\Theta_{s+1}} \theta_{n,\Theta_{s+1}}\frac{n-1}{n}v^n\right)$$
$$= \int_0^1 \sum_{n\in\Theta_{s+1}} \theta_{n,\Theta_{s+1}}\frac{n-1}{n}v^n dv = \sum_{n=s+1}^{m} \theta_{n,\Theta_{s+1}}\frac{n-1}{n(n+1)}.$$

In the above calculation, we exchange the order of sums and integrations. If $m < +\infty$, then the exchange is straightforward. If $m = +\infty$, the exchange is also feasible according to the property of term-by-term integration.

Let $g(n) = \frac{n-1}{n(n+1)}$, then $\Gamma(s) = \sum_{n=s}^{m} \theta_{n,\Theta_s} g(n)$ and $\Gamma(s+1) = \sum_{n=s+1}^{m} \theta_{n,\Theta_{s+1}} g(n)$. Therefore,

$$\Gamma(s+1) - \Gamma(s) = \sum_{n=s+1}^{m} \left(\theta_{n,\Theta_{s+1}} - \theta_{n,\Theta_s}\right) g(n) - \theta_{s,\Theta_{s+1}} g(s). \tag{7}$$

Using the fact that $\sum_{n=s+1}^{m} \theta_{n,\Theta_{s+1}} = \sum_{n=s}^{m} \theta_{n,\Theta_s} = 1$ and then $\sum_{n=s+1}^{m} \left(\theta_{n,\Theta_{s+1}} - \theta_{n,\Theta_s}\right) = \theta_{s,\Theta_{s+1}}$, we simplify Equation (7) to be $\Gamma(s+1) - \Gamma(s) = \sum_{n=s+1}^{m} (\theta_{n,\Theta_{s+1}} - \theta_{n,\Theta_s})(g(n) - g(s))$. According to Equation (1), $\theta_{n,\Theta_{s+1}} \geq \theta_{n,\Theta_s}$ holds for any $n \geq s+1$. Moreover, for any $n \geq s+1 \geq 3$, $g(n) \leq g(s+1) \leq g(3)$ holds because $g(n) = \frac{n-1}{n(n+1)}$ strictly decreases as $n \geq 3$ increases. Thus, $\Gamma(s+1) - \Gamma(s) \leq 0$. □

Theorem 3.3 indicates that when a participant knows the existence of more competitors (the larger $s \geq 2$), his expected effort is lower. This result is rather intuitive In order to maximize the expected payoff of a participant, he should increase the probability of his winning and decrease his payment. When he knows there are at least $s-1$ competitors, his probability of his winning $\sum_{n\in\Theta_s} \theta_{n,\Theta_s} F^{n-1}(v) = \sum_{n=s}^{m} \theta_{n,\Theta_s} F^{n-1}(v)$ strictly decreases as $s$ increases. Because he must pay for his effort whether he wins or not, he decreases his effort to increase his expected payoff.

## 4. **Optimal Pre-Commitments.**

In this section, we suppose that the contest designer can commit to always conceal or reveal the number of participants before he learns the exact value of that number. The timing of the game is as follows:

(1) The contest designer makes a public commitment about his revelation policy;

(2) Nature chooses the number of participants;

(3) The contest designer observes the realized number of participants and reveals this information to participants if and only if he has committed to reveal it;

(4) Participants choose their individual efforts simultaneously and the participant with the highest effort is picked as the winner.

If the contest designer commits to conceal his information, every participant only knows that there is at least one participant. In other word each participant has information $\xi \in \Theta = \{1, 2, \ldots, m\}$. If the contest designer commits to reveal his information, every participant exactly knows the realized number of participants. Which one is preferable for the contest designer, concealing or revealing the number of participants? In the following sections, we address this question under the goal of maximizing the expected aggregate efforts and under the goal of maximizing the expected highest effort, respectively. For feasibility of theoretical analyses, participants' valuations are assumed to follow the uniform distribution over the interval $[0,1]$ in the later analyses.

4.1. **Maximizing the expected highest effort.** By Equation (1) and Equation (2), a participant with valuation $v$ exerts the effort $b(v,\Theta) = \sum_{n=1}^{m} \theta_{n,\Theta} \frac{n-1}{n} v^n = \sum_{n=1}^{m} \frac{p_n}{1-p_0} \frac{n-1}{n} v^n$ when the contest designer commits to conceal his information. If there are $k$ participants, then the expected highest effort is $E\left(b\left(v_k^{(1)},\Theta\right)\right) = \sum_{n=1}^{m} \frac{p_n}{1-p_0} \frac{k(n-1)}{n(n+k)}$, where $v_k^{(1)}$ is the highest valuation of $k$ participants. Therefore, when the contest designer commits to conceal his information, the ex-ante expected highest effort is

$$\Pi_c = \sum_{k=0}^{m} p_k \times \left( \sum_{n=1}^{m} \frac{p_n}{1-p_0} \frac{k(n-1)}{n(n+k)} \right) = \sum_{k=1}^{m} p_k \times \left( \sum_{n=2}^{m} \frac{p_n}{1-p_0} \frac{k(n-1)}{n(n+k)} \right). \qquad (8)$$

According to Equation (5), a participant with valuation $v$ exerts an effort $b_k(v) = \frac{k-1}{k} v^k$ and then the expected highest effort is $Eb_k\left(v_k^{(1)}\right) = E\left(\frac{k-1}{k}\left(v_k^{(1)}\right)^k\right) = \frac{k-1}{2k}$ if the contest designer commits to reveal his information and there are $k$ participants. Therefore, when the contest designer commits to reveal his information, the ex-ante expected highest effort is

$$\Pi_r = 0 \times p_0 + \sum_{k=1}^{m} p_k \times \frac{k-1}{2k} = \sum_{k=2}^{m} p_k \times \frac{k-1}{2k}. \qquad (9)$$

**Theorem 4.1.** *If participants' valuations are distributed uniformly over the interval* $[0,1]$*, then the contest designer who wants to maximize the expected highest effort should ex-ante commit to reveal his information.*

**Proof:** Let $\Delta = \Pi_c - \Pi_r$ and $z(n,k) = \frac{k(n-1)}{n(n+k)}$. Since $1 - p_0 = \sum_{n=1}^{m} p_n$, we have

$$\begin{aligned}
\Delta = \Pi_c - \Pi_r &= \frac{1}{1-p_0}\left[\sum_{k=1}^{m}\sum_{n=2}^{m} p_k p_n z(n,k) - \sum_{k=2}^{m}\sum_{n=1}^{m} p_k p_n z(k,k)\right] \\
&= \frac{1}{1-p_0}\left\{\sum_{n=2}^{m} p_1 p_n [z(n,1) - z(n,n)] + \sum_{k,n=2}^{m} p_1 p_n [z(n,k) - z(k,k)]\right\} \\
&= \frac{1}{1-p_0}\left\{\sum_{n=2}^{m} p_1 p_n \frac{-(n-1)^2}{2n(n+1)} + \sum_{\substack{k,n=2 \\ k\neq n}}^{m} p_1 p_n \frac{-(n-k)^2}{2nk(n+k)}\right\} < 0.
\end{aligned}$$

□

In a winner-pay auction where only the winner needs to pay and the auctioneer gets the highest bid, McAfee and McMillan [30] pointed out that it is undifferentiated for the auctioneer to commit to conceal or reveal the number of participants. However, we find that this result is invalid in an all-pay auction and committing to reveal the number of participants is more beneficial to the contest designer.

4.2. **Maximizing the expected aggregate effort.** By Equation (1) and Equation (2), the expected effort of a participant is $E(b(v,\Theta)) = \sum_{n=1}^{m} \theta_{n,\Theta}\frac{n-1}{n}E(v^n) = \sum_{n=1}^{m} \frac{p_n}{1-p_0}\frac{n-1}{n(n+1)}$ when the contest designer commits to conceal his information. Therefore, if the contest designer commits to conceal his information, then the ex-ante expected aggregate effort is

$$\Pi_c = \sum_{k=0}^{m} p_k \left( k \sum_{n=1}^{m} \frac{p_n}{1-p_0}\frac{n-1}{n(n+1)} \right) = \sum_{k=1}^{m} kp_k \times \Lambda, \tag{10}$$

where $\Lambda = \sum_{n=1}^{m} b_n g(n) = b_k g(k) + \sum_{\substack{n=1 \\ n\neq k}}^{m} b_n g(n)$ is the weighted average of $g(n) = \frac{n-1}{n(n+1)}$ and the weight is $b_n = \frac{p_n}{1-p_0}$.

According to Equation (5), the expected effort of a participant is $E(b_k(v)) = \frac{k-1}{k(k+1)}$ when the contest designer commits to reveal his information and there are $k$ participants. Therefore, if the contest designer commits to reveal his information, then the ex-ante expected aggregate effort is

$$\Pi_r = \sum_{k=1}^{m} p_k \times k \times \frac{k-1}{k(k+1)} = \sum_{k=1}^{m} kp_k \times g(k). \tag{11}$$

Unfortunately, it is impossible to directly compare $\Lambda = \sum_{n=1}^{m} b_n g(n) = b_k g(k) + \sum_{\substack{n=1 \\ n\neq k}}^{m} b_n g(n)$ with $g(k)$ so that $\Pi_r$ and $\Pi_c$ are not directly comparable. The following examples show whether the contest designer should commit to reveal his information depends on the prior distribution of the number of participants.

**Example 4.1.** *Let $m = 4$ and $p_0 = 0$. We consider the following two cases.*

*Case 1: The prior distribution of the number of participants is $p_1 = p_2 = 0.1$, $p_3 = 0.3$, $p_4 = 0.5$. Note that $g(1) < \Lambda < g(4) < g(2) = g(3)$. That is, only when the realized number of participant is one, the contest designer benefits from concealing the number of participants. However, the probability of having only one participant in the contest is very small. As a result, $\Pi_c = 0.45333 < \Pi_r = 0.48333$. Therefore, the contest designer should commit to reveal his information.*

*Case 2: The prior distribution of the number of participants is $p_1 = p_3 = 0$, $p_2 = p_4 = 0.5$. We have $g(2) > \Lambda > g(4)$ and $\Pi_c = 0.475 > \Pi_r = 0.4667$. Therefore, the contest designer should commit to conceal his information.*

**Example 4.2.** *Assume that each potential participant participates in the contest with a probability $0 < p < 1$. In other word, the prior distribution of the number of participants is the binomial distribution with parameters $p$ and $m$. For different values of $p$ and $m$, Table 1 lists the difference between the expected aggregate effort under concealing policies and one under revealing policies.*

From Table 1, we can see how the contest designer chooses his information strategy depends on the values of $p$ and $m$. Especially, if the participation probability of each potential participant $p$ is very small (e.g., $p = 0.1$) or the number of potential participants $m$ is very small (e.g., $m = 3$), then the contest designer should always commit to reveal his information.

The results of these two examples are rather intuitive. In Case 1 of Example 4.1, it is likely to have fewer or no competitors in the contest. Similarly, when $p$ is very small or $m$ is very small in Example 4.2, the competition pressure of a participant is small and

TABLE 1. $\Delta = \Pi_c - \Pi_r$ as a function of $p$ and $m$

| $\Delta \backslash p$ / $m$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|---|---|---|---|---|---|---|---|---|---|
| 3 | −0.0043 | −0.0147 | −0.0272 | −0.0382 | −0.0446 | −0.0443 | −0.0365 | −0.0227 | −0.0077 |
| 4 | −0.007 | −0.0241 | −0.0394 | −0.0474 | −0.0453 | −0.0341 | −0.0183 | −0.0039 | 0.0028 |
| 5 | −0.012 | −0.0329 | −0.0469 | −0.0472 | −0.0349 | −0.0167 | −0.0006 | 0.0076 | 0.0068 |
| 6 | −0.016 | −0.0402 | −0.0494 | −0.0400 | −0.0199 | −0.0001 | 0.0116 | 0.0133 | 0.0081 |
| 7 | −0.021 | −0.0456 | −0.0473 | −0.0289 | −0.0047 | 0.0125 | 0.0185 | 0.0156 | 0.0084 |
| 8 | −0.025 | −0.0488 | −0.0419 | −0.0162 | 0.0084 | 0.0211 | 0.0220 | 0.0163 | 0.0082 |
| 9 | −0.029 | −0.0499 | −0.0339 | −0.0037 | 0.0187 | 0.0263 | 0.0235 | 0.0162 | 0.0079 |
| 10 | −0.034 | −0.0493 | −0.0247 | 0.0077 | 0.0262 | 0.0293 | 0.0239 | 0.0157 | 0.0075 |
| 11 | −0.038 | −0.0469 | −0.0149 | 0.0174 | 0.0314 | 0.0307 | 0.0236 | 0.0151 | 0.0071 |
| 12 | −0.041 | −0.0431 | −0.0052 | 0.0253 | 0.0347 | 0.0311 | 0.0230 | 0.0145 | 0.0067 |
| 13 | −0.044 | −0.0382 | −0.0040 | 0.0315 | 0.0368 | 0.0310 | 0.0223 | 0.0139 | 0.0064 |
| 14 | −0.046 | −0.0326 | 0.0125 | 0.0362 | 0.0378 | 0.0304 | 0.0215 | 0.0132 | 0.0061 |
| 15 | −0.048 | −0.0264 | 0.0200 | 0.0396 | 0.0381 | 0.0297 | 0.0207 | 0.0127 | 0.0058 |
| 16 | −0.049 | −0.0199 | 0.0266 | 0.0419 | 0.0379 | 0.0288 | 0.0199 | 0.0121 | 0.0055 |
| 17 | −0.0499 | −0.0132 | 0.0322 | 0.0435 | 0.0375 | 0.0280 | 0.0191 | 0.0116 | 0.0053 |
| 18 | −0.0503 | −0.0065 | 0.0368 | 0.0444 | 0.0369 | 0.0271 | 0.0184 | 0.0111 | 0.0051 |
| 19 | −0.0502 | 0 | 0.0407 | 0.0448 | 0.0361 | 0.0263 | 0.0177 | 0.0107 | 0.0049 |
| 20 | −0.0497 | 0.0062 | 0.0438 | 0.0449 | 0.0353 | 0.0254 | 0.0171 | 0.0103 | 0.0047 |

he believes that it is of great possibility to have no competitors. Therefore, it is possible for a participant to exert a lower effort and win. At this time, revealing the number of participants can eliminate this fluke mind of a participant and impel them to exert higher efforts.

While in Case 2 of Example 4.1, a participant knows there are one or three competitors. According to Theorem 3.3, explicitly knowing the existence of more competitors discourages a participant and impels him to exert a lower effort. Similarly, if both the number of potential participants $m$ and the participation probability of each potential participant $p$ are large in Example 4.2, then the competition is fierce and it is of great possibility to have many competitors. Comparing with revealing the number of participants which makes the fierce competition clear, concealing it can alleviate this competition pressure by giving a participant a hope that it is still likely to have fewer competitors.

5. **Strategic Revelation of the Contest Designer.** In this section, the contest designer is assumed to strategically decide whether to reveal his information or not after he learns the number of participants. The timing of the game is as follows:

(1) Nature chooses the number of participants;

(2) The contest designer observes the realized number of participants and then decides whether and how to reveal his information;

(3) Participants choose their individual efforts simultaneously and the participant with the highest effort wins the prize.

Obviously, his decision depends on the realized number of participants, which is denoted by $k$. According to Theorem 3.2, the contest designer fully conceals his information if and only if $k = 1$. Thus, in the following analyses, we assume $k \geq 2$. Then, should the contest designer reveal the exact number of $k$ or a range in which $k$ lies in?

We use $\xi \in \Omega$ to denote information released by the contest designer. Obviously, $\Omega$ is a subset of the set $\{2, 3, \dots, m\}$. Moreover, $k \in \Omega$ because information released by the contest designer must be true. Note that information released by the contest designer is not always the information held by participants because participants are aware of the contest designer's strategic behavior and they can infer more accurate information about the number of participants from information released by the contest designer. For example, the participant realizes that he is the only participant if the contest designer keeps silent (or does not reveal any additional information). Therefore, when the contest designer decides his revelation strategy, he should take participants' responses into account. We still use $\xi \in \Theta$ to denote information held by participants at the time they choose efforts. Obviously, $\Theta \subset \Omega$.

5.1. **Maximizing the expected highest effort.** By Equation (2), a participant with valuation $v$ exerts an effort $b(v,\Theta) = \sum\limits_{n\in\Theta} \theta_{n,\Theta}\frac{n-1}{n}v^n$. Therefore, the expected highest effort is $E\left(b\left(v_k^{(1)},\Theta\right)\right) = \int_0^1 \left(\sum\limits_{n\in\Theta}\theta_{n,\Theta}\frac{n-1}{n}v^n\right)kv^{k-1}dv = \sum\limits_{n\in\Theta}\theta_{n,\Theta}\frac{k(n-1)}{n(n+k)}$, where $v_k^{(1)}$ is the highest valuation of the $k$ participants. Let $h(n) = \frac{k(n-1)}{n(n+k)}$. Therefore, the expected highest effort can be rewritten to be

$$E\left(b\left(v_k^{(1)},\Theta\right)\right) = \theta_{k,\Theta}h(k) + \sum_{n\in\Theta,n\neq k}\theta_{n,\Theta}h(n). \tag{12}$$

In other word, the expected highest effort $E\left(b\left(v_k^{(1)},\Theta\right)\right)$ is a weighted average of $h(n)$. Thus, endowing a bigger weight $\theta_{n,\Theta}$ to the bigger function value $h(n)$ can increase the highest effort.

Notice that $h(n) = \frac{k(n-1)}{n(n+k)}$ is a monotonic increasing function of $n < n^*$ and is a monotonic decreasing function of $n > n^*$. Here, $n^* = \left[\frac{1+\sqrt{5+4k}}{2}\right] + 1$ and $\left[\frac{1+\sqrt{5+4k}}{2}\right]$ is the greatest integer which is no more than $\frac{1+\sqrt{5+4k}}{2}$. Especially, if $k = 2$ or 3, then $n^* = 3$. If $k \geq 4$, then $3 \leq n^* < k$.

**Theorem 5.1.** *If the contest designer who wants to maximize the expected highest effort decides his revelation strategy after he knows the number of participants, then the exact number of participants is always revealed.*

**Proof:** When $k = 3$, $h(n)$ reaches the maximum at $n^* = 3$. That is, $h(3) > h(n)$ holds for any $n \neq 3$. According to Equation (12),

$$E\left(b\left(v_k^{(1)},\Theta\right)\right) = \theta_{3,\Theta}h(3) + \sum_{n\in\Theta,n\neq 3}\theta_{n,\Theta}h(n) \leq h(3).$$

Obviously, only when $\Theta = \{3\}$, the greatest function value $h(3)$ is endowed the whole weight 1 and then the expected highest effort reaches the maximum $h(3)$.

Therefore, the contest designer must reveal the exact number of participants if $k = 3$. As a result, rational participants must know that there is zero probability with three participants when they are informed that $\xi \in \Omega$, where $\Omega \neq \{3\}$.

When $k = 4$, $h(4) > h(n)$ holds for any $n \neq 3, 4$. Information revealed by the contest designer is $\xi \in \Omega$ and $4 \in \Omega$; thus information held by participants must be $\xi \in \Theta = \Omega\backslash\{3\}$. According to Equation (12),

$$E\left(b\left(v_k^{(1)},\Theta\right)\right) = \theta_{4,\Theta}h(4) + \sum_{n\in\Theta,n\neq 4}\theta_{n,\Theta}h(n) \leq h(4).$$

Similarly, when $\Theta = \{4\}$, the greatest function value $h(4)$ is endowed the whole weight 1 and then the expected highest effort reaches the maximum $h(4)$.

Therefore, the contest designer must also reveal the exact number of participants if $k = 4$. Then, rational participants know that there is zero probability with three or four participants when they are informed that $\xi \in \Omega$, where $\Omega \neq \{3\}$ and $\Omega \neq \{4\}$.

When $k = 2$, $h(2) > h(n)$ holds for any $n \geq 5$ and information held by participants must be $\xi \in \Theta = \Omega \backslash \{3, 4\}$. Therefore, according to Equation (12),

$$E\left(b\left(v_k^{(1)}, \Theta\right)\right) = \theta_{2,\Theta} h(2) + \sum_{n \in \Theta, n \neq 2} \theta_{n,\Theta} h(n) \leq h(2).$$

Similarly, when $\Theta = \{2\}$, the expected highest effort can reach the maximum $h(2)$.

Applying the same logic to $k = 5, 6, \ldots, m$ and using the fact that $h(k) > h(n)$ holds for any $n > k$, we can complete the remaining proof. □

From the proof of Theorem 5.1, it is easy to see that the contest designer must reveal the exact value of $k$ if $k = 3$. For any $k \neq 3$, seemingly revealing a wider range in which the number of participants lies may yield a bigger expected highest effort than revealing the exact number of participants. For example, assume $k = 4$. Seemingly, the expected highest effort $\theta_{4,\Theta} h(4) + \theta_{3,\Theta} h(3)$ when revealing $\xi \in \{3, 4\}$ is larger than the expected highest effort $h(4)$ when revealing $\xi \in \{4\}$ because $h(4) < h(3)$. However, it is really no use because rational participants know that the exact value of $k$ certainly is not 3. Because $h(4) > h(n)$ holds for any $n \neq 3, 4$, revealing other range rather than $\xi \in \{4\}$ will transfer the weight from the largest function value $h(4)$ to the smaller function value $h(n)$, which will lower the expected highest effort. Therefore, revealing the exact number of participants is the best choice for the contest designer. Applying the same reasoning, we can see that no matter what $k$ is, the exact number of participants is always revealed.

5.2. **Maximizing the expected aggregate effort.** By Equation (2), the expected effort of each participant is $E(b(v, \Theta)) = \sum_{n \in \Theta} \theta_{n,\Theta} \frac{n-1}{n} E(v^n) = \sum_{n \in \Theta} \theta_{n,\Theta} \frac{n-1}{n(n+1)}$ and then the expected aggregate effort is $kE(b(v, \Theta)) = k \sum_{n \in \Theta} \theta_{n,\Theta} \frac{n-1}{n(n+1)}$. Therefore, maximizing the expected aggregate effort is equivalent to maximizing the expected effort of each participant $E(b(v, \Theta))$. Let $g(n) = \frac{n-1}{n(n+1)}$, and then the expected effort of a participant is simplified to be

$$E(b(v, \Theta)) = \theta_{k,\Theta} g(k) + \sum_{n \in \Theta, n \neq k} \theta_{n,\Theta} g(n). \tag{13}$$

That is, the expected effort of a participant $E(b(v, \Theta))$ is a weighted average of $g(n)$. Endowing a bigger weight $\theta_{n,\Theta}$ to the bigger function value $g(n)$ can yield a higher expected effort. Using the fact that $g(2) = g(3) > g(s) > g(t)$ holds for any $3 < s < t$, we can get the following theorem.

**Theorem 5.2.** *If the contest designer who wants to maximize the expected aggregate effort decides his revelation strategy after he knows the number of participants, then the exact number of participants is always revealed.*

**Proof:** The proof is similar to that of Theorem 5.1. According to Equation (13),

$$E(b(v, \Theta)) = \theta_{k,\Theta} g(k) + \sum_{n \in \Theta, n \neq k} \theta_{n,\Theta} g(n) \leq g(2) = g(3).$$

When $k = 2$ or 3, obviously revealing the exact value of $k$ (i.e., $\xi \in \Omega = \{k\}$) or revealing $\xi \in \Omega = \{2, 3\}$ can endow the largest function value $g(k)$ with the whole weight 1 and then maximize the expected effort of each participant $E(b(v, \Theta))$.

Therefore, the contest designer must reveal $\xi \in \Omega = \{k\}$ or $\xi \in \Omega = \{2, 3\}$ if $k = 2$ or 3. As a result, rational participants know that there is zero probability with two or three participants when they are informed that $\xi \in \Omega$, where $\Omega \neq \{2\}$, $\Omega \neq \{3\}$ and $\Omega \neq \{2, 3\}$.

When $k = 4$, $g(4) > g(n)$ holds for any $n > 4$. Information revealed by the contest designer is $\xi \in \Omega$ and $4 \in \Omega$; thus information held by participants must be $\xi \in \Theta = \Omega \backslash \{2, 3\}$. According to Equation (13),

$$E(b(v, \Theta)) = \theta_{4,\Theta} g(4) + \sum_{n \in \Theta, n \neq 4} \theta_{n,\Theta} g(n) \leq g(4).$$

Obviously, when $\Theta = \{4\}$, the greatest function value $g(4)$ is endowed the whole weight 1 and then the expected effort of a participant reaches the maximum $g(4)$. Therefore, the contest designer must also reveal the exact number of participants if $k = 4$.

Using the fact that $g(n)$ is a strictly decreasing function of $n \geq 4$ and applying the same logic to $k = 5, 6, \dots, m$ recursively, we can obtain Theorem 5.2. □

Theorem 5.1 and Theorem 5.2 indicate that the inability to commit harms the contest designer. He cannot conceal his private information and then cannot benefit from his information advantage at this time. The exact number of participants is always known by participants, no matter the contest designer wants to maximize the expected aggregate effort or the expected highest effort.

## 6. Conclusions.

Whether to reveal the number of participants is an important theoretical and practical problem in the design of a contest. Under two different goals (i.e., maximizing the expected aggregate effort and maximizing the expected highest effort), this paper not only studies the optimal pre-commitment of the contest designer, but also analyzes his strategic revelation. Our results show that: (1) the contest designer benefits from his commitment ability if he has it. He can adjust his optimal pre-commitment according to his goal and prior information about the number of participants; (2) the inability to commit harms the contest designer because he cannot achieve his goal by strategically revealing or concealing information and the exact number of participants is always known by participants. Thus, it is important for the contest designer to strengthen his commitment ability. Our result may provide some managerial insights for the design of a contest.

In this paper, we assume that participants' valuations for the prize are uniformly distributed. Relaxing this assumption and testing the validity of our results are one of our future works. Moreover, this paper assumes that the contest designer can know the number of participants. However, in some contests the contest designer does not know the exact number of participants. For example, in a procurement competition, the buyer may only know that the suppliers who have good cooperation relationship with him will attend the competition and does not know whether there are other suppliers. In this case, whether the contest designer should reveal his information is also our future work.

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