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$l_2 - l_\infty$ FILTERING FOR NETWORKED SWITCHED SYSTEMS WITH MULTIPLE PACKET DROPOUTS VIA RANDOM SWITCHED LYAPUNOV FUNCTION

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Abstract. *This paper examines an $l_2 - l_\infty$ filtering problem for a class of discrete-time networked switched systems with multiple packet dropouts, while the data packets include both the measurement output signal and the switching signal. The multiple packet dropouts phenomenon is described by Bernoulli binary sequences with known probabilities. Then, by constructing a novel switched Lyapunov function with the random missing of switching signal taken into account, a switched filter is designed such that the filtering error system is exponentially stable and satisfies the $l_2 - l_\infty$ disturbance attenuation level. The parameters of the filter are obtained by solving an LMI. Finally, a practical example is presented to verify the effectiveness of the proposed approach.*
Keywords: Networked switched systems, $l_2 - l_\infty$ filtering, Multiple packet dropouts, Linear matrix inequalities (LMIs), Random switched Lyapunov function

1. **Introduction.** Switched system is a special form of hybrid systems, where continuous operation and discrete behavior exist simultaneously in the systems [1]. Generally, switched systems are composed of a family of subsystems and a switching rate governing the switching among these subsystems. Accordingly, plenty of theoretical results that concentrated on switched systems have been published in literature, such as stability analysis [2,3], dynamical output feedback control [4,5], and especially the state estimation [6,7]. In traditional switched systems, a common Lyapunov function for all subsystems is constructed for stability analysis and filter design [8,9]. With such design methods, the filter has only one set that leads to high conservatism. With the wide spread development of networked technologies, networked switched system is formed by connecting switched system with network. Due to the limited bandwidth of the channels and other reasons, the measurement out signal will be lost, leading to the result that the measurement information obtained by the filter contains only noise signals. The missing of important output or control information will degrade the filtering performance.

During the past several years, the filtering problem for networked switched systems with missing measurements has been extensively studied [10-13]. To design a less conservative filter, a switched Lyapunov function was proposed, which relays on the switching rate, and the filter designed by this method corresponds to each subsystem. Recently, various studies on networked switched systems have been carried out with subject to the missing of the measurement signal only; while the switching signal is still available at the filter side. For instance, dwell-time-dependent asynchronous H_∞ filtering problem for discrete-time switched systems with missing measurements is addressed in [14]. In [15], H_∞ filter

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is designed for discrete-time switched fuzzy systems in presence of time-varying delay and packet dropouts. Distributed filtering for switched nonlinear positive systems with missing measurements over sensor networks is investigated in [16], and this filtering problem with missing measurements is extended to linear switched system in [17].

However, in the network environment, the measurement signal and the switching signal in the same packet output from the system are transmitted to the filter through the network simultaneously. Due to network defects, the switching signal received by the filter will be randomly lost like the measurement signal. When the switching signal is lost, it is hard for networked switched system to use the switching signal to design a controller (or filter) that depends on the switching rate to reduce the design conservativeness [12,14]. The phenomenon of packet dropouts is quite common in some practical systems, such as networked control systems, computer controlled systems, chemical process and aircraft control systems [16]. Therefore, studying the filtering problem for networked switched system with multiple packet dropouts of both the switching signal and measurement signal are of theoretical and practice significance. The loss of the switching signal makes the problem more practical and it brings new challenge to the analysis of networked switched systems. The single packet dropout of the switching signal has been studied in [18,19]. To the best of the authors' knowledge, the filtering for networked switched systems with multiple packet dropouts of real-time switching and measurement signal simultaneously has not been fully investigated. The motivation of our work is based on its practical significance.

In this paper, we aim to address the $l_2 - l_\infty$ filtering problem for a class of discrete-time switched systems with randomly occurring multiple packet dropouts of both the switching signal and measurement signal. In order to design a desired filter when the switching signal is lost, a novel switched Lyapunov function which consists of the random change of the switching signal is proposed in this paper. Bernoulli random sequences with known probabilities are used to describe the multiple packet dropouts phenomenon. Based on the proposed switched Lyapunov functional method, sufficient conditions for the desired filter are established, which ensure the filtering error system is exponentially stable in the sense of mean square with prescribed $l_2 - l_\infty$ performance.

The rest of the paper is outlined as follows. Section 2 formulates the problem and presents some preliminary results. A novel switched Lyapunov function is introduced in Section 3. The analysis of filtering performance and $l_2 - l_\infty$ filter design is given in Section 4. Section 5 presents a numerical simulation to demonstrate the application of the proposed method. Finally, the paper is concluded in Section 6.

Notations: The notation used in this paper is fairly standard. R^i, $i = \{n, r, q, p\}$ denotes the i-dimensional Euclidean space. $l_2[0, \infty)$ is the space of square integrable vectors. The notation $P > 0$ ($P \geq 0$) means that matrix P is symmetric and positive (semi-positive) definite. In symmetric block matrices or complex matrix expressions, we use the symbol "*" as an ellipsis for the terms that are introduced by symmetry.

2. Problem Statement and Preliminaries.

2.1. **Switched model of linear plant and stochastic variables.** As a subclass of hybrid systems, many practical systems can be effectively described using switched system models, such as networked control systems (NCSs), power electronic systems, aircraft control systems and single-link robot arm system [15,17]. Consider the following discrete-time switched systems:

$$\begin{cases} x(k+1) = \sum_{i=1}^{N} \alpha_i(k)(A_i x(k) + B_i w(k)), \\ y(k) = \sum_{i=1}^{N} \alpha_i(k)(C_i x(k) + D_i w(k)), \\ z(k) = \sum_{i=1}^{N} \alpha_i(k)(L_i x(k)), \end{cases} \tag{1}$$

where $x(k) \in R^n$ is the state, $y(k) \in R^p$ is the measured output, $z(k) \in R^q$ is the signal to be estimated, $w(k) \in R^r$ is the distributed input which belongs to $l_2\,[0, \infty)$. N is the number of the subsystems. A_i, B_i, C_i, D_i, L_i $(i \in I)$, $I = \{1, \ldots, N\}$ are system matrices with compatible dimensions. $\alpha_i(k)$ is a switching signal satisfying

$$\alpha_i : Z^+ \to \{0, 1\}\,, \ \sum_{i=1}^{N} \alpha_i(k) = 1, \ k \in Z^+ = \{0, 1, \ldots\},$$

where $\alpha_i(k) = 1$ means the ith subsystem will be activated at the kth time.

Considering the multiple packet dropouts, the measurement signal and the switching signal are described as follows:

$$\begin{bmatrix} \hat{y}(k) \\ \hat{\alpha}(k) \end{bmatrix} = \beta(k) \begin{bmatrix} y(k) \\ \alpha(k) \end{bmatrix} + (1 - \beta(k)) \begin{bmatrix} \hat{y}(k-1) \\ \hat{\alpha}(k-1) \end{bmatrix}, \tag{2}$$

where $\hat{y}(k)$ and $\hat{\alpha}(k)$ are the measurement signal and switching signal received by the filter respectively, $\hat{y}(k-1)$ and $\hat{\alpha}(k-1)$ are the last arrival data; $\beta(k) \in \{0, 1\}$ is a stochastic variable to model the transmission process of the data. If the measurement signal and switching signal do not have multiple packet dropouts, $\beta(k) = 1$, otherwise, $\beta(k) = 0$. Assuming that the packet loss probability satisfies Bernoulli distribution, the stochastic variable $\beta(k)$ satisfies the following distribution law:

$$\begin{cases} P\{\beta(k) = 1\} = E\{\beta(k)\} = \bar{\beta}, \\ P\{\beta(k) = 0\} = 1 - E\{\beta(k)\} = 1 - \bar{\beta}, \end{cases} \tag{3}$$

where $\bar{\beta}$ is the successful transmission rate, and usually, it is available.

2.2. **Filtering error system.** In this paper, we are interested in considering the following switched filter:

$$\begin{cases} \hat{x}(k+1) = A_{f\hat{\alpha}} \hat{x}(k) + B_{f\hat{\alpha}} \hat{y}(k), \\ \hat{z}(k) = C_{f\hat{\alpha}} \hat{x}(k), \end{cases} \tag{4}$$

where $\hat{x}(k) \in R^k$ is the state signal and $\hat{z}(k) \in R^q$ is the estimation of the filter, $A_{f\hat{\alpha}}$, $B_{f\hat{\alpha}}$, $C_{f\hat{\alpha}}$ are filter parameters to be determined.

Because the switching signal received by the filter is lost, filter parameters $A_{f\hat{\alpha}}$, $B_{f\hat{\alpha}}$, $C_{f\hat{\alpha}}$ will vary with the random loss of the switching signal, they can be written as:

$$\begin{cases} A_{f\hat{\alpha}} = A_f(\beta(k)\alpha(k) + (1-\beta(k))\hat{\alpha}(k-1)) = \beta(k)A_f(\alpha(k)) + (1-\beta(k))A_f(\hat{\alpha}(k-1)) \\ B_{f\hat{\alpha}} = B_f(\beta(k)\alpha(k) + (1-\beta(k))\hat{\alpha}(k-1)) = \beta(k)B_f(\alpha(k)) + (1-\beta(k))B_f(\hat{\alpha}(k-1)) \\ C_{f\hat{\alpha}} = C_f(\beta(k)\alpha(k) + (1-\beta(k))\hat{\alpha}(k-1)) = \beta(k)C_f(\alpha(k)) + (1-\beta(k))C_f(\hat{\alpha}(k-1)) \end{cases} \tag{5}$$

As mentioned above, the switching signal $\alpha(k)$ satisfies $\sum_{i=1}^{N}\alpha_i(k) = 1$, $k \in Z^+ = \{0, 1, \ldots\}$. From (2), (5) and $\beta(k)(1-\beta(k)) = 0$, $\beta(k)^2 = \beta(k)$, $(1-\beta(k))^2 = 1-\beta(k)$, (4) can be rewritten as:

$$\begin{cases} \hat{x}(k+1) = \sum_{i=1}^{N}\alpha_i(k)\sum_{j=1}^{N}\hat{\alpha}_j(k-1)\{\beta(k)A_{fi}\hat{x}(k) + \beta(k)B_{fi}C_ix(k) \\ \qquad + (1-\beta(k))A_{fj}\hat{x}(k) + (1-\beta(k))B_{fj}\hat{y}(k-1) + \beta(k)B_{fi}D_iw(k)\}, \\ \hat{z}(k) = \sum_{i=1}^{N}\alpha_i(k)\sum_{j=1}^{N}\hat{\alpha}_j(k-1)\{[\beta(k)C_{fi} + (1-\beta(k))C_{fj}]\}\hat{x}(k). \end{cases} \tag{6}$$

Defining $x_{cl}(k) = \begin{bmatrix} x(k)^T & \hat{x}(k)^T & \hat{y}(k-1)^T \end{bmatrix}^T$, $e(k) = z(k) - \hat{z}(k)$, using system (1) and (6), the corresponding filtering error system can be obtained:

$$\begin{cases} x_{cl}(k+1) = \sum_{i=1}^{N}\alpha_i(k)\sum_{j=1}^{N}\hat{\alpha}_j(k-1)(A_{cl}x_{cl}(k) + B_{cl}w(k)), \\ e(k) = \sum_{i=1}^{N}\alpha_i(k)\sum_{j=1}^{N}\hat{\alpha}_j(k-1)C_{cl}x_{cl}(k), \end{cases} \tag{7}$$

where $A_{cl} = \beta(k)A_{cl1} + (1-\beta(k))A_{cl2}$, $B_{cl} = \beta(k)B_{cl1} + (1-\beta(k))B_{cl2}$, $C_{cl} = \beta(k)C_{cl1} + (1-\beta(k))C_{cl2}$. And $A_{cl1} = \begin{bmatrix} A_i & 0 & 0 \\ B_{fi}C_i & A_{fi} & 0 \\ C_i & 0 & 0 \end{bmatrix}$, $A_{cl2} = \begin{bmatrix} A_i & 0 & 0 \\ 0 & A_{fj} & B_{fj} \\ 0 & 0 & I \end{bmatrix}$, $B_{cl1} = \begin{bmatrix} B_i \\ B_{fi}D_i \\ D_i \end{bmatrix}$, $B_{cl2} = \begin{bmatrix} B_i \\ 0 \\ 0 \end{bmatrix}$, $C_{cl1} = \begin{bmatrix} L_i & -C_{fi} & 0 \end{bmatrix}$, $C_{cl2} = \begin{bmatrix} L_i & -C_{fj} & 0 \end{bmatrix}$.

In order to analyze the performance of the filtering error system (7), a novel switched Lyapunov function will be proposed in the next section.

3. Random Switched Lyapunov Function.

Switched Lyapunov function, with a set of matrices P_i, $i \in I$ relaying on the switching rate, is applicable for switched systems in presence of communication and missing measurements [15,17,19]. However, when the switching signal has multiple packet dropouts, traditional switched Lyapunov function cannot be adopted directly. In this paper, a novel switched Lyapunov function concerning random packet dropouts of the switching signal is constructed as

$$V(k) = x_{cl}^T(k)P(\hat{\alpha}(k-1))x_{cl}(k), \tag{8}$$

and

$$V(k+1) = x_{cl}^T(k+1)P(\hat{\alpha}(k))x_{cl}(k+1) \tag{9}$$

where

$$P(\hat{\alpha}(k)) = \beta(k)P(\alpha(k)) + (1-\beta(k))P(\hat{\alpha}(k-1)). \tag{10}$$

Remark 3.1. *Different from the traditional switched Lyapunov function with a set of matrices P_i, $i \in I$ relaying on the switching rate, matrices $P(\hat{\alpha}(k))$ and $P(\hat{\alpha}(k-1))$ in (10) are both time-varying positive definite matrices, and the switching rate $\hat{\alpha}(k)$ in matrix P is also random lost and it satisfies $\hat{\alpha}(k) = \beta(k)\alpha(k) + (1-\beta(k))\hat{\alpha}(k-1)$. $\beta(k) \in \{0, 1\}$ is a stochastic variable with a known mathematical expectation. When the switching signal is lost, the same P_i, $i \in I$ may not exist or are difficult to find. Therefore, a novel switched Lyapunov function shown in (8) and (9) is proposed in this paper.*

First, before designing a switched full-order filter, it is necessary to introduce the following definition which is essential for the follow-up filtering procedures.

Definition 3.1. *The filtering error system (7) is exponentially mean square stable with predefined $l_2 - l_\infty$ performance γ, if the two requirements are satisfied: Q1) (Exponentially stability) The filtering error system (7) is exponentially stable in the sense of mean square when $w(k) = 0$. Q2) ($l_2 - l_\infty$ performance) Under zero initial conditions, for any non-zero disturbance $w(k) \in l_2[0,\infty)$, the estimation error $e(k)$ satisfies*

$$E\left\{\left\|e(k)_\infty^2\right\|\right\} < \gamma^2 E\left\{\left\|w(k)_2^2\right\|\right\},\ \forall w(k) \neq 0, \tag{11}$$

where

$$\left\|e(k)_\infty^2\right\| = \sup_k \left\{e^T(k)e(k)\right\},\ \left\|w(k)_2^2\right\| = \sum_{k=0}^{\infty} w^T(k)w(k). \tag{12}$$

Next, we will study the exponentially stability and $l_2 - l_\infty$ performance of the filtering error system (7) and a switched filter will also be designed.

4. Main Results.

4.1. Stability and $l_2 - l_\infty$ performance analysis.

We have the following important results.

Theorem 4.1. *Given a scalar $\gamma > 0$, the filtering error system (7) is exponentially stable with a given $l_2 - l_\infty$ performance γ, if there exist matrices $\left\{P_i = P_i^T > 0\right\}_{i=1}^N$ for any $\{i,j\} \in I = \{1,\dots,N\}$ such that the following matrix inequality holds:*

$$\begin{bmatrix} -P_j & A_{cl1}^T & A_{cl2}^T \\ * & -\left(\bar{\beta}P_i\right)^{-1} & 0 \\ * & * & -\left(\left(1-\bar{\beta}\right)P_j\right)^{-1} \end{bmatrix} < 0. \tag{13}$$

Proof: Firstly, the exponential stability of the filtering system (7) is proved. When $w(k) = 0$, system (7) becomes

$$x_{cl}(k+1) = \sum_{i=1}^{N} \alpha_i(k) \sum_{j=1}^{N} \hat{\alpha}_j(k-1) A_{cl} x_{cl}(k). \tag{14}$$

Let $\Delta V(k) = V(k+1) - V(k)$, it follows that for the particular case $\alpha_i(k) = 1$, $\alpha_{r\neq i}(k) = 0$, $\hat{\alpha}_j(k-1) = 1$, $\hat{\alpha}_{r\neq j}(k-1) = 0$, then, we have

$$\begin{aligned} E\{\Delta V(k)\} &= E\{V(k+1)|V(k)\} - E\{V(k)\} \\ &= E\left\{x_{cl}^T(k+1)P(\hat{\alpha}(k))x_{cl}(k+1)\right\} - x_{cl}^T(k)P(\hat{\alpha}(k-1))x_{cl}(k) \\ &= \sum_{i=1}^{N} \alpha_i(k) \sum_{j=1}^{N} \hat{\alpha}_j(k-1)\left\{x_{cl}^T(k)\Pi x_{cl}(k)\right\}, \end{aligned} \tag{15}$$

where

$$\Pi = \bar{\beta} A_{cl1}^T P_i A_{cl1} + \left(1-\bar{\beta}\right) A_{cl2}^T P_j A_{cl2} - P_j. \tag{16}$$

Applying the Schur complement to matrix inequality (13), $\Pi < 0$ holds. Clearly, $\Delta V(k) < 0$. Hence, the exponential stability of the filtering error system (7) under the case of $w(k) = 0$ is guaranteed. This completes the proof.

Theorem 4.2. *Given a scalar $\gamma > 0$, the filtering error system (7) is exponentially stable with a given $l_2 - l_\infty$ performance γ, if there exist matrices $\left\{P_i = P_i^T > 0\right\}_{i=1}^N$ for any $\{i, j\} \in I = \{1, \ldots, N\}$ such that the following matrix inequality holds:*

$$\begin{bmatrix} -P_j & * & * & * \\ 0 & -I & * & * \\ A_{cl1} & B_{cl1} & -\left(\bar{\beta} P_i\right)^{-1} & * \\ A_{cl2} & B_{cl2} & 0 & -\left(\left(1-\bar{\beta}\right) P_j\right)^{-1} \end{bmatrix} < 0 \tag{17}$$

$$\begin{bmatrix} -P_j & * & * \\ C_{cl1} & -\gamma^2 \bar{\beta}^{-1} I & * \\ C_{cl2} & 0 & -\gamma^2 \left(1-\bar{\beta}\right)^{-1} I \end{bmatrix} < 0 \tag{18}$$

Proof: Define

$$J = E\{V(k)\} - E\left\{\sum_{l=0}^{k-1} w^T(l) w(l)\right\}. \tag{19}$$

For any nonzero $w(k) \in l_2[0, \infty)$ and zero initial condition, one has

$$J = E\{V(k)\} - E\{V(0)\} - E\left\{\sum_{l=0}^{k-1} w^T(l) w(l)\right\} = \sum_{l=0}^{k-1} \left\{\Delta V(l) - w^T(l) w(l)\right\}. \tag{20}$$

It is noted that

$$\begin{aligned} &\Delta V(k) \\ &= V(k+1) - V(k) \\ &= \sum_{i=1}^N \alpha_i(k) \sum_{j=1}^N \hat{\alpha}_j(k-1) \Big\{ \Big[\beta(k)(A_{cl1} x_{cl}(k) + B_{cl1} w(k))^T P_i (A_{cl1} x_{cl}(k) + B_{cl1} w(k))\Big] \\ &\quad + \Big[(1-\beta(k))(A_{cl2} x_{cl}(k) + B_{cl2} w(k))^T P_j (A_{cl2} x_{cl}(k) + B_{cl2} w(k))\Big] - x_{cl}^T(k) P_j x_{cl}(k) \Big\}. \end{aligned} \tag{21}$$

Using (20) and (21) yields

$$\begin{aligned} J &= \sum_{l=1}^{k-1} \left(\Delta V(l) - w^T(l) w(l)\right) \\ &= \sum_{i=1}^N \alpha_i(k) \sum_{j=1}^N \hat{\alpha}_j(k-1) \sum_{l=1}^{k-1} \left\{ \begin{bmatrix} x_{cl}(l) \\ w(l) \end{bmatrix}^T \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} \begin{bmatrix} x_{cl}(l) \\ w(l) \end{bmatrix} \right\}, \end{aligned} \tag{22}$$

where $Q_1 = \bar{\beta} A_{cl1}^T P_i A_{cl1} + \left(1-\bar{\beta}\right) A_{cl2}^T P_j A_{cl2} - P_j$, $Q_2 = \bar{\beta} A_{cl1}^T P_i B_{cl1} + \left(1-\bar{\beta}\right) A_{cl2}^T P_j B_{cl2}$, $Q_3 = \bar{\beta} B_{cl1}^T P_i B_{cl1} + \left(1-\bar{\beta}\right) B_{cl2}^T P_j B_{cl2} - I$.

It follows from (17) and Schur complement formula that $J < 0$, which is

$$E\{V(k)\} - E\left\{\sum_{l=0}^{k-1} w^T(l) w(l)\right\} < 0, \tag{23}$$

$$E\left\{\sum_{i=1}^N \alpha_i(k) \sum_{j=1}^N \hat{\alpha}_j(k-1) \left[x_{cl}^T(k) P_j x_{cl}(k)\right]\right\} < E\left\{\sum_{l=0}^{k-1} w^T(l) w(l)\right\}. \tag{24}$$

Similarly, it follows from (18) and Schur complement formula that

$$\bar{\beta} C_{cl1}^T C_{cl1} + \left(1-\bar{\beta}\right) C_{cl2}^T C_{cl2} < \gamma^2 P_j. \tag{25}$$

When $k > 0$, one has

$$\begin{aligned} E\left\{e^T(k)e(k)\right\} &= \sum_{i=1}^{N} \alpha_i(k) \sum_{j=1}^{N} \hat{\alpha}_j(k-1) \left\{x_{cl}^T(k) \left[\bar{\beta}C_{cl1}{}^T C_{cl1} \right.\right. \\ &\quad \left.\left. + \left(1-\bar{\beta}\right) C_{cl2}{}^T C_{cl2}\right] x_{cl}(k)\right\} \\ &< \sum_{i=1}^{N} \alpha_i(k) \sum_{i=1}^{N} \hat{\alpha}_j(k-1) \left[\gamma^2 x_{cl}^T(k) P_j x_{cl}(k)\right] \\ &< \gamma^2 E\left\{\sum_{i=1}^{N} \alpha_i(k) \sum_{i=1}^{N} \hat{\alpha}_j(k-1) \left[x_{cl}^T(k) P_j x_{cl}(k)\right]\right\} \\ &< \gamma^2 E\left\{\sum_{l=0}^{k-1} w^T(l)w(l)\right\} \le \gamma^2 E\left\{\sum_{l=0}^{\infty} w^T(l)w(l)\right\}. \end{aligned} \tag{26}$$

From (26) leads to

$$\sup_k \left\{E\left\{e^T(k)e(k)\right\}\right\} < \gamma^2 E\left\{\sum_{l=0}^{\infty} w^T(i)w(i)\right\}, \tag{27}$$

which also implies that

$$E\left\{\|e(k)\|_\infty^2\right\} < \gamma^2 E\left\{\|w(k)\|_2^2\right\}, \tag{28}$$

for any nonzero $w(k) \in l_2[0,\infty)$. This clearly shows the l_2-l_∞ performance of the filtering system (7) is obtained. This completes the proof.

4.2. **Filter design.** In this subsection, the sufficient conditions for the existence of a desired switched filter will be presented in the form of linear matrix inequalities.

Theorem 4.3. *Given a scalar $\gamma > 0$, the filtering error system (7) is exponentially stable with a given l_2-l_∞ performance γ, if there exist matrix G and matrices $\left\{P_i = P_i^T > 0\right\}_{i=1}^N$ for any $\{i,j\} \in I = \{1,\ldots,N\}$ such that the following matrix inequality holds:*

$$\begin{bmatrix} -P_j & * & * & * \\ 0 & -I & * & * \\ G^T A_{cl1} & G^T B_{cl1} & \bar{\beta}^{-1}\left(P_i - G - G^T\right) & * \\ G^T A_{cl2} & G^T B_{cl2} & 0 & \left(1-\bar{\beta}\right)^{-1}\left(P_j - G - G^T\right) \end{bmatrix} < 0 \tag{29}$$

$$\begin{bmatrix} -P_j & * & * \\ C_{cl1} & -\gamma^2\bar{\beta}^{-1} I & * \\ C_{cl2} & 0 & -\gamma^2\left(1-\bar{\beta}\right)^{-1} I \end{bmatrix} < 0 \tag{30}$$

For the proof of the theorem, it is similar to that given in [18] and hence is omitted.

Theorem 4.4. *Given a scalar $\gamma > 0$, the filtering error system (7) is exponentially stable with a given $l_2 - l_\infty$ performance γ, if there exist matrices V_1, V_2, V_3, V_4 and matrices $\left\{P_{11i} = P_{11i}^T > 0\right\}_{i=1}^N$, $\{P_{12i} > 0\}_{i=1}^N$, $\{P_{13i} > 0\}_{i=1}^N$, $\left\{P_{22i} = P_{22i}^T > 0\right\}_{i=1}^N$, $\{P_{23i} > 0\}_{i=1}^N$, $\left\{P_{33i} = P_{33i}^T > 0\right\}_{i=1}^N$, $\{A_{Fi}(t)\}_{i=1}^N$, $\{B_{Fi}(t)\}_{i=1}^N$, $\{C_{Fi}(t)\}_{i=1}^N$ for any $\{i,j\} \in I = \{1,\ldots,N\}$ such that the following matrix inequality holds:*

$$\begin{bmatrix} -\Psi_1 & * & * & * \\ 0 & -I & * & * \\ \Psi_3 & \Psi_5 & \bar{\beta}^{-1}\Psi_{2i} & * \\ \Psi_4 & \Psi_6 & 0 & \left(1-\bar{\beta}\right)^{-1}\Psi_{2j} \end{bmatrix} < 0 \tag{31}$$

$$\begin{bmatrix} -\Psi_1 & * & * \\ \Psi_7 & -\gamma^2\bar{\beta}^{-1}I & * \\ \Psi_8 & 0 & -\gamma^2\left(1-\bar{\beta}\right)^{-1}I \end{bmatrix} < 0 \tag{32}$$

where

$$\Psi_1 = \begin{bmatrix} P_{11j} & * & * \\ P_{12j}^T & P_{22j} & * \\ P_{13j}^T & P_{23j}^T & P_{33j} \end{bmatrix},$$

$$\Psi_{2i} = \begin{bmatrix} P_{11i} - V_1 - V_1^T & * & * \\ P_{12i}^T - V_2^T E^T - V_3^T & P_{22i} - V_2^T - V_2 & * \\ P_{13i}^T & P_{23i}^T & P_{33i} - V_4 - V_4^T \end{bmatrix},$$

$$\Psi_{2j} = \begin{bmatrix} P_{11j} - V_1 - V_1^T & * & * \\ P_{12j}^T - V_2^T E^T - V_3^T & P_{22j} - V_2^T - V_2 & * \\ P_{13j}^T & P_{23j}^T & P_{33j} - V_4 - V_4^T \end{bmatrix},$$

$$\Psi_3 = \begin{bmatrix} V_1^T A_i + EB_{Fi}C_i & EA_{Fi} & 0 \\ V_3^T A_i + B_{Fi}C_i & A_{Fi} & 0 \\ V_4^T C_i & 0 & 0 \end{bmatrix},$$

$$\Psi_4 = \begin{bmatrix} V_1^T A_i & EA_{Fj} & EB_{Fj} \\ V_3^T A_i & A_{Fj} & B_{Fj} \\ 0 & 0 & V_4^T \end{bmatrix},$$

$$\Psi_5 = \begin{bmatrix} V_1^T B_i + EB_{Fi}D_i \\ V_3^T B_i + B_{Fi}D_i \\ V_4^T D_i \end{bmatrix},$$

$$\Psi_6 = \begin{bmatrix} V_1^T B_i \\ V_3^T B_i \\ 0 \end{bmatrix},$$

$$\Psi_7 = \begin{bmatrix} L_i & -C_{Fi} & 0 \end{bmatrix},\ \Psi_8 = \begin{bmatrix} L_i & -C_{Fj} & 0 \end{bmatrix}.$$

Moreover, if the LMIs (31), (32) are feasible, then the parameters of the filter in (4) can be calculated by

$$A_{fi} = V_2^{-1}A_{Fi},\ B_{fi} = V_2^{-1}B_{Fi},\ C_{fi} = C_{Fi}. \tag{33}$$

Proof: Select matrix G as the following form [18,19]:

$$G = \begin{bmatrix} V_1 & V_3 M^{-1} G_{22} & 0 \\ ME^T & G_{22} & 0 \\ 0 & 0 & V_4 \end{bmatrix}, \tag{34}$$

where

$$E = \begin{bmatrix} I_{k\times k} \\ 0_{(n-k)\times k} \end{bmatrix},\ V_1 \in R^{n\times n},\ V_3 \in R^{n\times k},\ M \in R^{k\times k},\ G_{22} \in R^{k\times k}. \tag{35}$$

Define

$$V_2 = M^T G_{22}^{-1} M,\ J_1 = \begin{bmatrix} I & 0 & 0 \\ 0 & G_{22}^{-1}M & 0 \\ 0 & 0 & I \end{bmatrix},\ J_1^T P_i J_1 = \begin{bmatrix} P_{11i} & * & * \\ P_{12i}^T & P_{22i} & * \\ P_{13i}^T & P_{23i}^T & P_{33i} \end{bmatrix},$$

$$J_1^T P_j J_1 = \begin{bmatrix} P_{11j} & * & * \\ P_{12j}^T & P_{22j} & * \\ P_{13j}^T & P_{23j}^T & P_{33j} \end{bmatrix},\ A_{Fi} = M^T A_{fi} G_{22}^{-1},\ B_{Fi} = M^T B_{fi},\ C_{Fi} = C_{fi} G_{22}^{-1} M.$$

Pre- and post-multiplying (29) by $T_1 = diag\left\{J_1, I, J_1, J_1\right\}$ and $T_1{}^T = diag\left\{J_1{}^T, I, J_1{}^T, J_1{}^T\right\}$, we can derive (29) from (31). Similarly, pre- and post-multiplying (30) by $T_2 = diag\left\{J_1, I, I\right\}$ and $T_2{}^T = diag\left\{J_1{}^T, I, I\right\}$, we can also derive (30) from (32). From the filtering transfer function:

$$\begin{aligned} T_{Fi} &= C_{fi}(zI - A_{fi})^{-1}B_{fi} \\ &= C_{Fi}M^{-1}G_{22}\left(zI - M^{-T}A_{Fi}M^{-1}G_{22}\right)^{-1}M^{-T}B_{Fi} \\ &= C_{Fi}\left(zM^TG_{22}^{-1}M - A_{Fi}\right)^{-1}B_{Fi} \\ &= C_{Fi}\left(zI - V_2^{-1}A_{Fi}\right)^{-1}V_2^{-1}B_{Fi} \end{aligned} \tag{36}$$

then the filtering parameters can be obtained from (33). This completes the proof.

5. **A Simulation Example.** A real PWM (Pulse-Width-Modulation)-driven boost converter model presented in [13] is considered. Its circuit system is shown in Figure 1. The system model can be transformed into a discrete-time switched system with two subsystems:

$$A_1 = \begin{bmatrix} 0.94 & 0.10 & 0.06 \\ -0.30 & 0.95 & -0.30 \\ -0.25 & -0.06 & 0.63 \end{bmatrix}, \ A_2 = \begin{bmatrix} 0.93 & 0.08 & 0.07 \\ -0.14 & 0.66 & -0.20 \\ -0.16 & -0.40 & 0.66 \end{bmatrix}.$$

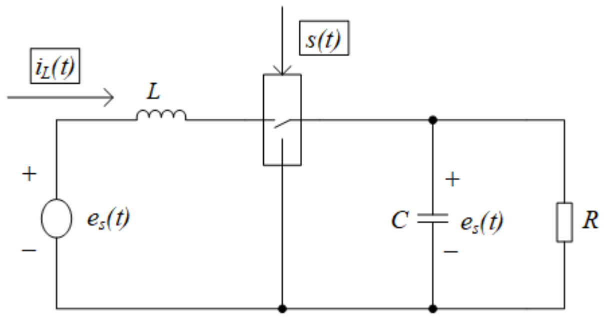

FIGURE 1. PWM-driven boost converter

Suppose that other system matrices are given by

$$\begin{aligned} &B_1 = \begin{bmatrix} -0.3 & 0.2 & 0.1 \end{bmatrix}^T, \ B_2 = \begin{bmatrix} -0.4 & -0.3 & 0.2 \end{bmatrix}^T, \\ &C_1 = \begin{bmatrix} 0.1 & -0.1 & 0.1 \end{bmatrix}, \ C_2 = \begin{bmatrix} 0.3 & -0.4 & 0.1 \end{bmatrix}, \\ &D_1 = 0.4, \ D_2 = -0.5, \ L_1 = \begin{bmatrix} 0.7 & 0 & 0.3 \end{bmatrix}, \\ &L_2 = \begin{bmatrix} 0.2 & 0 & 0.4 \end{bmatrix}, \ w(k) = e^{-0.4k}\sin(0.2\pi k). \end{aligned}$$

Also, we assume the successful transmission rate $\bar{\beta} = 0.8$, then the following filter parameters are obtained by solving LMIs in Theorem 4.4:

$$\begin{aligned} &A_{f1} = \begin{bmatrix} 0.8444 & 0.1398 & 0.1048 \\ -0.1197 & 0.9233 & -0.1612 \\ -0.0667 & -0.1369 & 0.5590 \end{bmatrix}, \ B_{f1} = \begin{bmatrix} 0.5963 & 0.1828 & -0.5158 \end{bmatrix}^T, \\ &C_{f1} = \begin{bmatrix} -0.6330 & -0.0121 & -0.2838 \end{bmatrix}, \\ &A_{f2} = \begin{bmatrix} 0.9527 & 0.1237 & 0.1039 \\ -0.5061 & 0.5180 & -0.7238 \\ -0.2832 & -0.2832 & 0.4943 \end{bmatrix}, \ B_{f2} = \begin{bmatrix} -0.3382 & -0.1356 & -0.1356 \end{bmatrix}^T, \end{aligned}$$

$$C_{f2} = \begin{bmatrix} -0.1203 & 0.0205 & -0.3448 \end{bmatrix},$$

with the corresponding optimal $l_2 - l_\infty$ performance $\gamma = 0.3305$.

Figure 2 shows the values of the random data packet dropouts variable $\beta(k)$ which is a stochastic variable and takes the values of 0 and 1. Figure 3 depicts the switching signal $\alpha(k)$ of original system and the corresponding switching signal $\hat{\alpha}(k)$ used by filter. It can be seen from Figure 3 that the switching signal received by the filter is randomly lost. The state response curves of the original signal $z(k)$ that is estimated by the system and filter signal $\hat{z}(k)$ are plotted in Figure 4 with the initial condition $x(0) = \begin{bmatrix} 0.1 & -0.1 & 0 \end{bmatrix}^T$ and $\hat{x}(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$, respectively. The filtering error response is given in Figure 5. As can be seen from Figures 4 and 5, the estimated signal of the filter tracks the original signal of the system, and the filtering error eventually tends to zero. From the above results, we can conclude that the estimation performance of the designed filter is good, and the optimized $l_2 - l_\infty$ performance can be guaranteed. In other words, the designed filter can estimate the state of the original system in a required $l_2 - l_\infty$ performance index. The filter design technique in this paper can be used to estimate signals in power electronic circuits, such as inductance current, capacitance voltage.

In order to compare the conservativeness of various methods, under a certain probability of the data packet dropouts $1 - \bar{\beta} = 0.2$, Table 1 gives a comparison of minimum $l_2 -$

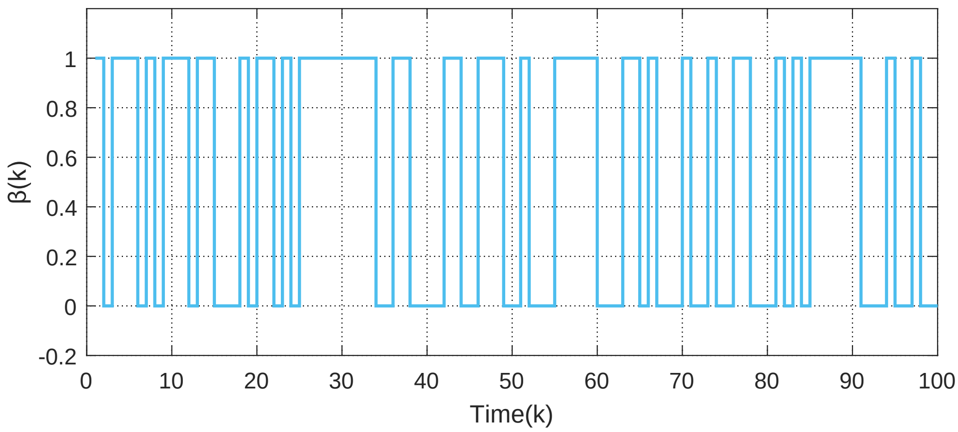

FIGURE 2. The value of $\beta(k)$

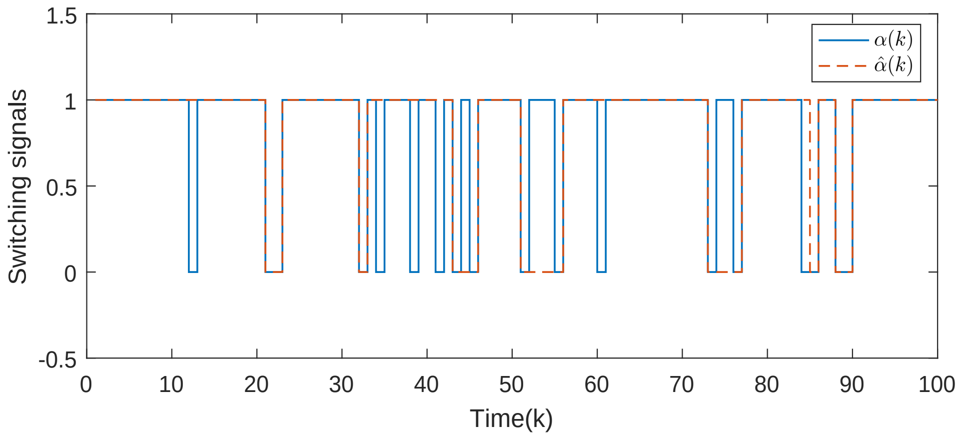

FIGURE 3. Switching signals

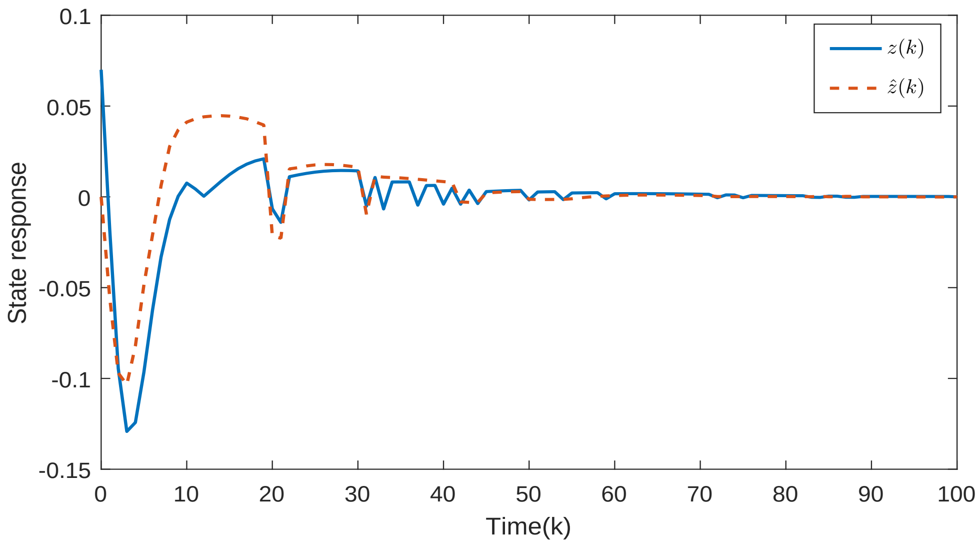

FIGURE 4. State responses of $z(k)$ and $\hat{z}(k)$

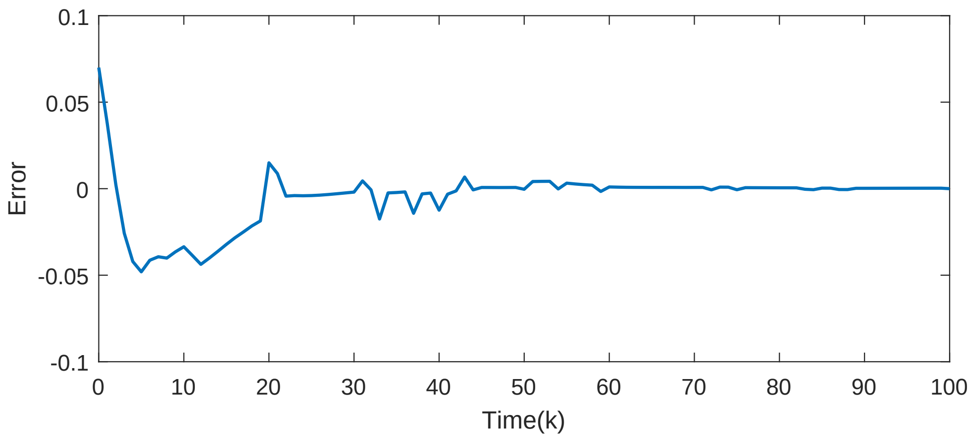

FIGURE 5. Filtering error response

TABLE 1. The values of the $l_2 - l_\infty$ performance index under different Lyapunov function methods

Method	Filter	Value of γ
Common Lyapunov function method	full-order	1.3192
Traditional switched Lyapunov function method	full-order	0.7856
The proposed switched Lyapunov function	full-order	0.3305

l_∞ performances among the common Lyapunov function, traditional switched Lyapunov function and the proposed switched Lyapunov function. When there are no variables i and j in matrix P in Theorem 4.4, then Theorem 4.4 becomes the existing condition of the filter based on common Lyapunov function method. Similarly, when the variables i and j in matrix P in Theorem 4.4 are equal, then Theorem 4.4 becomes the existing condition of the filter based on traditional switched Lyapunov function method. Using the Matlab LMI Toolbox to solve LMIs (31), (32), the values of the $l_2 - l_\infty$ performance index under common Lyapunov function, traditional switched Lyapunov function and the proposed switched Lyapunov function can be obtained, respectively. By comparing the results of γ, we can find which method is less conservative.

6. **Conclusions.** In this paper, the issue of filtering problem for a class of discrete-time switched systems with randomly occurring multiple packet dropouts is studied. The missing of the switching signal and measurement signal are taken into consideration simultaneously. Sufficient conditions for the exponential stability as well as desired $l_2 - l_\infty$ performance of the filtering error system are given with the aid of a novel switched Lyapunov function. As a result, a desired switched filter satisfying the performance requirements is designed. The filter parameters can be achieved by solving a set of LMIs. Finally, the developed theoretical results are validated by an illustrative example. The parameters of the designed filter in this paper vary with the random change of the switching signal, which reduces the conservativeness of the design.

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