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H_∞ FUZZY INTEGRAL CONTROLLER FOR NONLINEAR DESCRIPTOR SYSTEMS

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Abstract. *This paper examines the design procedure of an H_∞ fuzzy integral controller for a class of nonlinear descriptor systems described by a Takagi-Sugeno (TS) fuzzy model. Based on a linear matrix inequality (LMI) approach, the TS fuzzy model is employed to model the fuzzy descriptor system instead of the nonlinear descriptor system, and the H_∞ fuzzy controller is developed to achieve a set of sufficient conditions for overcoming the exogenous input disturbance. Since the interaction of fast and slow dynamic modes in the nonlinear descriptor system is a cause of ill-conditioned LMI results, the proposed technique with ε-independent can alleviate the ill-conditioned LMI. Moreover, integral control is used to enhance the equilibrium rapidity and the stable performance by low steady-state errors. Finally, a numerical example is presented to illustrate the performance results of the controller design.*
Keywords: H_∞ fuzzy controller, Integral controller, Linear matrix inequality (LMI), Takagi-Sugeno (TS) fuzzy model, Descriptor system

1. **Introduction.** Descriptor systems that are known as singularly perturbed systems have been considered in many control engineering fields for the past four decades. Descriptor systems are systems with "small" parasitic parameters or multiple time-scales such as masses, capacitances, inductances, and time constants. An example of defining such system parameters can be applied in other studies, i.e., the enzyme quantity in biochemistry, the transients of voltage regulators in power systems, the time constants of drives and actuators in industrial control systems, the time-scale characteristics of the longitudinal motion of an airplane in control systems, and convection-diffusion equations in semiconductor physics. The problems that researchers face are the effect of the existence of small parasitic parameters, and some theorems or applications cannot avoid that effect. These parasitic parameters can cause the systems to have high dimensionality and ill-conditioning from the interaction of fast and slow dynamic modes.

To alleviate these problems, the mathematical framework of descriptor systems is used to model such systems with small parasitic parameters, which are defined as ε and play an important role as separators of fast and slow modes in the state-space model. Thus, this approach is called the reduction technique. Many researchers have intensively researched the descriptor system approach for control systems [1-8].

Over the past decade, an H_∞ control design has been developed for linear descriptor systems [6-8]. Most of the uncertain systems and systems with state measurements have been examined for linear descriptor systems, but only a few researchers have considered

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nonlinear descriptor systems. The problem of a nonlinear descriptor system occurs because the systems are not separated between fast and slow subsystems. The H_∞ control design for nonlinear descriptor systems has been successfully designed only for slow dynamic modes [9-11]. Therefore, research on H_∞ control design for nonlinear descriptor systems requires more development.

Recently, nonlinear descriptor systems have been described by the TS fuzzy model [12-16]. The TS fuzzy model can substitute the nonlinear descriptor system with the relative of linear models, which are combined with fuzzy membership functions. Moreover, the extremely complex nonlinear system can estimate the global behaviors by a fuzzy linear model [12-29]. The TS fuzzy model and the robust H_∞ control designs are combined for overcoming nonlinear uncertain descriptor system influences [28]. The state-feedback and output-feedback control design are introduced by considering the relation between the H_∞ performance index of the system and the range of the valid ε values. In addition, parallel distributed compensation (PDC), as a fuzzy controller approach, has been employed to design the controller for the nonlinear tunnel diode circuit [29]. It is the fact that TS fuzzy model is still the most successful method for estimating general nonlinear systems; however, the nonlinear descriptor systems are too complex for good results to be achieved by using the parallel distributed compensation (PDC) method from the TS fuzzy control.

To date, the integral action method has been applied to solving the problem in control engineering areas [30-38]. The integral sliding-mode control (ISMC) has been used to remove a restrictive assumption of fuzzy ISMC by solving a set of linear matrix inequalities, and it can guarantee asymptotically stability [30,31]. Moreover, the robust H_∞ integral (RHFI) controller has succesfully overcome the approximate rule in the fuzzy system effect of a doubly-fed induction generator (DFIG) wind energy systems [32]. However, the class of nonliner descriptor system still needs to be considered because the complexity of a system with exogenous input disturbance cannot be solved by the integral action method.

Therefore, the main contribution of this paper is to solve the problem of a class of nonlinear descriptor systems by the H_∞ fuzzy integral controller method. First, this paper describes the nonlinear descriptor system mathematics model by using the TS fuzzy model in the problem statement and preliminaries section. Second, the H_∞ fuzzy controller based on an LMI approach is employed to achieve a set of sufficient conditions for overcoming the exogenous input disturbance. The main purpose of the H_∞ fuzzy controller is to guarantee that the L_2-gain of mapping from the regulated output energy to the exogenous input disturbance energy is less than or equal to the defined value. Moreover, the integral controller is used to improve the asymptotic stability performance of the nonlinear descriptor system. In main result section, the Lyapunov function is applied to proving the achieved condition of the H_∞ fuzzy integral controller.

The ill-conditioned LMI that occurs in the interaction of fast and slow dynamic mode is considered in the main results section. The ill-conditioned LMI has been separated into ε-independent LMI and ε-dependent LMI. When ε tends to zero, ε-dependent LMI also tends to zero. Thus, ε-independent LMI is solvable. The proposed controller is demonstrated through an example in the numerical example section. Finally, the overall results are illustrated in the conclusion section.

2. **Problem Statement and Preliminaries.**

The nonlinear descriptor system described by the TS fuzzy model is presented as follows:

Plant Rule i:

IF $v_1(t)$ is M_{i1} and … and $v_p(t)$ is M_{ip} THEN

$$\dot{x}_1(t) = A_{11_i}x_1(t) + A_{12_i}x_2(t) + B_{1_i}u(t) + B_{w_1}w(t) \tag{1}$$

$$\begin{aligned}\varepsilon\dot{x}_2(t) &= A_{21_i}x_1(t) + A_{22_i}x_2(t) + B_{2_i}u(t) + B_{w_2}w(t) &(2)\\ y(t) &= C_{y_{1_i}}x_1(t) + C_{y_{2_i}}x_2(t) &(3)\\ z(t) &= C_{z_{1_i}}x_1(t) + C_{z_{2_i}}x_2(t) &(4)\end{aligned}$$

where $i = 1, 2, 3, \ldots, r$, r is the IF-THEN rules, M_{iv} ($v = 1, 2, 3, \ldots, p$) are the fuzzy set, $v_1(t), \ldots, v_p(t)$ are the premise variables, ε ($\varepsilon > 0$) is the parasitic parameter, $x_1(t) \in \Re^{n_1}$ and $x_2(t) \in \Re^{n_2}$ are the state vectors, $u(t) \in \Re^{n_m}$ is the input, $w(t) \in \Re^{n_s}$ is the disturbance, $y(t) \in \Re^{n_p}$ is the measured output, $z(t) \in \Re^{n_q}$ is the controlled output, and the matrices A_{11_i}, A_{12_i}, A_{21_i}, A_{22_i}, B_{1_i}, B_{2_i}, B_{w_1}, B_{w_2}, $C_{y_{1_i}}$, $C_{y_{2_i}}$, $C_{z_{1_i}}$, $C_{z_{2_i}}$ are the appropriate matrices.

Let

$$\varpi_i(v(t)) = \prod_{v=1}^{n} M_{iv}(v_v(t))$$

and

$$\mu_i(v(t)) = \frac{\varpi_i(v(t))}{\sum_{i=1}^{r}\varpi_i(v(t))}$$

$M_{iv}(v_v(t))$ is the grade of membership of $v_v(t)$ in M_{iv}. It is assumed in this paper that

$$\begin{aligned}\varpi_i\left(v(t)\right) &\geq 0, \quad i = 1, 2, \ldots, n;\\ \sum_{i=1}^{r}\varpi_i\left(v(t)\right) &> 0, \quad i = 1, 2, \ldots, r;\end{aligned}$$

for all t. Therefore,

$$\begin{aligned}\mu_i(v(t)) &\geq 0, \quad i = 1, 2, \ldots, n;\\ \sum_{i=1}^{r}\mu_i(v(t)) &= 1, \quad i = 1, 2, \ldots, r;\end{aligned}$$

for all t. For the expediency of notation, let $\varpi_i(v) = \varpi_i(v(t))$ and $\mu_i(v) = \mu_i(v(t))$. The TS fuzzy model is inferred as follows:

$$\begin{aligned}\dot{x}_1(t) &= \sum_{i=1}^{r}\mu_i(v)\left(A_{11_i}x_1(t) + A_{12_i}x_2(t) + B_{1_i}u(t)\right) + B_{w_1}w(t) &(5)\\ \varepsilon\dot{x}_2(t) &= \sum_{i=1}^{r}\mu_i(v)\left(A_{21_i}x_1(t) + A_{22_i}x_2(t) + B_{2_i}u(t)\right) + B_{w_2}w(t) &(6)\\ y(t) &= \sum_{i=1}^{r}\mu_i(v)\left(C_{y_{1_i}}x_1(t) + C_{y_{2_i}}x_2(t)\right) &(7)\\ z(t) &= \sum_{i=1}^{r}\mu_i(v)\left(C_{z_{1_i}}x_1(t) + C_{z_{2_i}}x_2(t)\right) &(8)\end{aligned}$$

Next, let us recall the following definition.

Definition 2.1. *Given a positive real number, systems (5) and (6) are said to have $L_2[0, T_f]$ gain less than or equal to γ if*

$$\int_0^{T_f} z^T(t)z(t)dt \leq \gamma^2\left[\int_0^{T_f} w^T(t)w(t)\right]dt \tag{9}$$

for all $T_f \geq 0$ and $w(t) \in L_2[0, T_f]$.

3. **Main Results.** This section illustrates how to design an H_∞ fuzzy integral control based on an LMI approach. The form of the H_∞ fuzzy integral controller is inferred as follows:

$$u(t) = \sum_{j=1}^{r} \mu_i(v) \left(K_{1_j} x_1(t) + K_{2_j} x_2(t) + K_{I_j} q(t) \right) \tag{10}$$

where $x_1(t)$ and $x_2(t)$ are the state vectors, $q(t)$ is the state integral vector, K_{1_j} and K_{2_j} are the controller gains of state feedback, and K_{I_j} is the controller gain of state integral feedback. Thus, the TS fuzzy model (5)-(8) with the H_∞ fuzzy integral controller (10) is rewritten as follows:

$$E_\varepsilon \dot{\tilde{x}}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(v)\mu_j(v) \left(\tilde{A}_{ij} \tilde{x}(t) \right) + \tilde{B}_w w(t) \tag{11}$$

$$\tilde{y}(t) = \sum_{i=1}^{r} \mu_i(v) \left(\tilde{C}_{y_i} \tilde{x}(t) \right) \tag{12}$$

$$\tilde{z}(t) = \sum_{i=1}^{r} \mu_i(v) \left(\tilde{C}_{z_i} \tilde{x}(t) \right) \tag{13}$$

where

$$\tilde{A}_{ij} = \begin{bmatrix} A_{11_i} + B_{1_i} K_{1_j} & A_{12_i} + B_{1_i} K_{2_j} & B_{1_i} K_{I_j} \\ A_{21_i} + B_{2_i} K_{1_j} & A_{22_i} + B_{2_i} K_{2_j} & B_{2_i} K_{I_j} \\ C_{y1_i} & C_{y2_i} & 0 \end{bmatrix}, \quad \tilde{B}_w = \begin{bmatrix} B_{w_1} \\ B_{w_2} \\ 0 \end{bmatrix},$$

$$\tilde{C}_{y_i} = \begin{bmatrix} C_{y1_i} & C_{y2_i} & 0 \end{bmatrix}, \quad \tilde{C}_{z_i} = \begin{bmatrix} C_{z1_i} & C_{z2_i} & 0 \end{bmatrix},$$

$$\tilde{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ q(t) \end{bmatrix}, \quad E_\varepsilon = \begin{bmatrix} I & 0 & 0 \\ 0 & \varepsilon I & 0 \\ 0 & 0 & I \end{bmatrix}$$

An H_∞ fuzzy integral controller can reach the sufficient conditions in Definition 2.1 by using the Lyapunov approach. The following lemma illustrates the derivatives of these sufficient conditions. For the symmetric block matrices, a symbol $(*)$ is used as an ellipsis for terms that are induced by symmetry. The fuzzy system (11) with the controller (10), which is shown by a state space model, is considered as follows.

Lemma 3.1. *Consider systems (11)-(13). Given a prescribed H_∞ performance $\gamma > 0$, the inequality (9) holds if there exist a matrix $P_\varepsilon = P_\varepsilon^T$ and matrices Y_j, $j = 1, 2, \ldots, r$ that satisfy the following ε-dependent linear matrix inequalities:*

$$P_\varepsilon > 0 \tag{14}$$

$$\Omega_{ii}(\varepsilon) < 0, \quad i = 1, 2, \ldots, r \tag{15}$$

$$\Omega_{ij}(\varepsilon) + \Omega_{ji}(\varepsilon) < 0, \quad i < j \le r \tag{16}$$

where

$$\Omega_{ij}(\varepsilon) = \begin{bmatrix} \tilde{A}_{ij} E_\varepsilon^{-1} P_\varepsilon + E_\varepsilon^{-1} P_\varepsilon \tilde{A}_{ij}^T & (*)^T & (*)^T \\ E_\varepsilon^{-1} \tilde{B}_w^T & -\gamma^2 I & (*)^T \\ \tilde{C}_{z_i} P_\varepsilon & 0 & -I \end{bmatrix} \tag{17}$$

with

$$\tilde{A}_{ij}(\varepsilon) = \begin{bmatrix} A_{11_i} P_\varepsilon + B_{1_i} Y_{1_j}(\varepsilon) & A_{12_i} P_\varepsilon + B_{1_i} Y_{2_j}(\varepsilon) & B_{1_i} Y_{I_j}(\varepsilon) \\ A_{21_i} P_\varepsilon + B_{2_i} Y_{1_j}(\varepsilon) & A_{22_i} P_\varepsilon + B_{2_i} Y_{2_j}(\varepsilon) & B_{2_i} Y_{I_j}(\varepsilon) \\ C_{y1_i} P_\varepsilon & C_{y2_i} P_\varepsilon & 0 \end{bmatrix},$$

$$\tilde{B}_w = \begin{bmatrix} B_{w_1} \\ B_{w_2} \\ 0 \end{bmatrix}, \quad \tilde{C}_{z_i} = \begin{bmatrix} C_{z_{1_i}} & C_{z_{2_i}} & 0 \end{bmatrix}$$

Moreover, the suitable alternative of the fuzzy controller is as follows:

$$u(t) = \sum_{j=1}^{r} \mu_i(v) \left(K_{1_j}(\varepsilon) x_1(t) + K_{2_j}(\varepsilon) x_2(t) + K_{I_j}(\varepsilon) q(t) \right) \tag{18}$$

where

$$K_j(\varepsilon) = Y_j(\varepsilon) P_\varepsilon^{-1} E_\varepsilon$$

with

$$K_j(\varepsilon) = \begin{bmatrix} K_{1_j}(\varepsilon) & K_{2_j}(\varepsilon) & K_{I_j}(\varepsilon) \end{bmatrix}$$

Proof: Consider the following Lyapunov function:

$$V(\tilde{x}(t)) = \tilde{x}^T(t) Q_\varepsilon \tilde{x}(t) \tag{19}$$

where $Q_\varepsilon = P_\varepsilon^{-1}$. Differentiating $V(\tilde{x}(t))$ along the system with the controller (18) yields

$$\dot{V}(\tilde{x}(t)) = \dot{\tilde{x}}^T(t) Q_\varepsilon \tilde{x}(t) + \tilde{x}^T(t) Q_\varepsilon \dot{\tilde{x}}(t)$$

$$\dot{V}(\tilde{x}(t)) = \begin{bmatrix} \dot{x}_1^T(t) & \dot{x}_2^T(t) & \dot{q}^T(t) \end{bmatrix} Q_\varepsilon \begin{bmatrix} x_1(t) \\ x_2(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} x_1^T(t) & x_2^T(t) & q^T(t) \end{bmatrix} Q_\varepsilon \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{q}(t) \end{bmatrix}$$

$$\begin{aligned} \dot{V}(\tilde{x}(t)) = {} & \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(v)\mu_j(v) \tilde{x}^T(t) E_\varepsilon^{-1} \tilde{A}_{ij}^T Q_\varepsilon \tilde{x}(t) + \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(v)\mu_j(v) \tilde{x}^T(t) Q_\varepsilon \tilde{A}_{ij} E_\varepsilon^{-1} \tilde{x}(t) \\ & + w^T(t) E_\varepsilon^{-1} \tilde{B}_w^T Q_\varepsilon \tilde{x}(t) + \tilde{x}^T(t) Q_\varepsilon \tilde{B}_w E_\varepsilon^{-1} w(t) \end{aligned} \tag{20}$$

Adding and subtracting $-\tilde{z}^T(t)\tilde{z}(t) + \gamma^2 w^T(t) w(t)$ to and from (20) yields

$$\begin{aligned} \dot{V}(\tilde{x}(t)) = {} & \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(v)\mu_j(v) \begin{bmatrix} \tilde{x}^T(t) & w^T(t) \end{bmatrix} \\ & \times \begin{bmatrix} E_\varepsilon^{-1} \tilde{A}_{ij}^T Q_\varepsilon + Q_\varepsilon \tilde{A}_{ij} E_\varepsilon^{-1} + \tilde{C}_{z_i}^T \tilde{C}_{z_i} & Q_\varepsilon \tilde{B}_w E_\varepsilon^{-1} \\ E_\varepsilon^{-1} \tilde{B}_w^T Q_\varepsilon & -\gamma^2 I \end{bmatrix} \\ & \times \begin{bmatrix} \tilde{x}(t) \\ w(t) \end{bmatrix} - \tilde{z}^T(t)\tilde{z}(t) + \gamma^2 w^T(t) w(t) \end{aligned} \tag{21}$$

Pre and post multiplying (21) by $\begin{bmatrix} P_\varepsilon & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$ yields

$$\begin{bmatrix} \tilde{A}_{ij} E_\varepsilon^{-1} P_\varepsilon + E_\varepsilon^{-1} P_\varepsilon \tilde{A}_{ij}^T & (*)^T & (*)^T \\ E_\varepsilon^{-1} \tilde{B}_w^T & -\gamma^2 I & (*)^T \\ \tilde{C}_{z_i} P_\varepsilon & 0 & -I \end{bmatrix} < 0 \tag{22}$$

Applying the Schur complement on (22) and rewriting the equation as follows yield

$$\begin{bmatrix} \tilde{A}_{ij} E_\varepsilon^{-1} P_\varepsilon + E_\varepsilon^{-1} P_\varepsilon \tilde{A}_{ij}^T & \tilde{B}_w E_\varepsilon^{-1} \\ E_\varepsilon^{-1} \tilde{B}_w^T & -\gamma^2 I \end{bmatrix} - \begin{bmatrix} E_\varepsilon^{-1} \tilde{C}_{z_i}^T \\ 0 \end{bmatrix} [-I] \begin{bmatrix} \tilde{C}_{z_i} E_\varepsilon^{-1} & 0 \end{bmatrix} < 0 \tag{23}$$

(23) is less than zero; thus, because $\mu_i(v(t)) \geq 0$ and $\sum_{i=1}^{r} \mu_i(v(t)) = 1$, (21) becomes

$$\dot{V}(\tilde{x}(t)) \leq -\tilde{z}^T(t)\tilde{z}(t) + \gamma^2 w^T(t) w(t) \tag{24}$$

Substituting the value $\tilde{C}_{z_i} = \begin{bmatrix} C_{z_{1_i}} & C_{z_{2_i}} & 0 \end{bmatrix}$ in $\tilde{z}(t)$, then (24) can be rewritten as follows:

$$\dot{V}(\tilde{x}(t)) \leq -z^T(t)z(t) + \gamma^2 w^T(t)w(t) \tag{25}$$

Integrating both sides of (25) yields

$$\int_0^{T_f} \dot{V}(\tilde{x}(t))dt \leq \int_0^{T_f} \left(-z^T(t)z(t) + \gamma^2 w^T(t)w(t)\right) dt \tag{26}$$

$$V(\tilde{x}(T_f)) - V(\tilde{x}(0)) \leq \int_0^{T_f} \left(-z^T(t)z(t) + \gamma^2 w^T(t)w(t)\right)dt \tag{27}$$

Using the fact that $x(0) = 0$ and $V(x(T_f)) \geq 0$ for all, $T_f \neq 0$, then (27) becomes

$$\int_0^{T_f} z^T(t)z(t)dt \leq \gamma^2 \int_0^{T_f} w^T(t)w(t)dt \tag{28}$$

Therefore, the inequality (9) of system (11) holds.

Remark 3.1. *In Lemma 3.1, the ill-condition has occurred when solving LMI. This problem occurs because of the existence of the small parameter (ε), in which there is a chance for this ill-condition to occur in nonlinear descriptor systems. Therefore, the theorem with ε-independent can alleviate the ill-condition LMI as follows.*

Theorem 3.1. *Consider systems (11)-(13). Given a prescribed H_∞ performance $\gamma > 0$ if there exist a matrix P and matrices Y_j, $j = 1, 2, \ldots, r$ that satisfy the following ε-independent linear matrix inequalities:*

$$P > 0 \tag{29}$$

$$\Omega_{ii} < 0, \quad i = 1, 2, \ldots, r \tag{30}$$

$$\Omega_{ij} + \Omega_{ji} < 0, \quad i < j \leq r \tag{31}$$

$$MPM + NPN + OPO > 0 \tag{32}$$

where

$$\Omega_{ij} = \begin{bmatrix} \begin{pmatrix} \Psi_{11_{ij}} & (*)^T & (*)^T \\ \Psi_{21_{ij}} & \Psi_{22_{ij}} & (*)^T \\ \Psi_{31_{ij}} & \Psi_{32_{ij}} & \Psi_{33_{ij}} \end{pmatrix} & (*)^T & (*)^T \\ \tilde{B}_w^T & -\gamma^2 I & (*)^T \\ \tilde{C}_{z_i} P & 0 & -I \end{bmatrix} \tag{33}$$

$$MPM = MP^T M \tag{34}$$

$$NPN = NP^T N \tag{35}$$

$$OPO = OP^T O \tag{36}$$

with

$$P = \begin{bmatrix} P_1 & 0 & P_3 \\ P_2 & P_1 & P_2 \\ P_3 & 0 & P_1 \end{bmatrix}, \; M = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; N = \begin{bmatrix} 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{bmatrix}$$

$$\begin{aligned} \Psi_{11_{ij}} &= A_{11_i} P_1 + P_1 A_{11_i}^T + A_{12_i} P_2 + P_2 A_{12_i}^T + B_{1_i} Y_{1_j} + Y_{1_j}^T B_{1_i}^T \\ \Psi_{21_{ij}} &= A_{21_i} P_1 + P_1 A_{12_i}^T + A_{22_i} P_2 + B_{2_i} Y_{1_j} + Y_{2_j}^T B_{1_i}^T \\ \Psi_{22_{ij}} &= A_{22_i} P_1 + P_1 A_{22_i}^T + B_{2_i} Y_{2_j} + Y_{2_j}^T B_{2_i}^T \end{aligned}$$

$$\begin{aligned}
\Psi_{31_{ij}} &= C_{y_{1_i}} P_1 + C_{y_{2_i}} P_2 + P_2 A_{12_i}^T + P_3 A_{11_i}^T + Y_{3_j}^T B_{1_i}^T \\
\Psi_{32_{ij}} &= C_{y_{2_i}} P_1 + P_2 A_{22_i}^T + P_3 A_{21_i}^T + Y_{3_j}^T B_{2_i}^T \\
\Psi_{33_{ij}} &= C_{y_{2_i}} P_2 + P_2 C_{y_{2_i}}^T + C_{y_{1_i}} P_3 + P_3 C_{y_{1_i}}^T \\
\tilde{B}_w^T &= \begin{bmatrix} B_{w_1}^T & B_{w_2}^T & 0 \end{bmatrix} \\
\tilde{C}_{z_i} P &= \begin{bmatrix} \left(C_{z_{1_i}} P_1 + C_{z_{2_i}} P_2\right) \\ C_{z_{2_i}} P_1 \\ \left(C_{z_{1_i}} P_3 + C_{z_{2_i}} P_2\right) \end{bmatrix}^T
\end{aligned}$$

Then, there exists a sufficiently small $\hat{\varepsilon} > 0$ that the inequality (9) holds for $\varepsilon \in (0, \hat{\varepsilon}]$. Furthermore, a suitable alternative of the fuzzy controller is as follows:

$$u(t) = \sum_{j=1}^{r} \mu_i(v) \left(K_{1_j} x_1(t) + K_{2_j} x_2(t) + K_{I_j} q(t)\right) \tag{37}$$

where

$$K_j = Y_j P^{-1}$$

with

$$K_j = \begin{bmatrix} K_{1_j} & K_{2_j} & K_{I_j} \end{bmatrix}$$

Proof: If there is a matrix P that holds the inequality (32)-(36), P is as follows:

$$P = \begin{bmatrix} P_1 & 0 & P_3 \\ P_2 & P_1 & P_2 \\ P_3 & 0 & P_1 \end{bmatrix} \tag{38}$$

with $P_1 = P_1^T > 0$, $P_3 = P_3^T > 0$ and $P_1 > P_3$. Let

$$P_\varepsilon = E_\varepsilon \left(P + \varepsilon \tilde{P}\right) \tag{39}$$

with

$$\tilde{P} = \begin{bmatrix} 0 & P_2 & 0 \\ 0 & 0 & 0 \\ 0 & P_2 & 0 \end{bmatrix} \tag{40}$$

Substituting (38) and (40) into (39) yields

$$P_\varepsilon = \begin{bmatrix} P_1 & \varepsilon P_2 & P_3 \\ \varepsilon P_2 & \varepsilon P_1 & \varepsilon P_2 \\ P_3 & \varepsilon P_2 & P_1 \end{bmatrix} \tag{41}$$

Clearly, $P_\varepsilon = P_\varepsilon^T$ and there is a sufficiently small $\hat{\varepsilon}$ such that for $\varepsilon \in (0, \hat{\varepsilon}]$, $P_\varepsilon > 0$. Using the matrix inversion lemma, the knowledge discovered is

$$P_\varepsilon^{-1} = \left(P^{-1} + \varepsilon M_\varepsilon\right) E_\varepsilon^{-1} \tag{42}$$

where

$$M_\varepsilon = -P^{-1} \tilde{P} \left(I + \varepsilon P^{-1} \tilde{P}\right)^{-1} P^{-1} \tag{43}$$

Let us consider the following Lyapunov function

$$V(\tilde{x}(t)) = \tilde{x}^T(t) E_\varepsilon Q_\varepsilon \tilde{x}(t) \tag{44}$$

where $Q_\varepsilon = (P^{-1} + \varepsilon M_\varepsilon)$. Using the matrix inversion lemma, it can be shown simply as $E_\varepsilon Q_\varepsilon = Q_\varepsilon^T E_\varepsilon$, and there is a sufficiently small $\hat{\varepsilon}$ such that for $\varepsilon \in (0, \hat{\varepsilon}]$, $P_\varepsilon > 0$, $E_\varepsilon Q_\varepsilon > 0$. Differentiating $V(\tilde{x}(t))$ along the system with the controller (11)-(13) yields

$$\dot{V}(\tilde{x}(t)) = \dot{\tilde{x}}^T(t) E_\varepsilon Q_\varepsilon \tilde{x}(t) + \tilde{x}^T(t) E_\varepsilon Q_\varepsilon \dot{\tilde{x}}(t)$$

$$\dot{V}(\tilde{x}(t)) = \dot{\tilde{x}}^T(t)E_\varepsilon Q_\varepsilon \tilde{x}(t) + \tilde{x}^T(t)Q_\varepsilon^T E_\varepsilon \dot{\tilde{x}}(t)$$

$$\dot{V}(\tilde{x}(t)) = \begin{bmatrix} \dot{x}_1^T(t) & \varepsilon\dot{x}_2^T(t) & \dot{q}^T(t) \end{bmatrix} Q_\varepsilon \begin{bmatrix} x_1(t) \\ x_2(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} x_1^T(t) & x_2^T(t) & q^T(t) \end{bmatrix} Q_\varepsilon^T \begin{bmatrix} \dot{x}_1(t) \\ \varepsilon\dot{x}_2(t) \\ \dot{q}(t) \end{bmatrix}$$

$$\begin{aligned}\dot{V}(\tilde{x}(t)) = &\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(v)\mu_j(v)\tilde{x}^T(t)\tilde{A}_{ij}^T Q_\varepsilon \tilde{x}(t) + \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(v)\mu_j(v)\tilde{x}^T(t)Q_\varepsilon^T \tilde{A}_{ij}\tilde{x}(t) \\ &+ w^T(t)\tilde{B}_w^T Q_\varepsilon \tilde{x}(t) + \tilde{x}^T(t)Q_\varepsilon^T \tilde{B}_w w(t)\end{aligned} \quad (45)$$

Adding and subtracting $-\tilde{z}^T(t)\tilde{z}(t) + \gamma^2 w^T(t)w(t)$ to and from (45), one obtains the following:

$$\begin{aligned}\dot{V}(\tilde{x}(t)) = &\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(v)\mu_j(v)\begin{bmatrix} \tilde{x}^T(t) & w^T(t) \end{bmatrix} \times \begin{bmatrix} \tilde{A}_{ij}^T Q_\varepsilon + Q_\varepsilon^T \tilde{A}_{ij} + \tilde{C}_{z_i}^T \tilde{C}_{z_i} & (*)^T \\ \tilde{B}_w^T Q_\varepsilon & -\gamma^2 I \end{bmatrix} \\ &\times \begin{bmatrix} \tilde{x}(t) \\ w(t) \end{bmatrix} - \tilde{z}^T(t)\tilde{z}(t) + \gamma^2 w^T(t)w(t)\end{aligned} \quad (46)$$

Using the fact $Q_\varepsilon = \left(P + \varepsilon\tilde{P}\right)$ and $M_\varepsilon = -P^{-1}\tilde{P}\left(I + \varepsilon P^{-1}\tilde{P}\right)^{-1}P^{-1}$, the equation is rewritten as follows:

$$\begin{aligned}\dot{V}(\tilde{x}(t)) = &\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(v)\mu_j(v)\begin{bmatrix} \tilde{x}^T(t) & w^T(t) \end{bmatrix} \times \begin{bmatrix} \tilde{A}_{ij}^T Q + Q^T \tilde{A}_{ij} + \tilde{C}_{z_i}^T \tilde{C}_{z_i} & (*)^T \\ \tilde{B}_w^T Q & -\gamma^2 I \end{bmatrix} \\ &+ \varepsilon\Delta\Omega_{ij} \times \begin{bmatrix} \tilde{x}(t) \\ w(t) \end{bmatrix} - \tilde{z}^T(t)\tilde{z}(t) + \gamma^2 w^T(t)w(t)\end{aligned} \quad (47)$$

where

$$\varepsilon\Delta\Omega_{ij} = \begin{bmatrix} \tilde{A}_{ij}^T M_\varepsilon + M_\varepsilon^T \tilde{A}_{ij} & (*)^T \\ \tilde{B}_w^T M_\varepsilon & 0 \end{bmatrix} \quad (48)$$

When the parasitic parameter (ε) is less than zero, the term variable (48) will disappear by ε-multiplying. Next, recall Theorem 3.1 and substitute $K_j = Y_j P^{-1}$, so the equation (33) is rewritten as follows:

$$\Omega_{ij} = \begin{bmatrix} \begin{pmatrix} \Psi_{11_{ij}} & (*)^T & (*)^T \\ \Psi_{21_{ij}} & \Psi_{22_{ij}} & (*)^T \\ \Psi_{31_{ij}} & \Psi_{32_{ij}} & \Psi_{33_{ij}} \end{pmatrix} & (*)^T & (*)^T \\ \tilde{B}_w^T & -\gamma^2 I & (*)^T \\ \tilde{C}_{z_i} P & 0 & -I \end{bmatrix} \quad (49)$$

where

$$\begin{aligned}\Psi_{11_{ij}} &= A_{11_i}P_1 + P_1 A_{11_i}^T + A_{12_i}P_2 + P_2 A_{12_i}^T + B_{1_i}\left(K_{1_j}P_1 + K_{2_j}P_2 + K_{I_j}P_3\right) \\ &\quad + \left(P_1 K_{1_j} + P_2 K_{2_j} + P_3 K_{I_j}\right)B_{1_i}^T \\ \Psi_{21_{ij}} &= A_{21_i}P_1 + P_1 A_{12_i}^T + A_{22_i}P_2 + B_{2_i}\left(K_{1_j}P_1 + K_{2_j}P_2 + K_{I_j}P_3\right) + P_1 K_{2_j} B_{1_i}^T \\ \Psi_{22_{ij}} &= A_{22_i}P_1 + P_1 A_{22_i}^T + B_{2_i}K_{2_j}P_1 + P_1 K_{2_j} B_{2_i}^T\end{aligned}$$

$$\begin{aligned}
\Psi_{31_{ij}} &= C_{y_{1_i}} P_1 + C_{y_{2_i}} P_2 + P_2 A_{12_i}^T + P_3 A_{11_i}^T + \left(P_3 K_{1_j} + P_2 K_{2_j} + P_1 K_{I_j}\right) B_{1_i}^T \\
\Psi_{32_{ij}} &= C_{y_{2_i}} P_1 + P_2 A_{22_i}^T + P_3 A_{21_i}^T + \left(P_3 K_{1_j} + P_2 K_{2_j} + P_1 K_{I_j}\right) B_{2_i}^T \\
\Psi_{33_{ij}} &= C_{y_{2_i}} P_2 + P_2 C_{y_{2_i}}^T + C_{y_{1_i}} P_3 + P_3 C_{y_{1_i}}^T \\
\tilde{B}_w^T &= \begin{bmatrix} B_{w_1}^T & B_{w_2}^T & 0 \end{bmatrix} \\
\tilde{C}_{z_i} P &= \begin{bmatrix} \left(C_{z_{1_i}} P_1 + C_{z_{2_i}} P_2\right) \\ C_{z_{2_i}} P_1 \\ \left(C_{z_{1_i}} P_3 + C_{z_{2_i}} P_2\right) \end{bmatrix}^T
\end{aligned}$$

Pre and post multiplying (49) by $\begin{bmatrix} Q & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$ yields

$$\begin{bmatrix} \tilde{A}_{ij}^T Q + Q^T \tilde{A}_{ij} & (*)^T & (*)^T \\ \tilde{B}_w^T Q & -\gamma^2 I & (*)^T \\ \tilde{C}_{z_i} & 0 & -I \end{bmatrix} < 0 \tag{50}$$

Apply the Schur complement on (50), and rewrite the equation as follows:

$$\begin{bmatrix} \tilde{A}_{ij}^T Q + Q^T \tilde{A}_{ij} & (*)^T \\ \tilde{B}_w^T Q & -\gamma^2 I \end{bmatrix} - \begin{bmatrix} \tilde{C}_{z_i}^T \\ 0 \end{bmatrix} [-I] \begin{bmatrix} \tilde{C}_{z_i} & 0 \end{bmatrix} < 0 \tag{51}$$

(51) is less than zero; then, using the fact that $\mu_i(v(t)) \geq 0$ and $\sum_{i=1}^r \mu_i(v(t)) = 1$, (47) becomes

$$\dot{V}(\tilde{x}(t)) \leq -\tilde{z}^T(t)\tilde{z}(t) + \gamma^2 w^T(t) w(t) \tag{52}$$

Subtituting the value $\tilde{C}_{z_i} = \begin{bmatrix} C_{z_{1_i}} & C_{z_{2_i}} & 0 \end{bmatrix}$ in $\tilde{z}(t)$, (52) can be rewritten as follows:

$$\dot{V}(\tilde{x}(t)) \leq -z^T(t) z(t) + \gamma^2 w^T(t) w(t) \tag{53}$$

Integrating both sides of (53) yields

$$\int_0^{T_f} \dot{V}(\tilde{x}(t)) dt \leq \int_0^{T_f} \left(-z^T(t) z(t) + \gamma^2 w^T(t) w(t)\right) dt \tag{54}$$

$$V(\tilde{x}(T_f)) - V(\tilde{x}(0)) \leq \int_0^{T_f} \left(-z^T(t) z(t) + \gamma^2 w^T(t) w(t)\right) dt \tag{55}$$

Using the fact that $x(0) = 0$ and $V(x(T_f)) \geq 0$ for all, $T_f \neq 0$, then (55) becomes

$$\int_0^{T_f} z^T(t) z(t) dt \leq \gamma^2 \int_0^{T_f} w^T(t) w(t) dt \tag{56}$$

Therefore, the inequality (9) holds for $\varepsilon \in (0, \hat{\varepsilon}]$.

4. **Numerical Example.** This example is the tunnel diode circuit shown in Figure 1, which is an appropriate application for illustrating an H_∞ fuzzy integral control result [28]. The tunnel diode is characterized as follows:

$$i_{TD}(t) = 0.2 v_D(t) - 0.01 v_D^3(t) \tag{57}$$

Defining the state equation variables, $x_1(t) = v_C(t)$ and $x_2(t) = i_L(t)$ are the state vectors, and $\varepsilon = L$ is the parasitic parameter. Therefore, the state equation describing the tunnel diode circuit in Figure 1 can be shown as follows:

$$C\dot{x}_1(t) = -0.2 x_1(t) + 0.01 x_1^3(t) + x_2(t) \tag{58}$$

$$\varepsilon \dot{x}_2(t) = -x_1(t) - R x_2(t) + u(t) + 0.1 w(t) \tag{59}$$

$$y(t) = x_2(t) \tag{60}$$
$$z_1(t) = x_1(t) \tag{61}$$
$$z_2(t) = x_2(t) \tag{62}$$

where $u(t)$ is the controlled input, $w(t)$ is the disturbance, $y(t)$ is the measured output, and $z_1(t)$ and $z_2(t)$ are the controlled outputs. The variables i_L, i_C, i_{TD}, v_C, v_{TD}, R, L, C and TD are the inductor current, the capacitor current, the tunnel diode current, the capicitor voltage, the tunnel diode voltage, the resistance, the inductance, the capasitance and the tunnel diode respectively.

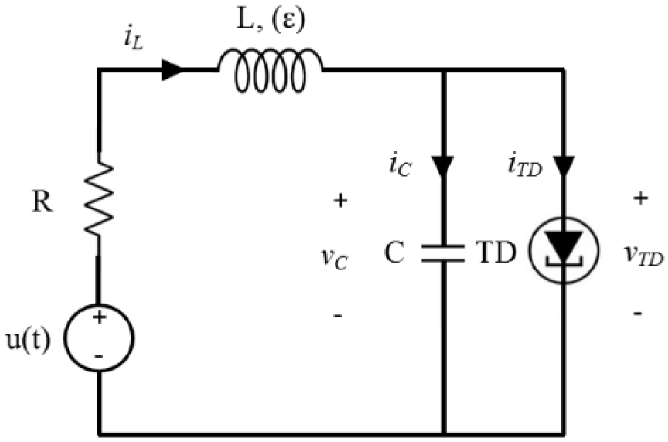

FIGURE 1. Tunnel diode circuit

Given the parameters in the tunnel diode circuit in Figure 1 by $R = 1\Omega$ and $C = 100$mF, Equations (58)-(62) can be rewritten as follows:

$$\dot{x}_1(t) = -2x_1(t) + 0.1x_1^3(t) + 10x_2(t) \tag{63}$$
$$\varepsilon\dot{x}_2(t) = -x_1(t) - x_2(t) + u(t) + 0.1w(t) \tag{64}$$
$$y(t) = x_2(t) \tag{65}$$
$$z_1(t) = x_1(t) \tag{66}$$
$$z_2(t) = x_2(t) \tag{67}$$

The tunnel diode circuit is the nonlinear system. Therefore, the TS fuzzy model can be used to describe Equations (63)-(67) by assuming that $|x_1(t)| \leq 3$. The TS fuzzy model of the tunnel diode circuit can be described as follows:

Plant rule 1: If $x_1(t)$ is $M_1(x_1(t))$ then

$$\begin{aligned} E_\varepsilon\dot{x}(t) &= A_1x(t) + B_1u(t) + B_ww(t) \\ z(t) &= C_zx(t) \\ y(t) &= C_yx(t) \end{aligned}$$

Plant rule 2: If $x_1(t)$ is $M_2(x_1(t))$ then

$$\begin{aligned} E_\varepsilon\dot{x}(t) &= A_2x(t) + B_2u(t) + B_ww(t) \\ z(t) &= C_zx(t) \\ y(t) &= C_yx(t) \end{aligned}$$

where

$$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix},$$

$$E_\varepsilon = \begin{bmatrix} I & 0 \\ 0 & \varepsilon \end{bmatrix}, \quad z(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix},$$
$$A_1 = \begin{bmatrix} -2 & 10 \\ -1 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2.9 & 10 \\ -1 & -1 \end{bmatrix},$$
$$B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_w = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix},$$
$$C_y = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad C_z = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

For the tunnel diode TS fuzzy model, the nonlinear system can be defined by the TS fuzzy model, and the membership functions can be chosen as in Figure 2. Let the X-axis be the state $x_1(t)$, the solid line be the first fuzzy set $M_1(x_1(t))$, and the dashed line be the second fuzzy set $M_2(x_1(t))$.

$$M_1(x_1(t)) = \frac{3 - x_1(t)}{3} \text{ and } M_2(x_1(t)) = \frac{x_1(t)}{3}$$

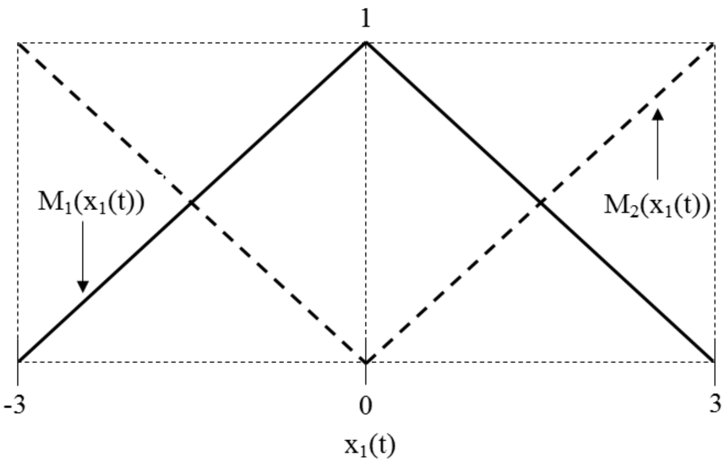

FIGURE 2. Membership functions of example

Applying Theorem 3.1 on the nonlinear tunnel diode example and the Matlab LMI solver, the results of LMI optimization with $\varepsilon = 0.01$ and $\gamma = 1$ are shown as follows:

$$P = \begin{bmatrix} 3.4898 & 0 & 0.2876 \\ -0.0757 & 3.4898 & -0.0757 \\ 0.2876 & 0 & 3.4898 \end{bmatrix},$$
$$Y_1 = \begin{bmatrix} 3.0490 & -8.3624 & 0.1730 \end{bmatrix},$$
$$Y_2 = \begin{bmatrix} 3.0467 & -8.3607 & 0.1738 \end{bmatrix},$$
$$K_1 = \begin{bmatrix} 0.8275 & -2.3963 & -0.0706 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} 0.8268 & -2.3958 & -0.0703 \end{bmatrix}$$

The resulting fuzzy controller is

$$u(t) = \sum_{j=1}^{2} \mu_i(v) \left(K_{1_j} x_1(t) + K_{2_j} x_2(t) + K_{I_j} q(t) \right) \tag{68}$$

where

$$\mu_1(v) = M_1(x_1(t))$$

$$\mu_2(v) = M_2(x_1(t))$$

Remark 4.1. *To achieve an H_∞ condition in Definition 2.1, the proposed controller (68) must control the system, which has the square root of ratio of the controlled output energy to the disturbance energy less than or equal to the prescribed value, γ. γ was defined as 1 in LMI optimization, and the disturbance $w(t)$ used in this example is presented in Figure*

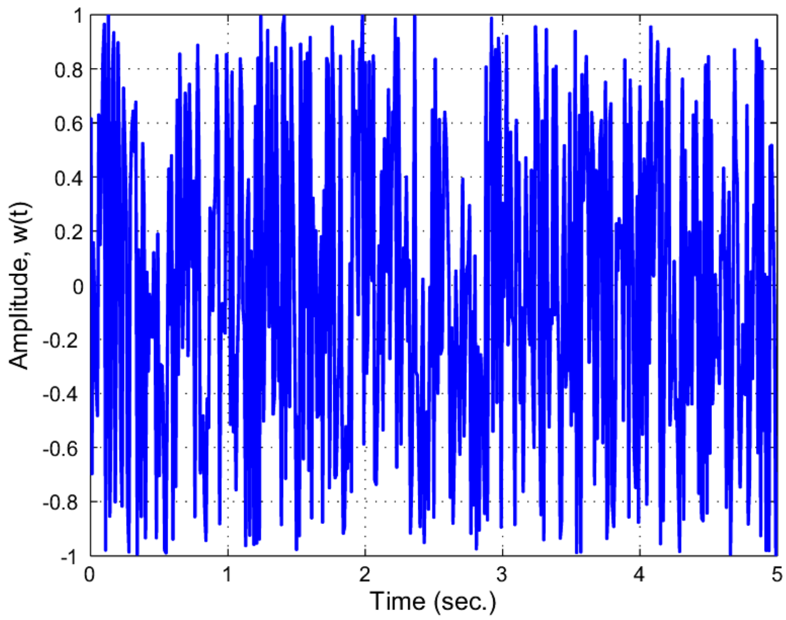

FIGURE 3. The disturbance $w(t)$

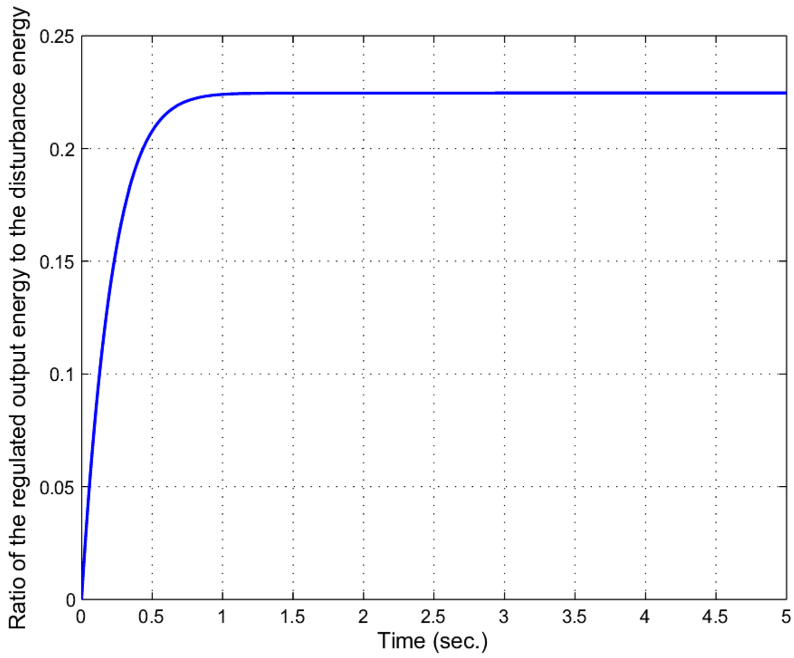

FIGURE 4. The ratio of the regulated output energy to the disturbance energy ($\varepsilon = 0.01$)

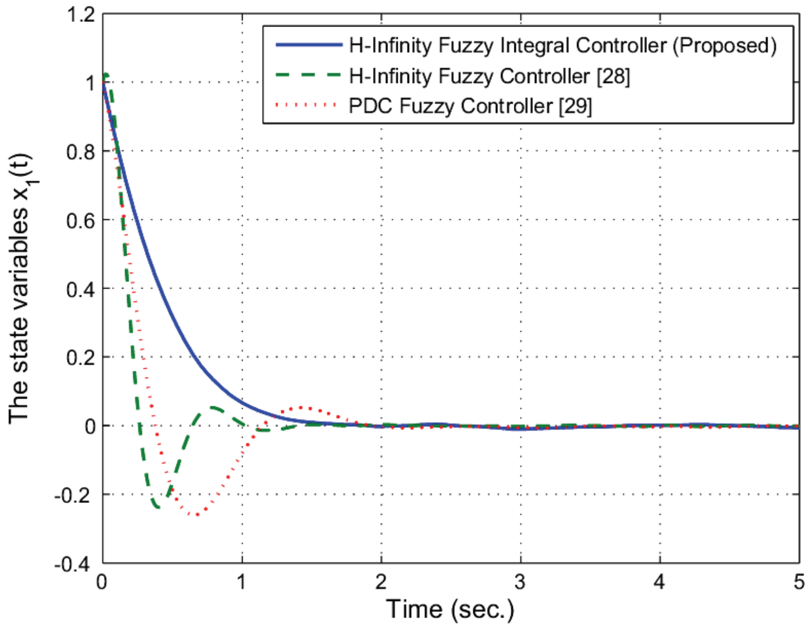

FIGURE 5. The state variable $x_1(t)$ ($\varepsilon = 0.01$)

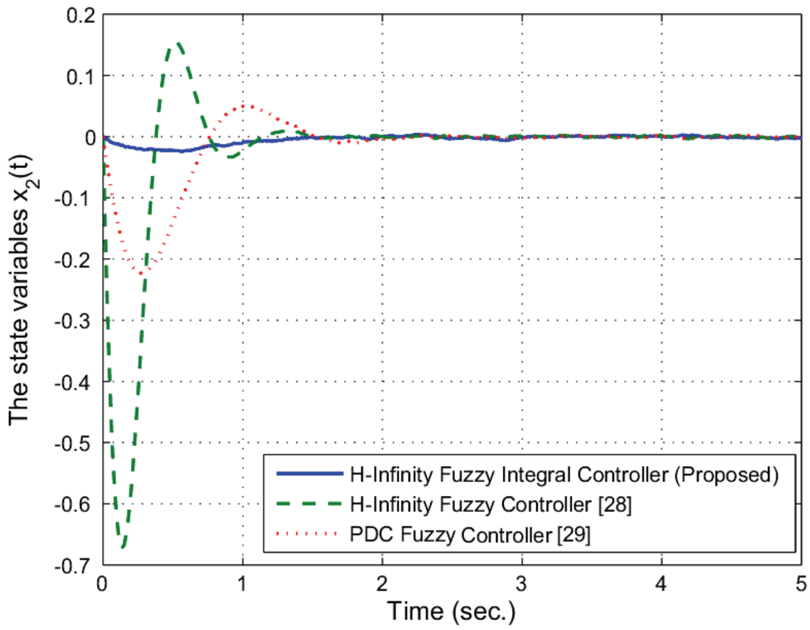

FIGURE 6. The state variable $x_2(t)$ ($\varepsilon = 0.01$)

3. The result of the ratio of the regulated output energy to the disturbance energy with $\varepsilon = 0.01$ *can illustrate the value of the ratio tends to the equilibrium point, which is 0.230 at 5 seconds. Therefore, the square root of the ratio of the controlled output energy to the disturbance energy in Figure 4 is 0.480, which guarantees the* H_∞ *condition. Moreover, the proposed controller can achieve rapid equilibrium and overcome the disturbance, as shown in Figure 5 and Figure 6. In these figures, the responses of the nonlinear tunnel diode system with the proposed state-feedback design controller are compared with [28] and [29]. The state variable* $x_1(t)$ *is the capacitor voltage,* $v_C(t)$*, and the state variable* $x_2(t)$ *is the inductor current,* $i_L(t)$ *of such a system. The reults show that the performance of*

Table 1. The ability of achieving the positive-definition condition and the H_∞ performance index with different values of ε

ε	γ	Positive-definition condition of P (Theorem 3.1)	P_ε (Lemma 3.1)
0.0001	0.482	Passed	Passed
0.001	0.491	Passed	Passed
0.01	0.480	Passed	Passed
0.10	0.484	Passed	Failed
0.50	0.403	Passed	Failed
1.00	0.370	Passed	Failed

the proposed controller can effectively reduce the overshoot and the settling time compared with [28] and [29] controllers.

In Table 1, the proposed controller (68) is shown to control the system with all ε *in Table 1 and guarantee the* L_2*-gain of the mapping from the regulated output energy to the disturbance energy to be less than or equal to the prescribed value,* γ*. Moreover, matrix* P *in Theorem 3.1 can reach the positive-definition condition with all* ε*, while matrix* P_ε *in Lemma 3.1 can reach this condition with some* ε*. Therefore, Theorem 3.1 with* P *is suitable to control the system that has* ε*.*

5. **Conclusions.** This paper demonstrated the H_∞ fuzzy integral control design for a nonlinear descriptor system described by the TS fuzzy model. Based on an LMI approach, the controller can achieve the H_∞ condition that guarantees L_2-gain and the controlled output energy to the disturbance energy to be less than or equal to the prescribed values. In addition, the H_∞ fuzzy integral controller is successful to improve the performance of rapid equilibrium, low steady-state errors and low overshoot. The example has demonstrated the effectiveness of the controller, which overcomes the disturbance, parasitic parameter (ε), and steady-state errors. However, some characteristics of nonlinear uncertain descriptor system do not meet the desired objectives, i.e., the rise time, the settling time, and transient oscillations due to poor transient responses. Thus, the robust H_∞ fuzzy integral controller with D-stablility constraints for the nonlinear uncertain descriptor system can be considered in the future research work.

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