International Journal of Innovative Computing, Information and Control
Volume **15**, Number **2**, April **2019**
ICIC International ©2019 ISSN 1349-4198
pp. 519–534

OBSERVER-BASED MULTI-AGENT SYSTEM FAULT UPPER BOUND ESTIMATION AND FAULT-TOLERANT CONSENSUS CONTROL

Mengyang Xu, Pu Yang, Yuxia Wang and Qibao Shu
College of Automation
Nanjing University of Aeronautics and Astronautics
No. 29, Jiangjun Avenue, Jiangning District, Nanjing 210016, P. R. China
ppyang@nuaa.edu.cn

Received May 2018; revised September 2018

Abstract. *This paper investigates the consistency of multi-agent system with actuator fault. By constructing an appropriate observer, an adaptive algorithm for the upper bound of actuator fault factor is proposed. Subsequently, a fault-tolerant control law was proposed by using the relative state information between agents and the estimated value of the fault upper bound. Moreover, by the related theory of Lyapunov, we prove the theoretical feasibility of the algorithm in realizing the consistency of multi-agent system with actuator fault and external disturbance. Finally, a numerical simulation example verifies the effectiveness of the proposed algorithm and a comparative experiment demonstrates the superiority of the proposed algorithm.*
Keywords: Multi-agent system, State observer, Fault-tolerant control, Adaptive control

1. **Introduction.** In recent years, the single agent system has gradually been replaced by the multi-agent system (MAS) due to its low work efficiency and weak reliability. On the one hand, the MAS is often connected by complex communication topologies, which overcomes the dependence of the entire system on a single agent. On the other hand, multiple agents work together to greatly improve overall work efficiency. Therefore, the MAS has applications in many fields, such as mobile robot networks [1,2], sensor networks, and unmanned aerial vehicle (UAV) formation [3].

In addition, on the one hand, we know that the most fundamental problem for the research of MAS is the realization of consistency [4-6] and it has very important applications in life sciences, technology and engineering. On the other hand, the superiority of MAS is reflected in its ability to cope with unpredictable and abruptly changing environment, which requires that agents can be consistent with changeable environment. Thus, the consistency problem has the important practical significance and theoretical value as the basis of cooperation and coordination control between agents. At present, there is much related literature about the consistency problems. For example, [7] studied the consistency problem for MAS with switching topologies and stochastic delays governed by Markov chains. [8] researched the consistency of second-order MAS with directed switching topologies, and also considered the issue of communication delays. [9] utilized frequency-domain correlation theory to give the consistency conditions for second-order MAS with communication delay problems. Moreover, a control algorithm for achieving finite-time consistency is proposed for the second-order MAS with disturbances by using the integral sliding mode method in [10]. Therefore, we can see that these related documents focus on the realization of the consistency problem.

DOI: 10.24507/ijicic.15.02.519

Furthermore, we know that it may go wrong for one or more agents in a complex MAS. In fact, the most common types of fault are actuator faults [11,12] and sensor faults [13]. However, for a faulty agent, the original actuator driver is often insufficient to achieve a consistent goal. Therefore, we need to redesign a control law, so that the agent can still achieve the consistency goal in the event of actuator fault. This is the basis for our idea about designing fault-tolerant control (FTC) laws. For example, [14] proposed a distributed adaptive update strategy for some parameters to compensate for the fault and external uncertain factors effects on the consistency control of MAS. In [15], by introducing the virtual parameter estimation error, a robust adaptive FTC method was proposed to solve the consistency problem of MAS with uncertain external disturbances and undetectable actuator faults. [16] introduced the concept of virtual parameter estimation error to deal with time-varying and uncertain control gains, and finally proposed a robust adaptive fault-tolerant finite-time consistency solution. [17] introduced a fault estimator based on consistency protocol, and finally showed that the consistency algorithm can make the consistency error of all agents within a small set around the origin under the appropriate parameter selection. In general, the fault-tolerant controllers were designed in these papers to compensate for the insufficient actuator drive caused by fault. Thus, these papers also reflect this view.

Moreover, we know that many states of the actual system are often unavailable or require expensive sensors to measure it. Thus, the observer-based methods are widely used in many papers. [18] built a model-based state observer to design an adaptive FTC law, and ultimately achieved state consistency in a MAS with actuator failure. [19] evaluated the actuator failure factor by constructing a suitable state observer. Then, it finally used the adaptive law constructed by the estimation value of actuator fault factor to achieve the consistency goal of the MAS. However, there are still relatively few papers that use the state observer's estimation error to assist the design of algorithms about relevant parameters.

Therefore, this article mainly has the following contributions. Firstly, according to a suitable state observer, an adaptive rate of change for the square of fault upper bound is proposed. Then, we use the estimation value of square of fault upper bound to construct an FTC algorithm which achieves the consistency of the MAS with actuator fault.

The rest of this article is organized as follows. In Section 2, some related graph theory and some theorems will be introduced at first. Then, system description and some assumptions will be introduced. Combined with the state observer, an adaptive law for the estimation of square of fault upper bound is proposed and the corresponding proof is given in Section 3. Then, in Section 4, we construct an FTC algorithm and prove the rationality of the algorithm. Finally, we use four-rotor aircraft model to verify the effectiveness of the algorithm and add comparative simulation to reflect the superiority of our proposed algorithm in Section 5. In Section 6, we give conclusions about this paper and future work.

The symbols used in this paper have the following meanings. R^n denotes an n-dimensional vector; $R^{n\times m}$ is an n-row and m-column real matrix; I_n represents an n-order identity matrix; $\boldsymbol{1}_n$ is an n-order vector with all 1 elements; $\bigotimes$ denotes Kronecker product; $\|\cdot\|_1$ is the 1 norm of a matrix; $\|\cdot\|$ represents the Euclidean norm of a matrix; $col\{a_1,\ldots,a_n\}$ denotes $\left[a_1^T,\ldots,a_n^T\right]^T$.

2. Problem Statement and Preliminaries.

2.1. Basic graph theory knowledge.

In this section, basic graph theory knowledge will be introduced for the later analysis process.

Considering that a MAS is composed of n following agents and 1 leader agent, we can use the directed graph $\bar{G} = \{\bar{v}, \bar{\varepsilon}\}$ with a set of nodes $\bar{v} = \{0, 1, 2, \dots, n\}$ and the directed edges $\bar{\varepsilon} \subseteq \bar{v} \times \bar{v}$ to describe the topological relationships among multi-agents. For communication topology between the following agents, we can use $G = \{\nu, \varepsilon\}$ to describe. Let $\bar{A} = (a_{ij}) \in R^{n\times n}$ denote the weight matrix of the directed graph G. In addition, if agent i can receive the communication information of agent j, which reflects the directed segment from node j to node i in graph G, then $a_{ij} = 1$ $(i \neq j)$. In contrast, $a_{ij} = 0$. Define the neighbor node of node i as $N_i = \{j \in \nu | (i, j) \in \varepsilon, i \neq j\}$. Moreover, the Laplacian matrix $L = (l_{ij}) \in R^{n\times n}$ of graph G is defined as $l_{ij} = -a_{ij}$, $(j \neq i)$ or $l_{ij} = \sum_{j=1}^{n} a_{ij}$, $(j = i)$.

Meanwhile, we use $\bar{D} = diag(d_1, \dots, d_n)$ to represent the communication relationship between the following agent and the leader. It is noteworthy that the $d_i > 0$ denotes the agent i can obtain the information from the leader, and $d_i = 0$ otherwise. In the directed graph G, the directed path from node i to node j can be composed of a series of directed edges $(i, i_1), (i_1, i_2), \dots, (i_l, j)$, where $(\cdot, \cdot) \in \varepsilon$. If there is a directed path from node i to node j, then node j is said to be reachable from node i. Furthermore, if there is a directed path from node i to every other node in graph G, then the node i is a globally reachable node. According to [20,21], it is easy to know that the directed graph G has a globally reachable node if and only if it has a spanning tree. For the connected undirected graph, all nodes are globally reachable.

Lemma 2.1. *If there is a directed spanning tree with the leader as the root node in graph $\bar{G}$, then matrix $L + \bar{D}$ is invertible.*

Lemma 2.2. *For a continuous derivative function $V(t)$, $V(t) \geq 0$ is satisfied for any $t > 0$. If*

$$\dot{V}(t) \leq -\alpha V(t) + \beta \tag{1}$$

where α and β are all positive constants, then

$$V(t) \leq e^{-\alpha(t-t_0)} V(t_0) + \frac{\beta}{\alpha}\left(1 - e^{-\alpha(t-t_0)}\right) \tag{2}$$

Lemma 2.3. *Let A and B be $m \times m$ and $n \times n$ matrices respectively, and their eigenvalues are $\lambda_1, \dots, \lambda_m$ and $\mu_1, \dots, \mu_n$ respectively. Then, the mn eigenvalues of $A \otimes B$ are $\lambda_i \mu_j$ $(i = 1, \dots, m;\ j = 1, \dots, n)$.*

Lemma 2.4. *For any matching real matrices X, Y and any positive scalar ϖ, the following inequality holds:*

$$2X^T Y \leq \varpi X^T X + \frac{1}{\varpi} Y^T Y \tag{3}$$

2.2. **System description and model parameters assumptions.** We know that for a complex agent system, it often receives external disturbance and actuator fault. Therefore, for n following agents, we can use the following state space model to describe.

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t) + Df(x_i) \\ y_i(t) = Cx_i(t) \end{cases} \tag{4}$$

where $x_i(t) \in R^m$, $y_i(t) \in R^q$, $u_i(t) \in R^p$ denote the system's state variables, output variables, and input variables, respectively. $f(x_i) \in R^m$ represents the external disturbance to the system and satisfies the Lipschitz continuity condition. A, B, C, D are real matrices with appropriate dimensions.

Here we only consider the actuator fault, so the actuator output model after fault can be given by the following model

$$u_i^F(t) = (I_m - \rho_i(t))u_i(t) \tag{5}$$

where we give an upper bound of the fault factor to facilitate the subsequent algorithm research. Thus, we assume that there is a positive constant $\bar{\rho}_i$ $(i = 1, \ldots, n)$ that satisfies the following inequality

$$\mathbf{0} \leq \rho_i(t) \leq \bar{\rho}_i I_m \leq I_m \tag{6}$$

For the leader agent, we assume that its actuator has no failure but is affected by external disturbances. Thus, the model of the leader agent can be represented by the following state space expressions

$$\begin{cases} \dot{x}_0(t) = Ax_0(t) + Bu_0(t) + Df(x_0) \\ y_0(t) = Cx_0(t) \end{cases} \tag{7}$$

where $x_0(t) \in R^m$, $y_0(t) \in R^q$, $u_0(t) \in R^p$ denote state variables, output variables, and input variables of the leader agent, respectively. Moreover, $u_0(t)$ meet with $\|u_0(t)\|_\infty \leq \overline{u}$, and $\overline{u}$ is a positive scalar.

Remark 2.1. *We know that the energy output of an actual physical device is always limited. Thus, for the actuator output of the leader agent, we can assume that it has a positive upper bound $\bar{u}$. This assumption is consistent with the actual engineering situation. It has been adopted by many papers.*

Assumption 2.1. *Matrix pairs (A, B) are stabilizable, and (A, C) are detectable.*

Assumption 2.2. *In graph G, there is a spanning tree with the leader agent as the root node.*

Lemma 2.5. *If the nonlinear function $f(x)$ satisfies the Lipschitz continuity condition, then the following inequality will be satisfied.*

$$\|f(x_1) - f(x_2)\| \leq l \|x_1 - x_2\| \tag{8}$$

where point $(x_1, f(x_1))$ and point $(x_2, f(x_2))$ are two different points. l is a positive scalar.

Thus, this section first introduces the most basic graph theory knowledge in MAS research, and gives some basic lemmas that will be used in the later proof process. Then, the state space models of leader and followers in the MAS are given respectively. Finally, some reasonable assumptions commonly used in the research of MAS are given. In general, by the introduction of the basic theory and system model in this section, it provides a theoretical basis for the construction of a suitable state observer in the third section and theoretical derivation in later section.

3. Observer-Based Adaptive Upper Bound Estimation of Fault Factor.

According to the system model of the following agent, we can construct a suitable observer model as follows

$$\begin{cases} \dot{\hat{x}}_i(t) = A\hat{x}_i(t) + Bu_i(t) + E\left(y_i(t) - \hat{y}_i(t)\right) + Df(x_i) \\ \hat{y}_i(t) = C\hat{x}_i(t) \end{cases} \tag{9}$$

where $\hat{x}_i(t)$, $\hat{y}_i(t)$ denote the estimation of the system state $x_i(t)$ and output $y_i(t)$, respectively. The E is the observer gain to be designed later. Let $\tilde{x}_i(t) = \hat{x}_i(t) - x_i(t)$ and $\tilde{y}_i(t) = \hat{y}_i(t) - y_i(t)$ as state estimation error and output error, respectively.

We can let $\xi_i = \bar{\rho}_i^2$. $\hat{\xi}_i$ is the estimation of ξ_i. Thus, the adaptive update law of evaluation of the square of the fault upper bound is designed as:

$$\dot{\hat{\xi}}_i = \varepsilon_i \eta_i \left\| u_i \right\|_2^2 - \frac{1}{2}\hat{\xi}_i \tag{10}$$

where ε_i and η_i are positive constant scalar that will be designed later.

According to (4) and (9), we can get the dynamic equation of state estimation error as follows

$$\dot{\tilde{x}}_i(t) = (A - EC)\tilde{x}_i(t) + B\rho_i(t)u_i(t) \tag{11}$$

Theorem 3.1. *If there is a positively definite symmetric real matrix P, an appropriate dimension of the matrix K, and the positive real scalar ε, such that the following linear matrix inequality and corresponding matrix relations holds*

$$E = P^{-1}K \tag{12}$$

$$\begin{bmatrix} PA + A^TP - KC - C^TK^T & B \\ * & -\varepsilon \end{bmatrix} < 0 \tag{13}$$

the system's state evaluation error $\tilde{x}_i(t)$ and the estimation error $\hat{\xi}_i$ of fault upper square are uniformly ultimately bounded under adaptive law (10), namely, $\|\tilde{x}_i\| \le \sqrt{\frac{\beta_i}{\alpha_i \lambda_{\min}(P)}}$, $\left\|\hat{\xi}_i - \xi_i\right\| \le \sqrt{\frac{2\eta_i\beta_i}{\alpha_i}}$, where $\alpha_i = \min\left\{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}, 1\right\}$, $\beta_i = \frac{1+2\varepsilon_i\eta_i\|u_i\|_2^2}{2\eta_i}$.

Proof: Choose the Lyapunov candidate function as below

$$V_i(t) = \tilde{x}_i^T P \tilde{x}_i + \frac{\left(\hat{\xi}_i - \xi_i\right)^2}{2\eta_i} \tag{14}$$

Then the time derivative of (14) can be obtained:

$$\begin{aligned}
\dot{V}_i(t) &= 2\tilde{x}_i^T P \dot{\tilde{x}}_i + \frac{\left(\hat{\xi}_i - \xi_i\right)}{\eta_i}\dot{\hat{\xi}}_i \\
&= \tilde{x}_i^T \left[P(A - EC) + (A - EC)^T P\right]\tilde{x}_i + 2\tilde{x}_i^T B\rho_i u_i + \frac{\left(\hat{\xi}_i - \xi_i\right)}{\eta_i}\dot{\hat{\xi}}_i \\
&\le \tilde{x}_i^T \left[P(A - EC) + (A - EC)^T P + \frac{1}{\varepsilon_i}BB^T\right]\tilde{x}_i + \varepsilon_i\xi_i \left\|u_i\right\|_2^2 + \frac{\left(\hat{\xi}_i - \xi_i\right)}{\eta_i}\dot{\hat{\xi}}_i \\
&= -\tilde{x}_i^T Q \tilde{x}_i + \varepsilon_i\xi_i \left\|u_i\right\|_2^2 + \frac{\left(\hat{\xi}_i - \xi_i\right)}{\eta_i}\left(\varepsilon_i\eta_i \left\|u_i\right\|_2^2 - \frac{1}{2}\hat{\xi}_i\right) \\
&\le -\tilde{x}_i^T Q \tilde{x}_i - \frac{\left(\hat{\xi}_i - \xi_i\right)^2}{2\eta_i} - \frac{\hat{\xi}_i\xi_i}{2\eta_i} + \frac{\xi_i^2}{2\eta_i} + \varepsilon_i\hat{\xi}_i\|u_i\|_2^2 \\
&\le -\tilde{x}_i^T Q \tilde{x}_i - \frac{\left(\hat{\xi}_i - \xi_i\right)^2}{2\eta_i} + \frac{1}{2\eta_i} + \varepsilon_i\hat{\xi}_i\|u_i\|_2^2 \\
&\le -\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}\tilde{x}_i^T P \tilde{x}_i - \frac{\left(\hat{\xi}_i - \xi_i\right)^2}{2\eta_i} + \frac{1}{2\eta_i} + \varepsilon_i\hat{\xi}_i\|u_i\|_2^2 \\
&\le -\alpha_i V_i + \beta_i
\end{aligned} \tag{15}$$

where $-Q = P(A - EC) + (A - EC)^T P + \frac{1}{\varepsilon} BB^T$ and $\varepsilon = \max\{\varepsilon_i\}$, $(i = 1, \ldots, n)$. In addition, ε_i and η_i are positive constant scalars that will be designed later. Thus, for $\dot{V}_i(t) \le -\alpha_i V_i(t) + \beta_i$, according to Lemma 2.2, we can obtain

$$V_i(t) \le e^{-\alpha_i(t-t_0)} V(t_0) + \frac{\beta_i}{\alpha_i} \le \frac{\beta_i}{\alpha_i} \tag{16}$$

where $\alpha_i = \min\left\{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}, 1\right\}$ and $\beta_i = \frac{1+2\varepsilon_i\eta_i\|u_i\|_2^2}{2\eta_i}$.

Therefore, according to algebraic relations, we can get

$$\|\tilde{x}_i\| \le \sqrt{\frac{\beta_i}{\alpha_i \lambda_{\min}(P)}}, \qquad \left\|\hat{\xi}_i - \xi_i\right\| \le \sqrt{\frac{2\eta_i\beta_i}{\alpha_i}} \tag{17}$$

Namely, $\tilde{x}_i(t)$ and $\hat{\xi}_i - \xi_i$ are uniformly ultimately bounded. Furthermore, since both α_i and β_i are adjustable variables, the bounds of $\tilde{x}_i(t)$ and $\hat{\xi}_i - \xi_i$ can be adjusted as small as possible.

4. **Main Results.** In this section, we will introduce the description of the consensus algorithm goal. Then, the control law is constructed by using the estimation of the upper bound of actuator fault factor. Finally, the Lyapunov theory is used to prove the feasibility of the control law.

First, for MAS (4)-(7), the consistency goal can be described by the following expression

$$\lim_{t\to\infty} \|x_i - x_0\| = 0, \quad (i = 1, \ldots, n) \tag{18}$$

Thus, we define the relative output error information for the i-th follower as

$$\theta_i = \sum_{j\in N_i} a_{ij}(y_i - y_j) + d_i(y_i - y_0) \tag{19}$$

Let $e_i = x_i - x_0$, $e = col\{e_1, \ldots, e_n\}$. Then, we can rewrite (19) as a compact form in the following

$$\theta = \left[(L + \bar{D}) \otimes C\right] e \tag{20}$$

where $\theta = col\{\theta_1, \ldots, \theta_n\}$.

Theorem 4.1. *In the case of Assumption 2.1 and Assumption 2.2, if there is a positive definite matrix M, matrix R with the proper dimension, and the parameter τ that satisfy the following expression relationship*

$$\begin{cases} RC = -B^T M \\ (L + \bar{D}) \otimes (MA + A^T M) + tI < 0 \\ \tau \ge \bar{u} \end{cases} \tag{21}$$

where $t = 2l \cdot \lambda_{\max}\left[(L + \bar{D}) \otimes PD\right]$. Then, the MAS (4)-(7) under the control law $u_i = \omega_i^{-1}\tau \mathrm{sgn}(R\theta_i)$, where $\omega_i = 1 - \sqrt{\hat{\xi}_i}$, can achieve the consistency of the state in the event of actuator fault.

Proof: Choose the Lyapunov candidate function as below

$$V(t) = e^T\left[(L + \bar{D}) \otimes M\right] e \tag{22}$$

Then the time derivative of (22) can be obtained:

$$\begin{aligned} \dot{V}(t) &= 2e^T\left[(L + \bar{D}) \otimes M\right] \dot{e} \\ &= 2e^T\left[(L + \bar{D}) \otimes MA\right] e + 2e^T\left[(L + \bar{D}) \otimes MB\right](I - \rho)u \\ &\quad - 2e^T\left[(L + \bar{D}) \otimes MB\right](\mathbf{1}_n \otimes u_0) + 2e^T\left[(L + \bar{D}) \otimes MD\right] F \end{aligned} \tag{23}$$

where $u = col\{u_1, \ldots, u_n\}$, $\rho = diag\{\rho_1, \ldots, \rho_n\}$, $F_i = f_i - f_0$ and $F = col\{f_1 - f_0, \ldots, f_n - f_0\}$.

We know $x^T \mathrm{sgn}(x) = \|x\|_1$, then

$$\begin{aligned}
&2e^T \left[\left(L+\bar{D}\right) \otimes MB\right] (I-\rho)\, u - 2e^T \left[\left(L+\bar{D}\right) \otimes MB\right] (\mathbf{1}_n \otimes u_0) \\
&\le -\sum_{i=1}^{n} 2\left(1-\sqrt{\hat{\xi}_i}\right)\left(R\sum_{j\in N_i} a_{ij}\left(y_i-y_j\right)+d_i\left(y_i-y_0\right)\right)^T u_i \\
&\quad +\sum_{i=1}^{n} 2\bar{u}\left\| R\sum_{j\in N_i} a_{ij}\left(y_i-y_j\right)+d_i\left(y_i-y_0\right)\right\|_1 \\
&= \sum_{i=1}^{n} 2\left(\bar{u}-\tau\right)\|R\theta_i\|_1
\end{aligned} \tag{24}$$

Moreover, according to Lemma 2.4, we can obtain

$$\begin{aligned}
2e^T \left[\left(L+\bar{D}\right) \otimes MD\right] F &\le 2\lambda_{\max}\left[\left(L+\bar{D}\right) \otimes MD\right] \sum_{i=1}^{n} {e_i}^T F_i \\
&\le 2l \cdot \lambda_{\max}\left[\left(L+\bar{D}\right) \otimes MD\right] \sum_{i=1}^{n} {e_i}^T e_i
\end{aligned} \tag{25}$$

Combining (23), (24) and (25), we can get

$$\begin{aligned}
\dot{V}(t) &= 2e^T \left[\left(L+\bar{D}\right) \otimes M\right] \dot{e} \\
&\le e^T \left[\left(L+\bar{D}\right) \otimes \left(MA+A^T M\right)\right] e - \sum_{i=1}^{n} 2\tau\left(1-\sqrt{\hat{\xi}_i}\right)\omega_i^{-1}\|K\theta_i\|_1 \\
&\quad + \sum_{i=1}^{n} 2\bar{u}\|K\theta_i\|_1 + 2\lambda_{\max}\left[\left(L+\bar{D}\right) \otimes MD\right] \sum_{i=1}^{n} {e_i}^T F_i \\
&\le e^T \left[\left(L+\bar{D}\right) \otimes \left(MA+A^T M\right)\right] e + \sum_{i=1}^{n} 2\left(\bar{u}-\tau\right)\|K\theta_i\|_1 \\
&\quad + 2l\lambda_{\max}\left(\left[\left(L+\bar{D}\right) \otimes MD\right]\right) \sum_{i=1}^{n} {e_i}^T e_i \\
&\le e^T \left[\left(L+\bar{D}\right) \otimes \left(MA+A^T M\right) + tI\right] e + \sum_{i=1}^{n} 2\left(\bar{u}-\tau\right)\|K\theta_i\|_1
\end{aligned} \tag{26}$$

Thus, as long as the variable relationship given by Theorem 4.1 is satisfied, the tracking error variable e will tend to zero within a certain time. Finally, in the case of actuator fault, the MAS (4)-(7) can achieve the goal of states consistency under our proposed control laws. The proof is completed.

5. **Simulation Verification.** In multi-aircraft systems, the aircraft uses the Qball-X4 and its physical map is shown in a in Figure 1.

Using one of the Qball-X4 as an example, the relationship of the thrust force F generated for each rotor can be expressed by the following first-order equation.

$$F = K_g \frac{\omega}{s+\omega} u \tag{27}$$

where u is the actuator input, ω is the actuator bandwidth, and K_g is the positive gain. Thus, we introduce a state variable v that represents actuator dynamics. The dynamic state space equation is

$$v = \frac{\omega}{s+\omega}u \qquad \dot{v} = -\omega v + \omega u \tag{28}$$

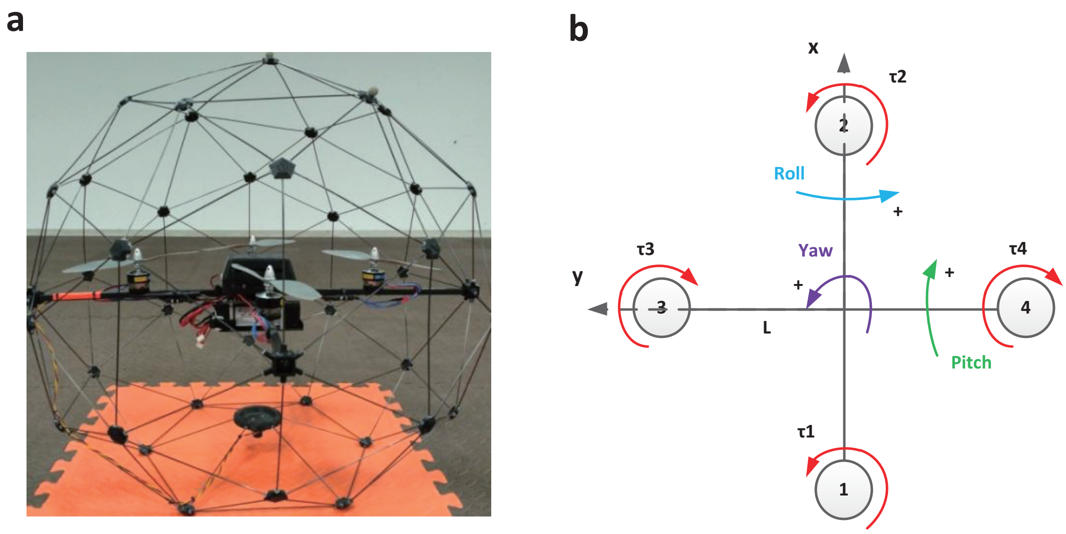

FIGURE 1. Physical image and structural diagram of the aircraft Qball-X4

With the Qball-X4 centroid as the origin, the body coordinate system is established, as shown in b in Figure 1. By combining the research topics, we can model the Qball-X4's linear motion in the x-axis. Namely, the aircraft moves forward or backward. Thus, we assume that the yaw angle is zero. When the Qball-X4 is flying along the x-axis, the body is affected by the thrust F and pitch angles θ, and the dynamic model on the x-axis is

$$M_g \ddot{X} = 4F\sin(\theta) \tag{29}$$

where M_g and X are expressed as the total mass of the body and the displacement of the body in the x-axis direction, respectively.

Considering that the pitch angle θ is small, there is $\theta \approx \sin(\theta)$. Therefore, we can get the state space expression as follows

$$\begin{bmatrix} \dot{X} \\ \ddot{X} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \dfrac{4K_g}{M_g}\theta \\ 0 & 0 & -\omega \end{bmatrix} \begin{bmatrix} X \\ \dot{X} \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix} u \tag{30}$$

We can select the state variable as $x(t) = \begin{bmatrix} X & \dot{X} & v \end{bmatrix}^T$ and control input $u(t)$. Therefore, Equation (30) can be written as follows

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{31}$$

where $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \frac{4K_g}{M_g}\theta \\ 0 & 0 & -\omega \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$.

By referring to the user manual of Quanser's Qball-X4 product, we can get the actual value of each parameter in (31) as shown in Table 1.

TABLE 1. Qball-X4 body parameters

Parameter	Value·Unit
K_g	120N
ω	15rad/sec
M_g	1.4kg

Suppose pitch angle $\theta = 0.025$rad in the x-axis position control stage. Thus, the matrix parameters in the model are $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 8.57 \\ 0 & 0 & -15 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 15 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$.

For the content we studied, we consider the MAS composed of four followers $i = 1, \ldots, 4$ and one lead 0. Further, the network communication topology among the agents of the MAS is shown in Figure 2 as follows.

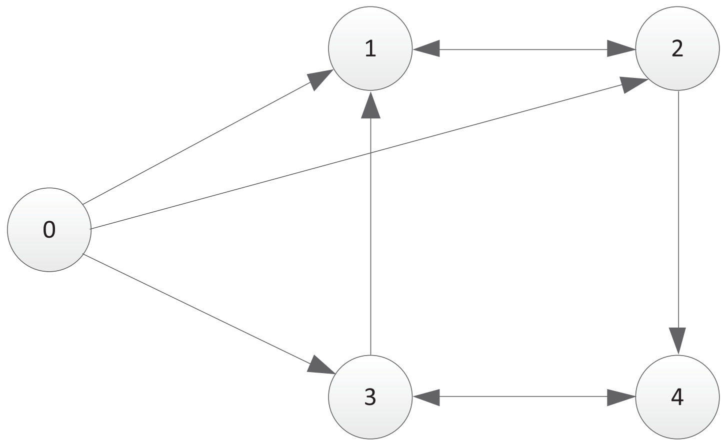

FIGURE 2. Multi-agent system communication topology network

For the Qball-X4's actual system model, we can get the kinematics equation of the leader aircraft by considering the external disturbance effect as follows

$$\begin{cases} \dot{x}_0(t) = Ax_0(t) + Bu_0(t) + Df(x_0) \\ y_0(t) = Cx_0(t) \end{cases} \tag{32}$$

Moreover, we can get the kinematics equation of the i-th ($i = 1, \ldots, 4$) follower aircraft by considering external disturbances and actuator faults as follows

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + B\left(I_3 - \rho_i(t)\right)u_i(t) + Df(x_i) \\ y_i(t) = Cx_i(t) \end{cases} \tag{33}$$

where we choose the perturbation distribution matrix D as $\begin{bmatrix} 0.2 & 1.5 & 0.8 \end{bmatrix}^T$.

We assume that the external disturbance function is $f(x) = 0.18\sin(0.3x) - 0.05x$, and the actuator failure occurs in the Qball-X4 aircraft 2, 3, 4. In addition, their actuator fault descriptions are as follows

$$\rho_2(t) = \begin{cases} 0I_3, & 0 \le t < 5\text{s} \\ (0.2\cos 0.02\pi t \sin 0.1t)\, I_3, & t \ge 5\text{s} \end{cases} \tag{34}$$

$$\rho_3(t) = \begin{cases} 0I_3, & 0 \le t < 3\text{s} \\ 0.15I_3, & t \ge 3\text{s} \end{cases} \tag{35}$$

$$\rho_4(t) = \begin{cases} 0I_3, & \text{others} \\ diag\left\{0.1\sin(0.2\pi t)\cos(0.5t) + 0.24, 0, 0.32e^{-0.24t}\right\}, & 20 \geq t \geq 8\text{s} \end{cases} \quad (36)$$

From the assumptions above, we can take $l = 0.12$. According to (12) and (13), using the linear matrix inequality tool in MATLAB, we can get the symmetric matrix P and the observer matrix E satisfying the conditions as shown below

$$P = \begin{bmatrix} 23.036 & 4.3670 & 2.4950 \\ 4.3670 & 1.0067 & 0.9585 \\ 2.4950 & 0.9585 & 4.7618 \end{bmatrix}, \quad E = \begin{bmatrix} 7.3472 & 4.0259 & 2.7146 \end{bmatrix}^T$$

Therefore, according to the adaptive law (10), we estimate the square of the upper bound of Qball-X4's actuator fault factors. The simulation results are shown in Figures 3-5.

Because the upper limit $\bar{\rho}_i$ of the fault $\rho_i(t)$ is a constant value, we can see from Figures 3, 4 and 5 that the final estimation error tends to a fixed value. In addition, we know that the estimation error can be made small by adjusting the parameters α_i and β_i.

For the multi-agent communication topology shown in Figure 2, we can get the Laplacian matrix L and the adjacency matrix $\bar{D}$ as shown below

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}, \quad \bar{D} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Set the initial state of each Qball-X4 aircraft as follows

$$x_0(0) = [0.362, -2.243, 1.270]^T, \quad x_1(0) = [-1.904, -0.860, -3, 104]^T,$$
$$x_2(0) = [2.728, -3.442, 4.412]^T, \quad x_3(0) = [-0.489, 1.294, 0.385]^T,$$
$$x_4(0) = [4.731, 4.704, 3.372]^T$$

The system's state variable we selected for model is $x(t) = \begin{bmatrix} X & \dot{X} & v \end{bmatrix}$. Moreover, we have previously pointed out that X represents the displacement of the Qball-X4 aircraft

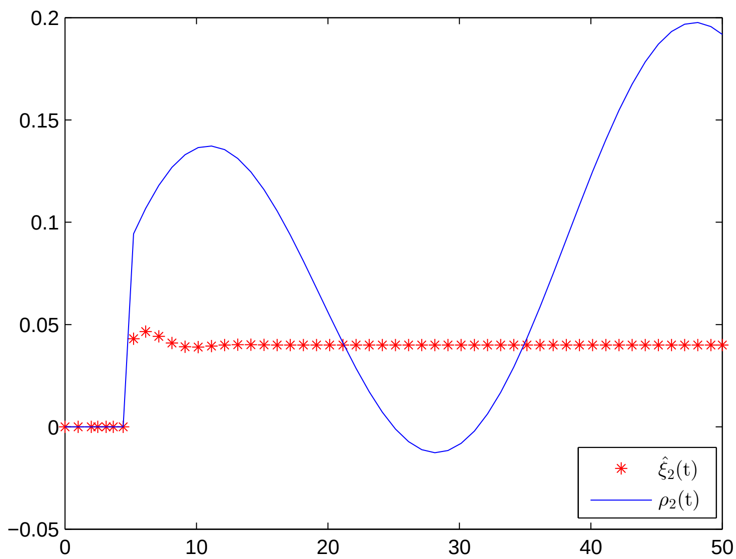

FIGURE 3. The actuator fault curve of the Qball-X4 aircraft 2 and the estimation of the square of its upper bound

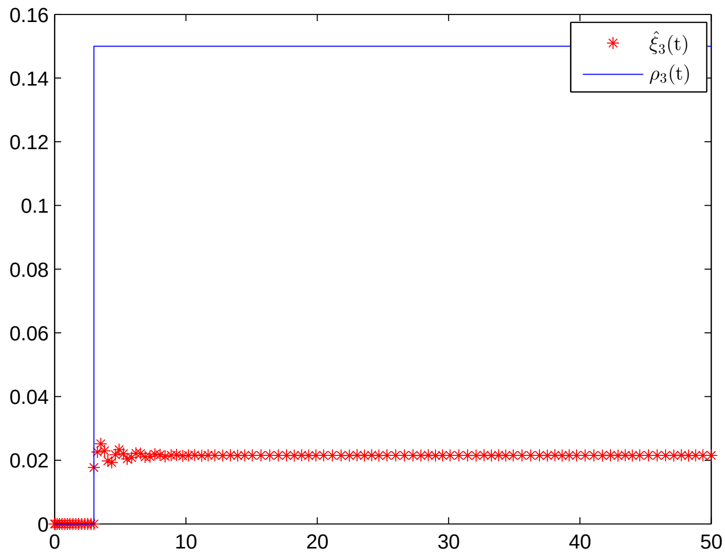

FIGURE 4. The actuator fault curve of the Qball-X4 aircraft 3 and the estimation of the square of its upper bound

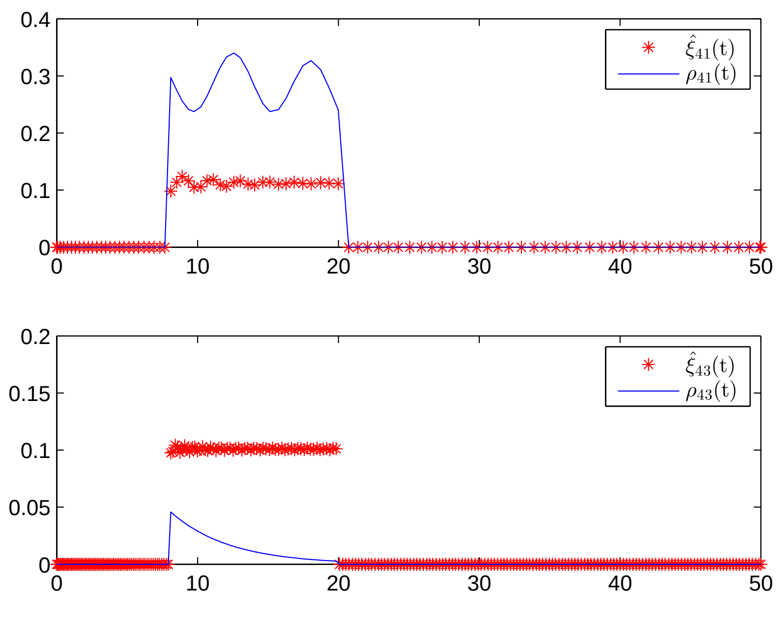

FIGURE 5. The actuator fault curve of the Qball-X4 aircraft 4 and the estimation of the square of its upper bound

in the x-axis direction. Thus, $\dot{X}$ represents the linear velocity of the aircraft in the x-axis direction. In addition, for v, we have also pointed out that it represents the actuator dynamics in (28).

If we want to achieve the consistency of MAS, we need to eventually make the state error variables between following agent and the leader agent go to zero. Thus, we define the state tracking error variable $e_{xi}(t) = x_i(t) - x_0(t)$, $i = 1, \ldots, 4$. Here the first component of $e_{xi}(t)$ is the position tracking error of the aircraft in the x-axis direction, the second component is the linear velocity tracking error of the aircraft in the x-axis direction, and the third component describes the tracking error of the actuator's dynamic behavior of the aircraft.

Thus, we choose the appropriate parameters $\bar{u} = 8$ and $\tau = 9.6$. The simulation results are as Figures 6-8.

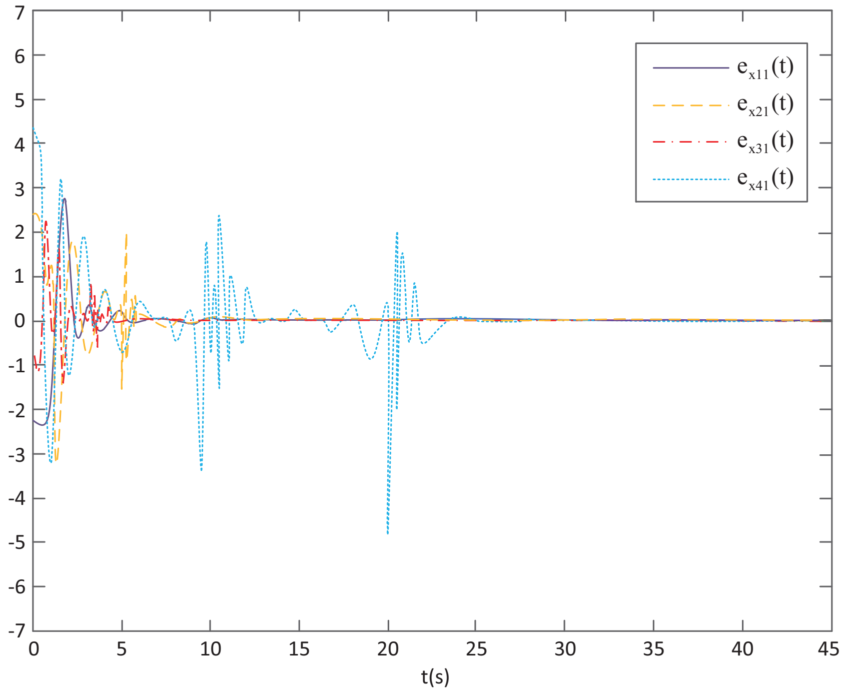

FIGURE 6. The position tracking error of each aircraft in the x-axis direction

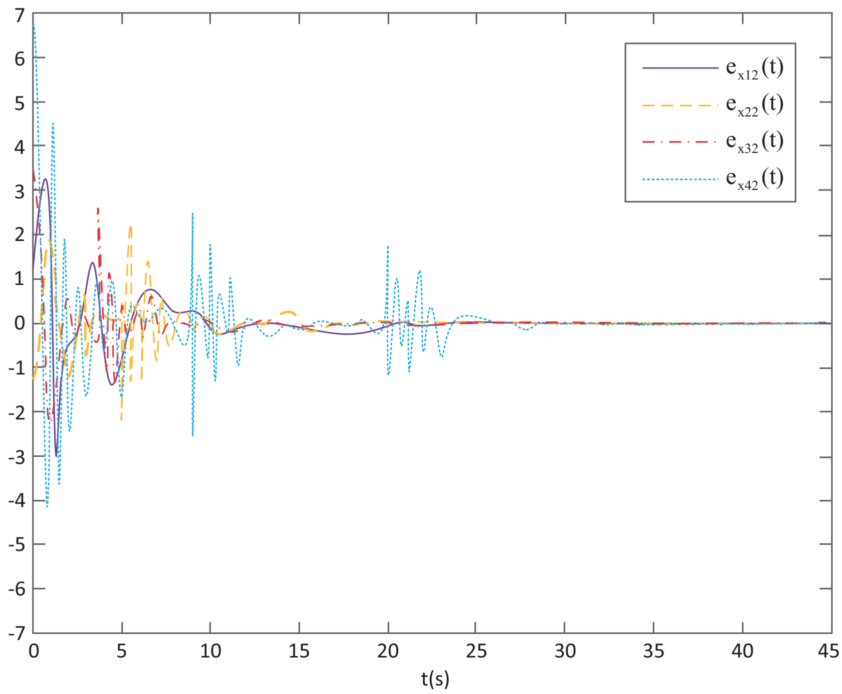

FIGURE 7. The linear velocity tracking error of each aircraft in the x-axis direction

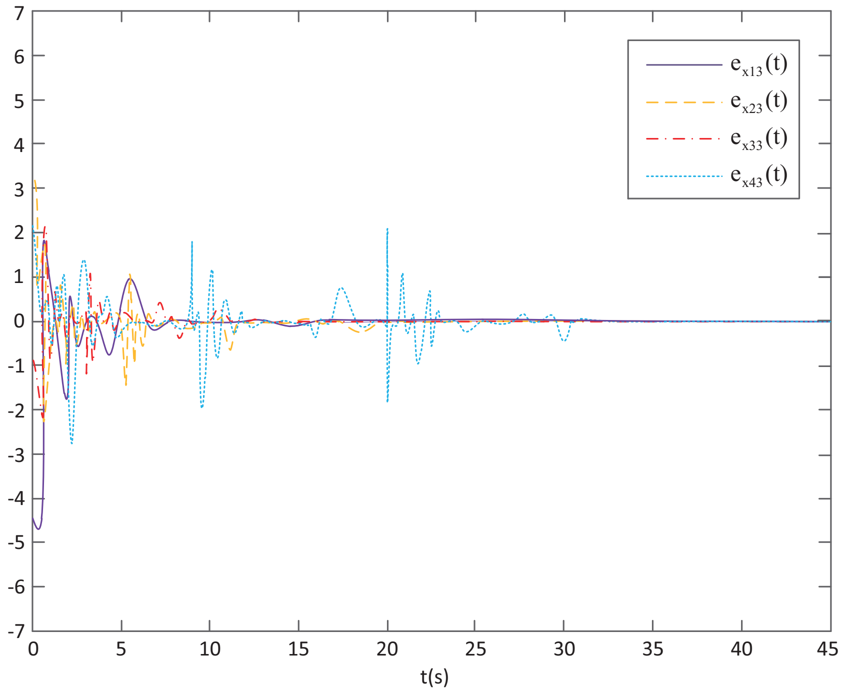

Figure 8. The tracking error of the actuator's dynamic behavior of the aircraft

From Figures 6, 7, and 8, we can see that although we set different initial conditions and different fault conditions, the motion state of each following aircraft eventually tends to the leader's state under the control law $u_i = \omega_i^{-1}\tau\text{sgn}\left(R\theta_i\right)$ in the motion direction we selected. It shows the good FTC effect of this algorithm.

In addition, we can see that the error tracking curves of the Qball-X4 aircraft 2, 3 and 4 fluctuate at 5th, 3rd, 8th and 20th seconds, respectively, which indicates that the occurrence of actuator fault has affected the approach of the error curve.

Further, we added a comparative experiment. At first, we make the parameter matrix $B = D$ in the agent state space model (4), and let $\alpha_k = 0$, $\beta_k = l = 0.12$ $(k = 1, \ldots, N)$ in the inequality (8) of [14]. Then, we select the same initial state as the above experiment for each agent. Finally, by setting parameters on the Qball-X4 platform control interface, we can get the simulation results as shown in Figures 9-11.

Thus, by using the control algorithm in [14] to simulate, we can get the consistency control effect curve as shown in Figures 9, 10 and 11. By comparing the simulation effect curve of the control algorithm proposed by our paper, on the one hand, we can see that the algorithm proposed in [14] can solve the consistency problem of the agent with the ordinary actuator fault. However, we can know from the state tracking error curve of the agent 4 that the control effect of the algorithm proposed in [14] is not good for the consistency realization of the agent with intermittent fault. On the other hand, it can be seen from the amplitude change of the state error tracking curve caused by the fault that the algorithm proposed by our paper has relatively high robustness against the fault effect. Thus, the fault-tolerant consistency control algorithm proposed by our paper is relatively more general and stronger robustness against MAS actuator failures and external disturbances.

6. **Conclusions.** This paper focuses on the fault-tolerance consistency of the multiple Qball-X4 aircraft system. First, we construct an appropriate state observer to estimate the upper square of the fault factor. Then, we design its adaptive law and use Lyapunov

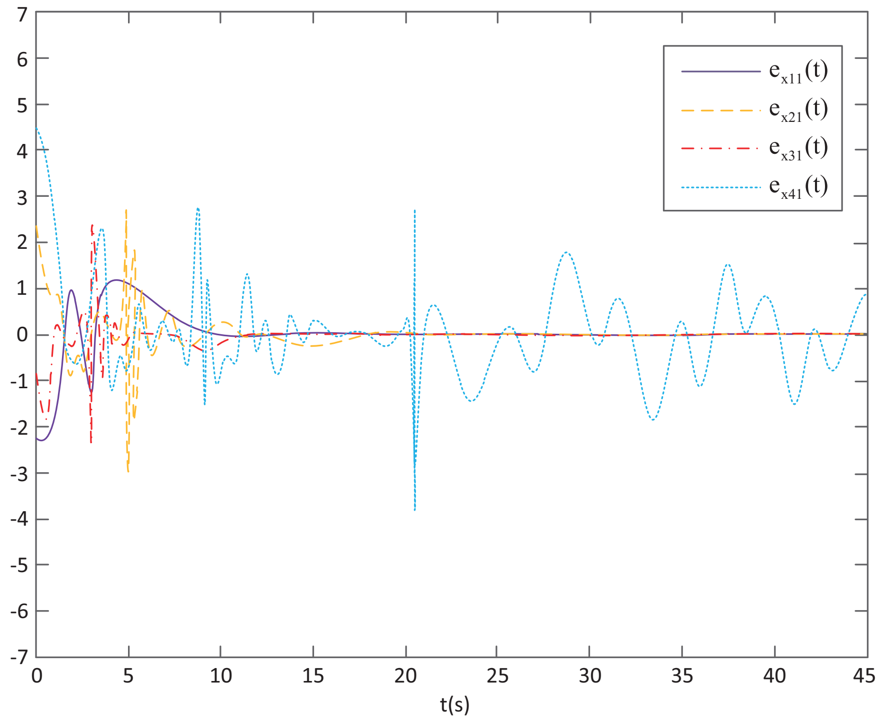

FIGURE 9. The position tracking error of each aircraft in the x-axis direction using the algorithm in [14]

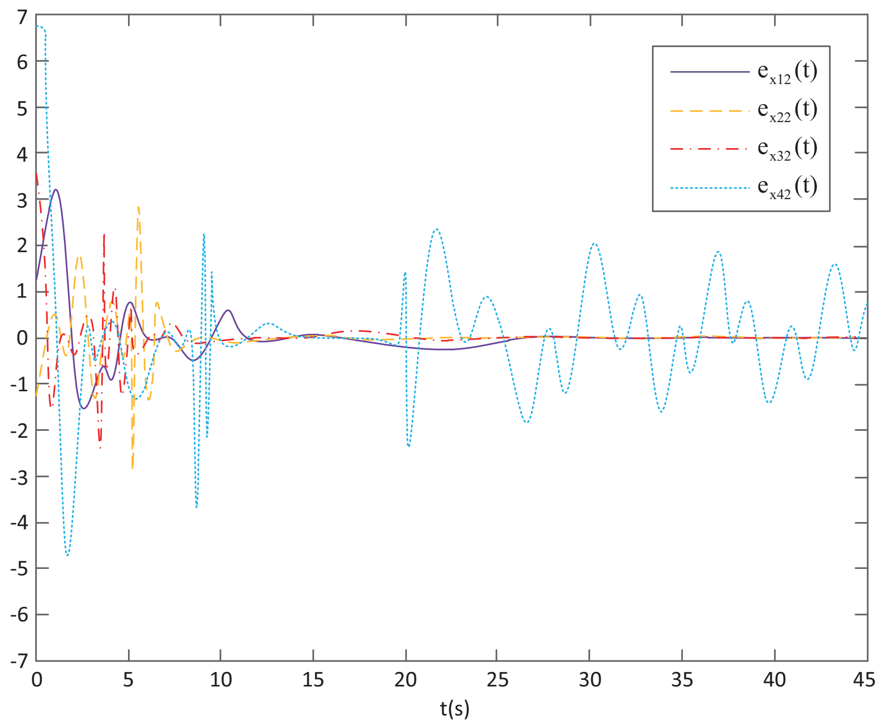

FIGURE 10. The linear velocity tracking error of each aircraft in the x-axis direction using the algorithm in [14]

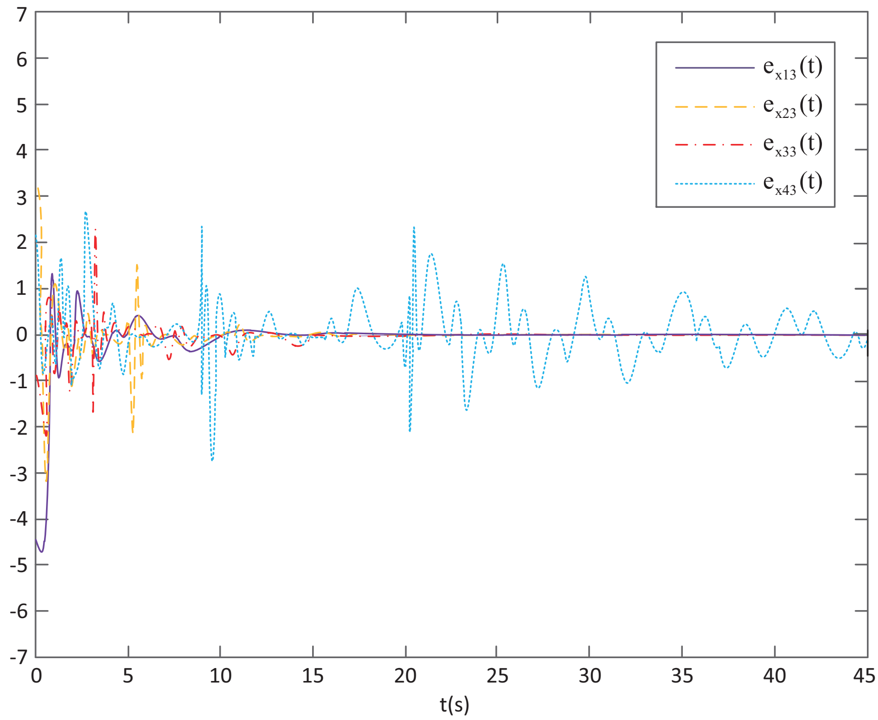

FIGURE 11. The tracking error of the actuator's dynamic behavior of the aircraft using the algorithm in [14]

correlation theory to prove the feasibility of the estimation method. Second, we utilize the estimated value of the upper bound of the fault factor to design the FTC law. Finally, we verified the effectiveness of the control method through simulation example.

However, in this paper, we only consider actuator's multiplicative fault and the leader aircraft limited input conditions. Therefore, in the future work, we can broaden these conditions to study the simultaneous occurrence of additive fault and multi-aircraft communication delay.

Acknowledgment. The authors thank the anonymous reviewers for their useful comments that improved the quality of the paper. This work is supported by the Fundamental Funds for the Central Universities No. NS2017018 and No. NJ20160025.

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