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RAMP CONTROL OF FREEWAY BY BALANCING TRAFFIC DENSITY

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Abstract. *In this paper, we develop a coordinated traffic responsive ramp control strategy, under which traffic density of freeway network can realize balanced. Firstly we propose a hybrid switched model of freeway network by using CTM; furthermore, we obtained the balanced density in free flow state case; then, on this basis, we design a feedback control law, under which the proposed control system can realize asymptotic stable; at last, we conduct a simulation on Beijing third ring freeway network. Simulation results show that the proposed ramp control strategy compares favorably against the well-known ALINEA control strategy in increasing the usage of freeway network and reducing delay of vehicles in on-ramps.*
Keywords: Freeway ramp metering, Coordinated control, Density balancing, State feedback control

1. **Introduction.** Ramp control, or ramp metering, has been recognized as one of the most effective ways for the control of freeway system. A common objective of ramp control is to regulate the amount of traffic entering a freeway from on-ramps during a certain time periods so that the flow on the freeway does not exceed its capacity. Ramp metering is implemented by placing a traffic light at the on-ramps that allows the vehicles to enter the freeway in a controlled way and thus reduces the disturbance of the traffic on the mainline.

From the viewpoint of system control, ramp metering is a typical set-point problem. A lot of control algorithms have been proposed, e.g., linear programming [1, 2]; PID (Proportion Integral Derivative)-like controller [3]; LQR (Linear Quadratic Regulator) [4]; model prediction control [5]; neural network and fuzzy control [6, 7]; optimal control theory [8, 9]; ILC (Iterative Learning Control) approach [10]; Reinforcement learning [11]. All of these algorithms can be further classified into two classes of strategies: fixed time strategies and real-time (traffic responsive) strategies. Fixed time strategies adopt fixed signals at specific times and have been plagued with low efficiency. Traffic responsive strategies determine ramp metering signals according to real-time traffic conditions.

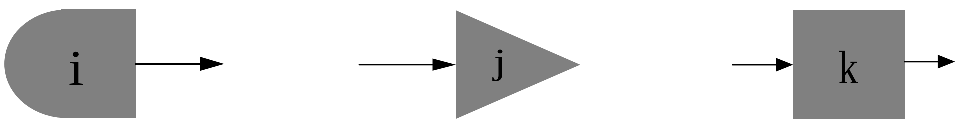
Traffic responsive strategies can be further categorized into the local and coordinated ramp metering strategies. Local ramp metering strategies employ measurements from the vicinity of a single controlled on-ramp while coordinated ramp metering strategies use measurements from an entire region of the network and are responsible for the concurrent operation of all included on-ramps. Different approaches to the design of local ramp metering strategies include feedforward control, that is, the demand-capacity strategy and its variations [3]; feedback control, that is, the ALINEA strategy and its variations [3, 12, 13, 14]; neural network control [15, 16]. A number of design approaches have been used for coordinated reactive ramp metering such as feedback control [17, 18] and optimal control [8, 9]. In this paper, we will focus our attention on the feedback coordinated ramp metering.

State balancing is studied in [19], which addresses the problem of averaging of a stored resource in batteries. A distributed algorithm was proposed for flow control, and the algorithm guarantees states asymptotically converge to the same value, which is equal to the average of initial values. However, for the problem of density balancing of freeway traffic system, we may represent vehicle density as the state of such a system while on-ramp demands are assumed for the inputs. The goal of density balancing for freeway traffic system is to find input flows that result in a uniform distribution of density. In practice, this uniform distribution can be understood as an equal inter-distance between vehicles. The equal inter-distance can be attractive to driver's point of view. It reduces the number and intensity of acceleration and deceleration events and therefore, it makes a travel more safe and comfortable, while decreasing emissions. In [20], authors investigate equilibrium sets for freeway system and give the necessary and sufficient conditions for the existence of equilibrium. However, they do not research the problem of designing ramp metering strategy. In this paper, therefore, we first derive the sets of balanced equilibrium points for freeway network. Then, by using feedback control theory, we design a state-feedback control law, under which the freeway system can realize asymptotic stable, which implies that densities of freeway can converge to a balanced equilibrium point. Lastly, we illustrate the obtained results by applying them to Beijing third ring road.

The rest of this paper is structured as follows. The traffic flow model of freeway network used for both simulation and control design purposes is described in Section 2. The sets of balanced equilibrium points of freeway network and the formulation of the control problem for ramp metering are presented in Section 3. Section 4 presents simulation results. And conclusions and further research topics are given in Section 5.

2. The Traffic Flow Model.

2.1. **Density-based cell transmission model.** In order to develop a ramp control strategy, we must construct a mathematical model of freeway network. Many models have been proposed in these years, for example, the well-known microscopic car-following models [21], macroscopic LWR model [22, 23, 24]. Daganzo [25, 26] proposed the Cell Transmission Model (CTM) by spatially and temporally discretizing the LWR model. CTM is an analytically simple model, and it can capture many important traffic phenomena, for example, queue build-up and dissipation, backward propagation of congestion waves. Therefore, CTM is chosen as underlying traffic flow model of freeway network. However, unlike the assumption of standard CTM [27], i.e., the link is divided into a collection of segments, each of which is called a cell, we call a link of having the same geometry and other conditions as a cell in our modeling framework. These cells are classified into three basic types: source cells, sink cells and internal cells, as shown in Figure 1.



(a) Source cells (b) Sink cells (c) Internal cells

FIGURE 1. Example of three types of cells

Figure 1(a) represents a source cell providing traffic demand to road network, Figure 1(b) represents a sink cell receiving the output from road network, and Figure 1(c) represents an internal cell.

2.2. **Dynamics of continuous state of the freeway system.** As shown in Figure 2, consider a cell i, its upstream cell $i-1$ and its downstream cell $i+1$. According to the literature [25, 26], continuous dynamic of cell i can be described as follows:

$$\rho_i(t+1) = \rho_i(t) + \frac{T}{L_i}\left(q_{i-1,i}(t) + d_i(t) - q_{i,i+1}(t) - o_i(t)\right),\ 0 \le \rho_i(t) \le \rho_{im}, \tag{1}$$

where $\rho_i(t)$ is the density of cell i at time instant t; ρ_{i0}, ρ_{im} are respectively the critical density and jam density, and they are constants in general; L_i is the length of cell i; T is the sampled period, and $v_iT \le L_i$; v_i is the free-flow speed of vehicles in cell i; C_i denotes the maximum flow that can enter or leave cell i in the time interval $[tT,(t+1)T]$. The relationship between these parameters is shown in Figure 3. $q_{i,i+1}(t)$ is the flow from cell i to $i+1$, and

$$q_{i,i+1}(t) = \min\left\{s_i(t), r_{i+1}(t)\right\}. \tag{2}$$

$s_i(t)$ is the traffic flow sent by cell i in the time interval $[tT,(t+1)T]$, and

$$\begin{aligned} s_i(t) &= \min\{C_i, v_i\rho_i\} \\ &= \begin{cases} v_i\rho_i(t), & \text{if } \rho_i(t) < \rho_{i0}; \\ C_i, & \text{if } \rho_i(t) \ge \rho_{i0}. \end{cases} \end{aligned} \tag{3}$$

$r_{i+1}(t)$ is the traffic flow received by cell $i+1$ in the time interval $[tT,(t+1)T]$, and

$$\begin{aligned} r_{i+1}(t) &= \min\{C_{i+1}, w_{i+1}(\rho_{i+1,\mathrm{m}} - \rho_{i+1})\} \\ &= \begin{cases} C_{i+1}, & \text{if } \rho_{i+1}(t) < \rho_{i+1,0}; \\ w_{i+1}(\rho_{i+1,\mathrm{m}} - \rho_{i+1}(t)), & \text{if } \rho_{i+1}(t) \ge \rho_{i+1,0}. \end{cases} \end{aligned} \tag{4}$$

where w_{i+1} is the backward wave speed of vehicles in cell $i+1$.

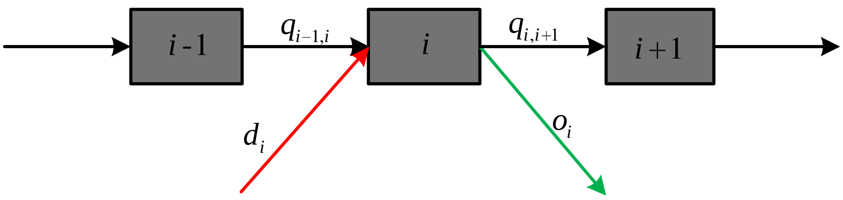


FIGURE 2. Cell i and its upstream cell $i-1$, downstream cell $i+1$

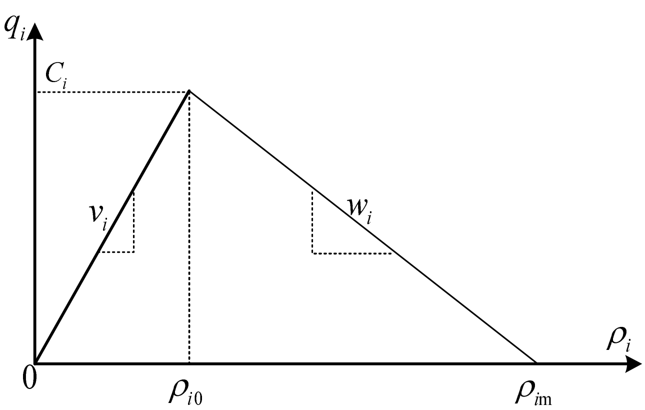


FIGURE 3. The triangle fundamental diagram

2.3. **Hybrid model of practical freeway network.** Consider a freeway network, which is divided into n cells, indexed $i = 1, \ldots, n$, as is shown in Figure 4. Each cell is assumed to be equipped with at most one on-ramp and one off-ramp, and $d_i(t)$, $i = 1, \ldots, m$, is the demand of on-ramp i at time instant t, $o_i(t)$, $i = 1, \ldots, p$, is the flow from upstream mainline cell i to the corresponding off-ramp at time instant t. Here assume that the off-ramp can receive enough vehicles, i.e., it is not congested, and

$$o_i(t) = \frac{\beta_i q_{i,i+1}}{\bar{\beta}_i}, \tag{5}$$

i.e., the flow entering the off-ramp from cell i is the β_i times of the flow leaving from the cell i, and $\bar{\beta}_i = 1 - \beta_i$. For the freeway system, we adopt the density-based cell transmission model described in previous section.

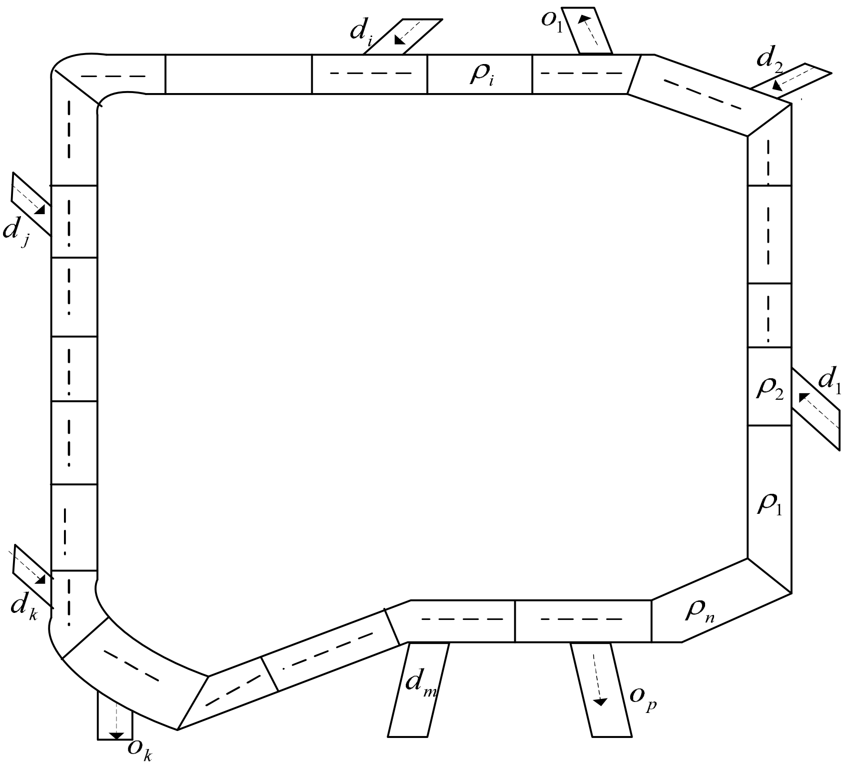


FIGURE 4. The diagram of freeway network

For the convenience of our further investigation, substituting (2)-(5) into (1), we can derive a switched control system as follows:

$$x(t+1) = A_s x(t) + B_s d(t) + F_s,\ x(t) \in \mathcal{D}_s,\ s \in M = \{1, \ldots, m\}, \tag{6}$$

where $s = 1, \ldots, S$, $x = [\rho_1, \ldots, \rho_n]^T \in \mathbb{R}^n$ denotes the traffic density vector of the road network, the input vector $d \in \mathbb{R}^m$ represents the on-ramp traffic demand of the road network. A_s and B_s are the system matrix and the input matrix, F_s is a vector, and these system parameters are composed of the parameters in the fundamental diagrams of all the road segments. The switching signal $\sigma(t)$ is determined by the convex polytopes $\mathcal{D}_s$, i.e., $\sigma(t) = s$ if and only if $x(t) \in \mathcal{D}_s$.

3. The Coordinated Ramp Control Strategy.

3.1. The steady equilibrium sets of freeway network. In this section, our goal is to analyze the equilibrium sets of freeway network. In particular, we will investigate sets of balanced equilibrium points. Before analyzing our problem, we first introduce some definitions as follows [20].

Definition 3.1. *The steady equilibrium set Ω for freeway network described by (6) is a set of pairs (x^*, d^*) that solve the following steady equilibrium equation:*

$$(A_s - I)x^* + B_s d^* = 0,\ x^* \in \mathcal{D}_s^*,\ s \in M = \{1, \dots, m\}, \tag{7}$$

where I stands for the identity matrix. Similarly, the switching signal $\sigma^*(t)$ is determined by the convex polytopes $\mathcal{D}_s^*$, i.e., $\sigma^*(t) = s$ if and only if $x*(t) \in \mathcal{D}_s^*$. According to (7), the steady equilibrium state is gained for each cell, i.e., the total flows entering and leaving cell are equal. We denote by Ω_x the set of steady equilibrium densities and Ω_d the set of steady equilibrium on-ramp flow respectively.

According to [20], we know that the system is in different modes under the different parameters. And the balanced equilibrium sets of densities only exist for the free flow case (Due to space limitations, more details can refer to [20]). Therefore, in this paper, we only consider the special case. The set of steady equilibrium densities Ω_{xB}^f and the set of steady equilibrium on-ramp flow Ω_{dB}^f in the free flow case are respectively described as follows:

$$\Omega_{xB}^f = \left\{ x^* \in \mathcal{X}, x^* = \phi 1 : 0 < \phi \leq \min(\{\rho_{i0}\}_{i=1}^n), \phi = \frac{d_1^*}{v_1 - \bar{\beta}_n v_n}, u^* \in \Omega_{dB}^f \right\}, \tag{8}$$

where 1 is a column vector for all-ones.

$$\Omega_{dB}^f = \left\{ d^* \in \mathcal{D}, d_1^* \leq \left(v_1 - \bar{\beta}_n v_n\right) \min\left(\{\rho_{i0}\}_{i=1}^n\right), d_i^* \leq \left(v_i - \bar{\beta}_{i-1} v_{i-1}\right) \min\left(\{\rho_{i0}\}_{i=2}^n\right), \right.$$
$$\left. \frac{d_1^*}{v_1 - \bar{\beta}_n v_n} = \frac{d_i^*}{v_i - \bar{\beta}_{i-1} v_{i-1}}, i = 2, \dots, n \right\}. \tag{9}$$

In the following, we will design a state feedback controller, under which the densities of freeway asymptotically converge to the balanced equilibrium point.

3.2. State-feedback controller design. From the previous section, we derive the balanced equilibrium density x^*, d^* is corresponding on-ramps flow of balanced equilibrium density, which has to satisfy:

$$x^* = Ax^* + Bd^*. \tag{10}$$

Let $\bar{x}(t) = x(t) - x^*$ and $u(t) = d(t) - d^*$ are state error and control input error respectively. Then we have:

$$\bar{x}(t+1) = A\bar{x}(t) + Bu(t), \tag{11}$$

where $\bar{x} = [\ \bar{\rho_1} \ \ \cdots \ \ \bar{\rho_n}\ ]$. Based on the feedback control theory, we design a distributed state feedback control law such that the system (11) can realize asymptotic stable, which is as follows:

$$u_i(t) = K_{ii} x_i(t) + K_{i,i-1} x_{i-1}(t) + K_{i,i+1} x_{i+1}(t),\ i = 1, \dots, n. \tag{12}$$

Let $u = [\ u_1 \ \ \cdots \ \ u_n\ ]$, then, the closed-loop freeway system resulting from (11) and (12) can be written as

$$\bar{x}(t+1) = A\bar{x}(t) + BK\bar{x}(t) = \bar{A}\bar{x}(t), \tag{13}$$

where $\bar{A} = A + BK$,

$$K = \begin{bmatrix} K_{11} & K_{12} & 0 & \cdots & \cdots & K_{1n} \\ K_{21} & K_{22} & K_{23} & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & K_{i,i-1} & K_{ii} & K_{i,i+1} & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & 0 \end{bmatrix}.$$

In order to ensure the densities of freeway converge to the balanced equilibrium, it is necessary that the system (13) asymptotically converges to 0. To achieve the goal, we give the following lemma.

Lemma 3.1. *The equilibrium $\bar{x} = 0$ of (13) is asymptotically stable if and only if the matrix $\bar{A}$ is Schur stable, i.e., the eigenvalues of $\bar{A}$ lie inside the interior of unit circle.*

Thus, from results in Lemma 3.1, the problem solving the feedback gain matrix K such that (13) can realize asymptotic stable, is transformed to the problem solving the matrix K such that the matrix $\bar{A} = A + BK$ in (13) is Schur stable, which is the standard problem of feedback stabilization of discrete-time linear time-invariant control systems. Then, Linear Matrix Inequality (LMI) methods [28] can be used to numerically solve the matrix K. Thus, we can obtain the following ramp metering rates:

$$d(t) = u(t) + d^*. \tag{14}$$

4. Simulation Example.

4.1. Parameters setting.

In this section, we report on simulation tests using the proposed control strategy in this paper. Beijing third ring freeway is approximately 48 km long and includes 62 on-ramps and 62 off-ramps. For the purpose of our study, only the counter-clockwise direction of ring road was modelled. And in order to reduce the computational complexity, we only consider a simplified version of Beijing third ring freeway. The simplified version third ring freeway is divided into $n = 24$ cells. Each cell contains an on-ramp, and there exists an off-ramp in cell $3, 7, 11, 15, 19, 23$ respectively. The split ratio are as follows: $\beta_3 = 0.31$, $\beta_7 = 0.39$, $\beta_{11} = 0.41$, $\beta_{15} = 0.38$, $\beta_{19} = 0.42$, $\beta_{23} = 0.37$. The time interval T is 10 s. The lengths of each link are 2.2754, 0.9139, 1.1128, 2.4992, 1.6475, 1.7283, 5.2097, 1.8588, 1.2185, 1.3304, 1.3304, 1.8215, 2.5643, 2.2381, 1.9769, 3.0587, 1.4423, 4.6813, 1.3242, 3.0152, 1.2993, 1.9459, 1.5107, 1.6324 km, respectively. The free flow speeds are 69.41, 83.77, 94.43, 73.25, 84.96, 91.38, 95.90, 68.26, 75.03, 84.56, 94.12, 69.39, 76.99, 87.92, 94.42, 68.41, 75.78, 84.40, 94.63, 63.52, 72.13, 84.68, 92.12, 59.36 km/h, respectively. Through Equations (8) and (9), the balanced equilibrium densities $x^* = 83.75 \cdot \mathbf{1}_{24}'$ are derived, where "$'$" is the transpose of a matrix or vector, $\mathbf{1}_{24}$ stands for all-ones row vector.

4.2. Simulation results.

As shown in Figure 5, the demand on on-ramps rises from low levels to high levels, then subsides. The performance of ramp metering is measured by the total number of vehicles using freeway network and ramp queue delay. The former is defined as $num = \sum_{t=1}^{N_m} \sum_{i=1}^{n} \rho_i(t) L_i$, and the latter is defined as $D = T \sum_{t=1}^{N_m} \sum_{i=1}^{n} \big(\bar{d}_i(t) - d_i(t)\big)$, where N_m is the length of the time horizon, $\bar{d}_i(t)$ is the traffic demand entering on-ramp i. Before presenting the simulation results, we describe briefly the ALINEA control law [3] so that the performance of the two types of controllers can be compared. It is shown as follows:

$$d(t+1) = d(t) + K_A(\tilde{\rho} - \rho(t)). \tag{15}$$

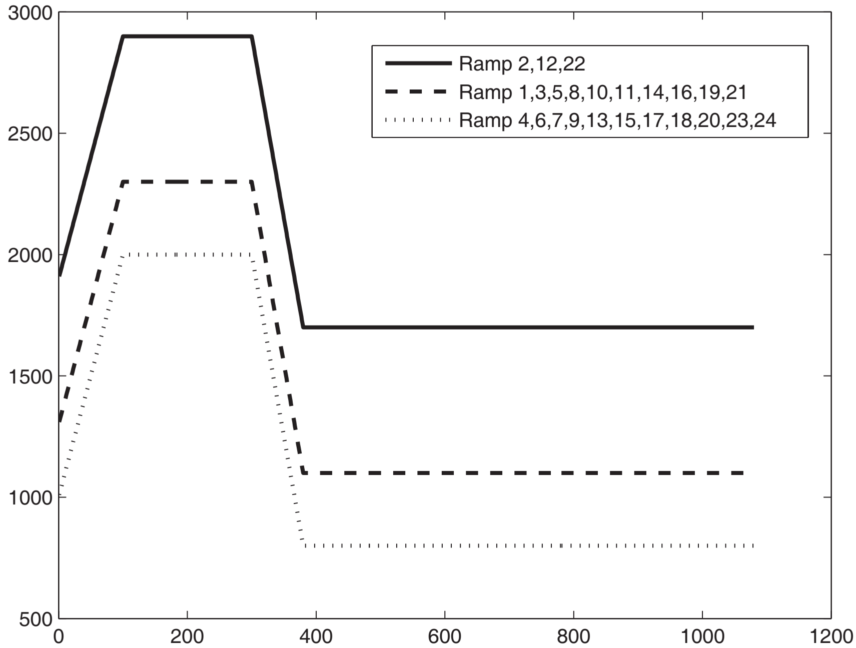
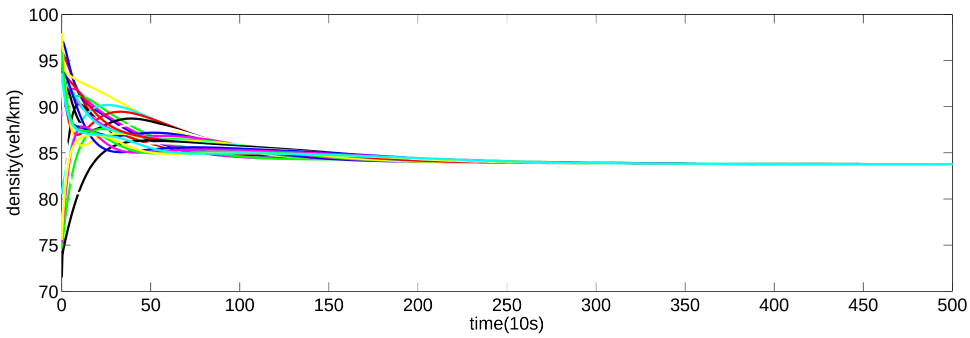
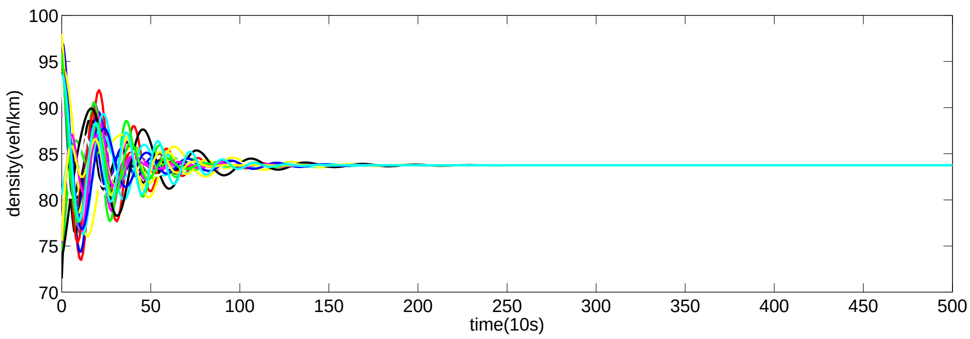


Figure 5. On-ramp demand



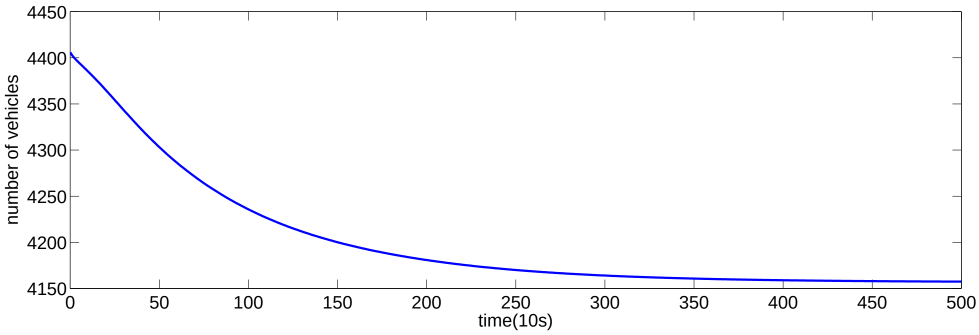
(a) Our proposed method



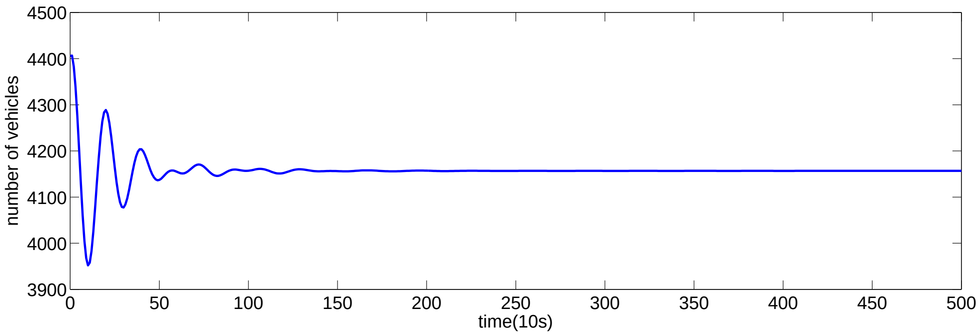
(b) ALINEA

Figure 6. The densities of freeway network

The key to design an ALINEA controller is to obtain the regulator parameter K_A and a desired value $\tilde{\rho}$ for the downstream density. Here, in order to compare with our method easily, we use $\tilde{\rho} = x^*$, and $K_A = 71$. The performance is calculated for two types of control algorithms; the simulation results are shown in Figures 6, 7 and 8.

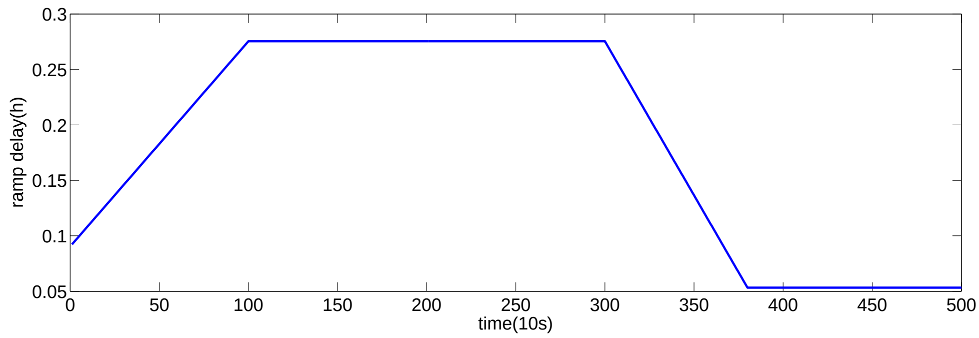


(a) Our proposed method

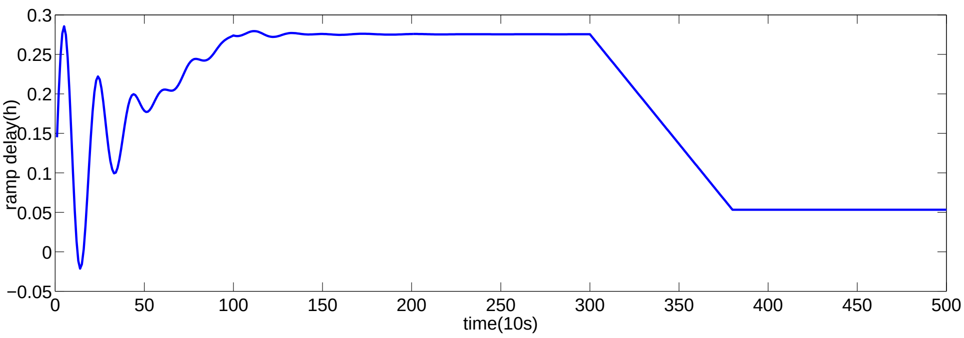


(b) ALINEA

FIGURE 7. The number of vehicles



(a) Our proposed method



(b) ALINEA

FIGURE 8. The ramp delay of freeway network

From Figure 6, we can see: using our designing method, the densities of all cells can asymptotically converge to the balanced equilibrium states, as is shown in Figure 6(a); the densities of cells also converge to balanced equilibrium state when ALINEA algorithm is applied, as is shown in Figure 6(b). The number of vehicles using the freeway in the sample time is described in Figure 7. It is apparent that the usage of freeway in Figure 7(a) is highest, i.e., when our design approach is applied, the total number of vehicles is 4.5155×10^6, yet it is only 4.4937×10^6 when ALINEA law is applied, which can show the proposed ramp control strategy compares favorably against the well-known ALINEA control strategy increasing the usage of freeway network. The delay, i.e., queues of vehicles in on-ramp in the time interval, is shown in Figure 8, by which, we can calculate the total delay in simulation horizon is 123.7515 veh $\cdot$ h, which is lower than 124.3793 veh $\cdot$ h when ALINEA is applied.

5. **Conclusions.** Ramp control has been an effective method for combating freeway congestion. It has produced a lot of ramp control algorithms in past research. We give the ideas about balancing of road traffic density distributions, and derive the numerical solution, on the basis of which, we design a coordinated control strategy based on state feedback control theory. Under free flow conditions, we conducted a simulation of third ring freeway in Beijing; simulation results show our method outperforms ALINEA law in increasing the total number of vehicles and reducing the total delay of on-ramp. In this paper, we only consider free flow condition; further research topics will focus on applications of our developed methods to the other conditions and compare our methods with existing methods proposed in the literature.

REFERENCES

[1] A. Alessandri, A. Di Febbraro, A. Ferrara et al., Optimal control of freeways via signalling and ramp metering, *Control Engineering Practice*, vol.6, no.6, pp.771-780, 1998.
[2] S. Cho and R. R. Rao, Coordinated ramp-metering control using a time-gap based traffic model, *IEEE the 80th Vehicular Technology Conference*, Vancouver, Canada, pp.1-6, 2014.
[3] M. Papageorgiou and H. Hadj-Salem, ALINEA: A local feedback control law for on-ramp metering, *Transportation Research Record*, vol.1320, 1991.
[4] H. M. Zhang, S. G. Ritchie and R. Jayakrishnan, Coordinated traffic-responsive ramp control via nonlinear state feedback, *Transportation Research Part C: Emerging Technologies*, vol.9, no.5, pp.337-352, 2001.
[5] A. Ferrara, S. Sacone and S. Siri, Distributed model predictive control for MLD systems: Application to freeway ramp metering, *IEEE 2014 American Control Conference*, Portland, USA, pp.5294-5299, 2014.
[6] A. H. Ghods, A. R. Kian and M. Tabibi, Adaptive freeway ramp metering and variable speed limit control: A genetic-fuzzy approach, *IEEE Intelligent Transportation Systems Magazine*, vol.1, no.1, pp.27-36, 2009.
[7] J. Xu, X. Zhao and D. Srinivasan, On optimal freeway local ramp metering using fuzzy logic control with particle swarm optimisation, *IET Intelligent Transport Systems*, vol.7, no.1, pp.95-104, 2013.
[8] G. Zhang and Y. Wang, Optimizing coordinated ramp metering: A preemptive hierarchical control approach, *Computer-Aided Civil and Infrastructure Engineering*, vol.28, no.1, pp.22-37, 2013.
[9] X. Wang, M. Hadiuzzaman, J. Fang et al., Optimal ramp metering control for weaving segments considering dynamic weaving capacity estimation, *Journal of Transportation Engineering*, vol.140, no.11, pp.04014057-1-04014057-10, 2014.
[10] Z. Hou, X. Xu, J. Yan et al., A complementary modularized ramp metering approach based on iterative learning control and ALINEA, *IEEE Trans. Intelligent Transportation Systems*, vol.12, no.4, pp.1305-1318, 2011.
[11] A. Fares and W. Gomaa, Freeway ramp-metering control based on reinforcement learning, *The 11th IEEE International Conference on Control and Automation (ICCA)*, Taichung, Taiwan, pp.1226-1231, 2014.

[12] M. Papageorgiou, H. Hadj-Salem and F. Middelham, ALINEA local ramp metering: Summary of field results, *Transportation Research Record: Journal of the Transportation Research Board*, vol.1603, pp.90-98, 1997.

[13] E. Smaragdis and M. Papageorgiou, Series of new local ramp metering strategies, *Transportation Research Record: Journal of the Transportation Research Board*, vol.1856, pp.74-86, 2003.

[14] E. Smaragdis, M. Papageorgiou and E. Kosmatopoulos, A flow-maximizing adaptive local ramp metering strategy, *Transportation Research Part B: Methodological*, vol.38, no.3, pp.251-270, 2004.

[15] H. M. Zhang and S. G. Ritchie, Freeway ramp metering using artificial neural networks, *Transportation Research Part C: Emerging Technologies*, vol.5, no.5, pp.273-286, 1997.

[16] D. I. S. Vukanovic and D. I. O. Ernhofer, Field evaluation of the fuzzy logic based ramp metering algorithm ACCEZZ, *IFAC Proceedings Volumes*, vol.39, no.12, pp.119-123, 1997.

[17] R. C. Carlson, I. Papamichail and M. Papageorgiou, Integrated feedback ramp metering and mainstream traffic flow control on motorways using variable speed limits, *Transportation Research Part C: Emerging Technologies*, vol.46, pp.209-221, 2014.

[18] S. Agarwal, P. Kachroo, S. Contreras et al., Feedback-coordinated ramp control of consecutive on-ramps using distributed modeling and Godunov-based satisfiable allocation, *IEEE Trans. Intelligent Transportation Systems*, vol.16, no.5, pp.2384-2392, 2015.

[19] M. Bariń and F. Borrelli, Distributed averaging with flow constraints, *IEEE 2011 American Control Conference*, San Francisco, USA, pp.4834-4839, 2011.

[20] D. Pisarski and C. Canudas-de-Wit, Analysis and design of equilibrium points for the cell-transmission traffic model, *IEEE 2012 American Control Conference*, Montreal, Canada, pp.5763-5768, 2012.

[21] D. C. Gazis, R. Herman and R. W. Rothery, Nonlinear follow-the-leader models of traffic flow, *Operations Research*, vol.9, no.4, pp.545-567, 1961.

[22] M. G. Lighthill and G. B. Whitham, On kinematic waves. II. A theory of traffic flow on long crowded roads, *Sciences*, vol.229, no.1178, pp.317-345, 1955.

[23] P. I. Richards, Shock waves on the highway, *Operations Research*, vol.4, no.1, pp.42-51, 1956.

[24] H. J. Payne, Models of freeway traffic and control, *Mathematical Models of Public Systems*, pp.51-61, 1971.

[25] C. F. Daganzo, The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory, *Transportation Research Part B: Methodological*, vol.28, no.4, pp.269-287, 1994.

[26] C. F. Daganzo, Requiem for second-order fluid approximations of traffic flow, *Transportation Research Part B: Methodological*, vol.29, no.4, pp.277-286, 1995.

[27] X. Sun, L. Muñoz and R. Horowitz, Mixture Kalman filter based highway congestion mode and vehicle density estimator and its application, *IEEE 2004 American Control Conference*, Boston, USA, pp.2098-2103, 2004.

[28] S. P. Boyd, L. E. I. Ghaoui, E. Feron et al., Linear matrix inequalities in system and control theory, *Philadelphia: Society for Industrial and Applied Mathematics*, 1994.