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EFFICIENT SERIAL CONCATENATION OF SYMBOL BY SYMBOL AND WORD BY WORD DECODERS

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Abstract. *Decoding algorithms try to find data transmitted over communication channel or stored in physical supports, which are generally sources of noise. The decoding problem is NP-hard and many algebraic and heuristic decoders are developed for detecting and correcting errors. The Hartman Rudolph algorithm (HR) permits to decode a word symbol by symbol, it uses a very large set of dual codewords in decoding and it has therefore high complexity. The Soft Permutation Decoding Algorithm (SPDA) uses a part of the automorphism group to move noised symbols by noise in the redundancy part to correct them by re-encoding. In this paper, we propose to use partially the Hartman Rudolph algorithm (PHR) only on few symbols of very low reliabilities and with a small number of dual codewords. The sequence partially corrected is transmitted on a second decoder to correct remain symbols. The simulation results of the proposed decoding scheme, over the Additive White Gaussian Noise channel (AWGN), show that it passes or reaches the error correcting performances of many competitors; they show also that the performances of the PHR chained with SPDA decoder largely pass those of PHR or SPDA without concatenation. The complexity of the proposed scheme is discussed and the studies of impacts of all parameters are made by simulation. These studies allow obtaining good performances in terms of bit error rate with low complexity. Obtained results prove clearly the huge success of the proposed idea which can be generalized to any symbol by symbol decoder chained with any word by word decoder.*

Keywords: Error correcting codes, Soft permutation decoding algorithm, Hartman Rudolph, Automorphism group, Communication channels

1. **Introduction.** Digital communication systems integrate new features more and more to protect information. Figure 1 describes a general model of digital communication process. In this model, the data source generates digital data which are encoded and modulated for communication over a transmit channel. At the other end of the channel, the digital data are demodulated, decoded, and sent to the receiving user. Each element of this model is mathematically described by theorems and modeled by equations that govern their performances [1].

When data are sent over a transmit channel, there is always a chance that the received data will contain errors. Errors correcting codes are often used to partially or completely

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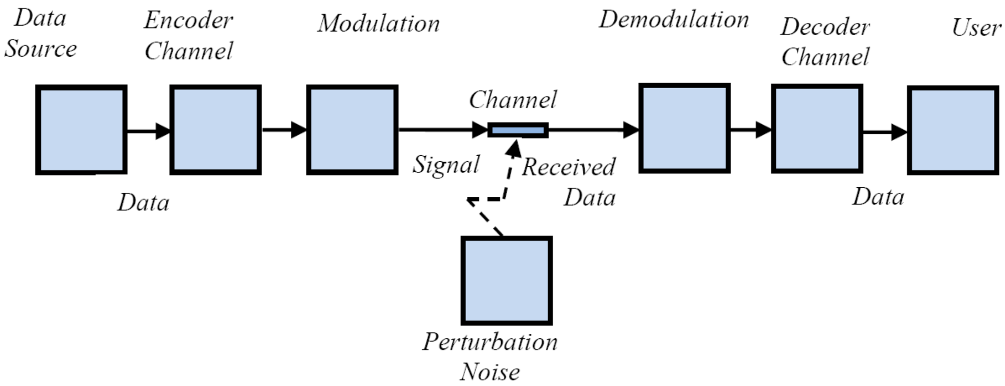

FIGURE 1. A simplified model of communication systems

correct these errors as much as possible, while at the same time ensure very high transmission speeds and low costs for the channel coder and decoder. The work of Shannon in 1948 [2] demonstrated that error correcting codes can ensure data transmission over a noisy channel, with an error probability as close to 0 as wants is possible, if the data source rate is less than a quantity called the channel capacity.

The channel coder is the first step in the error correction/detection process. The channel coder generates code words by addition of redundant symbols in the information part of original data, in a way that allows errors introduced into channel to be corrected. The channel is the way over which the information is transmitted. Examples of channels are, telephone lines, Internet cables, optical fiber lines, microwave link channels, high frequency channels, and cell phone channels. These are channels in which data is transmitted between two different positions. Data may also be conveyed between two different times, such as, by writing information onto a computer disk, and then retrieving it at a later time. CD-ROMs, DVDs, and Random Access memory are also other examples of channels.

In this work we focus our attention on improving Bit Error Rate performances (BER) of decoding process. We propose a novel decoding scheme, which minimizes the BER for data transmitted on channel that it is difficult to decode by the original version of the known Hartman Rudolph algorithm (HR).

There are actually two principal categories of decoders employed in communication systems: hard decision decoding and soft decision decoding. Hard decision decoding exploits binary inputs resulting from thresholding of the transmit channel output and usually it uses the Hamming distance as a measure. However, soft decision decoding exploits real inputs, i.e., the information in the transmit channel output is not thresholded and the measure used is often the Euclidean distance [1]. Decoding operation is an NP-hard problem and many algebraic and heuristic decoders are developed for detecting and correcting errors.

The maximum likelihood decoder is an optimal decoder because it minimizes the BER, it compares all codewords with the received sequence [1], but for a fixed coding rate, the number of comparisons rapidly becomes an obstacle when the length n of the code increases. Consequently, sub-optimal decoders with acceptable performance are appearing. Algebraic and probabilistic methods also have been used to improve BER, for example, the Ordered Statistics Decoding (OSD) [3], the Chase algorithm [4], the Berlekamp-Massey algorithm (BM) [5], and the Hartmann Rudolf algorithm [6] applied to linear block codes.

Recently, this problem has also been attacked with heuristic methods. For example, one idea used A* algorithm to decode linear block codes [7], another ones use genetic algorithms [8-13] and a third one uses simulated annealing method to decode linear block codes [14]. Deep learning is also used in decoding process [15] and authors of [16] used a local search algorithm for finding the correct transmitted version of the received sequence. In [17-20], authors have thought to use the hash techniques to accelerate the decoding process and to have therefore an acceptable run time complexity. In [21-23], authors have used a part of the automorphism group to move the noised symbols in the redundancy part of the sequence to decode and the most likely transmitted codeword is obtained by re-encoding. This idea will be exploited in this paper combined with the idea proposed in [6].

The remainder of this paper is structured as follows. In Section 2, we present some related works and the motivation of this work. Section 3 presents the proposed decoding scheme S2W2Dec. In Section 4 we give some simulation results of BER obtained by application of S2W2Dec, and we make a comparison with other decoders. In Section 5, we study the complexities of some decoders and compare them with that of S2W2Dec. Finally, a conclusion and a possible future direction of this research are outlined in Section 6.

2. **Related Works and Motivation.** Let $C(n,k)$ be a linear code of dimension k and length n. The Hartman Rudolph (HR) decoder [2] is a symbol by symbol soft decision optimal decoding algorithm. It maximizes the probability that a bit corresponding to a symbol r_j, of the sequence r to decode, is equal to 1 or 0. Hartman and Rudolph have shown that this probability depends on all the codewords of the dual code $C^{\perp}$ generated by a matrix H. This algorithm has a high complexity because it uses 2^{n-k} dual codewords; therefore, it can be applicable only in the case of linear codes with very high rate or small lengths. More precisely, for decoding a received word, it uses Formula (1) to decide if the m^{th} bit of the decoded word c' is equal to 1 or 0 from the received sequence r.

$$\begin{cases} c'_m = 0 & \text{if } \displaystyle\sum_{j=1}^{2^{n-k}} \prod_{l=1}^{n} \left(\frac{1-\Phi_l}{1+\Phi_l}\right)^{c^{\perp}_{jl} \oplus \delta_{ml}} > 0 \\ c'_m = 1 & \text{otherwise} \end{cases} \tag{1}$$

with the following notations:

- $\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$
- $\Phi_m = \Pr(r_m/1)/\Pr(r_m/0)$
- The bit $c^{\perp}_{jl}$ denotes the l^{th} bit of the j^{th} codeword of the code $C^{\perp}$.

When the HR decoder is used only in few symbols and uses just a part of M dual codewords from the dual code space, we call it the Partial Hartman-Rudolph decoder (PHR). These M codewords are generally chosen to have a small weight for improving the quality of decoding. Formula (1) becomes:

$$\begin{cases} c'_m = 0 & \text{if } Q = \displaystyle\sum_{j=1}^{M} \prod_{l=1}^{n} \left(\frac{1-\Phi_l}{1+\Phi_l}\right)^{c^{\perp}_{jl} \oplus \delta_{ml}} > 0 \\ c'_m = 1 & \text{otherwise} \end{cases} \tag{2}$$

Permutation decoding algorithm PDA developed by MacWilliams [21] can be used when a code has sufficiently many automorphisms to ensure the existence of a set of automorphisms called a PD-set. A PD-set for a code is a set of automorphisms of the code which is such that, if the code can correct t errors, then every possible error vector of weight t or less can be moved by some members of S out of the information positions. The SPDA decoder [22] is a soft decision version of the PDA decoder; it uses Euclidean distance instead of Hamming distance and it is decided by the closest codeword from the list of all codewords obtained by re-encoding. In [23] an iterative version of SPDA is developed for quadratic residue codes and yields good performances.

Let L be a list of stabilizers, h the hard decision version of rand. The decoder SPDA works as follows.

Inputs:
  - L a List of P stabilizers (a Part of the automorphism group of size P)
  - $r = (r_1, r_2, \ldots, r_n)$ the received word
  - The generator matrix G in its systematic form

Begin
$h \leftarrow$ hard version of the sequence r
dist $\leftarrow n$;
For $i = 1$ to P do
  $\pi \leftarrow$ the i^{th} permutation in L
  $h' \leftarrow \pi(h) = (h'_1, h'_2, \ldots, h'_k, h'_{k+1}, \ldots, h'_n)$
  New_Information_Vector $(h_1, h_2, \ldots, h_k)$
  $c \leftarrow$ New_Information_Vector*G
  $c' \leftarrow \pi^{-1}(c)$
  if (Euclidian_distance(r, c') < dist) then
    • dist = Euclidian_distance(r, c')
    • closest_word c'
  end if
end for
End
Outputs: closest_word

In the SPDA algorithm above the generator matrix G is presented in its systematic form to facilitate the process of obtaining correct symbols from the permuted version of the obtained closest codeword. When the code is cyclic, its generator polynomial can be used instead of the matrix G and can allow also systematic encoding.

In order to study the impact of the parameter M on the error correcting performances of the PHR decoder we have used this latest to decode QR$(31, 16, 7)$ code by varying M from 30 to 32767 according to the simulation parameters given in Table 1 and Formula (2). Figure 2 summarizes the obtained results. It shows that the use of only M dual codewords yields very bad performances when it is between 30 and 1000. The use of a large list of 5000 dual codewords improves relatively the performances. The use of a very large list of 15000 dual codewords improves considerably the performances and reaches those of the full list of $32767 = 2^{n-k} - 1$ dual codeword which corresponds to Formula (1).

The analysis of the results obtained in Figure 2 permits to conclude that the performances of PHR decoder are improved when the parameter M is increased. Sadly, the

Table 1. Simulation parameters of PHR for the QR$(31, 16, 7)$ code

Channel	AWGN
Modulation	BPSK
Minimum residual errors	200
Minimum transmitted blocks	10000

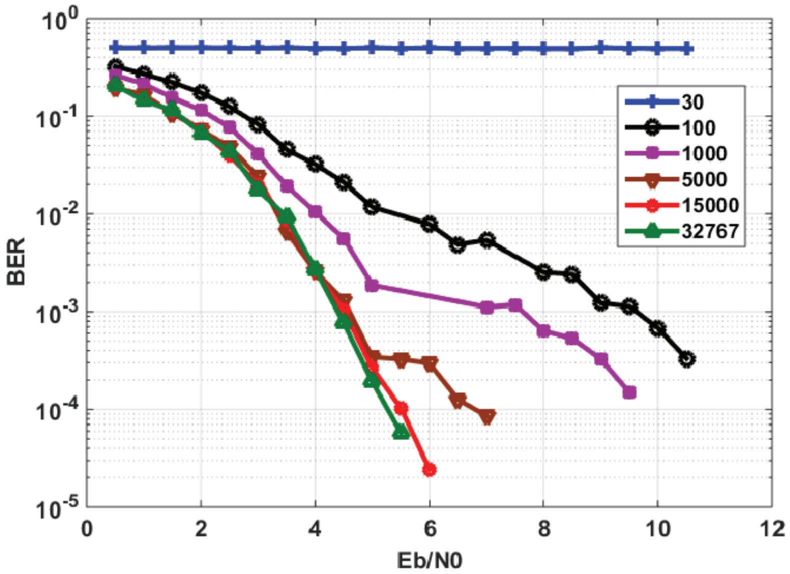

FIGURE 2. Impact of the number M of dual codewords on the PHR performances for the QR$(31, 16, 7)$ code

increase of M increases considerably the time complexity and the PHR cannot be used in practice to correct errors in real-time communication channels.

The analysis above has motivated us to ask the following question: is it possible to obtain good performance of the PHR decoder with only few dual codewords?

In this paper we will give an affirmative response to this question by combining PHR with SPDA to obtain the S2W2Dec decoder presented in the next section.

3. **The Proposed Decoding Scheme S2W2Dec.** The decoders of linear block codes can be classified in two families. The symbol by symbol decoders (S2 decoders) have the characteristic to decide if a symbol is correct or not without necessity to treat all the symbols of the received word. This first family contains as example the famous Hartman-Rudolph decoder [6]. On the contrary, the word by word decoders (W2 decoders) need to decide by a codeword entirely. This second family contains SPDA [22,23], Berlekamp-Massey [5], DDGA [11], SDGA [8], Maini [10], ARDecGA [24], AutDag [13], Chase-SDec [17] ande CGA-HSP [12] decoders.

In order to reduce the complexity of the PHR decoder, we introduce three parameters.

1) The number M of dual codewords of low weights to use in PHR (Formula (2)) instead of the 2^{n-k} dual codewords in Formula (1).
2) Threshold reliability S: In order to accelerate PHR, we propose to apply it only on symbols of small reliabilities, more precisely on symbols of reliability less than S.
3) Stop Threshold Reliability (STR): The quantity Q in (2) is computed practically by a loop which cumulates the results beginning from the first dual codeword to the M^{th} codeword. We have proposed that when the absolute value of the quantity Q becomes more than STR, the partial Hartman-Rudolph algorithm can decides the treated symbol and the loop can be stopped.
4) Stop Threshold Dimension (STD): Before stopping the PHR decoder when $|Q|$ becomes more than STR, it is important to ensure that a sufficient part of the set of M dual codewords is exploited in the computation of Q. So, we have proposed that PHR decoder can stop when at least $\lfloor STD * M \rfloor$ dual codewords are used in the computation of Q and when $|Q|$ becomes more than STR.

In this paper, we choose the symbol by symbol Partial Hartman-Rudolph decoder (PHR). For the word by word decoders we will study the cases of ARDecGA and SPDA.

The S2W2Dec works as follows.

Inputs:
- The sequence r of length n to decode
- The symbol by symbol decoder S2Dec with their parameters
- The word by word decoder W2Dec with their parameters
- Threshold realiability S

Outputs: The decided corrected word c

Begin

Step 1) For i from 1 to n do:
    if ($|r_i| < S$) then correct the symbol r_i by the S2Dec decoder
  End For

Step 2) Use the decoder W2Dec to decode the new version r' of the sequence r obtained in Step 1).

End

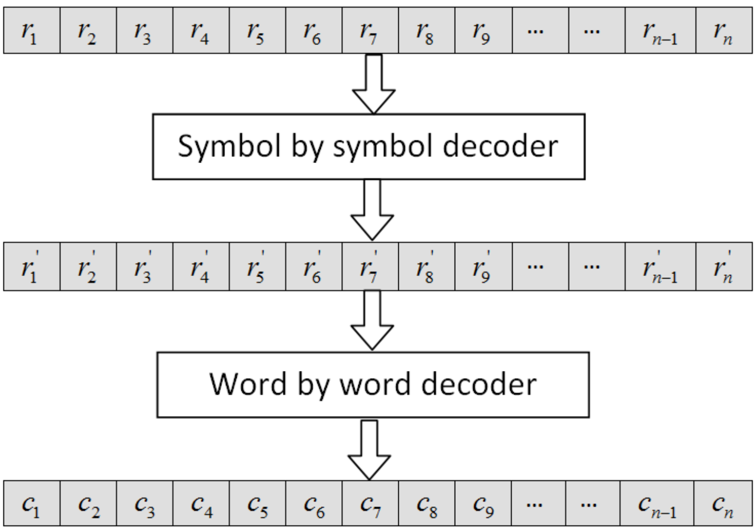

FIGURE 3. S2W2Dec diagram

Figure 3 presents a diagram of S2W2Dec and explains the steps given in the algorithm above.

In practice, the proposed decoder S2W2Dec can be used to correct errors in many telecommunication systems; for example, it can be used in satellite communication channels, in wireless communication, in telephone conversations and also in storage systems like the hard disk and the random-access memory.

In coding theory, the proposed concatenation represents a new way to improving the bit error rate performances by exploiting existing symbol by symbol and word by word decoders. This idea will open new perspectives in decoding process.

The famous known scheme that concatenates decoders for simple codes is that of Chase [4] which calls 2^t time a hard decision decoder with t being the error correction capability of the used code. When t increases, the complexity of the Chase scheme increases exponentially and it becomes inapplicable in practice. The novelty and the main gain in S2W2Dec are that it calls a symbol by symbol and a word by word decoder in just one time for each one.

4. **Simulation Results.** In order to show the error correcting performances of S2W2Dec, we do intensive simulations. With the exception of the cases where the simulation parameters are explicitly mentioned, the simulations were made with default parameters outlined in Table 2. The performances are given in terms of BER (Bit Error Rate) as a function of the signal to noise ratio SNR = E_b/N_0 (the energy per bit to noise power spectral density ratio). The simulation will be presented for Quadratic-Residue codes (QR).

TABLE 2. Default simulation parameters of S2W2Dec

Number of automorphisms P	1200
Threshold realiability S	0.5
Stop Threshold Realiability (STR)	0.4
Stop Threshold Dimension (STD)	0.4
M	2414
Default code	QR$(71, 36)$
Channel	AWGN
Modulation	BPSK
Minimum residual errors	200
Minimum transmitted blocks	10000

Figure 4 shows the S2W2Dec(PHR-ARDecGA) performances for the QR codes of lengths 47 and 71 and those of the BCH codes of length 63 and dimensions 51 and 39 obtained by using the simulation parameters given in Table 2. This figure proves the height quality of decoding in terms of BER performances of S2W2Dec(PHR-ARDecGA). Considering that without coding the BER = 10^{-5} is obtained at SNR = 9.6 dB, Figure

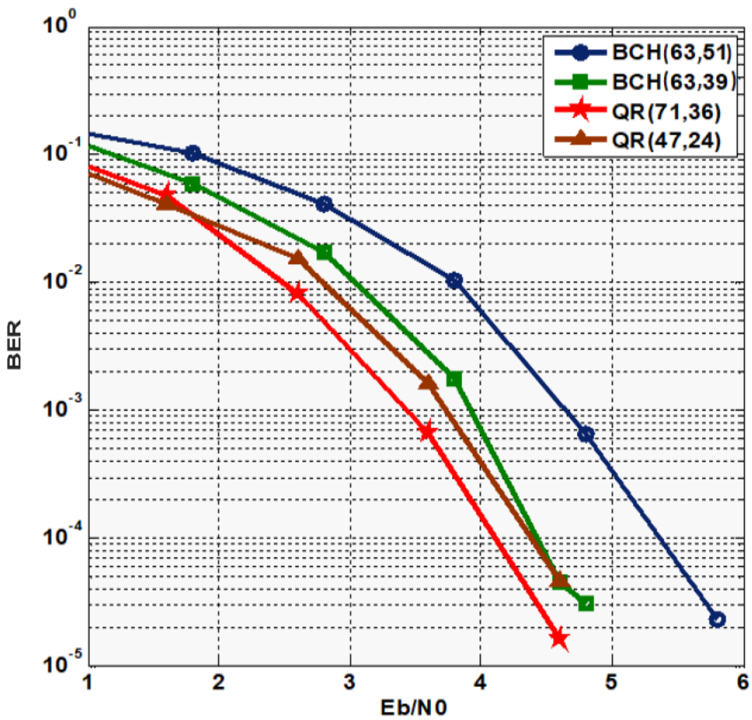

FIGURE 4. S2W2Dec(PHR-ARDecGA) performances for some QR and BCH codes

4 shows that a gain of coding of about 5.2 dB is obtained by applying S2W2Dec(PHR-ARDecGA) on the QR(71, 36) code. The first success of S2W2Dec is then their best BER performances.

Figure 5 shows the S2W2Dec(PHR-SPDA) performances for the QR codes of lengths 31, 47 and 71 codes. This figure proves the height quality of decoding in terms of BER performances of S2W2Dec(PHR-SPDA). It also shows that the gain of coding increases when the length of the QR code increases.

To show the impact of the permutations number P on the error correcting performances of S2W2Dec(PHR-SPDA) applied to the QR(71, 36) code, we give in Figure 6 the results obtained by varying P from 40 to 1200. This figure shows that the BER performances are improved when P increased. When P passes the value 900, the gain obtained when

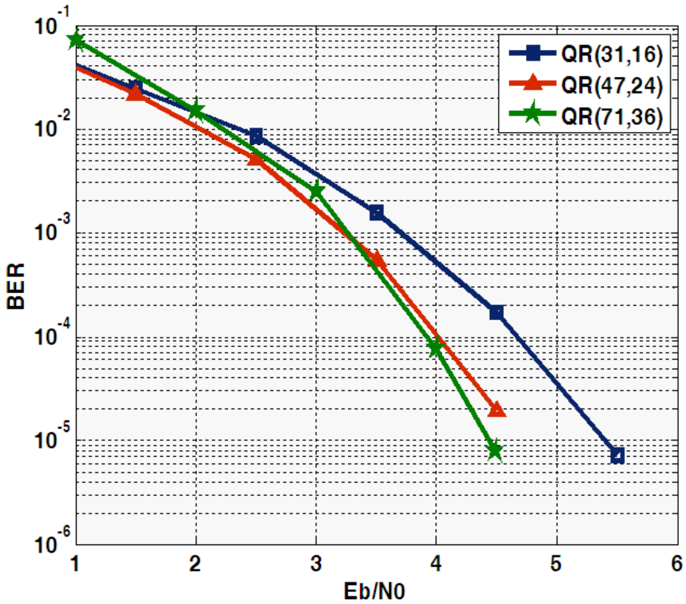

FIGURE 5. S2W2Dec(PHR-SPDA) performances for some QR codes

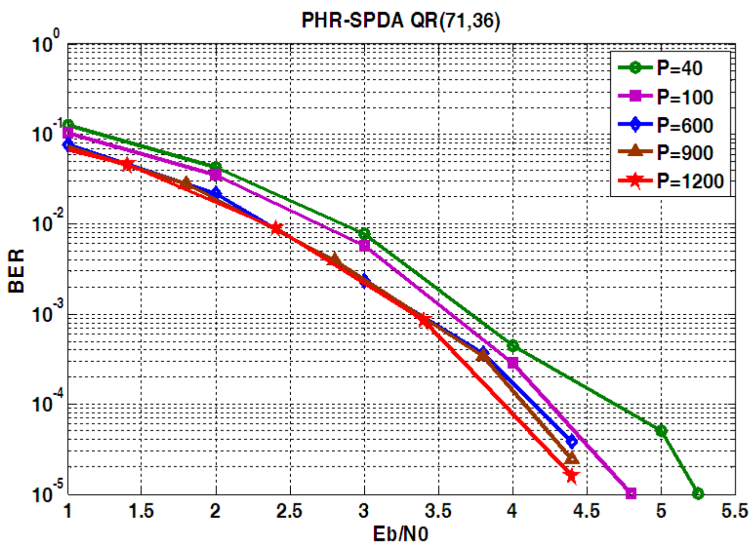

FIGURE 6. Impact of the permutations number P on S2W2Dec(PHR-SPDA) performances

it is increased to 1200 is practically negligible. The second success of S2W2Dec is that it yields good BER performances with only a few permutations.

To show the effect of the number of dual codewords M on the error correcting performances of S2W2Dec applied to the QR(71, 36) code, we give in Figure 7 the results obtained by varying M from 80 to 5000. This figure shows that the BER performances are improved when M increased. When M passes the value 2414, the gain obtained when it is increased to 5000 is practically negligible. The third success of S2W2Dec is that it yields good BER performances with only a few dual codewords.

To show the impact of the Threshold reliability S on the error correcting performances of S2W2Dec, applied to the QR(71, 36) code, we illustrate in Figure 8 the results obtained by varying S from 0 to 0.7. This figure shows that the BER performances are improved when S increased. When S passes the value 0.35, the gain obtained when it is increased

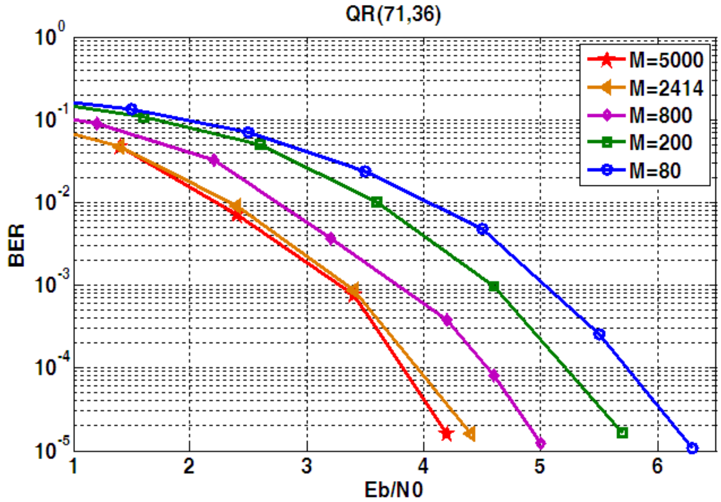

FIGURE 7. Impact of the parameter M on S2W2Dec performances for QR(71, 36) code

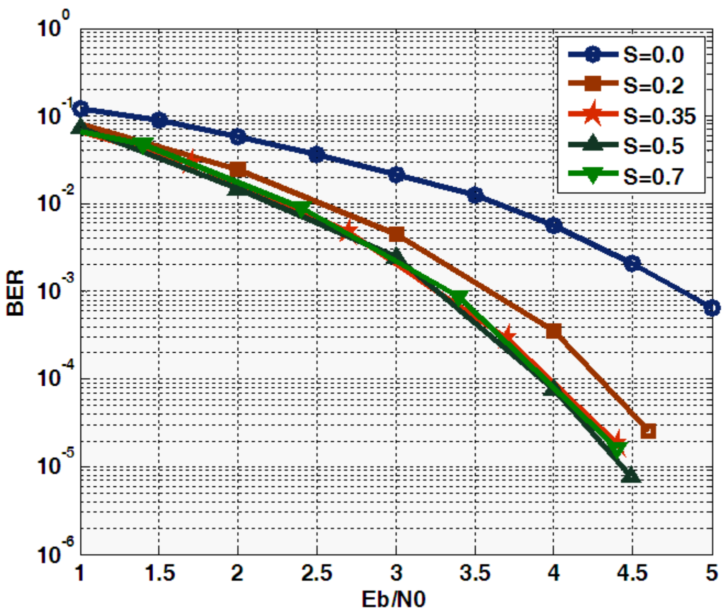

FIGURE 8. Impact of the parameter S on S2W2Dec performances for QR(71, 36) code

to 0.7 is practically negligible. The fourth success of S2W2Dec is that it yields good BER performances with applying PHR only on few symbols of reliability less than S.

To show the impact of the Stop Threshold Reliability (STR) on the error correcting performances of S2W2Dec(PHR-SPDA), applied to the QR(71, 36) code, we illustrate in Figure 9 the results obtained by varying STR from 0.1 to 0.95. This figure shows that the BER performances are not very affected when STR changes. When STR passes the value 0.1, the gain obtained when it is increased to 0.95 is practically negligible. The fifth success of S2W2Dec is that it yields good BER performances with stopping PHR when absolute value of the quantity Q in (2) becomes more than STR. This stop affects the rapidity of decoding and decreases the time complexity of S2W2Dec.

To show the impact of the Stop Threshold Dimension (STD) on the error correcting performances of S2W2Dec(PHR-SPDA), applied to the QR(71, 36) code, we illustrate in Figure 10 the results obtained by varying STD from 0.1 to 0.95. This figure shows that the BER performances are not very affected when STD changes. When STD passes the value 0.1, the gain obtained when it is increased to 0.95 is practically negligible. The

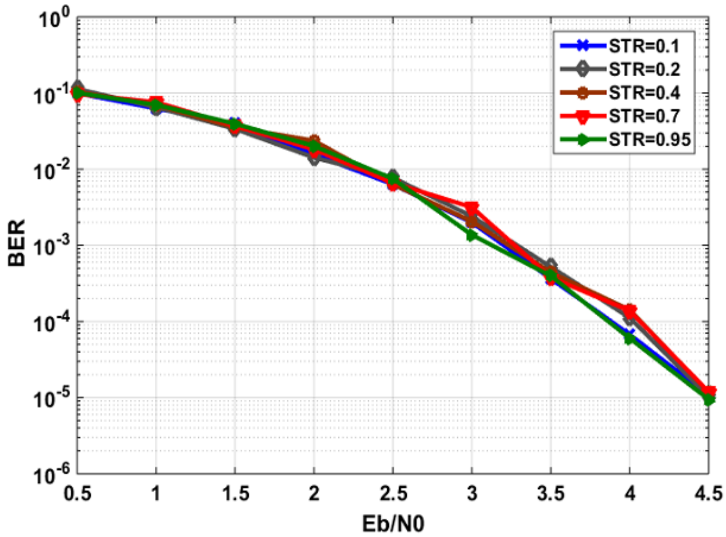

FIGURE 9. Impact of the parameter STR on S2W2Dec performances for QR(71, 36)

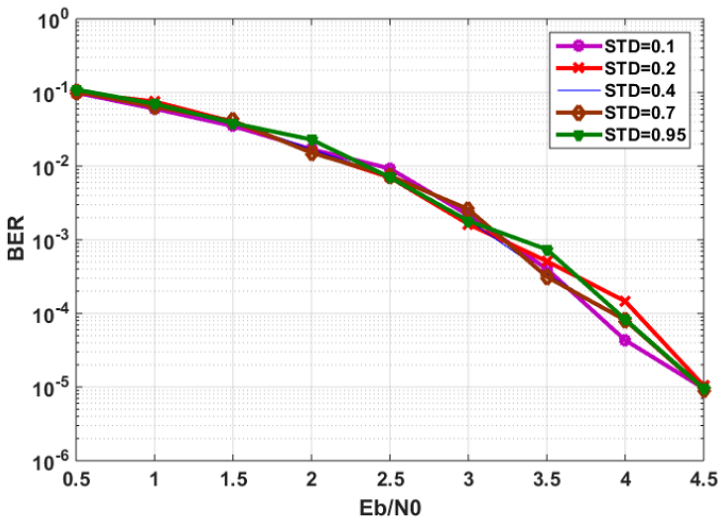

FIGURE 10. Impact of the parameter STD on S2W2Dec performances for QR(71, 36)

sixth success of S2W2Dec is that it yields good BER performances with stopping PHR when absolute value of the quantity Q in (2) becomes more than STR and when at least $\lfloor STD * M \rfloor$ dual codewords are used in the computation of Q. This stop affects the rapidity of decoding and decreases the time complexity of S2W2Dec.

Figure 11 presents a comparison between the error correcting performances of S2W2Dec (PHR-SPDA) and those of the S2W2Dec(PHR-ARDecGA) applied to the QR(71, 36) codes. This figure shows that S2W2Dec(PHR-ARDecGA) passes relatively S2W2Dec(PH-R-SPDA).

In order to show the importance of our proposed concatenation, we present in Figure 12 the BER performances of PHR without SPDA, those of SPDA without PHR and those of S2W2Dec(PHR-SPDA) for the QR(71, 36) code. PHR with 2414 dual codewords (applied on all symbols, S = 10, STD = 1, STR = 1) has bad performances. Even if the number of dual codewords is increased to 15000, the BER performances of PHR remain bad. SPDA with 1200 permutations has bad performances. Even if the number of permutations is

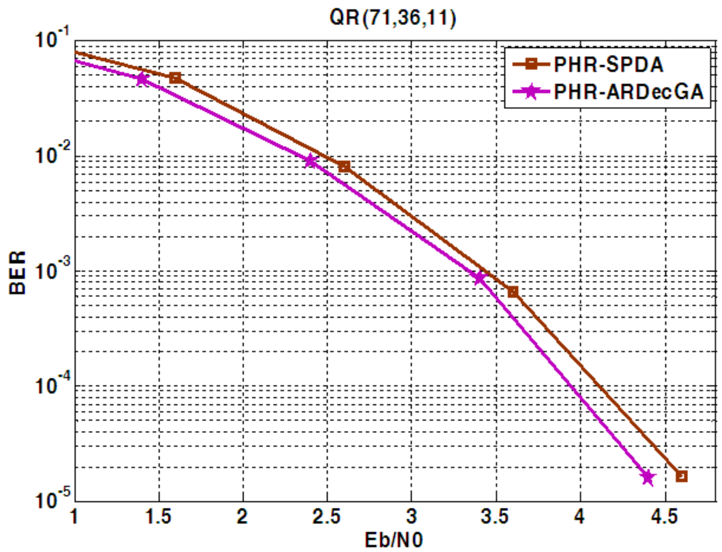

FIGURE 11. Comparison of BER performances of S2W2Dec(PHR-SPDA) and S2W2Dec(PHR-ARDecGA)

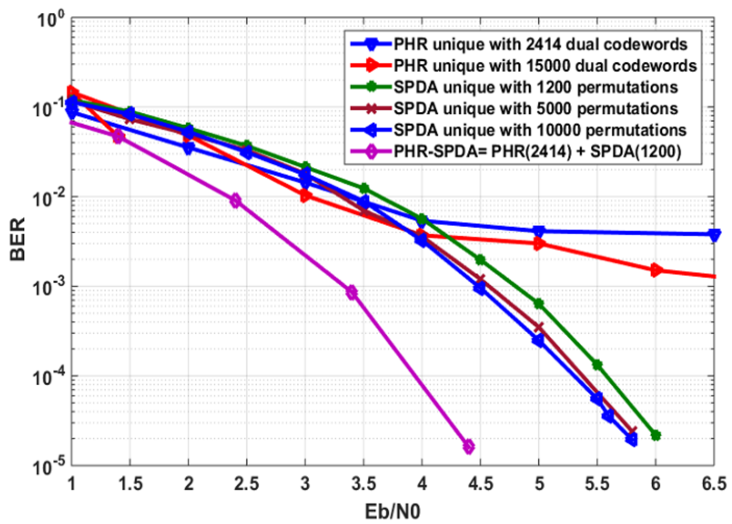

FIGURE 12. Comparison between PHR, SPDA and S2W2Dec(PHR-SPDA)

increased to 10000, the BER performances of SPDA remain bad but they are better than those of PHR.

The concatenation of PHR with SPDA yields good BER performances compared to SPDA and PHR. A gain of more than 1.5 dB is obtained compared to SPDA and a very large gain is obtained compared to PHR. This is the seventh and the main success of S2W2Dec.

Figure 13 presents a comparison between the error correcting performances of Chase-PDA [22], Maini [10] and S2W2Dec(PHR-SPDA) for the QR$(71, 36)$ code. This figure shows that the proposed decoding scheme passes the performances of Chase-PDA and it reaches approximately those of the Maini decoder. In the next section the complexities of these decoders will be discussed and the choice of good parameters of S2W2Dec(PHR-SPDA) decreases its temporal complexity to be significantly better than that of Maini.

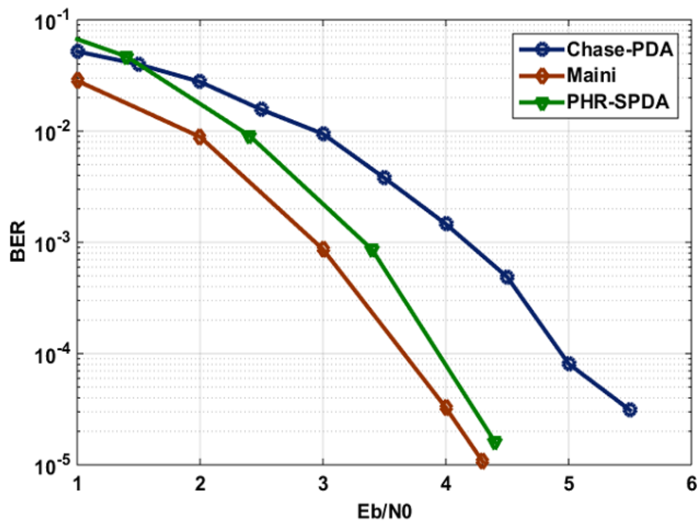

FIGURE 13. Comparison between the error correcting performances of Chase-PDA, Maini and S2W2Dec(PHR-SPDA)

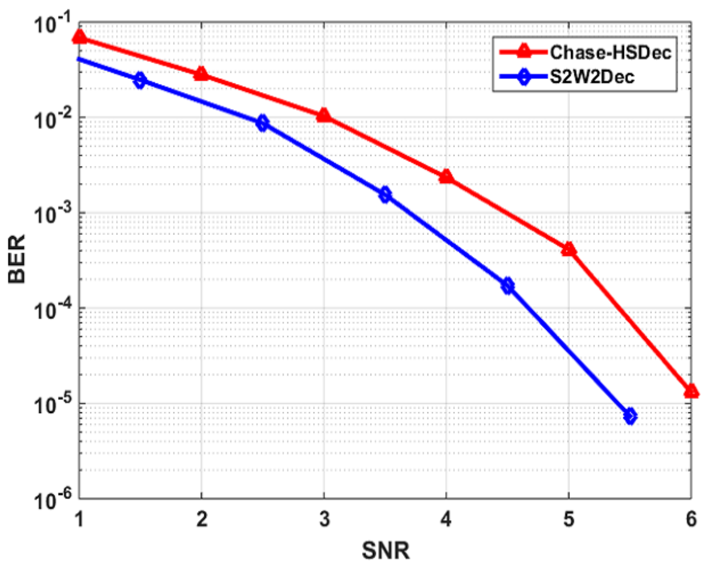

FIGURE 14. Comparison between the error correcting performances of Chase-HSDec and S2W2Dec(PHR-SPDA) for QR$(31, 16)$ code

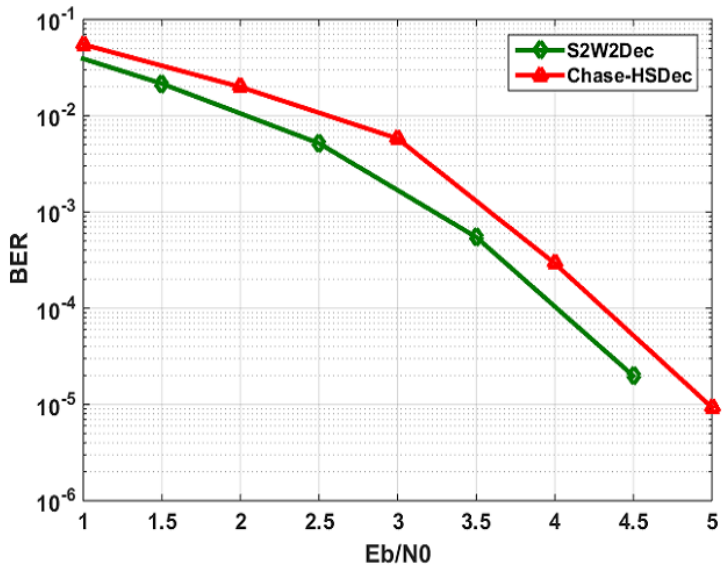

Figure 15. Comparison between the error correcting performances of Chase-HSDec and S2W2Dec(PHR-SPDA) for QR$(47, 24)$ code

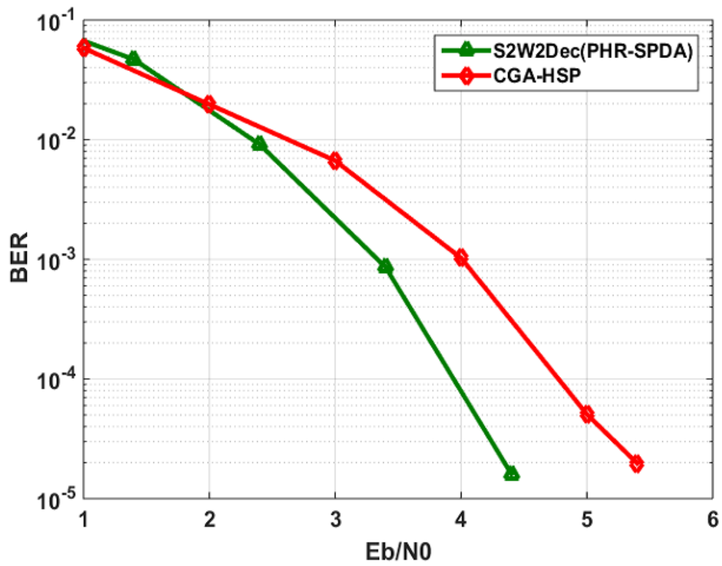

Figure 16. Comparison between BER performances of CGA-HSP and S2W2Dec(PHR-SPDA) for QR$(71, 36)$ code

Figures 14 and 15 present a comparison between the error correcting performances of Chase-HSDec [17] and S2W2Dec(PHR-SPDA) for the QR$(31, 16)$ and QR$(47, 24)$ codes. These figures show that the proposed decoding scheme passes significantly the performances of Chase-HSDec.

Figure 16 presents a comparison between the error correcting performances of CGA-HSP [12] and S2W2Dec(PHR-SPDA) for the QR$(71, 36)$ code. This figure shows that the proposed decoding scheme passes significantly the performances of CGA-HSP.

5. **Study of the Complexities.** The complexity of the decoder S2W2Dec(SS-WW) is equal to the sum of complexities of the symbol by symbol decoder SS and that of the word by word WW decoder used in the concatenation.

For example, for cyclic codes, when SS = PHR of complexity less than $O(M \cdot n^2)$ and WW = SPDA of complexity $O(P \cdot \log(n) \cdot \log(n-k))$, the complexity of S2W2Dec(PHR-SPDA) is less than: $O(M \cdot n^2 + P \cdot \log(n) \cdot \log(n-k))$.

For systematic linear codes that are not cyclic, the complexity of S2W2Dec(PHR-SPDA) is less than $O(M \cdot n^2 + P \cdot n \cdot (n-k))$.

The proposed decoder S2W2Dec(PHR-SPDA) has a low complexity compared to that of many competitors, and Table 3 shows a comparison among these complexities.

The three parameters STD, STR and S that we have introduced in the S2W2Dec(PHR-SPDA), precisely in the component PHR, allow us to reduce again its complexity.

Figure 17 plots the performances of S2W2Dec(PHR-SPDA) for the QR(71, 36, 11) code, with the parameters: S = 10, STR = 1 and STD = 1. In this case, PHR uses all the

TABLE 3. Complexity of some decoding algorithms

Decoding algorithm	**Type of binary block code**	**Temporal complexity**
S2W2Dec(PHR-SPDA)	Cyclic	Less than or equal to $O(M \cdot n^2 + P \cdot \log(n) \cdot \log(n-k))$
S2W2Dec(PHR-SPDA)	Linear and not cyclic	Less than or equal to $O(M \cdot n^2 + P \cdot n \cdot (n-k))$
AutDAG	Cyclic	Less than or equal to $O(Ni \cdot Ng(\log(n) \cdot \log(n-k)))$
AutDAG	Linear and not cyclic	Less than or equal to $O(Ni \cdot Ng \cdot k \cdot n)$
Maini algorithm	Cyclic or linear	$O(Ni \cdot Ng(k \cdot n + \log(Ni)))$
DDGA	Cyclic or linear	$O(Ni \cdot Ng(k \cdot (n-k) + \log(Ni)))$
SDGA	Cyclic or linear	$O(2^t \cdot (Ni \cdot Ng(k \cdot n^2 + k \cdot n + \log(Ni))))$
Chase2-HD	Binary block code	$O(2^t \cdot C(HD))$
Chana algorithm	Cyclic	$O(2^{m+1} \cdot k \cdot \log(n) \cdot (n + \log(n-k)))$
OSD of order m	Cyclic or linear	$O(n^{m+1})$

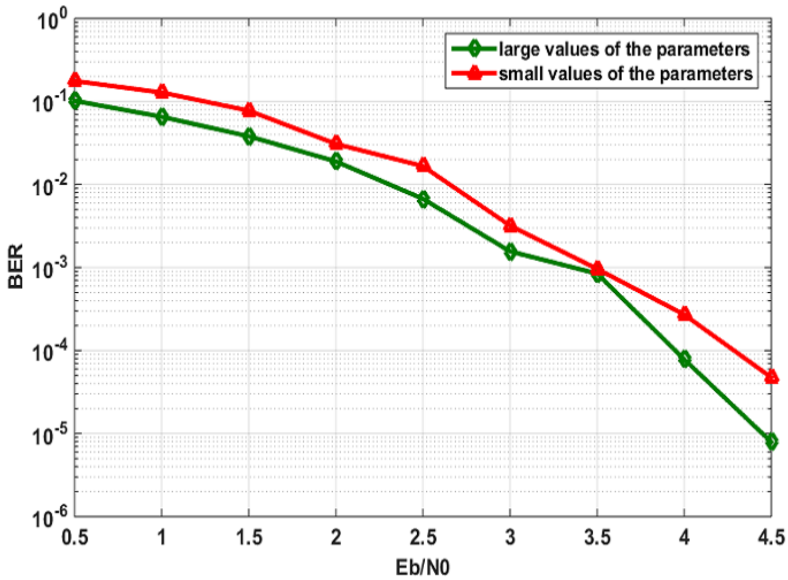

FIGURE 17. Comparison between performances of large and small value of parameters of S2W2Dec(PHR-SPDA) for QR(71, 36, 11) code

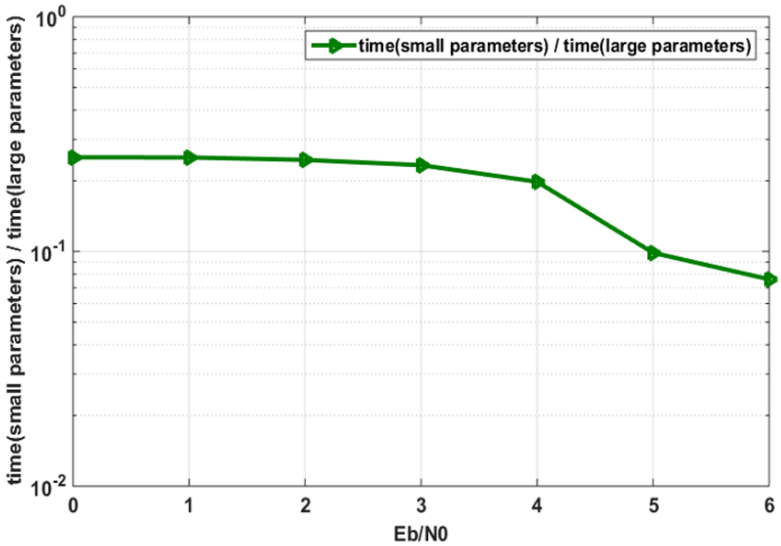

FIGURE 18. Comparison between the run time of S2W2Dec(PHR-SPDA) for large and small value of parameters

$M = 2414$ dual codewords to decide the value of each symbol. The complexity of PHR for one symbol is $O(M \cdot n)$, if it is repeated for all n symbols of the sequence to decode it become $O(M \cdot n^2)$ in total.

Figure 17 plots also the performances of S2W2Dec(PHR-SPDA) with the parameters: S = 0.5, STR = 0.4 and STD = 0.4. In this case, PHR uses only a small part of the $M = 2414$ dual codewords to decide the value of each symbol of reliability less than S because it stops when at least $\lfloor STD * M \rfloor$ dual codewords are used in the computation of Q and when $|Q|$ becomes more than STR. Only symbols of reliability less than S are processed by the decoder PHR, and their number is very small compared to the length n.

In order to show the reduction of complexity offered by the small value of parameters, we plot in Figure 18 the value of the run time of S2W2Dec(PHR-SPDA) for the QR$(71, 36, 11)$ code, with small values of parameters (S = 0.5, STR = 0.4 and STD = 0.4) for QR$(71, 36, 11)$ divided by the run time of the big values of parameters (S = 10, STR = 1, STD = 1) for SNR between 0 to 6 dB. This figure shows clearly that there is a reduction of more than 400% between 0 and 4 dB. For SNR = 6 dB, the reduction is more than 1300%.

6. **Conclusion and Perspectives.** In this paper we have proposed an efficient concatenation scheme S2W2Dec of symbol by symbol with word by word decoders. The simulation results show that the proposed concatenation gives good error correcting performances. The study of impact of different parameters has allowed reducing considerably the complexity of S2W2Dec without affecting significantly the bit error performances. Obtained results in both complexity run time and correction rate prove clearly the huge success of the proposed scheme and will open new horizons for the hybridization of decoding algorithms. S2W2Dec can be used as efficient decoder in many possible applications in telecommunication and storage systems. In the perspectives, we will apply the proposed scheme to other error correcting codes and other decoders.

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