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STABILIZATION FOR A CLASS OF DISCRETE-TIME SWITCHED SYSTEMS WITH STATE CONSTRAINTS AND QUANTIZED FEEDBACK

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Abstract. *This paper addresses the issue of stability analysis and controller gains design for a class of discrete-time linear switched systems with an average dwell-time, which involve constrained states and the quantization effect. The system requires a logarithmic quantizer and the quantized control input. The state constraints proposed in this paper are limited to a unit hypercube. Methods utilizing a quantization dependent Lyapunov function are introduced, which are applied to the design of the controller. Ultimately, some linear matrix inequalities (LMI) are given as the conditions to guarantee closed-loop systems are asymptotically stable. The accuracy of the theory is confirmed by a numerical example in the latter section.*
Keywords: Switched systems, Quantized feedback, State constraints, Average dwell-time, Iterative algorithms

1. **Introduction.** Frequently, continuous operation and discrete behavior exist simultaneously in practical systems which are known as hybrid systems [1]. Here, the switched system is composed of sets of continuous-time subsystems, discrete-time subsystems and a rule that supply the switching signal. Over the past two decades, the theory of switched systems has developed in many directions and a great number of studies report on its stability [2-5]. In addition, other results of switched systems have also been considered, such as H_∞ control [6], sliding mode control [7] and fault detection [8]. Several methods for investigating these issues have been proposed, such as, the common Lyapunov function method [9], multiple Lyapunov functions method [10], and average dwell time (ADT) method [11]. Also, the study of quantized feedback control systems has received growing attention in recent years, for which the rapid development of digital computers and bandwidth limit the transmission channel. There have been some results on its application in the switched systems reported in [12-15].

In addition, many systems subject to the constraints reside from subjective or objective factors in actual operation, such as state constraints, control constraints, and output constraints, which are also commonly found in the switched systems. Results on the research of switched systems with constraints can be found in [16-19]. A control strategy to switched discrete-time systems with constrained control variables by considering a static output feedback controller is provided by [20] and [21] proposes an improved ADT method, in which two different decay rates are used to design the Lyapunov function for a subsystem depending on whether the states are constrained. As the network transmission heavily relies on the quantization technique, the importance of the study on switched systems with constraints under the quantization effects is strengthened; however, few results attempt to solve this issue.

Inspired by the above discussion, this paper addresses the issue of stability analysis and stabilization for a class of discrete-time switched systems with state constraints by considering a logarithmic quantizer with an average dwell-time and constrained states limited to a unit hypercube. Seeing that the quantizer may cause error, the method in [22] will be used, which proposes an effective quantization dependent Lyapunov function method to impose stability analysis for the switched systems, especially concerning the quantization error of this Lyapunov function. In this paper, we investigate the logarithmic static quantizer. The aim is to design an appropriate state feedback for each subsystem so that switched systems with quantized feedback whose states are limited to a unit hypercube can be asymptotically stable. This will be followed by a description of the stabilization of the problem and a detailed presentation of how the required Lyapunov functions are defined.

The main contributions of this paper are introduced in the following two aspects. First, the quantized feedback and state constraints are considered simultaneously in switched system field for the first time. Second, this paper uses a kind of iterative algorithms to solve the LMI nonconvex problem caused by the simultaneous existence of quantization and state constraints.

The rest of this paper are composed as follows. Above all, Section 2 gives the problem formulation in detail. Section 3 involves stability analysis for switched systems with state constraint by considering the logarithmic quantizer and the quantized control input. The switched state-feedback gain is designed in Section 4. An iterative LMI algorithm is introduced in Section 5. A numerical evaluation is given in Section 6. Conclusions of this paper are given in the last part of this paper.

Notations: The notations used in this paper are as follows. A^T denotes the transpose of the matrix A. R^n represents the n-dimensional Euclidean space and notation $\mathbb{Z}^+$ refers to the set of negative integers and zero. The symbol $*$ expressed in a matrix stands for the symmetric elements. $\mathcal{C}^1$ represents the set of segmented continuous functions. Function $ceil(\cdot)$ means the smallest integer which is not less than the expression.

2. **Problem Formulation.** The control system is shown in Figure 1.

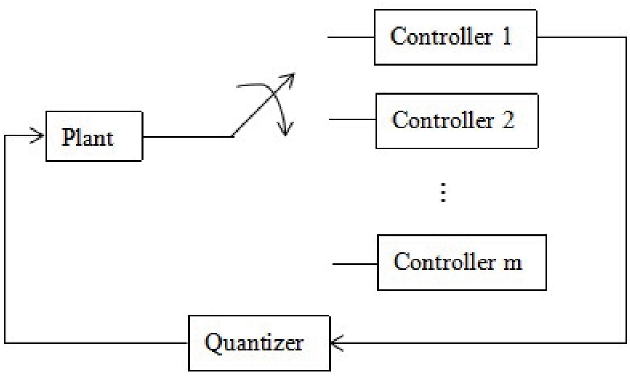

FIGURE 1. Control closed-loop system

Premeditate the discrete-time switched linear system as follows:

$$x(k+1) = h\left(A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k)\right) \quad (1)$$

where $x(k) = (x_1(k), x_2(k), \dots, x_n(k))^T \in R^n$ is the state vector, $u(k) \in R^p$ is the control input, and $\sigma(k)$ is a switching signal, which is selected from a finite set $\mathcal{P} = \{1, 2, \dots, m\}$. Also for a switching sequence $0 < k_1 < \dots < k_i < \dots$, k_i denotes the ith time to switch, when $\sigma(k) = p$, we mean the current operational subsystem is the pth subsystem, and p_i is referred as that when the system performs the ith switching, the pth subsystem is active. $A_{\sigma(k)}$ and $B_{\sigma(k)}$ are system matrices.

The function $h(\cdot)$ in (1) is given by:

$$h(\mathscr{A}\xi(k)) = [h_1(\mathscr{A}_1\xi(k)), \dots, h_n(\mathscr{A}_n\xi(t))]^T \tag{2}$$

for each $i \in \mathcal{N}$,

$$h_i(\mathscr{A}_i\xi(k)) = \begin{cases} \xi_i(k), & \text{if } |\xi_i(k)| = 1 \text{ and } (\mathscr{A}_i\xi_i(k) - \xi_i(k))\,\xi_i(k) > 0 \\ \mathscr{A}_i\xi(k), & \text{otherwise} \end{cases} \tag{3}$$

where matrix $\mathscr{A} = \left[\mathscr{A}_1^T, \dots, \mathscr{A}_n^T\right]^T \in R^{n\times n}$, and $\xi(k) = [\xi_1(k), \dots, \xi_n(k)]^T \in D^n$.

In this paper, we indicate the state feedback and the quantizer as follows:

$$\begin{aligned} u(k) &= q(v(k)) \\ v(k) &= K_p x(k) \end{aligned} \tag{4}$$

where K_p is the gain of the controller, $q(\cdot)$ is a quantizer with symmetric properties, we mean, $q(-v) = -q(v)$. Here, we concentrate on the logarithmic static quantizer and the set of quantized levels satisfies the following form:

$$\mathcal{V} = \{\pm\mu_i, \mu_i = \rho^i\mu_0, i = 0, \pm1, \pm2, \dots\} \cup \{0\}, \quad 0 < \rho < 1, \quad \mu_0 > 0 \tag{5}$$

The space is divided into a series of segments by $\mathcal{V}$ such that the quantizer maps each segment into one of the elements in $\mathcal{V}$ and μ_i denotes the ith quantized level. For the logarithmic quantizer whose quantized levels hold the form in (5) and the mapping relation q is in the following form [23,24]:

$$q(v) = \begin{cases} \mu_i & \text{if } \frac{1}{1+\delta}\mu_i < v \le \frac{1}{1-\delta}\mu_i,\ v > 0, \\ 0 & \text{if } v = 0, \\ -q(-v) & \text{if } v < 0, \end{cases} \tag{6}$$

where

$$\delta = \frac{1-\rho}{1+\rho}, \quad q(v(k)) = (1 + \Delta(k))v(k), \quad \Delta(k) \in [-\delta, \delta] \tag{7}$$

The definition of quantization density is proposed first in [23] and denoted by η_g. For a logarithmic quantizer, quantization density forms as $\eta_g = \frac{2}{\ln\frac{1}{\rho}}$, which infers the smaller the ρ is, the smaller the η_g is. In this sense, ρ can be called as quantization density equally, which can be seen in many studies. $\Delta(k)$ denotes the quantization error [24]. Thus, the final expression of the system is implied:

$$x(k+1) = h((A_p + (1 + \Delta(k))B_pK_p)x(k)) \tag{8}$$

Foremost purpose in this paper is to seek some suitable conditions such that the switched system in (8) achieves the asymptotic stability ultimately.

3. **Stability Analysis.** The aim of this section is to find out the conditions for system (8) to reach the asymptotic stability with average dwell-time. Above all, the definitions and lemmas used in the method of stability analysis and design of controller presented in this paper are given.

Definition 3.1. [11]. *For each switching signal $\sigma(k)$ and k_1, k_2 with the condition $k_2 \geq k_1 \geq 0$, let $N_{\sigma(k)}(k_2, k_1)$ be the number of interruption of $\sigma(k)$ when k belongs to (k_1, k_2). For a known N_0, $\tau_a > 0$,*

$$N_{\sigma(k)}(k_2, k_1) \leq N_0 + \frac{k_2 - k_1}{\tau_a}, \quad \forall k_2 \geq k_1 \geq 0$$

The constant τ_a is denoted as the average dwell-time and N_0 refers to the chatter bound.

Lemma 3.1. [11]. *Consider the discrete-time switched system $x(k+1) = f_{\sigma(k)}(x(k))$, where $\sigma(k) \in \mathcal{P}$, and let $0 < \lambda < 1$, $\mu > 1$ be given constants. Suppose that there exist positive definite $\mathcal{C}^1$ functions $V_{\sigma(k)} : \mathcal{R}^n \to \mathcal{R}$, and two class $\mathcal{K}_\infty$ functions κ_1, κ_2 satisfying following formulations for $\forall p \in \mathcal{P}$*

$$\kappa_1(\|x(k)\|) \leq V_p(x(k)) \leq \kappa_2(\|x(k)\|),$$

$$\Delta V_p(x(k)) \leq -\lambda V_p(x(k)),$$

and $\forall(\sigma(k_i) = p, \sigma(k_i^-) = q) \in \mathcal{P} \times \mathcal{P}$, $p \neq q$,

$$V_p(x(k_i)) \leq \mu V_q(x(k_i)),$$

then the system is global uniform asymptotic stability (GUAS) for any switching signal with average dwell-time

$$\tau_a \geq \tau_a^* = -\frac{\ln \mu}{\ln(1-\lambda)} \tag{9}$$

When a subsystem is active responding to the switching signal, we let k_i denote the starting time and k_{i+1} denote the ending time. With the interval $[k_i, k_{i+1})$, we utilize $\mathbb{T}_s(k_i, k_{i+1})$ and $\mathbb{T}_u(k_i, k_{i+1})$ to indicate the intervals, in which the state is saturated or not; meanwhile, denote $\mathcal{T}_s(k_i, k_{i+1})$ and $\mathcal{T}_u(k_i, k_{i+1})$ to be the length of them. Moreover, we let $\mathcal{T}_u(k_0, k) := \sum_{j=1}^{i} \mathcal{T}_u(k_{j-1}, k_j) + \mathcal{T}_u(k_i, k)$ and $\mathcal{T}_s(k_0, k) := \sum_{j=1}^{i} \mathcal{T}_s(k_{j-1}, k_j) + \mathcal{T}_s(k_i, k)$. In addition, for an active subsystem, denote β, α to be the decay rate of the Lyapunov function corresponding to the saturated interval or unsaturated interval respectively.

Lemma 3.2. [21]. *Consider the following discrete-time switched system with state constraints:*

$$x(k+1) = f_{\sigma(k)}(x(k)), \quad x(k) \in \Omega \subset R^n, \quad \sigma(k) = p \in \mathcal{P} \tag{10}$$

where $f_p(0) = 0$, $\forall p \in \mathcal{P}$. Suppose that all the trajectories are involved in Ω. Let α, β, and μ be given constants satisfying $1 > \alpha > \beta > 0$, $\mu \geq 1$. If there exist $\mathcal{C}^1$ functions $V_p(x(k)) : \Omega \to R$, for all $p \in \mathcal{P}$, such that

$$\varphi_1(\|x(k)\|) \leq V_p(x(k)) \leq \varphi_2(\|x(k)\|), \quad \forall x(k) \in \Omega \tag{11}$$

$$\Delta V_p(x(k)) \leq \begin{cases} -\alpha V_p(x(k)), & \forall k \in \mathcal{T}_u(k_i, k_{i+1}) \\ -\beta V_p(x(k)), & \forall k \in \mathcal{T}_s(k_i, k_{i+1}) \end{cases} \tag{12}$$

and $\forall(\sigma(k_i) = p,\ \sigma(k_i^-) = q) \in \mathcal{P} \times \mathcal{P}$, $p \neq q$

$$V_p(x(k_i)) \leq \mu V_q(x(k_i)),\ \forall x(k) \in \Omega \tag{13}$$

where φ_1 and φ_2 denote two class $\mathcal{K}_\infty$ functions. Then the switched system (10) with state constraints is GUAS for any switching signal with ADT:

$$\tau_a \geq \tau_a^* = ceil\left[-\frac{\ln \mu}{\ln(1-\zeta)}\right], \quad \frac{\mathcal{T}_u(k, k_0)}{\mathcal{T}_s(k, k_0)} \geq \frac{\ln(1-\beta) - \ln(1-\zeta)}{\ln(1-\zeta) - \ln(1-\alpha)} > 0, \quad \zeta \in (\beta, \alpha) \tag{14}$$

Lemma 3.3. [25]. *(Schur complements) Given the symmetric matrix* $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, *the following statements are equivalent:*

(1) $A < 0$;

(2) $A_{11} < 0$, $A_{22} - A_{12}^T A_{11}^{-1} A_{12} < 0$;

(3) $A_{22} < 0$, $A_{11} - A_{12} A_{22}^{-1} A_{12}^T < 0$.

Currently, the saturations can be handled with several methods. In this paper, we translate the saturation function $h(\cdot)$ into the vertex of a convex hull to solve the problem of the saturations as it said in [26].

Let D_n be the set of $n \times n$ diagonal matrices, that is, the diagonal elements of this kind of matrices are either 1 or 0, for example

$$D_2 = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\}$$

Obviously, there are 2^n elements involved in D_n. Let D_n be the elements of D_s, $s \in \mathcal{L} := 1, 2, \ldots, 2^n$, and denote $D_s^- = I - D_s$. Then, one can imply that D_s^- is also an element of D_n if $D_s \in D_n$. Then, one can obtain:

$$h(Ax) \in co\{D_s(Ax) + D_s^- G, s \in \mathcal{L}\}, \quad \forall x \in D^n \tag{15}$$

where $G = [g_{ij}] \in \mathcal{R}^{n \times n}$ is row diagonally dominant and the each element of the diagonal is negative, that is, the matrix G satisfies $|g_{ii}| > \sum_{j=1, j \neq i}^{n} |g_{ij}|$ and $g_{ii} < 0$, for all $i \in \mathcal{N}$. $co\{\}$ denotes convex hull, and for a group of points, $u^1, u^2, \ldots, u$, their convex hull is defined as,

$$co\{u^i : i \in [1, 2, 3, \ldots]\} := \left\{ \sum_{i=1} \alpha_i = 1, \alpha_i \geq 0 \right\}$$

The form of Lyapunov function selected in this paper is as follows [22]:

$$V_p(x(k)) = x^T(k) Q_p(\Delta(k)) x(k), \quad p \in \mathcal{P} \tag{16}$$

where $Q_p(\Delta(k))$ is time-variant and related to the quantization error $\Delta(k)$. Due to $\Delta(k) \in [-\delta, \delta]$, $Q_p(\Delta(k))$ is considered as the following form:

$$Q_p(\Delta(k)) = \frac{\delta - \Delta(k)}{2\delta} Q_{p1} + \frac{\delta + \Delta(k)}{2\delta} Q_{p2} \tag{17}$$

where Q_{p1} and Q_{p2} are positive definite matrices. The following theorem provides an adequate condition for the systems in (8) to be asymptotically stable in terms that the quantizer parameter δ is known.

Theorem 3.1. *Let* α, β *and* μ *be some given constants satisfying* $0 < \beta < \alpha < 1$, $\mu > 1$, *and* δ *is the quantizer parameter. If there exist positive symmetric matrices* Q_{p1}, Q_{p2}, *matrices* V_{p1}, V_{p2} *and row diagonally dominant matrices* G_p, $p \in \mathcal{P}$ *such that* $\forall (p, q) \in \mathcal{P} \times \mathcal{P}$, $p \neq q$, $s \in \mathcal{S}$, $D_s \neq I$

$$\begin{bmatrix} Q_{pi} - V_{pi} - V_{pi}^T & V_{pi}^T (A_p + (1 - \delta) B_p K_p) \\ * & -(1 - \alpha) Q_{p1} \end{bmatrix} < 0, \quad i \in \{1, 2\} \tag{18}$$

$$\begin{bmatrix} Q_{pi} - V_{pi} - V_{pi}^T & V_{pi}^T (A_p + (1 + \delta) B_p K_p) \\ * & -(1 - \alpha) Q_{p2} \end{bmatrix} < 0, \quad i \in \{1, 2\} \tag{19}$$

$$\begin{bmatrix} Q_{pi} - V_{pi} - V_{pi}^T & V_{pi}^T (D_s A_p + D_s (1 - \delta) B_p K_p + D_s^- G_p) \\ * & -(1 - \beta) Q_{p1} \end{bmatrix} < 0, \quad i \in \{1, 2\} \tag{20}$$

$$\begin{bmatrix} Q_{pi} - V_{pi} - V_{pi}^T & V_{pi}^T(D_sA_p + D_s(1+\delta)B_pK_p + D_s^-G_p) \\ * & -(1-\beta)Q_{p2} \end{bmatrix} < 0, \quad i \in \{1,2\} \tag{21}$$

$$Q_{pi} < \mu Q_{qi}, \quad i \in \{1,2\} \tag{22}$$

Then the switched system (8) is asymptotically stable for any switching signal with ADT satisfying (14).

Proof: Choose the Lyapunov functions as the form of (17).

When k belongs to the non-saturated period, i.e., $k \in \mathcal{T}_u(k_i, k_{i+1})$, $i \in \mathbb{Z}^+$. We have

$$\begin{aligned} &\Delta V_p(x(k)) + \alpha V_p(x(k)) \\ &= V_p(x(k+1)) - V_p(x(k)) + \alpha V_p(x(k)) \\ &= x^T(k)(A_p + (1+\Delta(k))B_pK_p)^T Q_p(\Delta(k+1))(A_p + (1+\Delta(k))B_pK_p)x(k) \\ &\quad - (1-\alpha)x^T(k)Q_p(\Delta(k))x(k) < 0 \end{aligned} \tag{23}$$

which is equivalent to

$$(A_p + (1+\Delta(k))B_pK_p)^T Q_p(\Delta(k+1))(A_p + (1+\Delta(k))B_pK_p - (1-\alpha)Q_p(\Delta(k))) < 0 \tag{24}$$

(24) → (18) and (19): In view of Schur complement, (24) can be translated into the following form

$$\begin{bmatrix} -Q_p(\Delta(k+1)) & Q_p(\Delta(k+1))(A_p + (1+\Delta(k))B_pK_p) \\ * & -(1-\alpha)Q_p(\Delta(k)) \end{bmatrix} < 0 \tag{25}$$

According to the range of $\Delta(k)$, concentrate on four circumstances as follows:

$$\begin{cases} (1) & \Delta(k) = -\delta, & \Delta(k+1) = -\delta \\ (2) & \Delta(k) = -\delta, & \Delta(k+1) = \delta \\ (3) & \Delta(k) = \delta, & \Delta(k+1) = -\delta \\ (4) & \Delta(k) = \delta, & \Delta(k+1) = \delta \end{cases}$$

For cases (1) and (2), from (25) we obtain

$$\begin{bmatrix} -Q_{pi} & Q_{pi}(A_p + (1-\delta)B_pK_p) \\ * & -(1-\alpha)Q_{p1} \end{bmatrix} < 0 \tag{26}$$

For cases (3) and (4), from (25) we obtain

$$\begin{bmatrix} -Q_{pi} & Q_{pi}(A_p + (1+\delta)B_pK_p) \\ * & -(1-\alpha)Q_{p2} \end{bmatrix} < 0 \tag{27}$$

Choosing $V_{pi}^T = V_{pi} = Q_{pi}$, we easily obtain (18) and (19). It appears clearly that if (24) holds, the positive symmetric matrices Q_{p1}, Q_{p2} and matrices V_{p1}, V_{p2} satisfying (18) and (19) can be found exactly.

(18) and (19) → (24): Suppose there exist positive symmetric matrices Q_{p1}, Q_{p2} and matrices V_{p1}, V_{p2} satisfying (18) and (19). As Q_{pi} is positive, we have $(V_{p1} - Q_{pi})^T Q_{pi}^{-1}(V_{pi} - Q_{pi}) \geq 0$, which concludes:

$$-V_{pi}^T Q_{pi}^{-1} V_{pi} \leq Q_{pi} - V_{pi}^T - V_{pi} \tag{28}$$

From (18), (19) and (28), one can obtain:

$$\begin{bmatrix} -V_{pi}^T Q_{pi}^{-1} V_{pi} & V_{pi}^T(A_p + (1-\delta)B_pK_p) \\ * & -(1-\alpha)Q_{p1} \end{bmatrix} < 0, \quad i \in \{1,2\} \tag{29}$$

$$\begin{bmatrix} -V_{pi}^T Q_{pi}^{-1} V_{pi} & V_{pi}^T(A_p + (1+\delta)B_pK_p) \\ * & -(1-\alpha)Q_{p2} \end{bmatrix} < 0, \quad i \in \{1,2\} \tag{30}$$

Performing congruence transformation to the (29) and (30) by diag $\{V_{pi}^{-1}, I\}$, $\{Q_{pi}, I\}$. Then, we have

$$\begin{bmatrix} -Q_{p1} & Q_{p1}(A_p + (1-\delta)B_pK_p) \\ * & -(1-\alpha)Q_{p1} \end{bmatrix} < 0 \tag{31}$$

$$\begin{bmatrix} -Q_{p2} & Q_{p2}(A_p + (1-\delta)B_pK_p) \\ * & -(1-\alpha)Q_{p1} \end{bmatrix} < 0 \tag{32}$$

$$\begin{bmatrix} -Q_{p1} & Q_{p1}(A_p + (1+\delta)B_pK_p) \\ * & -(1-\alpha)Q_{p2} \end{bmatrix} < 0 \tag{33}$$

$$\begin{bmatrix} -Q_{p2} & Q_{p2}(A_p + (1+\delta)B_pK_p) \\ * & -(1-\alpha)Q_{p2} \end{bmatrix} < 0 \tag{34}$$

For any $\Delta(k) \in [-\delta, \delta]$ and $\Delta(k+1) \in [-\delta, \delta]$, multiply $\frac{\delta - \Delta(k+1)}{2\delta}$ to both sides of (31) and multiply $\frac{\delta + \Delta(k+1)}{2\delta}$ to both sides of (32); then, totting up the results, we obtain

$$\begin{bmatrix} -Q_p(\Delta(k+1)) & Q_p(\Delta(k+1))(A_p + (1-\Delta(k))B_pK_p) \\ * & -(1-\alpha)Q_{p1} \end{bmatrix} < 0 \tag{35}$$

Similar to the above derivation, multiply $\frac{\delta - \Delta(k+1)}{2\delta}$ to both sides of (33) and $\frac{\delta + \Delta(k+1)}{2\delta}$ both sides of (34) respectively; totting up two results, we obtain

$$\begin{bmatrix} -Q_p(\Delta(k+1)) & Q_p(\Delta(k+1))(A_p + (1+\Delta(k))B_pK_p) \\ * & -(1-\alpha)Q_{p2} \end{bmatrix} < 0 \tag{36}$$

Moreover, multiply $\frac{\delta - \Delta(k)}{2\delta}$ to both sides of (35), multiply $\frac{\delta + \Delta(k)}{2\delta}$ to both sides of (36), and summing up, we obtain

$$\begin{bmatrix} -Q_p(\Delta(k+1)) & Q_p(\Delta(k+1))(A_p + (1+\Delta(k))B_pK_p) \\ * & -(1-\alpha)Q_p(\Delta(k)) \end{bmatrix} < 0 \tag{37}$$

By Schur complement, it can be translated into (24) easily.

On the other hand, when k belongs to the saturated period, i.e., $k \in \mathcal{T}_s[k_i, k_{i+1})$, $i \in \mathbb{Z}^+$, substitute $D_sA_p + D_s(1-\delta)B_pK_p + D_s^-G_p$ for $A_p + (1-\delta)B_pK_p$, and substitute $D_sA_p + D_s(1+\delta)B_pK_p + D_s^-G_p$ for $A_p + (1+\delta)B_pK_p$. Thus, if (20) and (21) hold, we can obtain $\Delta V_p(x(k)) + \beta V_p(x(k)) < 0$.

In addition, if $\forall(\sigma(k_i) = p, \sigma(k_i^-) = q) \in \mathcal{P} \times \mathcal{P}$, $p \neq q$

$$P_p \leq \mu P_q \tag{38}$$

We can obtain that $V_p(x(k_i)) \leq \mu V_q(x(k_i))$, we pre- and post-multiply (38) by X^{-1} and its transpose, we obtain $Q_p < \mu Q_q$, and consider the form of Q_p and four cases mentioned above, which implies to (22). Thus, according to Lemma 3.2, if (18), (19), (20), (21) and (22) are feasible, the closed-loop system in (8) is asymptotically stable, which completes the proof.

4. **Stabilization.** In Section 3, we obtain the stability conditions with the restriction of the controller, whose feedback gains are assumed to be known. However, in the practical system the feedback gains are unknown frequently. Thus, the most critical we need to do is to design suitable controller gains to guarantee switched systems are asymptotically stable exactly. In this section, we address the design of the state-feedback gain for the system in (8).

Theorem 4.1. *Let α, β and μ be some given constants satisfying $0 < \beta < \alpha < 1$, $\mu > 1$, and δ is the quantizer parameter. If there exist positive symmetric matrices $\bar{Q}_{p1}$, $\bar{Q}_{p2}$ and matrices V_p, $\bar{K}_p$ and row diagonally dominant matrices G_p, $p \in \mathcal{P}$ such that $\forall (p,q) \in \mathcal{P} \times \mathcal{P}$, $p \neq q$, $s \in \mathcal{S}$, $D_s \neq I$*

$$\begin{bmatrix} \bar{Q}_{pi} - V_p^T - V_p & A_pV_p + (1-\delta)B_p\bar{K}_p \\ * & -(1-\alpha)\bar{Q}_{p1} \end{bmatrix} < 0, \quad i \in \{1,2\} \tag{39}$$

$$\begin{bmatrix} \bar{Q}_{pi} - V_p^T - V_p & A_pV_p + (1+\delta)B_p\bar{K}_p \\ * & -(1-\alpha)\bar{Q}_{p2} \end{bmatrix} < 0, \quad i \in \{1,2\} \tag{40}$$

$$\begin{bmatrix} \bar{Q}_{pi} - V_p^T - V_p & D_sA_pV_p + D_s(1-\delta)B_p\bar{K}_p + D_s^-G_pV_p \\ * & -(1-\beta)\bar{Q}_{p1} \end{bmatrix} < 0, \quad i \in \{1,2\} \tag{41}$$

$$\begin{bmatrix} \bar{Q}_{pi} - V_p^T - V_p & D_sA_pV_p + D_s(1+\delta)B_p\bar{K}_p + D_s^-G_pV_p \\ * & -(1-\beta)\bar{Q}_{p2} \end{bmatrix} < 0, \quad i \in \{1,2\} \tag{42}$$

$$\bar{Q}_{pi} < \mu\bar{Q}_{qi}, \quad i \in \{1,2\} \tag{43}$$

Then the switched system (8) is asymptotically stable for any switching signal with ADT satisfying (14). Moreover, if there exist feasible solutions for (39), (40), (41), (42) and (43), the controller gains are given by

$$K_p = \bar{K}_pV_p^{-1} \tag{44}$$

Proof: Suppose there exist positive symmetric matrices $\bar{Q}_{p1}$, $\bar{Q}_{p2}$ and matrices V_p, $\bar{K}_p$, $p \in \mathcal{P}$,satisfying conditions proposed in Theorem 4.1. From (39), we can conclude $\bar{Q}_{pi} - V_p^T - V_p < 0$, which is equivalent to $V_p^T + V_p > \bar{Q}_{pi} > 0$, and thus V is nonsingular. Performing congruence transformation to (39), (40), (41) and (42) by diag $\{V_p^{-1}, V_p^{-1}\}$, then, we obtain

$$\begin{bmatrix} V_p^{-T}\bar{Q}_{pi}V_p^{-1} - V_p^{-T} - V_p^{-1} & V_p^{-T}A_p + (1-\delta)V_p^{-T}B_p\bar{K}_pV_p^{-1} \\ * & -(1-\alpha)V_p^{-T}\bar{Q}_{p1}V_p^{-1} \end{bmatrix} < 0, \quad i \in \{1,2\} \tag{45}$$

$$\begin{bmatrix} V_p^{-T}\bar{Q}_{pi}V_p^{-1} - V_p^{-T} - V_p^{-1} & V_p^{-T}A_p + (1+\delta)V_p^{-T}B_p\bar{K}_pV_p^{-1} \\ * & -(1-\alpha)V_p^{-T}\bar{Q}_{p2}V_p^{-1} \end{bmatrix} < 0, \quad i \in \{1,2\} \tag{46}$$

$$\begin{bmatrix} V_p^{-T}\bar{Q}_{pi}V_p^{-1} - V_p^{-T} - V_p^{-1} & V_p^{-T}\left(D_sA_p + D_s(1-\delta)B_p\bar{K}_pV_p^{-1} + D_s^-G_p\right) \\ * & -(1-\beta)V_p^{-T}\bar{Q}_{p1}V_p^{-1} \end{bmatrix} < 0, \quad i \in \{1,2\} \tag{47}$$

$$\begin{bmatrix} V_p^{-T}\bar{Q}_{pi}V_p^{-1} - V_p^{-T} - V_p^{-1} & V_p^{-T}\left(D_sA_p + D_s(1+\delta)B_p\bar{K}_pV_p^{-1} + D_s^-G_p\right) \\ * & -(1-\beta)V_p^{-T}\bar{Q}_{p2}V_p^{-1} \end{bmatrix} < 0, \quad i \in \{1,2\} \tag{48}$$

Defining $Q_{pi} = V_p^{-T}\bar{Q}_{pi}V_p^{-1}$, $V_{pi} = V_p^{-1}$, $K_p = \bar{K}_pV_p^{-1}$, the latest inequality is nothing than the stability condition (18), (19), (20), (21) and (22) to the closed-loop system (8). Hence, by Theorem 3.1, we can conclude the system (8) is asymptotically stable.

5. **Algorithm.** In this section, to solve the LMI nonconvex problem caused by the simultaneous existence of quantization and state constraints and check the sufficient conditions of Theorem 4.1, we will learn an iterative LMI algorithm from [26]. Suppose there is a set of n-dimensional row vectors with only one element, 1, and utilize $\mathcal{V}$ to represent this set. Denote υ_i, $i \in \mathcal{N}$, as a nonzero element of $\mathcal{V}$ that means, υ_i is 1. Let $\mathcal{W}_i$ be the set of n-dimensional column vectors composed of 1 and -1, especially, the ith element is 1. Represent ω_{ij} to be the elements of $\mathcal{W}_i$, $j \in \mathcal{M} := \{1, \ldots, 2^{n-1}\}$. Thus, we can conclude the following LMIs, which express the condition satisfying G_p is row diagonally and each element of the diagonal is negative:

$$\upsilon_i G_p \omega_{ij} < 0, \quad i \in \mathcal{N}, \quad j \in \mathcal{M}, \quad \forall p \in \mathcal{P}.$$

Algorithm.

Step 1. Choose a positive matrix W_p and solve $\bar{Q}_{pi}$, V_p, and $\bar{K}_p$ from the following Lyapunov formulas:

$$\begin{bmatrix} \bar{Q}_{pi} - V_p^T - V_p & A_p V_p + (1-\delta) B_p \bar{K}_p \\ * & -(1-\alpha)\bar{Q}_{p1} \end{bmatrix} < -W_p, \quad i \in \{1,2\}$$
$$\begin{bmatrix} \bar{Q}_{pi} - V_p^T - V_p & A_p V_p + (1+\delta) B_p \bar{K}_p \\ * & -(1-\alpha)\bar{Q}_{p2} \end{bmatrix} < -W_p, \quad i \in \{1,2\}$$

where $K_p = \bar{K}_p V_p^{-1}$, which guarantee that $A_p + (1 \pm \delta) B_p K_p$ are Hurwitz.

Set $k = 0$.

Step 2. Employ $\bar{Q}_{pi}$, V_p, and $\bar{K}_p$ obtained by the above to solve the following LMI optimization problem for G_p and γ:

$$\inf_{G_p} \quad \gamma$$
$$\textbf{s.t.} \begin{bmatrix} \bar{Q}_{pi} - V_p^T - V_p - \gamma V_p & D_s A_p V_p + D_s(1-\delta) B_p \bar{K}_p + D_s^- G_p V_p \\ * & -(1-\beta)\bar{Q}_{p1} \end{bmatrix} < 0, \quad i \in \{1,2\}$$
$$\begin{bmatrix} \bar{Q}_{pi} - V_p^T - V_p - \gamma V_p & D_s A_p V_p + D_s(1+\delta) B_p \bar{K}_p + D_s^- G_p V_p \\ * & -(1-\beta)\bar{Q}_{p2} \end{bmatrix} < 0, \quad i \in \{1,2\}$$
$$\upsilon_i G_p \omega_{ij} < 0, \quad s \in \mathcal{S}, \quad p \in \mathcal{P}, \quad i \in \mathcal{N}, \quad j \in \mathcal{M}$$

If $k = 0$ and $\gamma \le 0$, switch to Step 4. If $k > 0$, $\gamma \le 0$ or $\gamma > \gamma_k$, switch to Step 4. Otherwise, set $k = k + 1$, $\gamma_k = \gamma$, switch to the next.

Step 3. Employing G_p obtained previously, consider the following LMI optimization problem for $\bar{Q}_{pi}$, V_p, $\bar{K}_{pi}$, and γ:

$$\inf_{P_p > 0} \quad \gamma$$
$$\textbf{s.t.} \begin{bmatrix} \bar{Q}_{pi} - V_p^T - V_p - \gamma V_p & A_p V_p + (1-\delta) B_p \bar{K}_p \\ * & -(1-\alpha)\bar{Q}_{p1} \end{bmatrix} < 0, \quad i \in \{1,2\}$$
$$\begin{bmatrix} \bar{Q}_{pi} - V_p^T - V_p - \gamma V_p & A_p V_p + (1+\delta) B_p \bar{K}_p \\ * & -(1-\alpha)\bar{Q}_{p2} \end{bmatrix} < 0, \quad i \in \{1,2\}$$
$$\begin{bmatrix} \bar{Q}_{pi} - V_p^T - V_p - \gamma V_p & D_s A_p V_p + D_s(1-\delta) B_p \bar{K}_p + D_s^- G_p V_p \\ * & -(1-\beta)\bar{Q}_{p1} \end{bmatrix} < 0, \quad i \in \{1,2\}$$

$$\begin{bmatrix} \bar{Q}_{pi} - V_p^T - V_p - \gamma V_p & D_s A_p V_p + D_s(1+\delta) B_p \bar{K}_p + D_s^- G_p V_p \\ * & -(1-\beta)\bar{Q}_{p2} \end{bmatrix} < 0, \quad i \in \{1,2\}$$
$$s \in \mathcal{S}, \quad p \in \mathcal{P}$$

If $\gamma \leq 0$ or $\gamma > \gamma_k$, switch to Step 4. Otherwise, set $k = k+1$, $\gamma_k = \gamma$, and switch to Step 2.

Step 4. If $\gamma \leq 0$, we can deduce the system (8) is asymptotically stable. Moreover, the current K_p is the final result required gains of controller. Otherwise, come to nothing, and the algorithm needs to repeat from Step 1 with a different W_p.

6. **Numerical Example.** In this section, the correctness of the theory is confirmed by a numerical example. The matrices in (8) can be given by:

$$A_1 = \begin{bmatrix} 1.2 & -0.6 \\ -0.5 & 0 \end{bmatrix} \quad A_2 = \begin{bmatrix} 2.6 & -2.2 \\ 0.8 & -1 \end{bmatrix} \quad B_1 = B_2 = \begin{bmatrix} -0.0525 \\ -0.0082 \end{bmatrix}$$

The eigenvalues of A_1 are 1.4121, −0.2124; the eigenvalues of A_2 are 2.0166, −0.4166; thus, both two subsystems are unstable. In this paper, we choose the other parameters $\mu = 1.04$, $\alpha = 0.03$, $\beta = 0.02$, $\rho = 0.4$ and the initial states are 1.85, −0.9. The selection of parameters satisfies the ADT condition and shows obvious effects in the simulation. Then, the controller gains are obtained by solving the LMIs (39), (40), (41), (42) and (43),

$$K_1 = \begin{bmatrix} 25.5136 & -0.2924 \end{bmatrix} \qquad K_2 = \begin{bmatrix} 42.7371 & -31.4334 \end{bmatrix} \tag{49}$$

Figure 2 shows a group of switching signals satisfying ADT $\tau_a = 2$sec. The response effects of the states are shown in Figure 3, the dotted lines represent states without saturation and the solid lines represent states with saturation. It can be seen from Figure 3 that both states are reduced to the origin after a period of oscillation. The trajectories of the state with saturation or not are shown in Figure 4, which shows that states are constrained within a unit hypercube more intuitively and achieve asymptotic stability finally under the switching strategy designed with the method proposed in this paper satisfying ADT $\tau_a = 2$sec as shown in Figure 2. Furthermore, Figure 5 gives the contrast of the signal before and after quantization.

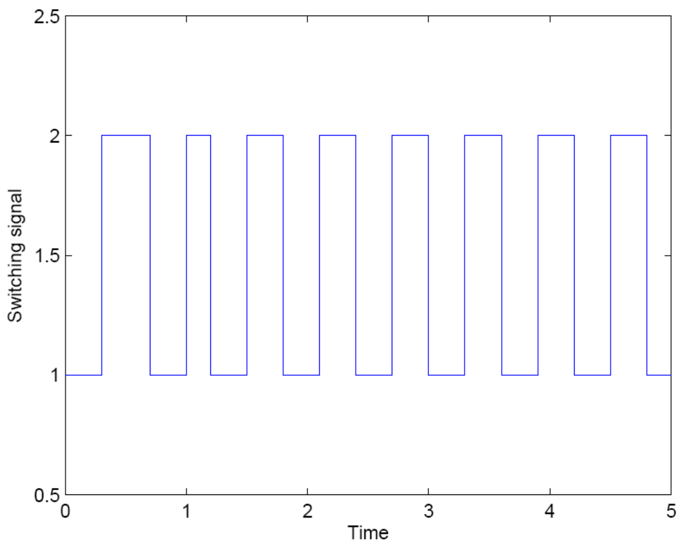

FIGURE 2. Switching signal with the ADT $\tau_a = 2$sec

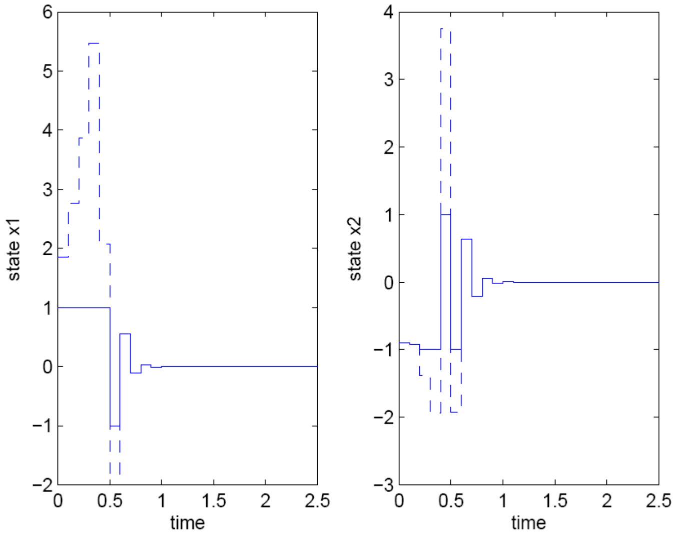

FIGURE 3. Response of the states under switching signal with $\tau_a = 2$sec

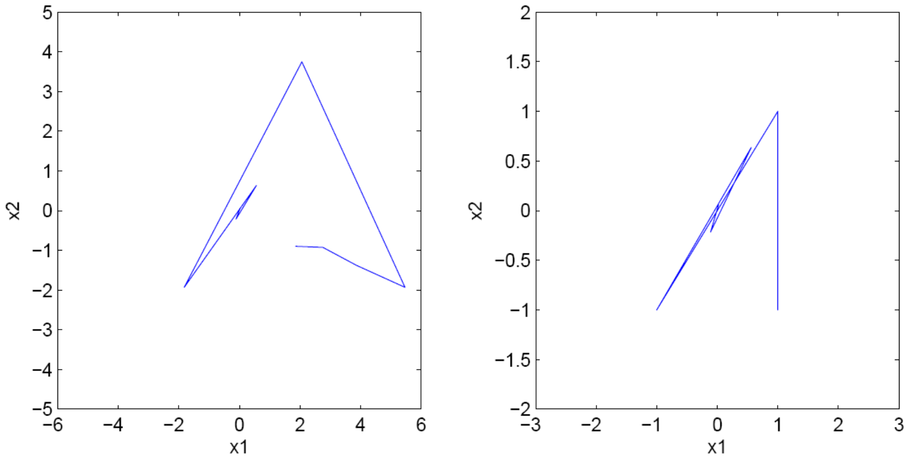

FIGURE 4. State trajectory with the ADT $\tau_a = 2$sec

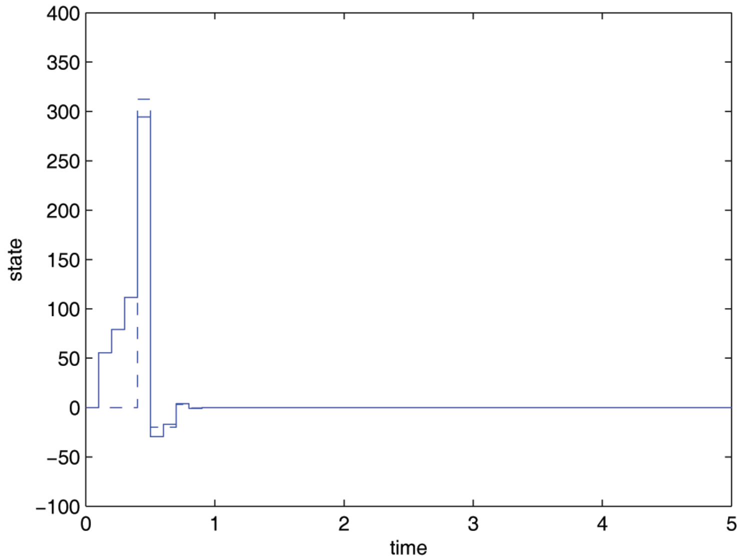

FIGURE 5. Contrast of the signal quantization

7. **Conclusions.** In this paper, the stability analysis and controller design for discrete-time switched linear systems with state constraints and quantized control input have been investigated. The quantizer utilized is a logarithmic quantizer. By the average dwell-time method and a quantization dependent Lyapunov function, suitable conditions are observed and we have obtained the controller gains. In fact, the final LMI is nonconvex and we used a kind of iterative algorithms to solve the problem. Furthermore, the theoretical developments have been illustrated by numerical simulations. Nevertheless, one of the most common situations, packet dropout, is not considered here and we expect to study more complicated and comprehensive problems on the basis of these results, such as packet dropout, dynamic quantized feedback.

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