RESEARCH ON DRIVER’S CHOICE BEHAVIOR
BASED ON EVOLUTIONARY GAME MODEL
OF IMPROVED REPLICATION DYNAMICS

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ABSTRACT. In the urban traffic system, due to the macroscopic characteristics of the road network information, the driver cannot fully grasp the real-time dynamic traffic information, which is bound to form a game with the induced information provided by the traffic manager. At the same time, there are also game conflicts among the group drivers. In this paper, evolutionary game theory is used to analyze the interaction between drivers’ path selection behavior under information-induced conditions and the game relationship between them. The evolutionary model of driver’s path selection behavior under induced conditions is established. And based on the improved replication dynamic equation, the intensity coefficient was introduced for quantitative analysis, which extended the changes of different drivers choosing different paths. The research shows that the evolution of the equilibrium point is related to the driver’s natural growth rate, the value of the natural growth rate is different, and finally different evolutionary steady states are determined. After a comprehensive discussion of various situations, some suggestions on how to flexibly implement practical induction strategies according to the distribution of evolutionary stability strategies are proposed, which provides theoretical support for alleviating traffic pressure to a certain extent.

Keywords: Traffic guidance, Evolutionary game, Replication dynamic, Intensity coefficient, Evolutionary stability strategy

1. Introduction. With the rapid economic development in our country, urban traffic congestion is becoming more and more serious. Traffic congestion and traffic accidents are even more frequent. More high-speed arterial roads, wider urban roads, metro and sea-crossing bridges can significantly improve traffic flow. The measures of the situation are inevitably constrained through the land resources. China’s urban traffic is facing more and more severe tests and challenges. It is imperative to establish an urban intelligent transportation system to comprehensively and effectively solve traffic congestion, traffic accidents, environmental protection and energy conservation. On the other hand, urban traffic is a large and complicated system with rich regional characteristics. In different regions, the driver’s properties, road network hardware and software conditions cannot be the same. Therefore, a common ATIS (Automatic Terminal Information Service) permeability cannot meet the actual needs. However, there are few studies on the driver’s
path selection behavior at home and abroad at present. Based on the evolutionary game theory, Li analyzed the travel path selection behavior under the condition of information induction [1], but did not find the concrete evolution stable state. Uchiyama and Taniguchi developed a route choice model that considered travel time reliability and traffic impediments, including traffic accidents [2]. Based on evolutionary game theory, they designed the model to identify a route that a dispatcher chooses when considering changes in daily route travel times and traffic impediments using measured data. Based on the characteristics of the bounded rationality of drivers [3], an evolutionary game model of travel-route selection behavior under information-induced conditions was established and the strategy of evolutionary stability was analyzed to some extent to reveal the behavior of drivers’ route selection under information-induced conditions. Guo and Jin established evolutionary game model analysis [4], and the choice of commuting methods is mainly affected by travel time, travel costs, comfort and other factors. When the urban road transport system cannot carry more and more private cars, commute by private cars is no longer the only optimal way. Wang et al. proposed a path selection prediction model that takes account of the vehicle navigation characteristics [5]. The model is proposed to prevent the delay in the release of guidance information and route planning due to inaccurate timing predictions of the traditional guidance systems. Li and Xu analysed the influence of traffic signs on the path selection behavior of pedestrians, and constructed MAKLINK diagram to study the impact of traffic signs on pedestrians’ proceeding decisions by a simulation technique [6]. With the MAKLINK diagram, the path plan under the guidance of traffic signs is formulated. Then, the optimal pedestrian path on the MAKLINK diagram considering the effect of traffic signs is optimized via the ant colony algorithm. Peng et al. established a vehicle route choosing model without guidance information. The model is based on a finite rational fuzzy game [7]. The game equilibrium results are obtained under various initial conditions. Numerical simulation represents that with no guidance information, the traffic flow distribution of the network eventually reaches a balance given by theoretical analysis.

The above studies mainly focus on the influence of induced information on the driver’s route selection behavior, but there are few studies on the driver’s route selection behavior from the macroscopic perspective. Based on the improved replication dynamic equation and the evolutionary game theory, this paper analyzes the interaction between different driver groups, and obtains the influence of the growth rate of drivers on the evolutionary stability strategy. These provide theoretical support for the formulation of traffic guidance strategies and the long-term evaluation of traffic guidance information systems. The article is divided into four parts altogether. The first three parts establish the evolutionary game model and obtain the drivers’ income matrix. Based on the traditional replicating dynamic equations, the evolution and stability of the system are analyzed. In the fourth part, the dynamic equation of replication is extended to compare the evolutionarily stable state of the new system with that of the previous one, and a more stable evolution strategy is obtained.

2. Model Establishment. Traffic system is a large system filled with lots of complex individual interactions. It is because of the existence of such interactions and uncertainties that the basis for the driver’s path selection evolution is formed [8]. We consider the simple path selection behavior shown in Figure 1.

In order to achieve a reasonable allocation of traffic flow, the traffic manager provides the driver with a recommended driving route according to some induction strategy. The driver will play a game at intersection A. There are two paths from intersection A to intersection B, line1 and line2 respectively. The guidance information given by the guidance
system is that line1 is better than line2 but line2 is shorter than line1. After receiving the route provided by the traffic manager, the driver applies his own knowledge and experience to making decisions [9]. We study the object of the entire driver group, not a single driver. In the face of induced information, some drivers choose line1 and others choose line2, remembering group a and group b respectively.

For the sake of convenience, we let:

- \( x \): the probability of the driver in group a choosing line1 (Then the probability that group a driver chooses line2 is \( 1 - x \));
- \( y \): the probability of the driver in group b choosing line1 (Then the probability that group a driver chooses line2 is \( 1 - y \));
- \( u_1, u_2 \): represent the normal pass payment for driver in group a for line1 and line2, respectively;
- \( v_1, v_2 \): represent the normal pass payment for driver in group b for line1 and line2, respectively;
- \( p_1 \): the probability of congestion occurring when both select line1;
- \( p_2 \): the probability of congestion occurring when both select line2;
- \( e_1, e_2 \): respectively, when the drivers of group a and the drivers of group b both select line1, the waiting payment due to congestion;
- \( f_1, f_2 \): respectively, when the drivers of group a and the drivers of group b both select line2, the waiting payment due to congestion;
- \( q_1 \): the probability of traffic accidents due to congestion when both select line1;
- \( q_2 \): the probability of traffic accidents due to congestion when both select line2;
- \( g_1, g_2 \): respectively, when the drivers of group a and the drivers of group b both select line1, the payment after the traffic accident caused by the congestion;
- \( h_1, h_2 \): respectively, when the drivers of group a and the drivers of group b both select line2, the payment after the traffic accident caused by the congestion.

The driver’s strategy selection behavior is faced with a dynamic environment. During the decision-making process, the driver can choose different strategies to obtain the corresponding benefits. After each decision, the driver accumulates experience through study, guesses the overall situation induced, dynamically adjusts his behavior strategy, and improves his effectiveness as much as possible [10]. After a period of evolution, the adoption of a strategic act will result in an increase or decrease in driver profitability [11], so that the driver’s strategic behavior distribution will evolve according to the principle of “survival of the fittest”. According to idea of the hawk-dove game, we have the driver’s path choice behavior game model. The game participants are drivers of two groups, each of them has two strategies to choose, line1 and line2. Therefore, we establish the following revenue matrix.
Table 1. Two groups of driver’s income matrix

<table>
<thead>
<tr>
<th>Game players</th>
<th>Income</th>
<th>Ratio</th>
<th>Drivers of group b</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Line1</td>
<td></td>
<td>Line1</td>
</tr>
<tr>
<td>Drivers of group b</td>
<td></td>
<td></td>
<td>Line2</td>
</tr>
<tr>
<td>Line2</td>
<td>1 - x</td>
<td></td>
<td>(u1 + p1e1 + q1g1,</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>v1 + p1e2 + q1g2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(u2, v1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(u2 + p2f1 + q2h1,</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>v2 + p2f2 + q2h2)</td>
</tr>
</tbody>
</table>

3. Analysis of Model.

3.1. Driver’s revenue analysis. Evolved by evolutionary game theory [12], when drivers of group a choose line1, its revenue is:

\[ w_1 = y(u_1 + p_1e_1 + q_1g_1) + (1 - y)u_1 \] 

(1)

When drivers of group a selects line2, the gain is:

\[ w_2 = yu_2 + (1 - y)(u_2 + p_2f_1 + q_2h_1) \] 

(2)

The average income of drivers of group a is:

\[ \bar{w} = xw_1 + (1 - x)w_2 \] 

(3)

Therefore, a group of driver’s replication dynamic equation is:

\[ \dot{x} = \frac{dx}{dt} = x(w_1 - \bar{w}) = x(1 - x)(w_2 - w_1) \]  

(4)

\[ \dot{y} = \frac{dy}{dt} = y(1 - y) [(p_1e_2 + q_1g_2 + p_2f_2 + q_2h_2) + (v_1 - v_2 - p_2f_2 - q_2h_2)] \] 

(5)

3.2. Evolutionary steady state analysis. For the sake of simplicity, we let

\[ P_1 = p_1e_1 + q_1g_1 + p_2f_1 + q_2h_1 \] 

(6)

\[ P_2 = u_1 - u_2 - p_2f_1 - q_2h_1 \] 

(7)

\[ Q_1 = p_1e_2 + q_1g_2 + p_2f_2 + q_2h_2 \] 

(8)

\[ Q_2 = v_1 - v_2 - p_2f_2 - q_2h_1 \] 

(9)

Making \( \dot{x} = 0, \dot{y} = 0 \), then the game has five balance points, namely \( E_1(0, 0), E_2(0, 1), E_3(1, 0), E_4(1, 1), E_5 \left( -\frac{Q_2}{Q_1}, -\frac{P_2}{P_1} \right) \). In (4) and (5), two types of game, the Jacobian matrix has a positive determinant. When the trace is negative, the equilibrium point is stable and the determinant is positive. When the trajectory is positive, the equilibrium point is not stable. When the determinant is negative, and the trace is any value, the balance point is saddle point [13]. Further analysis found that the equilibrium points became stable just under certain conditions: When both drivers choose line2, the payment for waiting due to congestion and payment for causing traffic accidents to drivers of two groups are relatively large, and the equilibrium point \( E_1(0, 0) \) is stability point. The balance point \( E_2(0, 1) \) is the stability point, when line2 is selected by both drivers, the payment for waiting due to congestion and the payment generated by traffic accidents to drivers of group a are relatively large. When both drivers of two groups choose line2, the payment for waiting due to congestion and the payment caused by traffic accidents to drivers of group b are relatively large, and the equilibrium point \( E_3(1, 0) \) is the stability point.
When both drivers choose line1 or line2, the payment for waiting due to congestion and payment for causing traffic accidents to drivers of two groups are relatively small, and the equilibrium point $E_1(1,1)$ is the stability point. However, balance point $E_5\left(-\frac{p_2}{q_1},-\frac{p_1}{q_1}\right)$ cannot be a stability point. When both drivers of group a and group b choose line1 or line2 at the same time, the payment for waiting due to congestion caused by both drivers and the traffic accidents caused by them are relatively large. The stable strategy of the model is (line1, line2). The drivers of group a and group b apply selected line1 and line2 respectively to reducing the payments.

In practice, the number of drivers of group a and group b is not fixed and will increase or decrease over time.

4. Improved Steady-State Analysis of Replication Dynamics.

4.1. Extended replication dynamic equation. According to the method provided in [13], we can deduce the dynamic equation of replication after the system is introduced into the natural growth rate.

The replication dynamic equation is:

$$\dot{x} = \lambda_1 x (1-x) \left\{ y \left[ p_1 e_1 + q_1 g_1 + \frac{\lambda_2}{\lambda_1} (p_2 f_1 + q_2 h_1) \right] + u_1 - \frac{\lambda_2}{\lambda_1} (u_2 + p_2 f_1 + q_2 h_1) \right\} \quad (10)$$

$$\dot{y} = \rho_1 y (1-y) \left\{ x \left[ p_1 e_2 + q_1 g_2 + \frac{\lambda_2}{\lambda_1} (p_2 f_2 + q_2 h_2) \right] + v_1 - \frac{\rho_2}{\rho_1} (v_2 + p_2 f_2 + q_2 h_2) \right\} \quad (11)$$

Here, $\lambda_1$, $\lambda_2$ were the natural growth rates when line1 and line2 were selected respectively for drivers of group a. $\rho_1$, $\rho_2$ were respectively the natural growth rates when line1 and line2 were selected by drivers of group b respectively, and let $\frac{\lambda_2}{\lambda_1} = q_{21}$, $\frac{\rho_2}{\rho_1} = p_{21}$. The probability $q_{21}$ and $p_{21}$ are relabeled the intensity coefficient.

For convenience, we choose another set of letters to replace the original letters, like the following:

- $u_1 + p_1 e_1 + q_1 g_1 = a$
- $v_1 + p_1 e_2 + q_1 g_2 = b$
- $u_1 = c$
- $v_2 = d$
- $u_2 = e$
- $v_1 = f$
- $u_2 + p_2 f_1 + q_2 h_1 = g$
- $v_2 + p_2 f_2 + q_2 h_2 = h$

4.2. Steady state evolution trend. Since the size of the parameter value is undetermined, the stability of the equilibrium point cannot be determined. In this case, there are four situations [13]. When the parameter values in the two drivers’ income matrix (see Table 1) satisfy the first three cases, the game that composed of two types of (10) and (11) has 4 equilibrium points, namely $E_1(0,0)$, $E_2(0,1)$, $E_3(1,0)$, $E_4(1,1)$. As time goes by, the final equilibrium point is unique, and the evolution of the equilibrium point is related to the intensity coefficient $\lambda_i$, $\rho_i$ $(i = 1, 2)$. However, when there are two paths available, the values of the drivers of two groups in the revenue matrix satisfy

$$a - c - q_{21} e + q_{21} g \neq 0, \quad b - f - p_{21} d + p_{21} h \neq 0$$

There are 9 kinds of situations that conclude a total of 36 cases [11], the game has 5 balance points, namely $E_1(0,0)$, $E_2(0,1)$, $E_3(1,0)$, $E_4(1,1)$, $E_5\left(\frac{f-p_{21} h}{f-p_{21} h + p_2 d - b}, \frac{c-q_{21} e}{c-q_{21} e + q_2 d - a}\right)$. 

\[\]
After the evolution process, there may not be a steady-state equilibrium point or a steady-state equilibrium point which may not be unique. The extent to which one of the two steady-state equilibrium points is biased or which state is the center point is also related to the intensity coefficient $\lambda_i, \rho_i$ ($i = 1, 2$).

The following discussion in the case $0 < f - p_{21} h < b - p_{21} d$, $E_1(0, 0), E_4(1, 1)$ are the unstable points, $E_2(0, 1), E_3(1, 0)$ are the stable points, and $E_5 \left( \frac{f - p_{21} h}{f - p_{21} h + p_{21} d - b}, \frac{c - q_{21} g}{c - q_{21} g + q_{21} e - a} \right)$ is the saddle point. At this point, the dynamic evolution of the system is shown in Figure 2 below:

$$x^* = \frac{f - p_{21} h}{f - p_{21} h + p_{21} d - b}, \quad y^* = \frac{c - q_{21} g}{c - q_{21} g + q_{21} e - a}$$

![Figure 2. Dynamic evolution diagram](image-url)

$E_1(0, 0), E_2(0, 1), E_3(1, 0), E_4(1, 1)$ for the vertices, composed of a quadrilateral, $E_5(x^*, y^*)$ is an internal point of the quadrangle, and the area of the quadrangle is

$$S = \frac{1 \times y^* + 1 \times (1 - x^*)}{2} = \frac{1}{2} \left( \frac{c - q_{21} g}{c - q_{21} g + q_{21} e - a} + \frac{f - p_{21} h}{f - p_{21} h + p_{21} d - b} \right)$$

due to

$$\frac{\partial s}{\partial q_{21}} = \frac{ag - ce}{2(c - q_{21} g + q_{21} e - a)^2} = \frac{(u_1 + p_1 e_1 + q_1 g_1) (u_2 + p_2 f_1 + q_2 h_1) - u_1 u_2}{2 (c - q_{21} g + q_{21} e - a)^2} > 0$$

So $s$ will increase when $q_{21}$ increases. That is, the greater the intensity coefficient of driver in a group is, the closer its equilibrium strategy is to $x = 1$.

On the other hand, the partial derivative

$$\frac{\partial s}{\partial p_{21}} = \frac{df - bh}{2(f - p_{21} h + p_{21} d - b)^2} = \frac{u_1 u_2 - (v_1 + p_1 e_1 + q_1 g_1) (v_2 + p_2 f_2 + q_2 h_2)}{2 (f - p_{21} h + p_{21} e - b)^2} < 0$$

we know that $s$ will decrease when $p_{21}$ increases, and the greater the $b$-driver’s strength coefficient is, the closer its equilibrium strategy is to $y = 0$. With the result that, the
system equilibrium point tends to steady state $E_3(1,0)$, on the contrary, it tends to steady state $E_2(0,1)$. That is, when the number of drivers of group a and group b increases rapidly, the choice between line1 and line2 at the same time will make the payment caused by congestion awaiting and traffic accidents to be large relatively. The driver’s path selection behavior is faced with a dynamic environment, and unable to pre-select the road to his own satisfaction. Only after each selection he can accumulate experience through learning to guess the overall situation induced and constantly modify his behavior strategy, the effectiveness can be improved as much as possible. Similarly, the information-based guidance center does not always need to issue guidance information to drivers on a road network. It can determine whether or not to induce it according to the distribution of the evolutionary-stable strategy, or re-release the guidance information after the driver accepts the guidance information to make a strategy adjustment. At this time the evolution game is at stable moment.

5. Conclusion. China’s traffic guidance information system is in the process of construction and development. Traffic guidance systems may not meet the increasing expectations of the traffic management department. What kind of guidance strategy and how to induce are the focus of research on traffic control system. Based on the improved duplicate dynamic equation, we introduce the intensity coefficient for quantitative analysis and expand the change of path selection behavior. To a certain extent, the evolvement rule of driver’s path selection behavior with the change of quantity under information-induced condition is revealed, that is, the evolution of equilibrium point is related to the natural growth rate of driver, and the values of different natural growth rates finally determine different evolutionary stability status. Through the analysis of the above evolutionary game model, we get the dynamic adjustment result of driver’s group behavior and put forward the suggestion of implementing inducement strategy based on the conclusion of the study. However, this study is limited to drivers of two groups with only two paths. It is a subject to be studied further for the steady state analysis of the evolution of multiple paths and multiple drivers of groups.

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