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DISTURBANCE-OBSERVER-BASED RELIABLE OUTPUT CONTROL FOR TIME-DELAY SYSTEMS WITH ACTUATOR FAULTS

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Abstract. *This paper is concerned with the disturbance-observer-based reliable robust static output controller design of linear uncertain time-delay systems with actuator faults. The estimation of exogenous disturbance is derived by a disturbance observer and used to compensate the negative effect caused by the disturbance. The disturbance-observer-based static output control scheme is constructed by the estimation and a linear controller based on system output. The design of the disturbance-observer-based reliable robust output controller follows Finsler lemma, where the sufficient conditions of the closed-loop system being asymptotically stable are established in a set of linear matrix inequalities. The effectiveness of the proposed approach is verified by a numerical example.*

Keywords: Disturbance observer, Static output control, Robust control, Reliable control, Time delay

1. **Introduction.** Time delay is a common phenomenon in a wide range of practical applications such as networked communication systems, and chemical dynamic process. As a significant source of system performance degradation and instability, time delay system has attracted a great deal of attention on stability analysis [1-6] and control synthesis [7-10] over the past decades. In most of the researches mentioned above, system models used for controller design are described with explicit knowledge of system parameters. However, real systems often contain some uncertainties which can be induced by environment change or measurement error. To deal with these uncertainties, robust control has been investigated extensively by researchers in [11-14]. Furthermore, many works regarding systems under the influence of both uncertain parameters and time delay can be found in [15-18]. To be specific, for time-delay uncertain systems, robust sliding-mode control is studied in [15]. [16] addresses robust H_∞ filtering issue of time-delay stochastic systems with sector-bounded nonlinearities. In [17], the robust stability analysis is developed for uncertain Markovian jump time-delay systems with polytopic parameter uncertainties. By choosing appropriate Lyapunov-Krasovskii functionals and using an improved inequality, some novel delay-dependent stability conditions are proposed for time-delay uncertain systems in [18].

On the other hand, external disturbances such as measurement noise also exist in most practical systems. As an efficient anti-disturbance mechanism, disturbance-observer-based (DOB) control has been developed to attenuate the negative impact of these disturbances. Typically, DOB control mechanism first constructs an observer to estimate the disturbance. Combining this estimation with a linear control law, a nonlinear control scheme is then presented to ensure stability of system under external disturbances. Regarding this

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scheme, some theoretical results can be referred in [19-22], and practical applications in robotic systems [23], missile systems [24] and so on. However, most of the above results are obtained under the assumption that full system state is accessible. As has mentioned, this is inapplicable due to the limitation of devices and cost in real situations. Consequently, the anti-disturbance control problem of nonlinear Markov jump systems is addressed via system output in [26], where the time delay and parameter uncertainties are not taken into account. Apart from this, the above mentioned works fail to consider that system also suffers from faulty operations of actuators, which is another instability factor of real systems [27, 28]. To the best knowledge of authors, few results considering reliable robust DOB output control strategy for time-delay uncertain systems with actuator faults have been addressed so far.

Motivated by the above observations, this paper investigates reliable robust DOB output control of time-delay uncertain systems with actuator faults. A DOB output control scheme consisting of a linear control law and the estimation of disturbance is employed to compensate the impact of disturbance while ensuring the asymptotic stability of the closed-loop system. With the help of Finsler lemma, sufficient conditions for obtaining the output controller and observer gains are formed via linear matrix inequalities (LMIs). A numerical example is given to demonstrate the effectiveness of the provided method.

The paper is organized as below. Problem preliminaries are given in Section 2, which are prepared to obtain the main results. Section 3 gives the sufficient LMI conditions for event-triggered non-fragile H_∞ filtering of linear Markov jump systems with unreliable communications. Section 4 produces a numerical example to show the approach effectiveness. Then, some conclusions are given in Section 5.

Notation. In this paper, X^T is used to mean the transpose of the matrix X. The notation $X > 0$ represents X is positive definite and symmetric. R^n denotes the n-dimensional Euclidean space and $R^{n\times m}$ is $n \times m$ real matrices. * stands for the symmetry. Finally, the symbol $He(X)$ equals $X + X^T$.

2. Problem Statement and Preliminaries.

Consider the following time delay uncertain system described as

$$\begin{cases} \dot{x}(t) = [A + \Delta A(t)]x(t) + [A_d + \Delta A_d(t)]x(t-\tau) + B(u(t) + d(t)) \\ x(t) = \phi(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^m$ is control input, $y(t)$ is output measurement, and $\phi(t)$ is the initial state vector of the system. A, A_d, B, C are known system matrices and the matrix B is full column rank, τ is the time delay, $\Delta A(t)$ and $\Delta A_d(t)$ are uncertain parameter matrices, and $d(t) \in \mathbb{R}$ is the system disturbance input which is described by

$$\begin{cases} \dot{\omega}(t) = W\omega(t) \\ d(t) = V\omega(t) \end{cases} \tag{2}$$

where $\omega(t) \in \mathbb{R}^n$, $d(t) \in \mathbb{R}^m$, W and V are known constant matrices.

To compensate the effect on systems induced by the disturbance $d(t)$, a disturbance observer with respect to $y(t)$ is used to estimate $d(t)$:

$$\begin{cases} \hat{d}(t) = V\hat{\omega}(t) \\ \hat{\omega}(t) = v(t) - Ly(t) \\ \dot{v}(t) = (W + LCBV)\hat{\omega}(t) + LCBu(t) \end{cases} \tag{3}$$

where $v(t)$ is the disturbance observer, $\hat{\omega}(t)$ and $\hat{d}(t)$ mean the estimation of $\omega(t)$ and $d(t)$, and L is the observer gain to be designed.

Remark 2.1. *Compared with the DOB state feedback control scheme used in* [20, 26] *where the full state of real systems is assumed to be accessible, system output is used to construct the DOB control scheme in this paper due to practical considerations.*

The control input with estimated disturbance $\hat{d}(t)$ and a linear control law is formulated as

$$u(t) = Ky(t) - \hat{d}(t), \tag{4}$$

where K is the controller gain to be designed.

Moreover, considering the actuator faults, the corresponding system model is captured by

$$\dot{x}(t) = [A + \Delta A(t)]x(t) + [A_d + \Delta A_d(t)]x(t-\tau) + B(I-\rho)(u(t) + d(t)), \tag{5}$$

with $\rho = diag(\rho_1, \dots, \rho_m)$ satisfying:

$$0 \le \underline{\rho}_k \le \rho_k \le \bar{\rho}_k < 1, \quad k = 1, 2, \dots, m, \tag{6}$$

where ρ_k ($k = 1, \dots, m$) represents the loss of effectiveness of the kth actuator. It is supposed that, the lower and upper bounds of ρ_k are known constants. Define $\underline{\rho} = diag\left(\underline{\rho}_1, \dots, \underline{\rho}_m\right)$, $\bar{\rho} = diag\left(\bar{\rho}_1, \dots, \bar{\rho}_m\right)$. Taking the upper and lower bounds $\left(\overline{\rho}_k, \underline{\rho}_k\right)$ into account, it gives:

$$N_\rho = \left\{\rho : \rho = diag\{\rho_1, \rho_2, \dots, \rho_m\}, \rho_k = \underline{\rho}_k \text{ or } \overline{\rho}_k, k = 1, 2, \dots, m\right\}, \tag{7}$$

where N_ρ contains a maximum of 2^m elements.

Remark 2.2. *Different from some existing results for DOB control issue in which the actuator is assumed perfect, this paper takes actuator faults into consideration by covering the normal case ($\rho = 0$) and faulty case ($\rho \neq 0$).*

Define $e(t) = \omega(t) - \hat{\omega}(t)$. From (5) and (3), one has

$$\begin{aligned}\dot{e}(t) &= \dot{\omega}(t) - \dot{\hat{\omega}}(t) = W\omega(t) - (\dot{v}(t) - L\dot{y}(t)) \\ &= (W + LCB(I-\rho)V)e(t) + LC(A + \Delta A)x(t) \\ &\quad + LC(A_d + \Delta A_d)x(t-\tau) - LCB\rho KCx(t).\end{aligned} \tag{8}$$

Based on (3), (5) and (8), the augmented DOB closed-loop system is established as

$$\dot{\xi}(t) = \bar{A}\xi(t) + \bar{A}_d x(t-\tau) \tag{9}$$

with $\xi(t) \triangleq \begin{bmatrix} x^T(t) & e^T(t) \end{bmatrix}^T$, and

$$\bar{A} = \begin{bmatrix} A + \Delta A + B(I-\rho)KC & B(I-\rho)V \\ LC(A+\Delta A) - LCB\rho KC & W + LCB(I-\rho)V \end{bmatrix}, \quad \bar{A}_d = \begin{bmatrix} A_d + \Delta A_d \\ LC(A_d + \Delta A_d) \end{bmatrix}.$$

Before ending this section, some assumptions and technical lemmas are given as below.

Assumption 2.1. [20]. *The time delay is continuous function which is bounded and satisfies*

$$0 \le \tau \le \tau_M, \quad \dot{\tau} \le \tau_D,$$

where τ_M and τ_D are given constants which mean the upper bound of the time delay and the derivative of the time delay, respectively.

Assumption 2.2. [20]. *The system uncertainty $\Delta A(t)$ and the time-varying uncertainty $\Delta A_d(t)$ satisfy the condition described as:*

$$\left[\begin{array}{cc} \Delta A(t) & \Delta A_d(t) \end{array} \right] = DF(t) \left[\begin{array}{cc} E_1 & E_2 \end{array} \right],$$

where E_1 and E_2 are known constant matrices with appropriate dimensions, and $F(t)$ is a time-varying uncertain matrix satisfying $F^T(t)F(t) \leq I$.

Lemma 2.1. *For any real constant $\alpha > 0$, the following inequality holds:*

$$X^TY + Y^TX \leq \alpha X^TX + \alpha^{-1}Y^TY.$$

Lemma 2.2. [29]. *Finsler Lemma. Letting $\upsilon \in \mathcal{R}^n$, $\mathcal{P} = \mathcal{P}^T \in \mathcal{R}^{n\times n}$, and $\mathcal{H} \in \mathcal{R}^{m\times n}$ such that $rank(\mathcal{H}) = r < n$, then the following statements are equivalent:*
1) $\upsilon^T\mathcal{P}\upsilon$, for all $\upsilon \neq 0$, $\mathcal{H}\upsilon = 0$;
2) $\mathcal{H}^{\perp T}\mathcal{P}\mathcal{H}^{\perp} < 0$;
3) $\exists\mathcal{S} \in \mathcal{R}^{n\times m}$ such that $\mathcal{P} + He(\mathcal{S}\mathcal{H}) < 0$.

3. **Main Results.** In this section, stability analysis and control synthesis conditions of the closed-loop system (9) are presented in Theorem 3.1 and Theorem 3.2, respectively.

Theorem 3.1. *For given positive constants α_i, $i \in (1, 2, 3, 4)$, if there exist positive definite matrices P_1, P_2, R and matrices $Q_1 \in \mathbb{R}^{n\times n}$, $Q_2 \in \mathbb{R}^{n\times n}$, $Q_3 \in \mathbb{R}^{n\times n}$, $Q_4 \in \mathbb{R}^{n\times n}$ such that the following inequality holds:*

$$\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} \\ * & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} \\ * & * & \Gamma_{33} & \Gamma_{34} \\ * & * & * & \Gamma_{44} \end{bmatrix} < 0, \tag{10}$$

where

$$\begin{aligned} \Gamma_{11} &= He(P_1(A + B(I - \rho)KC)) + (\alpha_1 + \alpha_2)PDD^TP \\ &\quad + (\alpha_1^{-1} + \alpha_3^{-1})E_1^TE_1 + P_1 - Q_1 - Q_1^T, \\ \Gamma_{12} &= P_1B(I - \rho)V + (P_2LCA - P_2LCB\rho KC)^T - Q_4, \\ \Gamma_{13} &= P_1A_d - Q_2 + Q_1^T,\ \Gamma_{14} = Q_1^T - Q_3, \\ \Gamma_{22} &= He(P_2(W + LCB(I - \rho)V)) + (\alpha_3 + \alpha_4)P_2LCDD^TC^TL^TP_2, \\ \Gamma_{23} &= P_2LCA_d + Q_4^T,\ \Gamma_{24} = Q_4^T,\ \Gamma_{34} = Q_2^T + Q_3,\ \Gamma_{44} = Q_3^T + Q_3, \\ \Gamma_{33} &= (\alpha_2^{-1} + \alpha_4^{-1})E_2^TE_2 + (\tau_D - 1)R + Q_2^T + Q_2, \end{aligned}$$

then the augmented DOB closed-loop system is said to be asymptotically stable.

Proof: Choose a candidate Lyapunov functional as

$$V(t) = \xi^T(t) \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \xi(t) + \int_{t-\tau}^{t} x^T(s)Rx(s)ds. \tag{11}$$

The derivation of (11) is computed as:

$$\begin{aligned} \dot{V} &= 2x^T(t)P_1\dot{x}(t) + 2e^T(t)P_2\dot{e}(t) + x^T(t)Rx(t) \\ &\quad + (\dot{\tau} - 1)x^T(t - \tau)Rx(t - \tau) + 2\left(\int_{t-\tau}^{t} \dot{x}(s)ds - x(t) + x(t - \tau)\right)^T \\ &\quad \times \left(Q_1x(t) + Q_2x(t - \tau) + Q_3\int_{t-\tau}^{t} \dot{x}(s)ds + Q_4e(t)\right). \end{aligned} \tag{12}$$

To deal with the nonlinear terms induced by $\dot{x}(t)$ and $\dot{e}(t)$ via using Lemma 2.1, it yields

$$\begin{aligned}
&2x^T(t)P_1DF(t)E_1x(t) \le \alpha_1 x^T(t)P_1DD^TPx(t) + \alpha_1^{-1}x^T(t)E_1^TE_1x(t),\\
&2x^T(t)P_1DF(t)E_2x(t-\tau) \le \alpha_2 x^T(t)P_1DD^TP_1x(t) + \alpha_2^{-1}x^T(t-\tau)E_2^TE_2x(t-\tau),\\
&2e^T(t)P_2LCDF(t)E_1x(t) \le \alpha_3 e^T(t)P_2LCDD^TC^TL^TP_2e(t) + \alpha_3^{-1}x^T(t)E_1^TE_1x(t),\\
&2e^T(t)P_2LCDF(t)E_2x(t-\tau) \le \alpha_4 e^T(t)P_2LCDD^TC^TL^TP_2e(t)\\
&\qquad\qquad\qquad + \alpha_4^{-1}x^T(t-\tau)E_2^TE_2x(t-\tau).
\end{aligned} \tag{13}$$

Substituting (13) into (12), one can get

$$\begin{aligned}
\dot{V} \le\ & 2x^T(t)P_1(A + B(I-\rho)KC)x(t) + 2x^T(t)P_1B(I-\rho)Ve(t)\\
&+ 2x^T(t)P_1A_dx(t-\tau) + \alpha_1x^T(t)P_1DD^TP_1x(t) + \alpha_1^{-1}x^T(t)E_1^TE_1x(t)\\
&+ \alpha_2x^T(t)P_1DD^TP_1x(t) + \alpha_2^{-1}x(t-\tau)^TE_2^TE_2x(t-\tau)\\
&+ x^T(t)Rx(t) + (\tau_D - 1)x^T(t-\tau)Rx(t-\tau)\\
&+ 2e^T(t)\left[P_2(W + LCB(I-\rho)V)e(t) + 2e^T(t)P_2(LCA - LCB\rho KC)\right]x(t)\\
&+ 2e^T(t)P_2LCA_dx(t-\tau) + \alpha_3e^T(t)P_2LCDD^TC^TL^TP_2e(t) + \alpha_3^{-1}x^T(t)E_1^TE_1x(t)\\
&+ \alpha_4e^T(t)P_2LCDD^TC^TL^TP_2e(t)\\
&+ \alpha_4^{-1}x^T(t-\tau)E_2^TE_2x(t-\tau) + 2\left(\int_{t-\tau}^t \dot{x}(s)ds - x(t) + x(t-\tau)\right)^T\\
&\times\left(Q_1x(t) + Q_2x(t-\tau) + Q_3\int_{t-\tau}^t \dot{x}(s)ds + Q_4e(t)\right),
\end{aligned} \tag{14}$$

which can be rewritten as

$$\dot{V} \le \zeta^T(t)\Gamma\zeta(t), \tag{15}$$

where $\zeta(t) = \begin{bmatrix} x^T(t) & e^T(t) & x^T(t-\tau) & \int_{t-\tau}^t \dot{x}^T(s)ds \end{bmatrix}^T$ and Γ is just given in (10).

To guarantee the asymptotic stability of system (9), one just needs $\dot{V}(t) < 0$, which can be ensured by (10). Thus, the proof is completed. □

The stability analysis condition derived in Theorem 3.1 is established in nonlinear form, which cannot be used to solve the corresponding controller and observer gains. Then, with the help of Finsler lemma, control synthesis conditions are formed in the framework of LMIs in Theorem 3.2.

Theorem 3.2. *For given positive constants b_1, b_2, μ, α_i, $i \in (1,2,3,4)$, if there exist symmetric matrices $P_1 > 0$, $P_2 > 0$, $R > 0$, matrices Q_1, Q_2, Q_3, Q_4, M, Z, N such that the following inequality holds for $\rho \in N_\rho$:*

$$\Lambda = \begin{bmatrix}
\Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} & \Lambda_{15} & \Lambda_{16} & 0\\
* & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} & \Lambda_{25} & 0 & \Lambda_{27}\\
* & * & \Lambda_{33} & \Lambda_{34} & 0 & 0 & 0\\
* & * & * & \Lambda_{44} & 0 & 0 & 0\\
* & * & * & * & \Lambda_{55} & 0 & 0\\
* & * & * & * & * & \Lambda_{66} & 0\\
* & * & * & * & * & * & \Lambda_{77}
\end{bmatrix} < 0, \tag{16}$$

where

$$\Lambda_{11} = He(P_1A + b_1B(I-\rho)NC) + \left(\alpha_1^{-1} + \alpha_3^{-1}\right)E_1^TE_1 + R - He(Q_1),$$

$\Lambda_{12} = P_1B(I-\rho)V + (MCA - b_2B\rho NC)^T - Q_4,\ \Lambda_{13} = P_1A_d - Q_2 + Q_1^T,$
$\Lambda_{14} = Q_1^T - Q_3,\ \Lambda_{15} = P_1B(I-\rho) - b_1B(I-\rho)Z + \mu(NC)^T,\ \Lambda_{16} = P_1D,$
$\Lambda_{22} = He(P_2W + MCB(I-\rho)V),\ \Lambda_{23} = MCA_d + Q_4^T,$
$\Lambda_{24} = Q_4^T,\ \Lambda_{25} = b_2B\rho Z - MCB\rho,\ \Lambda_{27} = MCD,$
$\Lambda_{33} = \left(\alpha_2^{-1} + \alpha_4^{-1}\right) E_2^TE_2 + (\tau_D - 1)R + He(Q_2),\ \Lambda_{34} = Q_2^T + Q_3,\ \Lambda_{44} = He(Q_3),$
$\Lambda_{55} = -\mu He(Z),\ \Lambda_{66} = -(\alpha_1+\alpha_2)^{-1}I,\ \Lambda_{77} = -(\alpha_3+\alpha_4)^{-1}I,$

then the augmented DOB closed-loop system is said to be asymptotically stable under $K = Z^{-1}N$ *and* $L = P_2^{-1}M$.

Proof: Based on (10) in Theorem 3.1, it results in

$$\mathcal{H}^{\perp^T}\mathcal{P}\mathcal{H}^{\perp} < 0, \tag{17}$$

where

$$\mathcal{P} = \begin{bmatrix} \Gamma & 0 \\ * & 0 \end{bmatrix},\ \mathcal{H}^{\perp} = \begin{bmatrix} \mathcal{H}_1^{\perp} \\ \mathcal{B}_{i2}^{\perp} \end{bmatrix},\ \mathcal{H}_1^{\perp} = diag\{I, I, I, I\},\ \mathcal{H}_2^{\perp} = \begin{bmatrix} KC & 0 & 0 & 0 \end{bmatrix}.$$

According to Lemma 2.2, (17) can be rewritten as

$$\mathcal{P} + He(\mathcal{M}\mathcal{H}) < 0, \tag{18}$$

where

$$\mathcal{H} = \begin{bmatrix} KC & 0 & 0 & 0 & -I \end{bmatrix},\ \mathcal{M} = \begin{bmatrix} \mathcal{M}_1^T & \mathcal{M}_2^T & 0 & 0 & \mu Z^T \end{bmatrix}^T,$$
$$\mathcal{M}_1 = b_1B(I-\rho)Z - P_1B(I-\rho),\ \mathcal{M}_2 = -b_2B\rho Z + MCB\rho,\ M = P_2L,$$
$$He(\mathcal{M}\mathcal{H}) = \begin{bmatrix} \Delta_{11} & \Delta_{12} & 0 & 0 & \Delta_{15} \\ * & 0 & 0 & 0 & \Delta_{25} \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & -\mu He(Z) \end{bmatrix},$$
$$\Delta_{11} = He(b_1B(I-\rho)NC - P_1B(I-\rho)KC),\ \Delta_{12} = (-b_2B\rho NC + MCB\rho),$$
$$\Delta_{25} = b_2B\rho Z - MCB\rho,\ \Delta_{15} = P_1B(I-\rho) - b_1B(I-\rho)Z + \mu(NC)^T.$$

Thus, $P + He(\mathcal{M}\mathcal{H}) < 0$ can be rewritten by

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & \Theta_{14} & \Theta_{15} \\ * & \Theta_{22} & \Theta_{23} & \Theta_{24} & \Theta_{25} \\ * & * & \Theta_{33} & \Theta_{34} & 0 \\ * & * & * & \Theta_{44} & 0 \\ * & * & * & * & \Theta_{55} \end{bmatrix} < 0, \tag{19}$$

with

$$\begin{aligned} \Theta_{11} &= He(P_1A + b_1B(I-\rho)NC) + (\alpha_1+\alpha_2)P_1DD^TP_1 \\ &\quad + \left(\alpha_1^{-1} + \alpha_3^{-1}\right) E_1^TE_1 + R - He(Q_1), \\ \Theta_{12} &= P_1B(I-\rho)V + (MCA - b_2B\rho NC)^T - Q_4, \\ \Theta_{13} &= P_1A_d - Q_2 + Q_1^T,\ \Theta_{14} = Q_1^T - Q_3, \\ \Theta_{15} &= P_1B(I-\rho) - b_1B(I-\rho)Z + \mu(NC)^T,\ \Theta_{25} = b_2B\rho Z - MCB\rho, \\ \Theta_{22} &= He(P_2W + MCB(I-\rho)V) + (\alpha_3+\alpha_4)P_2LCDD^TC^TL^TP_2, \\ \Theta_{23} &= MCA_d + Q_4^T,\ \Theta_{33} = (\alpha_2^{-1} + \alpha_4^{-1})E_2^TE_2 + (\tau_D - 1)R + He(Q_2), \end{aligned}$$

$$\Theta_{24} = Q_4^T,\ \Theta_{34} = Q_2^T + Q_3,\ \Theta_{44} = He(Q_3),\ \Theta_{55} = -\mu He(Z).$$

Then, (19) is equivalent to (16) by applying the Schur complement. □

Remark 3.1. *From (4), the presented reliable robust DOB output controller consists of linear control law $Ky(t)$ and the disturbance estimation $\hat{d}(t)$. The linear part $Ky(t)$ ensures the closed-loop system stability, and the estimation $\hat{d}(t)$ is adopted to offset the effect of disturbance.*

Remark 3.2. *If we choose $\rho = 0$, then Theorem 3.2 reduces to the conventional normal case, which is shown in the next corollary.*

Corollary 3.1. *For given positive constants b_1, b_2, μ, α_i, $i \in (1, 2, 3, 4)$, if there exist symmetric matrices $P_1 > 0$, $P_2 > 0$, $R > 0$, matrices Q_1, Q_2, Q_3, Q_4, M, Z, N such that the following inequality holds*

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & \Omega_{16} & 0 \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & 0 & 0 & \Omega_{27} \\ * & * & \Omega_{33} & \Omega_{34} & 0 & 0 & 0 \\ * & * & * & \Omega_{44} & 0 & 0 & 0 \\ * & * & * & * & \Omega_{55} & 0 & 0 \\ * & * & * & * & * & \Omega_{66} & 0 \\ * & * & * & * & * & * & \Omega_{77} \end{bmatrix} < 0, \tag{20}$$

where

$$\begin{aligned}
&\Omega_{11} = He(P_1A + b_1BNC)) + (\alpha_1^{-1} + \alpha_3^{-1})E_1^TE_1 - He(Q_1),\\
&\Omega_{12} = P_1BV + (MCA)^T - Q_4,\ \Omega_{13} = P_1A_d - Q_2 + Q_1^T,\\
&\Omega_{14} = Q_1^T - Q_3,\ \Omega_{15} = P_1B - b_1BZ + (b_2NC)^T,\\
&\Omega_{16} = P_1D,\ \Omega_{22} = He(P_2W + MCBV),\\
&\Omega_{23} = MCA_d + Q_4^T,\ \Omega_{24} = Q_4^T,\ \Omega_{27} = MCD,\\
&\Omega_{33} = (\alpha_2^{-1} + \alpha_4^{-1})E_2^TE_2 + (\tau_D - 1)R + Q_2^T + Q_2,\\
&\Omega_{34} = Q_2^T + Q_3,\ \Omega_{44} = He(Q_3),\ \Omega_{55} = He(-b_2Z),\\
&\Omega_{66} = -(\alpha_1 + \alpha_2)^{-1}I,\ \Omega_{77} = -(\alpha_3 + \alpha_4)^{-1}I,
\end{aligned}$$

then the augmented DOB closed-loop system is said to be asymptotically stable under $K = Z^{-1}N$ and $L = P_2^{-1}M$.

4. **Numerical Example.** The parameters of the time delay uncertain system are given as follows:

$$A = \begin{bmatrix} 2.8 & -1.7 \\ -1.5 & -1.2 \end{bmatrix},\ A_d = \begin{bmatrix} 0.9 & 0.7 \\ -0.5 & -1 \end{bmatrix},\ B = \begin{bmatrix} -6 \\ 2 \end{bmatrix},$$
$$D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ E_1 = \begin{bmatrix} 0.02 & -0.01 \\ -0.01 & 0.02 \end{bmatrix},\ E_2 = \begin{bmatrix} 0.03 & 0 \\ 0 & 0.03 \end{bmatrix},$$
$$W = \begin{bmatrix} 0 & 0.5 \\ -0.5 & 0 \end{bmatrix},\ V = \begin{bmatrix} 2 & 0 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

Other parameters are selected as $\tau_D = 0.2$, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 1$, $b_1 = 10$, $b_2 = 0.1$, $\mu = 1$. One possible actuator fault parameter is considered as $\bar{\rho} = 0.8$, $\underline{\rho} = 0$, which means the actuator can lose effectiveness up to 80%.

The output feedback control gain K and observer gain L are calculated by solving the Theorem 3.2 as

$$K = 22.5073, \qquad L = \begin{bmatrix} 0.0323 & 0.0012 \end{bmatrix}^T.$$

Given the initial condition as $x_0 = \begin{bmatrix} -1 & 0.5 \end{bmatrix}^T$ and $\rho = 0.75$, the curves of augmented closed-loop system state $x(t)$, $e(t)$ and the estimation error $d(t) - \hat{d}(t)$ of the disturbance observer with standard controller solved by Corollary 3.1 and reliable controller solved by Theorem 3.2 are given in Figures 1-6, respectively.

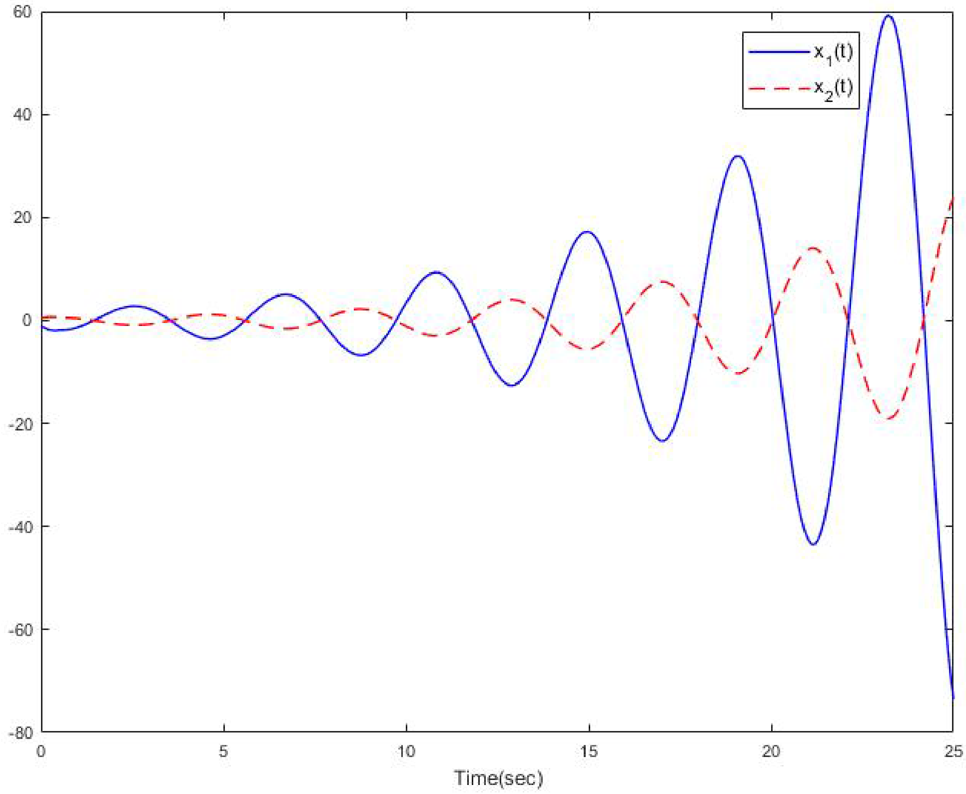

FIGURE 1. Curves of system state responses in Corollary 3.1

From Figure 1, with standard DOB control approach, the dynamics of the closed-loop system are not asymptotically stable when actuator works in faulty case. Figure 2 and Figure 3 illustrate that the standard DOB control approach fails to estimate the disturbance with actuator faults. However, it is observed in Figure 4 that the state of the closed-loop system with the designed reliable robust controller is asymptotically stable. From Figure 5 and Figure 6, it is also seen that the disturbance observer considering actuator fault can effectively approximate the external disturbance. Thus, these figures indicate that the proposed reliable control scheme in Theorem 3.2 is able to tolerate some actuator fault, while the standard DOB control approach may lose the effectiveness of guaranteeing the system stability when actuator fault happens.

5. **Conclusions.** In this article, the issue of reliable robust DOB output feedback control for time-delay uncertain systems with actuator faults is studied. First, a disturbance observer considering actuator fault is used to derive the estimation of the exogenous disturbance. Second, consisting of the estimation and a linear controller based on system output, the disturbance-observer-based static output control scheme is constructed. Third, with the help of Finsler lemma, sufficient conditions are formed in terms of LMIs

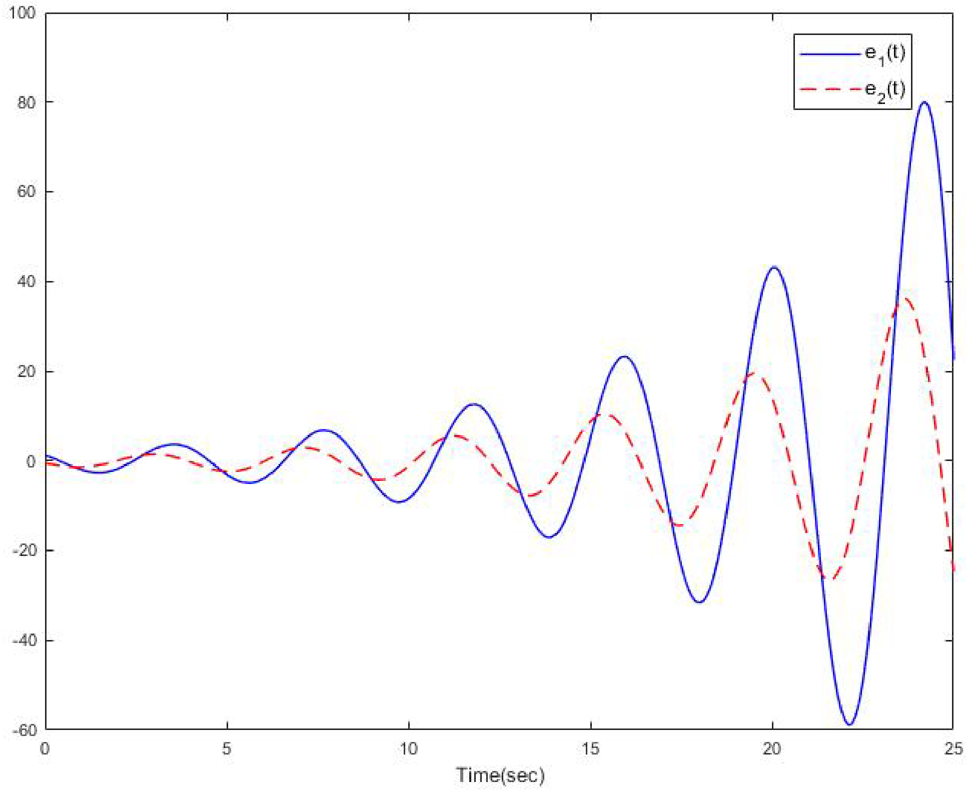

FIGURE 2. Curves of $e(t)$ in Corollary 3.1

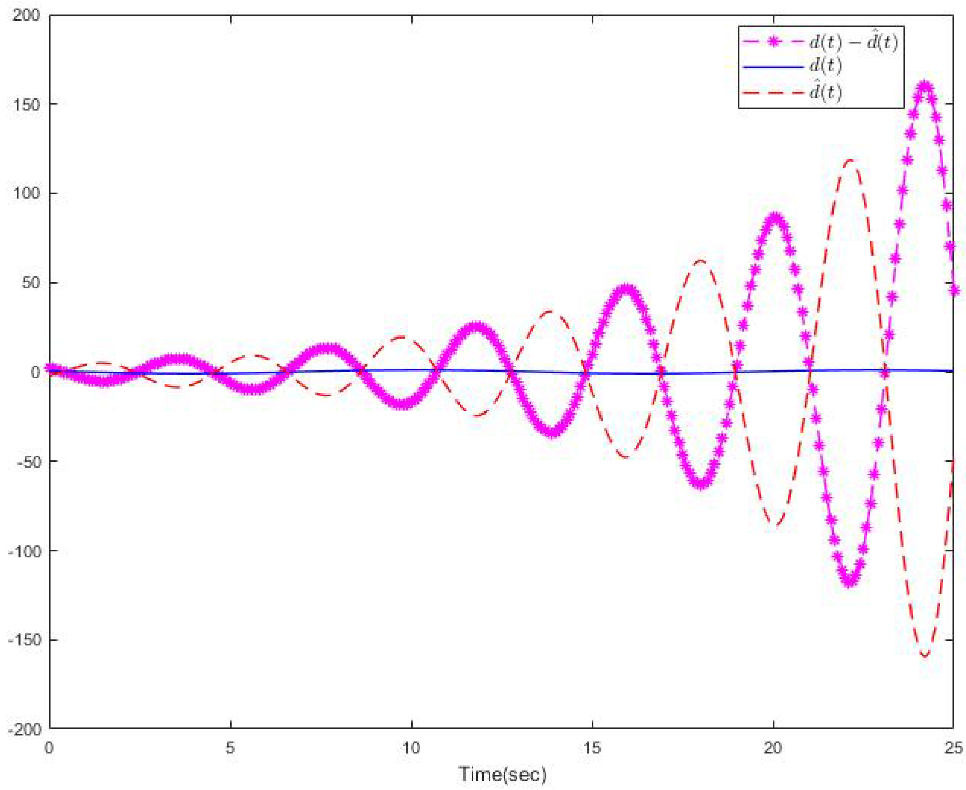

FIGURE 3. Curves of $d(t) - \hat{d}(t)$, $d(t)$, $\hat{d}(t)$ in Corollary 3.1

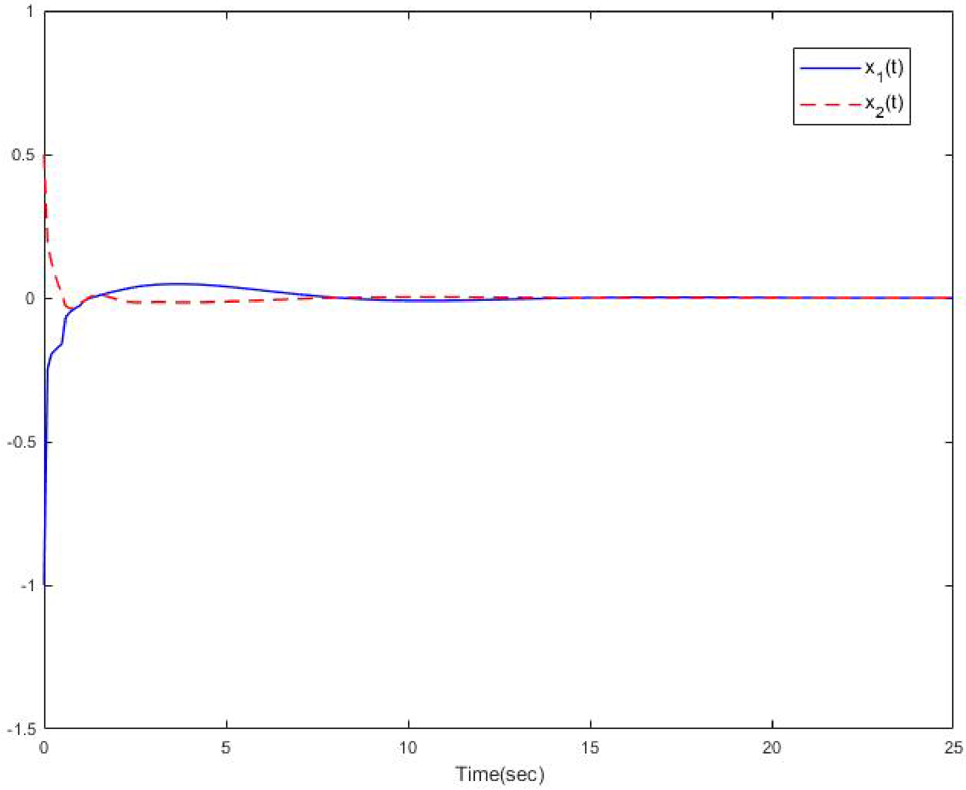

FIGURE 4. Curves of system state responses in Theorem 3.2

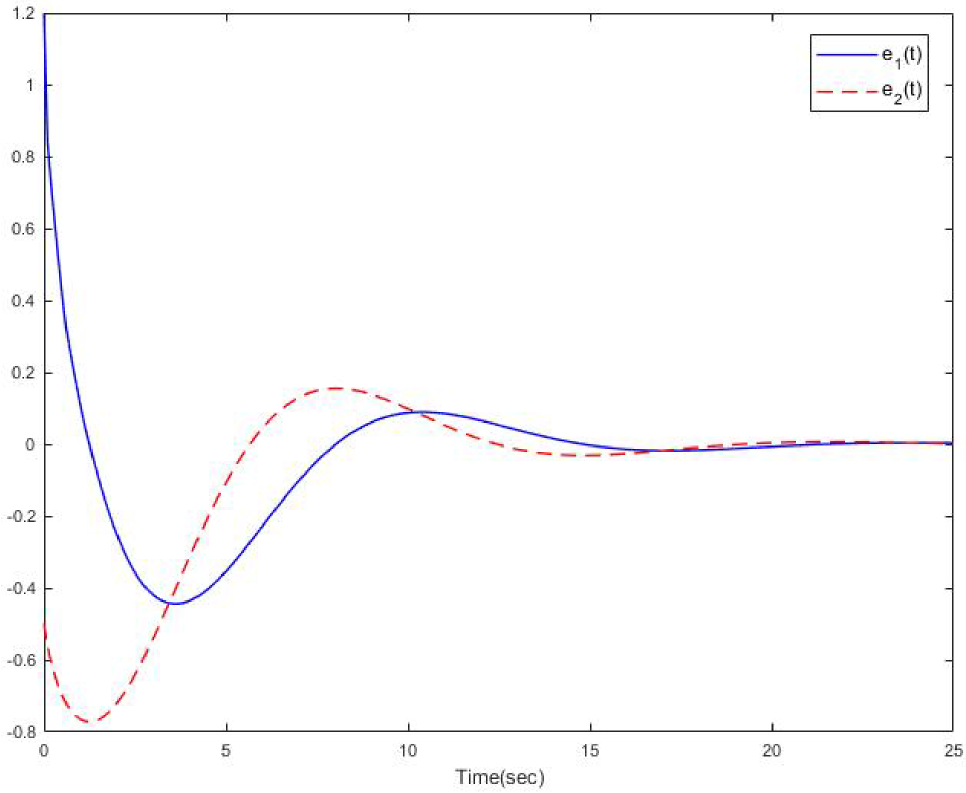

FIGURE 5. Curves of $e(t)$ in Theorem 3.2

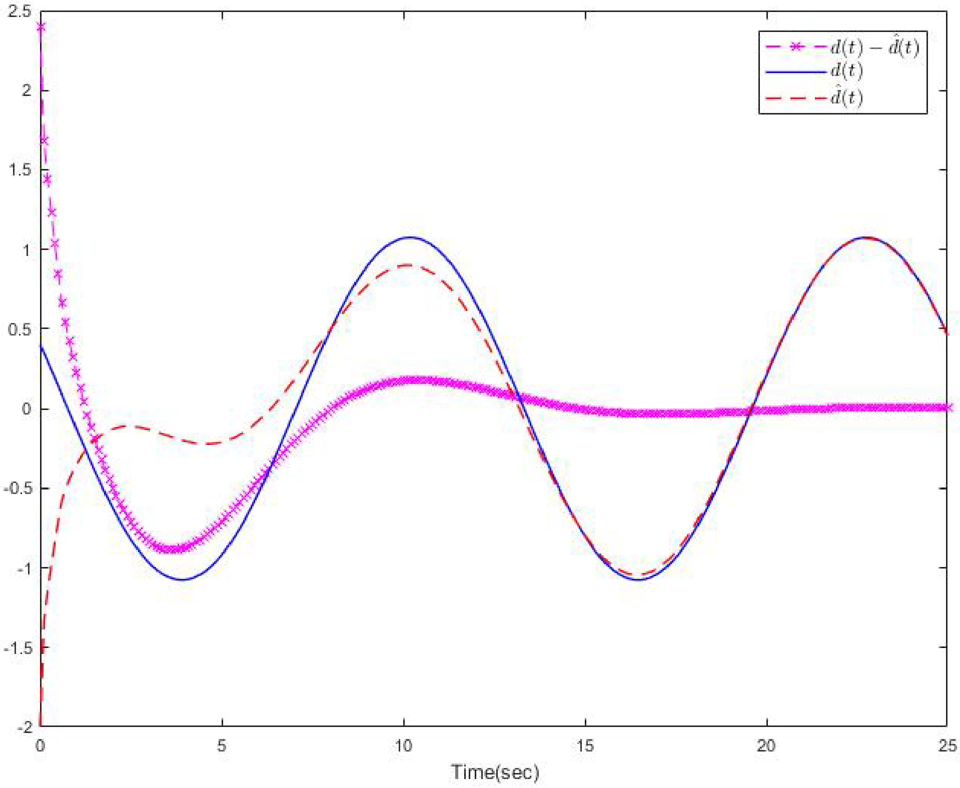

FIGURE 6. Curves of $d(t)-\hat{d}(t)$, $d(t)$, $\hat{d}(t)$ in Theorem 3.2

to ensure the asymptotic stability of the closed-loop system. A numerical example is simulated to show the validity of the presented method in the end. Note that faulty operations of sensors are not considered in this paper; further studies on control strategies for systems with both actuator and sensor faults will be carried out in the future.

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