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CONTROL OF TIME-VARYING DELAY SYSTEMS WITH UNCERTAIN PARAMETERS VIA FUZZY-MODELED PRESCRIBED PERFORMANCE CONTROL APPROACH

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Abstract. *This paper deals with the problem of robust control for time-varying delay systems with uncertain parameters and disturbances which have the inexactly measured state via the Fuzzy-Modeled Prescribed Performance Control (F-PPC) procedure. The system models are assumed to depend on the phenomena of uncertain parameters and disturbances which are frequently encountered in most real dynamical systems as well as a time-varying delay of systems. To describe the uncertain nonlinearities with time-varying delay systems in the Takagi-Sugeno (T-S) fuzzy model, the global behavior of a nonlinear system can be simply represented using the T-S plant rule models. Then, a novel controller is computed by the linear matrix inequality (LMI) conditions. The obtained controller guarantees the L_2-gain of the mapping from the exogenous input noise to the regulated output to be less than some prescribed value. The control design is applied to the nonlinear benchmark problems to illustrate the benefits and applicability of the proposed method. The results also show that the proposed F-PPC approach guarantees the fulfillment of both the asymptotic stability and the prescribed performance index.*
Keywords: Linear matrix inequality (LMI), Prescribed performance control, Takagi-Sugeno (T-S) fuzzy model, Time-varying delay systems, Uncertain parameter systems

1. Introduction.

Robust prescribed control namely H-infinity control is an effective and widely used technique for the control of nonlinear systems [1]. The aim of approach can be stated to find the controller of a dynamic system with the exogenous input noise and the measured output such that guarantee the L_2-gain of the mapping from the exogenous input noise to the regulated output to be less than or equal to some prescribed value [2]. Recently, [3] develops the finite-time H-infinity controller for a fractional-order hydraulic turbine governing system. Moreover, [4] shows the positive definiteness in LMI problem for H-infinity output feedback control problem. The coupling constraint is provided to strictly hold at optimal solutions. With most recent work in [5], the structured H-infinity controller for an uncertain deregulated power system is investigated. The approach gives sufficiently lower-order controllers and can be used in power system with nonlinearities like generation rate constraint, governor deadband, and time delay associated with communication network in the system.

Together with the phenomena of uncertain parameters and external disturbance noises, time varying delays are considered as a source of poor control performances and instabilities, for instance, the problem of time-varying networked induced delays [6], the packet

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losses in the network control systems [7], and the mixed mode phenomenon that occurs naturally in the switched time varying delay system [8]. Recently, the problem of nonlinear interconnected systems with interval time-varying delays are studied [9]. Moreover, the problem of complex economic phenomenons is investigated by using the non-parametric time-varying coefficient panel data models [10].

By using the T-S model approach, the nonlinearity can be explained in the T-S plant rules which can be combined as the nonlinear system model [11]. By taking the advantage of the T-S approach, a small number of rules of T-S fuzzy plant can model the higher order nonlinear systems which provide an effective representation of complex nonlinear systems in terms of fuzzy sets and fuzzy reasoning applied to a set of linear input-output sub-models [28, 29]. Thus, the powerful design tools of T-S fuzzy method can be applied to complex nonlinear systems for various engineering applications. For instance, [12] presents the proposed T-S fuzzy controller that can overcome the nonlinearities, the uncertain parameters and the disturbances of the dynamic behavior of the DFIG wind energy system. Furthermore, the controller design for the fuzzy adaptive output-feedback control and the reliable switched controller design for a class of discrete time with random delays have been examined using T-S fuzzy procedures [13, 14].

In the modern control theory, the studies of state-derivative feedback are dramatically proposed via many research works due to the fact that the use of accelerometer is as a part of control systems. In several practical applications, there exist some practical problems where the state feedback signals are not available for measurement or not possible to obtain with good precision. For instance, in the practical active-vibration control, the accelerometers are often used due to their simple structures and low operational costs. Therefore, it is possible to accurately compute velocity signals, but not the displacements [15, 16]. Furthermore, [17] proposes the state-derivative feedback technique using the information on velocities and accelerations which are less conservative than employing the information on displacements and velocities. Especially, a more realistic situation is the case where the states are inexactly available in the measurement of temperature inside a bauxite smelter due to the estimation errors [18]. Furthermore, [19] proposes the novel results by designing H-infinity control using T-S model approach and state-derivative feedback technique. Unfortunately, those results have only been applied to a nonlinear system; however, they do not consider the uncertainties with time-varying delay.

Concisely, from practical points of views, it is necessary to consider the control design approach for time-varying delay systems including uncertain parameters and disturbances with inexactly measured states due to involving in reality. Therefore, the following scenarios may cover in this research work.

- Firstly, the original nonlinear system involves with the uncertain parameters including the disturbances.
- Secondly, the dynamical system is considered with the time-varying delay.
- Thirdly, the states are not accurately measured for feedback.

To the best of our knowledge, the robust H-infinity controller design approach for time-varying delay systems including uncertain parameters and disturbances with no accurately measured states has not been previously investigated in literature. Additionally, in consideration of computation viewpoints, the proposed design approach is aggregated to examine a set of LMI in conjunction with the T-S fuzzy model approach. The convex optimization algorithm is employed to quickly solve the LMI problem [20]. The proposed approach can significantly mitigate the computational difficulties; therefore, it reduces the design costs associated with the practical use of theoretical outcomes due to the fact that the T-S fuzzy controller gains are easily acquired and are able to directly apply to the controller for such a system. Accordingly, the research on F-PPC design approach for time-varying delay systems with uncertain parameters and disturbances can be conducted on both the theoretical and practical point of view.

Motivated by the above observations, this paper aims at solving the problem of robust control for time-varying delay systems with uncertain parameters and disturbances which have the inexactly measured state. We assume that the system model is depended on the phenomena of uncertain parameters and disturbances which are frequently encountered in most real dynamical systems as well as the time-varying delay of systems. The main contributions of this research are summarized as follows.

1) The definitions of the H-infinity control problem and asymptotic stability are introduced for the system.
2) The T-S fuzzy model is applied to approximate nonlinear time-varying delay systems with uncertain parameters and disturbances.
3) The Lyapunov-Krasovskii functional [21], with the LMI approach is used to develop a means of designing a robust controller that satisfies for both performance and robustness specifications.

The proposed F-PPC design method consists of three stages. At the first stage, an uncertain time-varying delay system is modeled in accordance with T-S fuzzy procedure for explaining a behavior of plant. Then, the development of the proposed approach can guarantee the L_2-gain of the mapping from the exogenous input noise to the regulated output to be less than some prescribed value. Finally, the desired F-PPC controller is computed via an LMI condition to guarantee the fulfillment of both the asymptotic stability and the prescribed performance index. In a numerical example, the robust controller is designed by the proposed F-PPC method for the nonlinear benchmarks. The results demonstrate the applicability of the proposed F-PPC procedure. Additionally, the numerical examples are provided to illustrate the superiority of the proposed F-PPC design when compared to the conventional approaches.

This paper is organized as follows. The descriptions and the definitions are illustrated in Section 2. In Section 3, the F-PPC design methods are proposed for the time-varying delay system with uncertain parameter as described in Section 2. The results are demonstrated through the examples presented in Section 4. Finally, the conclusion is summarized in Section 5.

2. **System Descriptions and Definitions.**

In this paper, we focus on the following T-S fuzzy time-varying delay system with uncertain parameters and disturbances.

Plant Rule i: IF $\bar{\theta}_1(t)$ is $\bar{M}_{i1}(t)$ and ... and $\bar{\theta}_g(t)$ is $\bar{M}_{ig}(t)$ THEN

$$\dot{x}(t) = A_i x(t) + B_i u(t) + A_{d_i} x(t - \tau(t)) + B_w w(t), \tag{1}$$

$$z(t) = C_i x(t), \tag{2}$$

$$x(t) = \bar{\varphi}(t),\ t \in [-\tau, 0] \tag{3}$$

where $i = 1, 2, \ldots, l$, l is the number of IF-THEN rules; $\bar{M}_{ij}(t)$ and $\bar{\theta}_j(t)$ $(j = 1, 2, \ldots, g)$ are the fuzzy sets and the premise variables, respectively; $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the input, $w(t) \in \Re^q$ is the input disturbance which belongs to $L_2[0, \infty)$ and $z(t) \in \Re^p$ is the controlled output; A_i, B_i, B_w, A_{d_i} and C_i are known real matrices of the system; $\tau(t)$ is the time-varying delay which satisfies $0 \leq \tau(t) \leq \tau$ and $\dot{\tau}(t) \leq \tau_d < 1$ where τ is real positive constant; $\bar{\varphi}(t)$ is initial function on the interval $[-\tau, 0]$. For any specified state vector and the control input, the T-S fuzzy model is inferred as follows.

$$\dot{x}(t) = \sum_{i=1}^{l} \bar{h}_i\left(\bar{\theta}(t)\right)\left[(A_i + \Delta A_i)\,x(t) + (B_i + \Delta B_i)\,u(t)\right.$$
$$\left. + A_{d_i} x(t-\tau(t)) + (B_{w_i} + \Delta B_{w_i}) w(t)\right], \tag{4}$$
$$z(t) = \sum_{i=1}^{l} \bar{h}_i\left(\bar{\theta}(t)\right)\left[(C_i + \Delta C_i)\,x(t)\right], \;\; i = 1, 2, 3, \ldots, l \tag{5}$$

where $\bar{h}_i\left(\bar{\theta}(t)\right) = \varpi_i\left(\bar{\theta}(t)\right) \Big/ \sum_{i=1}^{l} \varpi_i\left(\bar{\theta}(t)\right)$, $\varpi_i\left(\bar{\theta}(t)\right) = \prod_{j=1}^{g} \bar{M}_{ij}\left(\bar{\theta}_j(t)\right)$.

$\bar{M}_{ij}\left(\bar{\theta}_j(t)\right)$ is the grade of membership of $\bar{\theta}_j(t)$ in $\bar{M}_{ij}$. It is assumed in this paper that

$$\bar{h}_i\left(\bar{\theta}(t)\right) \geq 0, \quad \sum_{i=1}^{l} \bar{h}_i\left(\bar{\theta}(t)\right) = 1, \quad i = 1, 2, \ldots, l, \tag{6}$$

for all t. To simplify the notations, we use $\bar{h}_i = \bar{h}_i\left(\bar{\theta}(t)\right)$. Thus, we can generalize the T-S fuzzy models as the weighted average of the following forms:

$$\dot{x}(t) = \sum_{i=1}^{l} \bar{h}_i[(A_i+\Delta A_i)x(t)+(B_i+\Delta B_i)u(t)+A_{d_i}x(t-\tau(t))+(B_{w_i}+\Delta B_{w_i})w(t)], \tag{7}$$
$$z(t) = \sum_{i=1}^{l} \bar{h}_i\left[(C_i + \Delta C_i)x(t)\right], \;\; i = 1, 2, 3, \ldots, l \tag{8}$$

where the matrices A_i, B_i, B_w and C_i are defined as in (1)-(3) and the matrices ΔA_i, ΔB_i, ΔB_{w_i} and ΔC_i represent the parameter uncertainties in the system and satisfy the following assumption.

Assumption 2.1.

$$\Delta A_i = F(x(t), t)H_{1_i}, \; \Delta B_{w_i} = F(x(t), t)H_{2_i},$$
$$\Delta B_i = F(x(t), t)H_{3_i}, \; \Delta C_i = F(x(t), t)H_{4_i}$$

where H_{j_i}, $j = 1, 2, 3, 4$ are known matrix functions that characterize the structure of the uncertainties. Furthermore, the following inequality holds:

$$\|F(x(t), t)\| \leq \rho$$

for any known positive constant ρ.

Next, let us recall the following definition and lemma.

Definition 2.1. *Suppose γ is a given positive real number. A system of form (7) is said to have an L_2 gain less than or equal to γ if*

$$\int_0^{T_f} z^T(t)z(t)dt \leq \gamma^2 \left[\int_0^{T_f} w^T(t)w(t)dt\right] \tag{9}$$

for all $T_f \geq 0$ and $w(t) \in L_2[0, T_f]$.

Lemma 2.1. [22] *Let $x_e = 0$ be an equilibrium for $\dot{x} = f(x)$. Let $V : R^n \longrightarrow R$ be a continuously differentiable function such that*

- *$V(0) = 0$ and $V(x) > 0$ for all $x \neq 0$.*
- *$\dot{V}(x) < 0$ for all $x \neq 0$, $\dot{V}(0) = 0$.*

Then, x_e is asymptotically stable and is the unique equilibrium point.

Note that for the symmetric block matrices, we denote (*) as the transposed element in the symmetric positions of a matrix.

3. **Main Results.** In this section, an F-PPC design method is proposed for the T-S fuzzy uncertain parameter system, which involves three steps. The first one is to construct a T-S system which is explained the problem of the control system. In the second, a novel F-PPC is developed to ensure the objectives of the proposed approach. Finally, an F-PPC controller is synthesized such that the asymptotic stability and the prescribed performance index can be guaranteed. Suppose that there exists an F-PPC controller of the term:
Controller Rule j: IF $x_{k_1}(t)$ is $M_{i1}(t)$ and ... and $x_{k_j}(t)$ is $M_{ij}(t)$ THEN

$$u(t) = -K_j\dot{x}(t), \quad \forall j = 1, 2, 3, \ldots, l \tag{10}$$

where $x(t)$ is state vector and K_j is the controller gain of an F-PPC. Figure 1 illustrates the structure diagram of T-S fuzzy model which uses a multi-model approach in which simple sub-models are fuzzily combined to describe the global behavior of a nonlinear system. According to (4)-(8) defined in previous section, the grade of membership and the weighted average can be generalized the T-S fuzzy models [11]. Consequently, the fuzzy controller shown in Figure 1 can be inferred as

$$u(t) = \sum_{j=1}^{l} \bar{h}_j\big(-K_j\dot{x}(t)\big), \quad \forall j = 1, 2, 3, \ldots, l. \tag{11}$$

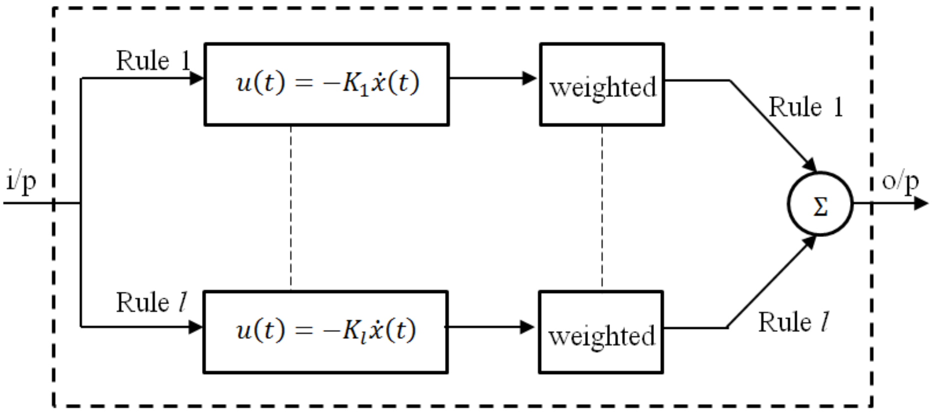

FIGURE 1. Fuzzy controller model

Figure 2 shows the closed-loop of robust fuzzy system associated with the fuzzy controller. The major implication of this approach is that the structure of the controller has to take into an account the effect of uncertainty and time-varying delay of the system. The problem addressed is the design of F-PPC controller such that it guarantees the L_2-gain of the mapping from the exogenous input noise to the regulated output to be less than some prescribed value. Using Assumption 2.1, the closed-loop fuzzy system (7)-(8) and the controller (11) can be expressed as follows:

$$\left[I + \sum_{i=1}^{l}\sum_{j=1}^{l} \bar{h}_i\bar{h}_j B_i K_j\right]\dot{x}(t) = \sum_{i=1}^{l}\sum_{j=1}^{l} \bar{h}_i\bar{h}_j\left[A_i x(t) + A_{d_i}x(t-\tau(t)) + \tilde{B}_{w_i}\tilde{w}(t)\right] \tag{12}$$

where $\tilde{B}_{w_i} = [\delta I \;\; I \;\; \delta I \;\; B_{w_i}]$ and the disturbance is

$$\tilde{w}(t) = \begin{bmatrix} \frac{1}{\delta}F(x(t),t)H_{1_i}E_{ij}x(t) \\ F(x(t),t)H_{2_i}w(t) \\ 0 \\ w(t) \end{bmatrix}.$$

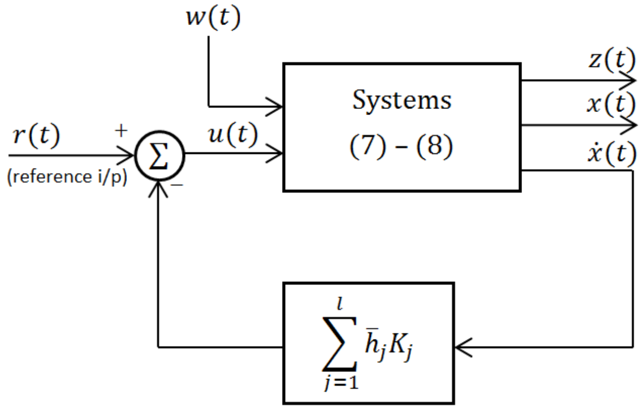

FIGURE 2. Closed-loop fuzzy system

Remark 3.1. *The issue is to obtain the F-PPC gains* K_j $(j = 1, 2, \ldots, l)$ *such that the following conditions hold:*

1) *Matrices* $(I + B_i K_j)$, $\forall i, j = 1, 2, 3, \ldots, l$ *have full rank.*
2) *The system (7)-(8) with the fuzzy controller (11) is asymptotically stable, and the H-infinity performance is satisfied for all admissible values based on the sufficient condition for a prescribed scalary* > 0.

To establish the proposed results and without loss of generality, we assume that the following assumption exists: rank $[I \mid B_i] = n$. *Thus, it is easy to know that if rank* $[I \mid B_i] = n$ *holds, then there exists* K_j *such that rank* $[I + B_i K_j] = n$*, that is, matrices* $(I + B_i K_j)$, $\forall i, j = 1, 2, 3, \ldots, l$ *have full rank.*

The conditions given in the Remark 3.1 may apply to the dynamic system (12) for stability and performance analysis [32, 33]. Since $A \in \Re^{n\times n}$ is the state matrix, $B \in \Re^{n\times m}$ is the input matrix, $K_j \in \Re^{m\times n}$ is the controller gain, and I is the $n \times n$ identity matrix. Throughout this paper, the following three assumptions are imposed on the system:

Assumption 3.1. *The matrix* B_i *is completely controllable.*

Assumption 3.2. *Rank* $B_i = m$.

Assumption 3.3. *The term* $(I + B_i K_j)$ *has full rank.*

In accordance with the Remark 3.1, Assumption 3.1, Assumption 3.2 and Assumption 3.3, the controller gain K_j is restricted to ensure that the term $(I + B_i K_j)$ is always nonsingular. Therefore, we define

$$E_{ij} = (I + B_i K_j)^{-1}. \tag{13}$$

(12) can be written as

$$\dot{x}(t) = \sum_{i=1}^{l}\sum_{j=1}^{l} \bar{h}_i \bar{h}_j \left[E_{ij} A_i x(t) + E_{ij} A_{d_i} x(t - \tau(t)) + E_{ij} \tilde{B}_{w_i} \tilde{w}(t) \right]. \tag{14}$$

Remark 3.2. *An LMI approach is applied to derive an F-PPC controller that stabilizes the system (14) and guarantees the disturbance rejection of level* $\gamma > 0$ *immediately. First, to design the F-PPC controller, the following design objectives are satisfied:*

(a) *The closed-loop system is asymptotically stable when* $w(t) = 0$;
(b) *Under the zero initial condition, the system (14) satisfies* $\|z\|_2 \le \gamma \|w\|_2$ *for any non-zero* $w(t) \in L_2[0, +\infty)$*, where* $\gamma > 0$ *is a prescribed constant.*

As aforementioned, the time-varying delay is assumed as a source of poor control performance. Correspondingly, the F-PPC for uncertain nonlinear systems with time-varying delay will be proposed in following Subsection 3.2. Specially, we also present the F-PPC for uncertain nonlinear systems without time-varying delay in Subsection 3.1. The both theorems provide sufficient conditions for the existence of a robust F-PPC. These sufficient conditions can be derived by the Lyapunov approach.

3.1. **Design for uncertain nonlinear systems without time-varying delay.** In most real practical systems, it is known that the uncertain parameters and the disturbances are found within the complexities of the design problems; for instance, the humanoid robots control system [30, 31, 34, 35]. Most quantities are available to measure for the closed loop system; however, some important parameters are not available. Therefore, [34] resorted to use the differentiation of an encoder in order to obtain the information for increasing the performance of robot control systems. From motivation as aforementioned, the robust F-PPC methods are designed to achieve the pre-prescribed performance index and the asymptotic stability. Therefore, we can generalize the T-S fuzzy models as the weighted average of the following forms:

$$\dot{x}(t) = \sum_{i=1}^{l} \bar{h}_i[(A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) + (B_{w_i} + \Delta B_{w_i})w(t)], \tag{15}$$

$$z(t) = \sum_{i=1}^{l} \bar{h}_i \left[(C_i + \Delta C_i)x(t)\right], \;\; i = 1, 2, 3, \ldots, l. \tag{16}$$

Theorem 3.1. *Consider the system (15)-(16). Given a prescribed H-infinity performance* $\gamma > 0$ *and a positive constant* δ *and if the inequality (9) holds, there exist symmetric matrices* $P > 0$ *and matrices* Y_j, $j = 1, 2, \ldots, l$, *satisfying the following linear matrix inequalities:*

$$\Gamma_{ii} < 0, \qquad i = 1, 2, \ldots, l, \tag{17}$$

$$\Gamma_{ij} + \Gamma_{ji} < 0, \qquad i < j \leq l \tag{18}$$

where

$$\Gamma_{ij} = \begin{pmatrix} \Phi_{ij} & (*)^T & (*)^T \\ \tilde{B}_{w_i}^T & -\gamma^2 I & (*)^T \\ \tilde{C}_i P + \tilde{C}_i Y_j^T B_i^T & 0 & -I \end{pmatrix} \tag{19}$$

with

$$\Phi_{ij} = PA_i^T + A_iP + B_iY_jA_i^T + A_iY_j^TB_i^T,$$
$$\tilde{C}_i = \left[\begin{array}{cccc} \frac{\gamma\rho}{\delta}H_{1_i}^T & 0 & \sqrt{2}\lambda\rho H_{3_i}^T & \sqrt{2}\lambda C_i^T \end{array}\right]^T,$$
$$\lambda = \left(1 + \rho^2 \sum_{i=1}^{l}\sum_{j=1}^{l}\left[\| H_{2_i}^T H_{2_j} \|\right]\right)^{1/2}.$$

Furthermore, the suitable choice of the fuzzy controller is

$$u(t) = \sum_{j=1}^{l} \bar{h}_j \left(-K_j \dot{x}(t)\right), \;\; \forall j = 1, 2, 3, \ldots, l \tag{20}$$

where

$$K_j = Y_j P^{-1}. \tag{21}$$

Proof: Refer to Appendix 1 for the proof. □

3.2. **Design for uncertain nonlinear systems with time-varying delay.** As the aforesaid, the time-varying delay is frequently a cause of instability and poor performance. They often occur in many dynamical systems such as metallurgical processing systems, biological systems, chemical systems, network systems and communication system. Therefore, the robust controller for an uncertain nonlinear system with time-varying delay will be proposed in this subsection in order to ensure that the closed-loop system is asymptotically stable and able to meet the pre-prescribed performance index.

Theorem 3.2. *Consider the system (7)-(8). Given a prescribed H-infinity performance $\gamma > 0$, a positive constant δ and $0 \leq \tau_d < 1$, if there exist symmetric matrices $P > 0$, $W > 0$ and matrices Y_j, $j = 1, 2, \ldots, l$, satisfying the following linear matrix inequalities:*

$$\Xi_{ii} < 0, \qquad i = 1, 2, \ldots, l, \tag{22}$$

$$\Xi_{ij} + \Xi_{ji} < 0, \qquad i < j \leq l \tag{23}$$

where

$$\Xi_{ij} = \begin{pmatrix} \Psi_{ij} & (*)^T & (*)^T & (*)^T \\ \tilde{B}_{w_i}^T & -\gamma^2 I & (*)^T & (*)^T \\ P + Y_j^T B_i^T & 0 & -(1-\tau_d)W & (*)^T \\ \tilde{C}_i P + \tilde{C}_i Y_j^T B_i^T & 0 & 0 & -I \end{pmatrix} \tag{24}$$

with

$$\Psi_{ij} = PA_i^T + A_i P + B_i Y_j A_i^T + A_i Y_j^T B_i^T + A_{d_i} W A_{d_i}^T,$$
$$\tilde{C}_i = \left[\begin{array}{cccc} \frac{\gamma\rho}{\delta} H_{1_i}^T & 0 & \sqrt{2}\lambda\rho H_{3_i}^T & \sqrt{2}\lambda C_i^T \end{array}\right]^T,$$
$$\lambda = \left(1 + \rho^2 \sum_{i=1}^{l} \sum_{j=1}^{l} \left[\| H_{2_i}^T H_{2_j} \|\right]\right)^{1/2}$$

for any delay $\tau(t)$ satisfying (1), then the inequality (9) holds. Furthermore, the suitable choice of the fuzzy controller is

$$u(t) = \sum_{j=1}^{l} \bar{h}_j\big(-K_j \dot{x}(t)\big), \quad \forall j = 1, 2, 3, \ldots, l \tag{25}$$

where

$$K_j = Y_j P^{-1}. \tag{26}$$

Proof: Refer to Appendix 2 for the proof. □

Remark 3.3. *In contrast to* [19]*, the criteria on F-PPC for time-varying delay system with uncertain parameter are considered. Theorems 3.1 and 3.2 synthesize the F-PPC controller such that the asymptotic stability and the prescribed performance index can be guaranteed. In addition, based on the explanations in this paper, it is known that the F-PPC controller using Theorem 3.1 is not suitable to system (7)-(8), since there is the time-varying delay in fuzzy model. By using Theorem 3.2, the underlying system (7)-(8) can be guaranteed to meet the pre-prescribed performance and ensure the closed-loop system is asymptotically stable.*

4. Numerical Examples.

4.1. **Example 1.** The problem of balancing an inverted pendulum on a cart is considered. The movement equations are [23]:

$$\begin{aligned}\dot{x}_1(t) &= x_2(t),\\ \dot{x}_2(t) &= \frac{f(x(t)) - a\cos(x_1(t))u(t)}{4(\iota+\Delta\iota)/3 + am(\iota+\Delta\iota)\cos^2(x_1(t))} + 0.01w(t),\\ z(t) &= \begin{bmatrix} 0.01x_1(t)\\ 0.01u(t)\end{bmatrix}\end{aligned} \tag{27}$$

where $f(x(t)) = g\sin(x_1(t)) - am(\iota+\Delta\iota)x_2^2(t)\sin(2x_1(t))/2$, $x_1(t)$ represents the angle from the vertical axis (in radians), $x_2(t)$ is the angular velocity of the pendulum, $u(t)$ is the control force applied to the cart (in Newtons), $z(t)$ is the regulated output, $w(t)$ is the disturbance, $g = 9.8$ m/s^2 is the gravity constant, M is the cart mass, 2ι is the pendulum length, m is the pendulum mass and $x_1(t) \in [-\pi/2, \pi/2]$. Define $M = 8$ kg, $m = 2$ kg, $2\iota = 1$ m, $a = 1/(m+M)$ and $\Delta\iota$ as an uncertain term that is bounded in $[0\ \ 0.10]$. Note that the system is uncontrollable when $x_1(t) = \pm\pi/2$; therefore, we linearize the system around 0° and 88° instead. Therefore, it is assumed that $x_1(t) \in [-88°, 88°]$. The nonlinear system plant can be approximated using two T-S fuzzy rules. Let us choose the membership functions of the fuzzy sets as follows:

$$M_1(x_1(t)) = 1 - \frac{2}{\pi}|x_1(t)| \quad \text{and} \quad M_2(x_1(t)) = \frac{2}{\pi}|x_1(t)|.$$

Note that $M_1(x_1(t))$ and $M_2(x_1(t))$ can be interpreted as the membership functions of the fuzzy sets shown in Figure 3.

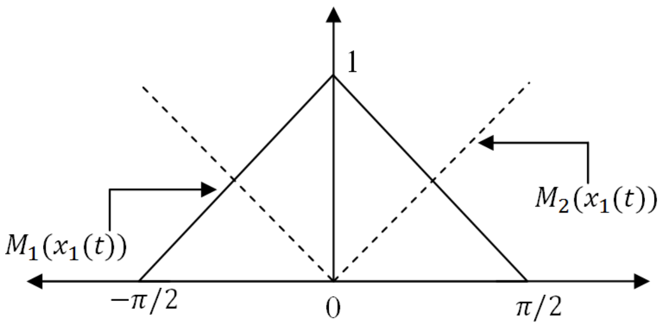

FIGURE 3. Membership functions for two fuzzy sets on Example 1

Using these two fuzzy sets, the uncertain nonlinear system can be represented by the following T-S fuzzy model:

Plant Rule 1: IF $x_1(t)$ is $M_1(x_1(t))$, THEN

$$\begin{aligned}\dot{x}(t) &= [A_1 + \Delta A_1]x(t) + B_1 w(t) + B_{2_1}u(t),\\ z(t) &= C_1 x(t),\end{aligned}$$

Plant Rule 2: IF $x_1(t)$ is $M_2(x_1(t))$, THEN

$$\begin{aligned}\dot{x}(t) &= [A_2 + \Delta A_2]x(t) + B_1 w(t) + B_{2_2}u(t),\\ z(t) &= C_1 x(t)\end{aligned}$$

where

$$A_1 = \begin{bmatrix} 0 & 1\\ \dfrac{g}{4\iota/3 - am\iota} & 0\end{bmatrix},\ A_2 = \begin{bmatrix} 0 & 1\\ \dfrac{2g}{\pi(4\iota/3 - am\iota\beta^2)} & 0\end{bmatrix},\ B_1 = \begin{bmatrix} 0\\ 0.01\end{bmatrix},$$

$$B_{2_1} = \begin{bmatrix} 0\\ -\dfrac{a}{4\iota/3 - am\iota}\end{bmatrix},\ B_{2_2} = \begin{bmatrix} 0\\ -\dfrac{a\beta}{4\iota/3 - am\iota\beta^2}\end{bmatrix},\ C_1 = \begin{bmatrix} 0.01 & 0\\ 0 & 0\end{bmatrix},$$

$$\beta = \cos(88^\circ),\ \Delta A_1 = F(x(t),t)H_{1_1} \text{ and } \Delta A_2 = F(x(t),t)H_{1_2},$$

and assuming that $\|F(x(t),t)\| \leq \rho = 1$, we have

$$H_{1_1} = \begin{bmatrix} 0 & 0 \\ 4.32 & 0 \end{bmatrix} \text{ and } H_{1_2} = \begin{bmatrix} 0 & 0 \\ 2.75 & 0 \end{bmatrix}.$$

Using the LMI optimization algorithm and Theorem 3.1 with $\gamma = 1$ and $\delta = 0.1$, we obtain

$$P = \begin{bmatrix} 0.6290 & -1.9610 \\ -1.9610 & 6.2995 \end{bmatrix},$$
$$Y_1 = \begin{bmatrix} 13.1895 & -32.9675 \end{bmatrix},\ Y_2 = \begin{bmatrix} 13.1901 & -32.9793 \end{bmatrix},$$
$$K_1 = \begin{bmatrix} 157.7724 & 43.8803 \end{bmatrix} \text{ and } K_2 = \begin{bmatrix} 157.6067 & 43.8269 \end{bmatrix}.$$

The resulting fuzzy controller is

$$u(t) = \sum_{j=1}^{2} \bar{h}_j \left(-K_j \dot{x}(t)\right) \tag{28}$$

where $\bar{h}_1 = M_1(x_1(t))$ and $\bar{h}_2 = M_2(x_1(t))$.

Remark 4.1. *The fuzzy controller (28) ensures that the inequality (9) holds. Figures 4, 5 and 6 illustrate the state variables, $x_1(t)$, $x_2(t)$, of the open-loop system, the proposed Theorem 3.1 and the disturbance input, $w(t)$, which was used during the simulation, respectively. As depicted in Figure 7, after 1.4 seconds, the ratio of the regulated output energy to the disturbance input noise energy approaches a constant value that is less than the prescribed value of 1. These results satisfy design requirements that ensure that the closed-loop system is asymptotically stable and that the pre-prescribed performance index is satisfied.*

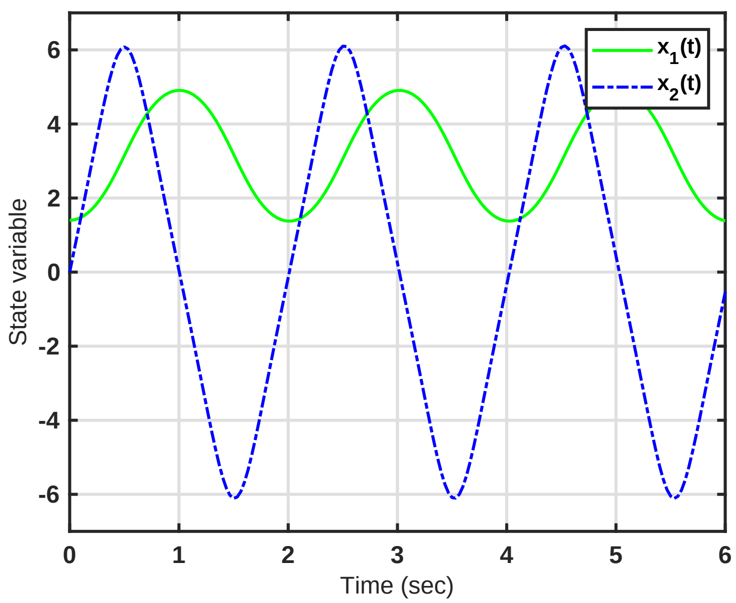

FIGURE 4. State variables of open-loop system

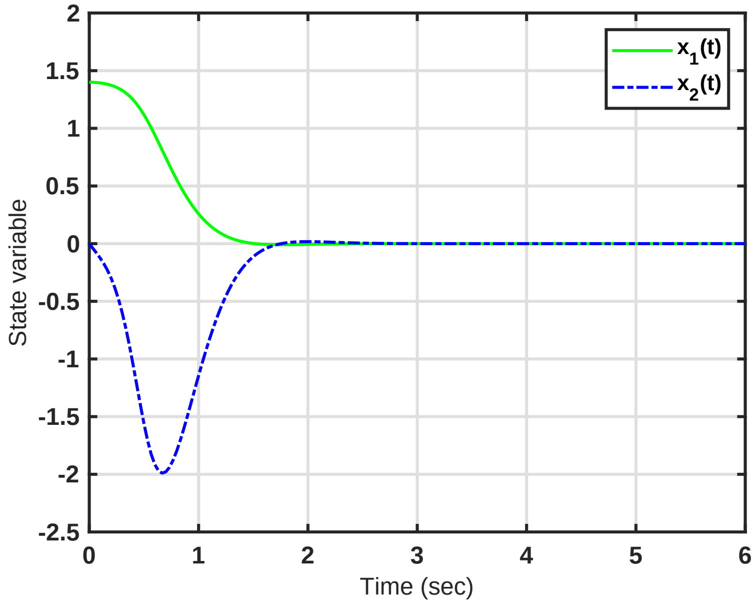

FIGURE 5. State variables of closed-loop system

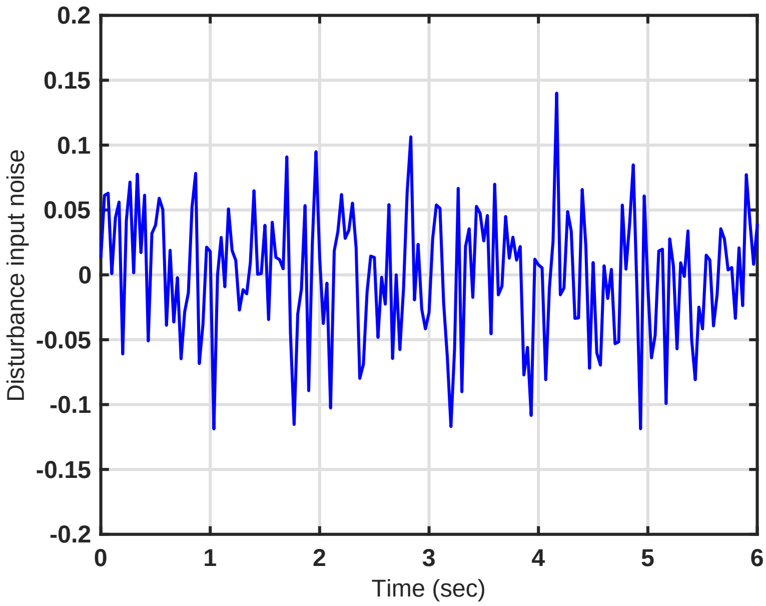

FIGURE 6. Disturbance input noise

Remark 4.2. *According to the robust state-feedback controller design of Theorem 1, used in* [24]*, the robust integral state-feedback controller design of Theorem 2, presented in* [12] *and the proposed Theorem 3.1 used in this paper, Figure 8 shows comparative results for the state variable $x_1(t)$ at the same $\gamma = 1$ for the allowed $\Delta\iota = 0.10$. Figure 8 illustrates that Theorem 3.1 used in this study generates the response 113% and 67% faster than that shown in* [24] *and* [12]*, respectively. This shows that the uncertain nonlinear system is effectively controlled using the F-PPC approach.*

Remark 4.3. *Compared with some existing work, the proposed F-PPC techniques have an advantage on* [19] *since it is seen that the parameter uncertainty problem is considered in*

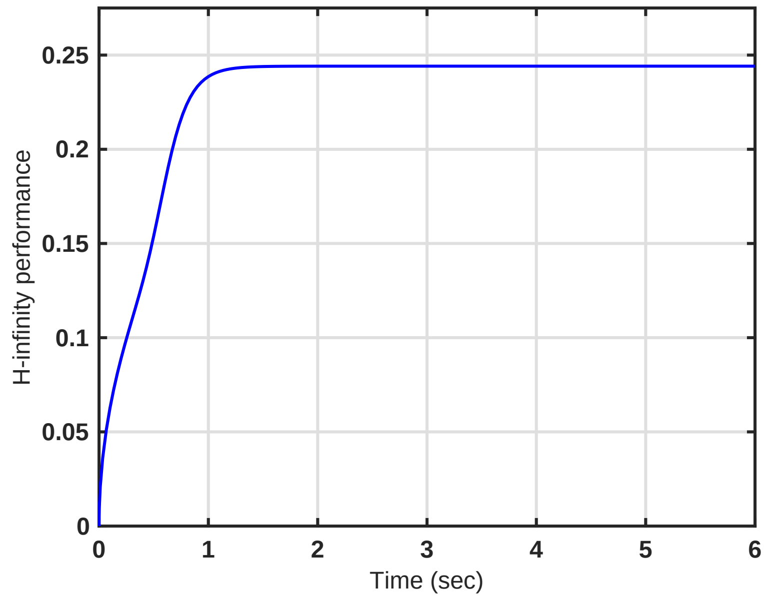

Figure 7. Performance of the proposed Theorem 3.1 on Example 1

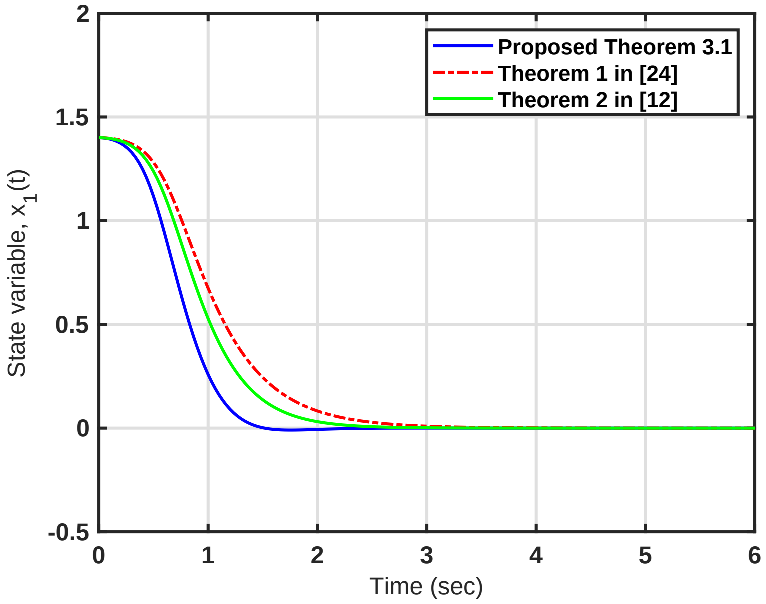

Figure 8. Comparison of state variable

this work. More practically, Example 1 studies a more general system where there are the uncertainties in systems. An inverted pendulum on a cart with parameter uncertainty is investigated using Theorem 3.1. As shown via the results, it is obvious that F-PPC is the effective control approach which can be easily solved and can directly apply the fuzzy gains to the controller for such a system. Generally, this pendulum problem could be applied to more scope of applications such as manipulator, larger humanoid robotic and other highly dynamic legged robot control system [34, 35, 36].

4.2. **Example 2.** The modified model of the truck-trailer backing-up control system with the time delay is investigated in this example [25]:

$$\begin{aligned}\dot{x}_1(t) &= -a\frac{vt}{(L+\Delta L)t_0}x_1(t) - (1-a)\frac{vt}{(L+\Delta L)t_0}x_1(t-\tau(t)) + \frac{vt}{\ell t_0}u_1(t) + w(t),\\ \dot{x}_2(t) &= a\frac{vt}{(L+\Delta L)t_0}x_1(t) + (1-a)\frac{vt}{(L+\Delta L)t_0}x_1(t-\tau(t)),\\ \dot{x}_3(t) &= \frac{vt}{t_0}\sin\left[x_2(t) + a\frac{vt}{2(L+\Delta L)}x_1(t) + (1-a)\frac{vt}{2(L+\Delta L)}x_1(t-\tau(t))\right]\end{aligned} \tag{29}$$

where $a = 0.7$, $v = -1.0$, $t = 2$, $t_0 = 0.5$, $L = 5.5$, $\ell = 2.8$ and $-0.2619 \leq \Delta L \leq 0.2895$.

Based on [26], set $d = 10t_0/\pi$ and employ the following membership function:

$$\begin{aligned}N_1(x_3(t)) &= \left(1 - \frac{1}{1+\exp(-3(x_2(t)-0.5\pi))}\right) \times \left(\frac{1}{1+\exp(-3(x_2(t)-0.5\pi))}\right),\\ N_2(x_3(t)) &= 1 - N_1(x_2(t)).\end{aligned} \tag{30}$$

Note that $N_1(x_3(t))$ and $N_2(x_3(t))$ can be interpreted as the membership functions of the fuzzy sets shown in Figure 9. Using these two fuzzy sets, the uncertain nonlinear system with time-varying delay can be represented by the following T-S fuzzy model:

Plant Rule 1: IF $x_3(t)$ is $N_1(x_3(t)) = x_2(t) + a\frac{vt}{2L}x_1(t) + (1-a)\frac{vt}{2L}x_1(t-\tau(t))$ is 0 THEN

$$\begin{aligned}\dot{x}(t) &= [A_1 + \Delta A_1]x(t) + A_{d_1}x(t-\tau(t)) + B_1 w(t) + B_2 u(t),\\ z(t) &= C_1 x(t),\end{aligned}$$

Plant Rule 2: IF $x_3(t)$ is $N_2(x_3(t)) = x_2(t) + a\frac{vt}{2L}x_1(t) + (1-a)\frac{vt}{2L}x_1(t-\tau(t))$ is $\pm\pi$ THEN

$$\begin{aligned}\dot{x}(t) &= [A_2 + \Delta A_2]\,x(t) + A_{d_2}x(t-\tau(t)) + B_1 w(t) + B_2 u(t),\\ z(t) &= C_1 x(t)\end{aligned}$$

where

$$A_1 = \begin{bmatrix} -a\frac{vt}{Lt_0} & 0 & 0\\ a\frac{vt}{Lt_0} & 0 & 0\\ -a\frac{v^2t^2}{2Lt_0} & \frac{vt}{t_0} & 0\end{bmatrix},\ A_2 = \begin{bmatrix} -a\frac{vt}{Lt_0} & 0 & 0\\ a\frac{vt}{Lt_0} & 0 & 0\\ -a\frac{dv^2t^2}{2Lt_0} & \frac{dvt}{t_0} & 0\end{bmatrix},\ B_1 = \begin{bmatrix} 0.1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},$$

$$A_{d_1} = \begin{bmatrix} -(1-a)\frac{vt}{Lt_0} & 0 & 0\\ (1-a)\frac{vt}{Lt_0} & 0 & 0\\ (1-a)\frac{v^2t^2}{2Lt_0} & 0 & 0\end{bmatrix},\ A_{d_2} = \begin{bmatrix} -(1-a)\frac{vt}{Lt_0} & 0 & 0\\ (1-a)\frac{vt}{Lt_0} & 0 & 0\\ (1-a)\frac{dv^2t^2}{2Lt_0} & 0 & 0\end{bmatrix},$$

$$B_2 = \begin{bmatrix} \frac{vt}{\ell t_0}\\ 0\\ 0\end{bmatrix},\ C_1 = \begin{bmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

with

$$\Delta A_1 = F(x(t),t)H_{1_1},\ \Delta A_2 = F(x(t),t)H_{1_2},$$

and assuming that

$$\|F(x(t),t)\| \leq \rho = 1,$$

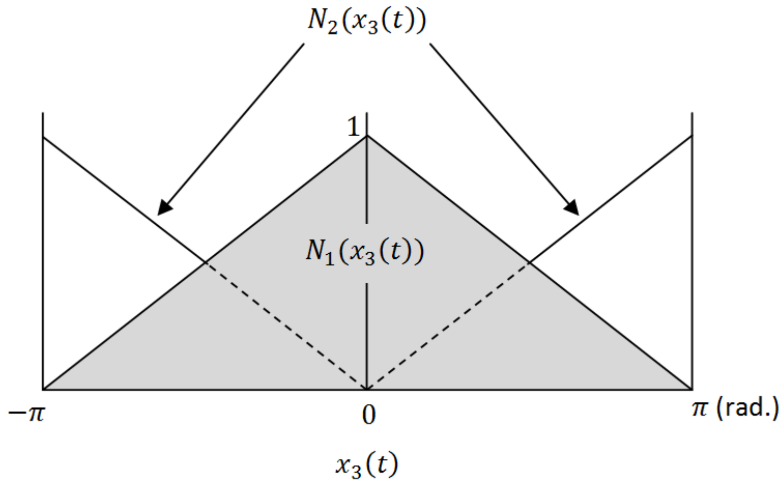

FIGURE 9. Membership functions for two fuzzy sets on Example 2

we have

$$H_{1_1} = \begin{bmatrix} -0.025 & 0 & 0 \\ 0.025 & 0 & 0 \\ 0.025 & 0 & 0 \end{bmatrix} \text{ and } H_{1_2} = \begin{bmatrix} -0.025 & 0 & 0 \\ 0.025 & 0 & 0 \\ 0.040 & 0 & 0 \end{bmatrix}.$$

Using the LMI optimization algorithm and Theorem 3.2 with $\gamma = 1$, $\tau_d = 0.5$ and $\delta = 0.1$, we obtain

$$P = \begin{bmatrix} 111.92 & -90.01 & 2.301 \\ -90.01 & 105.71 & -2.903 \\ 2.301 & -2.903 & 0.4355 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} 12028 & -13126 & 363 \end{bmatrix},\ Y_2 = \begin{bmatrix} 11486 & -12523 & 347 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} 24.2159 & -103.0456 & 18.7418 \end{bmatrix} \text{ and } K_2 = \begin{bmatrix} 23.4130 & -97.9918 & 19.9306 \end{bmatrix}.$$

The resulting fuzzy controller is

$$u(t) = \sum_{j=1}^{2} \bar{h}_j \left(-K_j \dot{x}(t)\right), \tag{31}$$

where $\bar{h}_1 = N_1(x_3(t))$ and $\bar{h}_2 = N_2(x_3(t))$.

Remark 4.4. *The fuzzy controller (31) ensures that the inequality (9) holds. Figures 10, 11 and 6 illustrate the state variables, $x_1(t)$, $x_2(t)$, and $x_3(t)$, of the open-loop system, the proposed Theorem 3.2 and the disturbance input, $w(t)$, which was used during the simulation, respectively. As depicted in Figure 12 after 1.8 seconds, the ratio of the regulated output energy to the disturbance input noise energy approaches a constant value that is less than the prescribed value of 1. These results satisfy design requirements that ensure that the closed-loop system is asymptotically stable and that the pre-prescribed performance index is satisfied.*

Remark 4.5. *According to the robust state-feedback controller design of Theorem 3.1, used in* [27]*, the robust delay control of Theorem 3, used in* [25] *and Theorem 3.2 used in this paper, Figure 13 presents comparative results for the state variable $x_3(t)$ at the same $\gamma = 1$ for the allowed delay $\tau = 2.00$ and $\Delta L = 0.15$. Figure 13 illustrates that Theorem 3.2 used in this study generates a response 40% and 50% faster than that shown in* [27] *and* [25]*. This shows that the uncertain nonlinear system with time-varying delays is effectively controlled using the proposed F-PPC approach.*

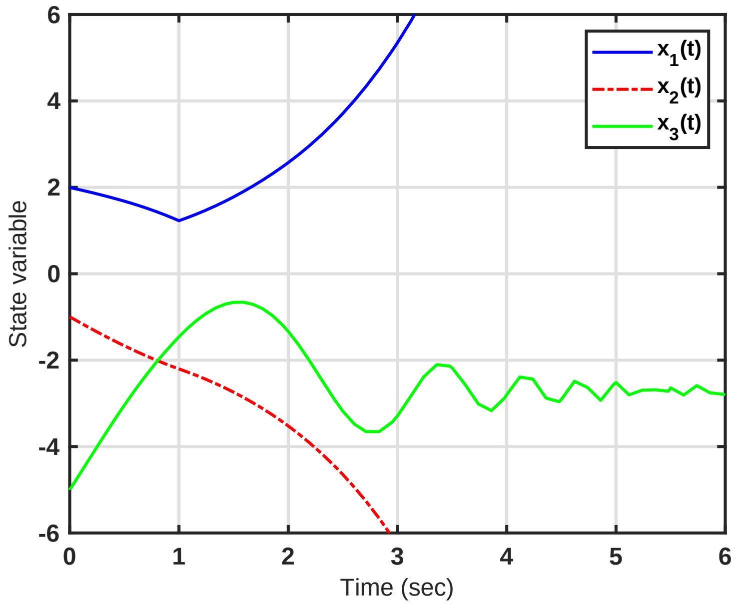

FIGURE 10. State variables of the open-loop system

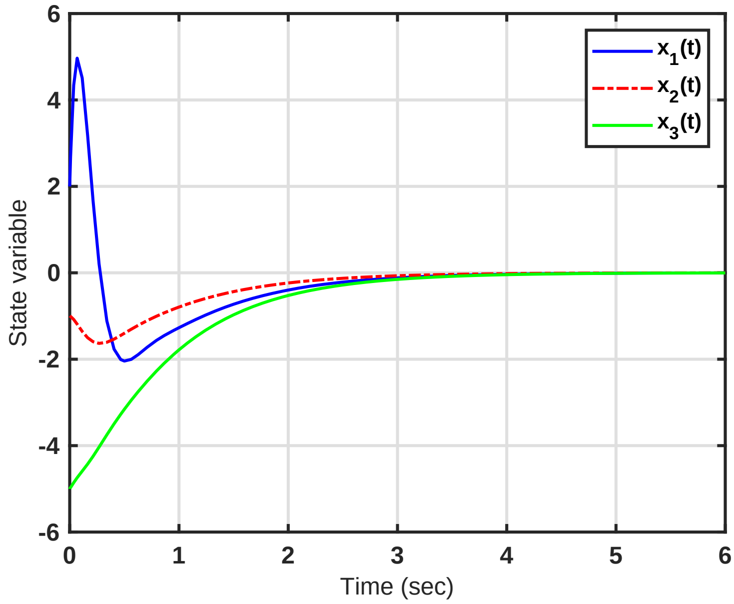

FIGURE 11. State variables of the closed-loop system

Remark 4.6. *The maximum allowable delay τ for the same $\gamma = 1$ and different bounds of ΔL for Example 2 are tabulated in Table 1. Under different values of ΔL, the second and third column of Table 1 show the case of $\Delta L = 0.0000$ and 0.0125, respectively. It is seen that except for [19], reference methods, including those proposed Theorem 3.2 in this paper, give the results no outstandingly better than others. When we consider the maximum allowed delay τ derived from [19, 25, 27] and the proposed Theorem 3.2 in this paper under ΔL value of 0.0425 and 0.0725, it is seen from Table 1 that the results of $\tau > 2.0000$ obtained from the proposed Theorem 3.2 are significantly better than those obtained from the other methods. Thus, this result shows that the uncertain nonlinear*

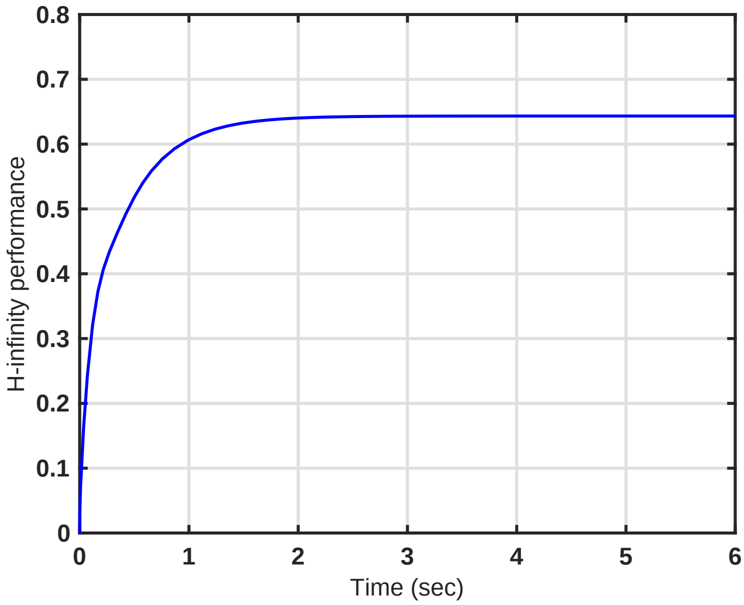

Figure 12. Performance of F-PPC on Example 2

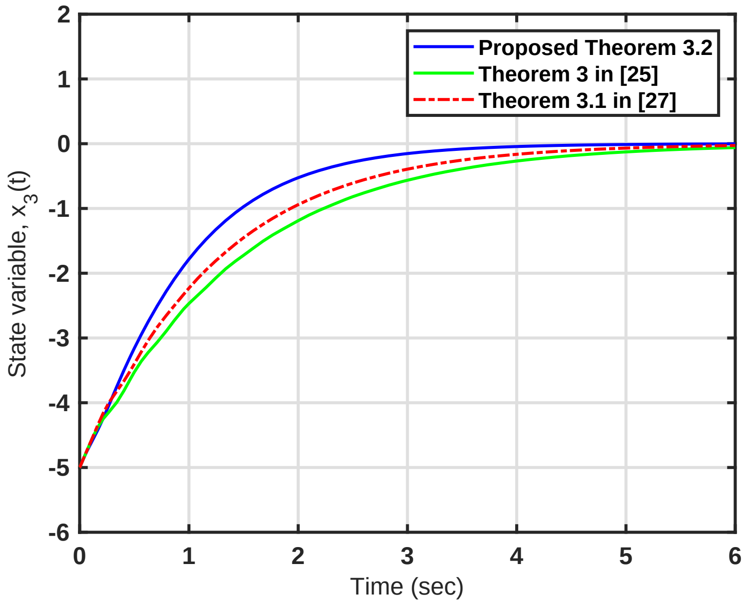

Figure 13. Comparison of state variable

system with time-varying delay has been effectively controlled using the proposed F-PPC design.

Remark 4.7. *As result of Example 2, the good effectiveness of F-PPC for time-varying delay system with uncertain parameter has been presented. Theorem 3.2 satisfies the design objective on both the pre-prescribed performance index and the closed-loop system be asymptotically stable. In computation viewpoints, the proposed design approach is aggregated to examine a set of LMI in conjunction with the T-S fuzzy model approach. The convex optimization algorithm is employed to quickly solve the LMI problem. Therefore,*

TABLE 1. Comparison of the maximum allowed delay on Example 2, τ

ΔL	0.0000	0.0125	0.0425	0.0725
τ via approach in [25]	2.0550	2.0421	1.8843	1.6054
τ via approach in [19]	1.9677	1.9650	1.2633	1.0262
τ via approach in [27]	2.1014	2.0556	1.9143	1.8891
τ via proposed Theorem 3.2	2.2898	2.2363	2.1907	2.0475

it reduces the design costs associated with the practical use of theoretical outcomes. In addition, the F-PPC gains are able to directly apply to the controller for such a system.

5. **Conclusion.** This paper has proposed F-PPC design procedure for the problem of robust control for time-varying delay systems with uncertain parameters and disturbances which have the inexactly measured state. Based on the LMI approach, the LMI-based sufficient conditions for the uncertain T-S fuzzy model with a prescribed performance are established. The main results prove that the proposed methodology guarantees that the asymptotical stability and the L_2 gain from an exogenous input to a regulated output is less than or equal to a prescribed value. The effectiveness of the proposed design is demonstrated via the nonlinear benchmark examples. The results have been illustrated the benefits and applicability of the proposed F-PPC. However, the failure of components with delay-dependent constraint can be easily found in many real physical control problems. Therefore, the robust F-PPC with D-stability constraints for an uncertain nonlinear system with time-varying delay can be investigated in future research work. Additionally, applications of the proposed approach to uncertain physical systems such as wind energy conversion control systems, photovoltaic control systems and communication systems, will be also examined in the future work.

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REFERENCES

[1] S. K. Nguang, W. Assawinchaichote and P. Shi, H_∞ filter for uncertain Markovian jump nonlinear systems: An LMI approach, *Circuits Syst. Signal Process.*, vol.26, no.6, pp.853-874, 2007.

[2] W. Assawinchaichote, A non-fragile H_∞ output feedback controller for uncertain fuzzy dynamical systems with multiple time-scales, *Int. J. Compu. Comm. Control*, vol.7, no.1, pp.8-19, 2012.

[3] L. Liu, B. Wang, S. Wang, Y. Chen, T. Hayat and F. E. Alsaadi, Finite-time H-infinity control of a fractional-order hydraulic turbine governing system, *IEEE Access*, vol.6, pp.57507-57517, 2018.

[4] H. Waki, Positive definiteness in linear matrix inequality problem for H-infinity output feedback control problem, *IEEE Trans. Control Syst. Letters*, vol.3, no.2, pp.446-451, 2019.

[5] A. D. Rosaline and U. Somarajan, Structured H-infinity controller for an uncertain deregulated power system, *IEEE Trans. Industry Appl.*, vol.55, no.1, pp.892-906, 2019.

[6] M. Khanesar, O. Kaynak, S. Yin and H. Gao, Adaptive indirect fuzzy sliding mode controller for networked control systems subject to time-varying network-induced time delay, *IEEE Trans. Fuzzy Syst.*, vol.23, no.1, pp.205-214, 2015.

[7] M. Hamdy, S. Abd-Elhaleem and M. A. Fkirin, Time-varying delay compensation for a class of nonlinear control systems over network via H_∞ adaptive fuzzy controller, *IEEE Trans. Syst. Man Cybern.: Syst.*, vol.47, no.8, pp.2114-2123, 2017.

[8] Q. Li and H. Lin, Effects of mixed-modes on the stability analysis of switched time-varying delay systems, *IEEE Trans. Autom. Control*, vol.61, no.10, pp.3038-3044, 2016.

[9] Y. Liu, F. Fang and J. H. Park, Decentralized dissipative filtering for delayed nonlinear interconnected systems based on T-S fuzzy model, *IEEE Trans. Fuzzy Syst.*, vol.27, no.4, pp.790-801, 2019.

[10] L. Yin and W. Wang, The comparison of semi-parametric trending panel data models and non-parametric time-varying coefficient panel data models, *ICIC Express Letters*, vol.12, no.11, pp.1163-1169, 2018.

[11] K. Tanaka and H. O. Wang, *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*, John Wiley and Sons, NY, USA, 2001.

[12] N. Chayaopas and W. Assawinchaichote, A novel approach to robust H_∞ integral control for TS fuzzy systems, *Comput. Appl. Math.*, vol.37, no.2, pp.954-977, 2018.

[13] H. Wang, X. Liu and P. Shi, Observer-based fuzzy adaptive output-feedback control of stochastic nonlinear multiple time-delay systems, *IEEE Trans. Cybern.*, vol.47, no.9, pp.2568-2578, 2017.

[14] S. Dong, Z. Wu, P. Shi, H. Su and R. Lu, Reliable control of fuzzy systems with quantization and switched actuator failures, *IEEE Trans. Syst. Man Cybern.: Syst.*, vol.47, no.8, pp.2198-2208, 2017.

[15] H. Yazici and M. Sever, L_2 gain state derivative feedback control of uncertain vehicle suspension system, *J. Vibrat. Control*, vol.24, no.16, pp.3779-3794, 2018.

[16] T. H. S. Abdelaziz, Robust pole assignment using velocity-acceleration feedback for second-order dynamical systems with singular mass matrix, *ISA Trans.*, vol.57, pp.71-84, 2015.

[17] J. M. Araujo, C. E. T. Dorea, L. M. G. Goncalves and B. N. Datta, State derivative feedback in second-order linear systems: A comparative analysis of perturbed eigenvalues under coefficient variation, *Mech. Syst. Signal Processing*, vols.76-77, pp.33-46, 2016.

[18] P. Albertos and I. Mareels, *Feedback and Control for Everyone*, Springer, Berlin, Heidelberg, 2010.

[19] N. Krewpraek and W. Assawinchaichote, H_∞ fuzzy state-feedback control plus state-derivative feedback control synthesis for the photovoltaic system, *Asian J. Control*, vol.18, no.4, pp.1441-1452, 2016.

[20] S. Ruangsang and W. Assawinchaichote, A novel robust H_∞ fuzzy state feedback plus state-derivative feedback controller design for nonlinear time-varying delay systems, *Neural Comput. Applic.*, vol.31, no.10, pp.6303-6318, 2019.

[21] C. Lin, Q. Wang, T. Lee and Y. He, Design of observer-based H_∞ control for fuzzy time-delay systems, *IEEE Trans. Fuzzy Syst.*, vol.16, no.2, pp.534-543, 2008.

[22] H. J. Marquez, *Nonlinear Control Systems Analysis and Design*, John Wiley and Sons, NJ, 2003.

[23] W. Assawinchaichote, S. K. Nguang, P. Shi and E. Boukas, H_∞ fuzzy state-feedback control design for nonlinear systems with D-stability constraints: An LMI approach, *Math. Comput. in Simu.*, vol.78, pp.514-531, 2008.

[24] W. Assawinchaichote, Synthesis of a robust H_∞ fuzzy controller for uncertain nonlinear dynamical systems, in *Fuzzy Controllers, Theory and Applications*, L. Grigorie (ed.), In-Tech Education and Publishing, Vienna, 2011.

[25] C. Peng, D. Yue and Y. Tian, New approach on robust delay dependent: H_∞ control for uncertain T-S fuzzy systems with interval time varying delay, *IEEE. Trans. Fuzzy Syst.*, vol.17, no.4, pp.890-900, 2009.

[26] Y. Cao and P. M. Frank, Stability analysis and synthesis of nonlinear time-delay systems via linear Takagi-Sugeno fuzzy models, *Fuzzy Sets Syst.*, vol.124, pp.213-229, 2001.

[27] W. Assawinchaichote, LMIs approach based robust H_∞ fuzzy controller design for nonlinear time-varying delay systems, in *Control Theory: Perspectives, Applications and Developments*, F. Miranda (ed.), Nova Science Publishers, NY, USA, 2015.

[28] W. Assawinchaichote and S. K. Nguang, Fuzzy observer-based controller design for singularly perturbed nonlinear systems: An LMI approach, *Proc. of the 41st IEEE Conf. Decision and Control*, Las Vegas, USA, pp.2165-2170, 2002.

[29] S. K. Nguang, W. Assawinchaichote, P. Shi and Y. Shi, H_∞ fuzzy filter design for uncertain nonlinear systems with Markovian jumps: An LMI approach, *Proc. of American Control Conf. (ACC)*, Portland, USA, pp.1799-1804, 2005.

[30] T. Samakwong and W. Assawinchaichote, PID controller design for electro-hydraulic servo valve system with genetic algorithm, *Proc. of Int. Electrical Engineering Congress (iEECON)*, Chiang Mai, Thailand, pp.91-94, 2016.

[31] A. Sungthonga and W. Assawinchaichote, Particle swam optimization based optimal PID parameters for air heater temperature control system, *Proc. of Int. Electrical Engineering Congress (iEECON)*, Chiang Mai, Thailand, pp.108-111, 2016.

[32] S. Fallah, A. Khajepour, B. Fidan, S.-K. Chen and B. Litkouhi, Vehicle optimal torque vectoring using state-derivative feedback and linear matrix inequality, *IEEE. Trans. Vehicular Technology*, vol.62, no.4, pp.1540-1552, 2013.

[33] W. Michiels, T. Vyhlidal, H. Huijberts and H. Nijmeijer, Stabilizability and stability robustness of state derivative feedback controllers, *SIAM J. Control Optim.*, vol.47, no.6, pp.3100-3117, 2009.
[34] H. Dallali, G. A. Medrano-Cerda, M. Focchi, T. Boaventura, M. Frigerio, C. Semini, J. Buchli and D. G. Caldwell, On the use of positive feedback for improved torque control, *Control Theory Tech.*, vol.13, no.3, pp.266-285, 2015.
[35] N. Rotella, S. Mason, S. Schaal and L. Righetti, Inertial sensor-based humanoid joint state estimation, *Proc. of IEEE Int. Conf. Robotics and Automation (ICRA)*, Stockholm, Sweden, pp.1825-1831, 2016.
[36] A. Herzog, N. Rotella, S. Mason, F. Grimminger, S. Schaal and L. Righetti, Momentum control with hierarchical inverse dynamics on a torque-controlled humanoid, *Autonomous Robots*, vol.40, no.3, pp.473-491, 2016.

Appendix 1. Proof of Theorem 3.1.

Proof: Firstly, for considering of the case of uncertainty systems without any delay, (14) can be rewritten as

$$\dot{x}(t) = \sum_{i=1}^{l}\sum_{j=1}^{l}\bar{h}_i\bar{h}_j\left[E_{ij}A_ix(t) + E_{ij}\tilde{B}_{w_i}\tilde{w}(t)\right]. \tag{32}$$

Let us consider a Lyapunov function

$$V(x(t)) = x^T(t)Qx(t) \tag{33}$$

where $Q = P^{-1} > 0$. Taking the derivative of $V(x(t))$ along the closed-loop system (32), we have

$$\begin{aligned}\dot{V}(x(t)) = \sum_{i=1}^{l}\sum_{j=1}^{l}\bar{h}_i\bar{h}_j\Big[&x^T(t)\left({A_i}^TE_{ij}^TQ + QE_{ij}A_i\right)x(t) \\ &+ \tilde{w}^T(t)\tilde{B}_{w_i}^TE_{ij}^TQx(t) + x^T(t)QE_{ij}\tilde{B}_{w_i}\tilde{w}(t)\Big].\end{aligned} \tag{34}$$

Adding and subtracting the following

$$-\tilde{z}^T(t)\tilde{z}(t) + \gamma^2\sum_{i=1}^{l}\sum_{j=1}^{l}\sum_{m=1}^{l}\sum_{n=1}^{l}\bar{h}_i\bar{h}_j\bar{h}_m\bar{h}_n\left[\tilde{w}^T(t)\tilde{w}(t)\right] \tag{35}$$

to and from (34), we acquire

$$\begin{aligned}\dot{V}(x(t)) = &\sum_{i=1}^{l}\sum_{j=1}^{l}\sum_{m=1}^{l}\sum_{n=1}^{l}\bar{h}_i\bar{h}_j\bar{h}_m\bar{h}_n\left[\begin{array}{cc} x^T(t) & \tilde{w}^T(t)\end{array}\right] \\ &\times\left(\begin{array}{cc}\left({A_i}^TE_{ij}^TQ + QE_{ij}A_i + \tilde{C}_i^T\tilde{C}_i\right) & (*)^T \\ \tilde{B}_{w_i}^TE_{ij}^TQ & -\gamma^2I\end{array}\right)\left[\begin{array}{c} x(t) \\ \tilde{w}(t)\end{array}\right] \\ &-\tilde{z}^T(t)\tilde{z}(t) + \gamma^2\sum_{i=1}^{l}\sum_{j=1}^{l}\sum_{m=1}^{l}\sum_{n=1}^{l}\bar{h}_i\bar{h}_j\bar{h}_m\bar{h}_n\left[\tilde{w}^T(t)\tilde{w}(t)\right]\end{aligned} \tag{36}$$

where

$$\tilde{z}(t) = \sum_{i=1}^{l}\sum_{j=1}^{l}\bar{h}_i\bar{h}_j\tilde{C}_ix(t). \tag{37}$$

Next, let us consider Theorem 3.1. By substituting Φ_{ij}, we rearrange with $K_j = Y_jP^{-1}$, then it yields

$$\left(\begin{array}{ccc}\left(\begin{array}{c}(I + B_iK_j)PA_i^T \\ +A_iP(I + B_iK_j)^T\end{array}\right) & (*)^T & (*)^T \\ \tilde{B}_{w_i}^T & -\gamma^2I & (*)^T \\ \tilde{C}_iP(I + B_iK_j)^T & 0 & -I\end{array}\right) < 0. \tag{38}$$

Referring to the Remark 3.1, and pre- and post-multiplying by $\begin{pmatrix} (I+B_iK_j)^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}$ and $\begin{pmatrix} (I+B_iK_j)^{-T} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}$, respectively, we get

$$\begin{pmatrix} \left(PA_i{}^T E_{ij}^T + E_{ij} A_i P \right) & (*)^T & (*)^T \\ \tilde{B}_{w_i}^T E_{ij}^T & -\gamma^2 I & (*)^T \\ \tilde{C}_i P & 0 & -I \end{pmatrix} < 0. \tag{39}$$

By multiplying both sides of (39) by $\begin{pmatrix} Q & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}$, we obtain

$$\begin{pmatrix} \left(A_i{}^T E_{ii}^T Q + Q E_{ii} A_i \right) & (*)^T & (*)^T \\ \tilde{B}_{w_i}^T E_{ii}^T Q & -\gamma^2 I & (*)^T \\ \tilde{C}_i & 0 & -I \end{pmatrix} < 0, \tag{40}$$

$i = 1, 2, 3, \ldots, l$ and

$$\begin{aligned} &\begin{pmatrix} \left(A_i{}^T E_{ij}^T Q + Q E_{ij} A_i \right) & (*)^T & (*)^T \\ \tilde{B}_{w_i}^T E_{ij}^T Q & -\gamma^2 I & (*)^T \\ \tilde{C}_i & 0 & -I \end{pmatrix} \\ &+ \begin{pmatrix} \left(A_j{}^T E_{ji}^T Q + Q E_{ji} A_j \right) & (*)^T & (*)^T \\ \tilde{B}_{w_i}^T E_{ji}^T Q & -\gamma^2 I & (*)^T \\ \tilde{C}_j & 0 & -I \end{pmatrix} < 0, \end{aligned} \tag{41}$$

$i < j \leq l$. Applying the Schur complement to (40)-(41) and rearranging them, we then have

$$\begin{pmatrix} \left(A_i{}^T E_{ii}^T Q + Q E_{ii} A_i + \tilde{C}_i^T \tilde{C}_i \right) & (*)^T \\ \tilde{B}_{w_i}^T E_{ii}^T Q & -\gamma^2 I \end{pmatrix} < 0, \tag{42}$$

$i = 1, 2, 3 \ldots, l$, and

$$\begin{aligned} &\begin{pmatrix} \left(A_i{}^T E_{ij}^T Q + Q E_{ij} A_i + \tilde{C}_i^T \tilde{C}_i \right) & (*)^T \\ \tilde{B}_{w_i}^T E_{ij}^T Q & -\gamma^2 I \end{pmatrix} \\ &+ \begin{pmatrix} \left(A_j{}^T E_{ji}^T Q + Q E_{ji} A_j + \tilde{C}_j^T \tilde{C}_j \right) & (*)^T \\ \tilde{B}_{w_i}^T E_{ji}^T Q & -\gamma^2 I \end{pmatrix} < 0, \end{aligned} \tag{43}$$

$i < j \leq l$. From the fact that

$$\sum_{i=1}^{l} \sum_{j=1}^{l} \sum_{m=1}^{l} \sum_{n=1}^{l} \bar{h}_i \bar{h}_j \bar{h}_m \bar{h}_n M_{ij}^T N_{mn} \leq \frac{1}{2} \sum_{i=1}^{l} \sum_{j=1}^{l} \bar{h}_i \bar{h}_j \left[M_{ij}^T M_{ij} + N_{ij} N_{ij}^T \right], \tag{44}$$

(42)-(43) become

$$\begin{pmatrix} \left(A_i{}^T E_{ij}^T Q + Q E_{ij} A_i + \tilde{C}_i^T \tilde{C}_i \right) & (*)^T \\ \tilde{B}_{w_i}^T E_{ij}^T Q & -\gamma^2 I \end{pmatrix} < 0 \tag{45}$$

where $i, j = 1, 2, \ldots, l$. Since (45) is less than zero and because $\bar{h}_n \geq 0$ and $\sum_{n=1}^{l} \bar{h}_n = 1$, then (36) becomes

$$\dot{V}(x(t)) \leq -\tilde{z}^T(t)\tilde{z}(t) + \gamma^2 \sum_{i=1}^{l}\sum_{j=1}^{l}\sum_{m=1}^{l}\sum_{n=1}^{l} \bar{h}_i\bar{h}_j\bar{h}_m\bar{h}_n[\tilde{w}^T(t)\tilde{w}(t)]. \tag{46}$$

Integrating both sides of (46) yields

$$V(x(T_f)) - V(x(0)) \leq \int_0^{T_f} \left[-\tilde{z}^T(t)\tilde{z}(t) + \gamma^2 \sum_{i=1}^{l}\sum_{j=1}^{l}\sum_{m=1}^{l}\sum_{n=1}^{l} \bar{h}_i\bar{h}_j\bar{h}_m\bar{h}_n \left[\tilde{w}^T(t)\tilde{w}(t)\right] \right] dt. \tag{47}$$

Because $V(x(0)) = 0$ and $V(x(T_f)) \geq 0$ for all $T_f \neq 0$, we obtain

$$\int_0^{T_f} \tilde{z}^T(t)\tilde{z}(t)dt \leq \gamma^2 \left[\int_0^{T_f} \sum_{i=1}^{l}\sum_{j=1}^{l}\sum_{m=1}^{l}\sum_{n=1}^{l} \bar{h}_i\bar{h}_j\bar{h}_m\bar{h}_n \left[\tilde{w}^T(t)\tilde{w}(t)\right] dt \right]. \tag{48}$$

By inserting $\tilde{z}(t)$, $\tilde{w}(t)$, with referring the fact (44) and because $\|F(x(t),t)\| \leq \rho$, we have

$$\begin{aligned} &\int_0^{T_f} \sum_{i=1}^{l}\sum_{j=1}^{l} \bar{h}_i\bar{h}_j \left(2\lambda^2 x^T(t)C_i^T C_i x(t) + 2\lambda^2\rho^2 x^T(t)H_{3_i}^T H_{3_i} x(t)\right) dt \\ &\leq \gamma^2 \left[\int_0^{T_f} \sum_{i=1}^{l}\sum_{j=1}^{l} \bar{h}_i\bar{h}_j \left[w^T(t)w(t)\right] dt + \rho^2 \int_0^{T_f} \sum_{i=1}^{l}\sum_{j=1}^{l} \bar{h}_i\bar{h}_j \left[w^T(t)H_{2_i}^T H_{2_i} w(t)\right] dt \right], \end{aligned} \tag{49}$$

and using $\lambda^2 = 1 + \rho^2 \sum_{i=1}^{l}\sum_{j=1}^{l} \left[\| H_{2_i}^T H_{2_j} \|\right]$, we obtain

$$\begin{aligned} &\int_0^{T_f} \sum_{i=1}^{l}\sum_{j=1}^{l} \bar{h}_i\bar{h}_j \left(2\lambda^2 x^T(t)C_i^T C_i x(t) + 2\lambda^2\rho^2 x^T(t)H_{3_i}^T H_{3_i} x(t)\right) dt \\ &\leq \gamma^2\lambda^2 \left[\int_0^{T_f} \left[w^T(t)w(t)\right] dt \right]. \end{aligned} \tag{50}$$

Adding and subtracting

$$\lambda^2 z^T(t)z(t) = \lambda^2 \sum_{i=1}^{l}\sum_{j=1}^{l} \bar{h}_i\bar{h}_j \left(x^T(t)(C_i + F(x(t),t)H_{3_i})^T (C_i + F(x(t),t)H_{3_i})x(t) \right) \tag{51}$$

to and from (50), one obtains

$$\begin{aligned} &\int_0^{T_f} \Bigg[\lambda^2 z^T(t)z(t) + \sum_{i=1}^{l}\sum_{j=1}^{l} \bar{h}_i\bar{h}_j \Big[\left(2\lambda^2 x^T(t)C_i^T C_i x(t) + 2\lambda^2\rho^2 x^T(t)H_{3_i}^T H_{3_i} x(t)\right) \\ &- \left(\lambda^2 \left(x^T(t)(C_i + F(x(t),t)H_{3_i})^T (C_i + F(x(t),t)H_{3_i})x(t) \right)\right) \Big] \Bigg] dt \\ &\leq \gamma^2\lambda^2 \left[\int_0^{T_f} \left[w^T(t)w(t)\right] dt \right]. \end{aligned} \tag{52}$$

Using the triangular inequality and the fact that $\|F(x(t),t)\| \leq \rho$, we have

$$\begin{aligned}&\lambda^2 \sum_{i=1}^{l}\sum_{j=1}^{l}\bar{h}_i\bar{h}_j\left[\left(x^T(t)(C_i+F(x(t),t)H_{3_i})^T(C_i+F(x(t),t)H_{3_i})x(t)\right)\right]\\ &\leq \sum_{i=1}^{l}\sum_{j=1}^{l}\bar{h}_i\bar{h}_j\left[2\lambda^2x^T(t)C_i^TC_ix(t)+2\lambda^2\rho^2x^T(t)H_{3_i}^TH_{3_i}x(t)\right].\end{aligned} \tag{53}$$

Substituting (53) into (52), we obtain

$$\int_0^{T_f} z^T(t)z(t)dt \leq \gamma^2\left[\int_0^{T_f} w^T(t)w(t)dt\right]. \tag{54}$$

Hence, the inequality (9) holds. When $w(t)=0$, (48) becomes $\dot{V}(t) \leq -z^T(t)z(t) \leq 0$. Therefore, the system (32) is asymptotically stable, and (b) in the Remark 3.2 is achieved. This completes the proof. □

Appendix 2. Proof of Theorem 3.2.

Proof: Let us choose the quadratic Lyapunov-Krasovskii functional $V(x(t))$:

$$V(x(t)) = x^T(t)Qx(t)+\beta\int_{t-\tau(t)}^{t} x^T(V)Sx(V)dV \tag{55}$$

where $Q=P^{-1}>0$, $S=W^{-1}>0$ and $\beta=\frac{1}{1-\tau_d}$. Taking the derivative of $V(x(t))$ along the closed-loop system (14), and because for any vector $x(t)$ and $x(t-\tau(t))$ and a matrix G,

$$\begin{aligned}&x^T(t)Gx(t-\tau(t))+x^T(t-\tau(t))G^Tx(t)\\ &\leq x^T(t)GR^{-1}G^Tx(t)+x^T(t-\tau(t))Rx(t-\tau(t))\end{aligned} \tag{56}$$

where R is a positive definite matrix, we have

$$\begin{aligned}\dot{V}(x(t)) &= \sum_{i=1}^{l}\sum_{j=1}^{l}\bar{h}_i\bar{h}_j\Big[x^T(t)\left(A_i{}^TE_{ij}^TQ+QE_{ij}A_i+\beta S\right)x(t)\\ &\quad + x^T(t-\tau(t))A_{d_i}^TE_{ij}^TQx(t)+x^T(t)QE_{ij}A_{d_i}x(t-\tau(t))\\ &\quad - \beta\left(1-\dot{\tau}(t)\right)x^T(t-\tau(t))Sx(t-\tau(t))\\ &\quad + \tilde{w}^T(t)\tilde{B}_{w_i}^TE_{ij}^TQx(t)+x^T(t)QE_{ij}\tilde{B}_{w_i}\tilde{w}(t)\Big]\\ &\leq \sum_{i=1}^{l}\sum_{j=1}^{l}\bar{h}_i\bar{h}_j\Big[x^T(t)\left(A_i{}^TE_{ij}^TQ+QE_{ij}A_i+\beta S\right)x(t)\\ &\quad + x^T(t)QE_{ij}A_{d_i}S^{-1}A_{d_i}^TE_{ij}^TQx(t)\\ &\quad + x^T(t-\tau(t))Sx(t-(t))-x^T(t-\tau(t))Sx(t-\tau(t))\\ &\quad + \tilde{w}^T(t)\tilde{B}_{w_i}^TE_{ij}^TQx(t)+x^T(t)QE_{ij}\tilde{B}_{w_i}\tilde{w}(t)\Big]\\ &= \sum_{i=1}^{l}\sum_{j=1}^{l}\bar{h}_i\bar{h}_j\Big[x^T(t)\big(A_i{}^TE_{ij}^TQ+QE_{ij}A_i\\ &\quad + QE_{ij}A_{d_i}S^{-1}A_{d_i}^TE_{ij}^TQ+\beta S\big)x(t)\\ &\quad + \tilde{w}^T(t)\tilde{B}_{w_i}^TE_{ij}^TQx(t)+x^T(t)QE_{ij}\tilde{B}_{w_i}\tilde{w}(t)\Big].\end{aligned} \tag{57}$$

Adding and subtracting $-\tilde{z}^T(t)\tilde{z}(t)+\gamma^2\sum\limits_{i=1}^{l}\sum\limits_{j=1}^{l}\sum\limits_{m=1}^{l}\sum\limits_{n=1}^{l}\bar{h}_i\bar{h}_j\bar{h}_m\bar{h}_n\left[\tilde{w}^T(t)\tilde{w}(t)\right]$ to and from (57), we obtain

$$\begin{aligned}\dot{V}(x(t)) &= \sum_{i=1}^{l}\sum_{j=1}^{l}\sum_{m=1}^{l}\sum_{n=1}^{l}\bar{h}_i\bar{h}_j\bar{h}_m\bar{h}_n\left[\begin{array}{cc} x^T(t) & \tilde{w}^T(t)\end{array}\right]\\ &\times\left(\begin{array}{cc}\left(\begin{array}{c} A_i^TE_{ij}^TQ+QE_{ij}A_i+\beta S\\ +QE_{ij}A_{d_i}S^{-1}A_{d_i}^TE_{ij}^TQ+C_i^TC_i\end{array}\right) & (*)^T\\ \tilde{B}_{w_i}^TE_{ij}^TQ & -\gamma^2 I\end{array}\right)\left[\begin{array}{c} x(t)\\ \tilde{w}(t)\end{array}\right]\\ &-\tilde{z}^T(t)\tilde{z}(t)+\gamma^2\sum_{i=1}^{l}\sum_{j=1}^{l}\sum_{m=1}^{l}\sum_{n=1}^{l}\bar{h}_i\bar{h}_j\bar{h}_m\bar{h}_n\left[\tilde{w}^T(t)\tilde{w}(t)\right]\end{aligned}\tag{58}$$

where

$$\tilde{z}(t)=\sum_{i=1}^{l}\sum_{j=1}^{l}\bar{h}_i\bar{h}_j\tilde{C}_ix(t).\tag{59}$$

Next, let us consider Theorem 3.2; we have

$$\left(\begin{array}{cccc}\left(\begin{array}{c} PA_i^T+A_iP+B_iY_jA_i^T\\ +A_iY_j^TB_i^T+A_{d_i}WA_{d_i}^T\end{array}\right) & (*)^T & (*)^T & (*)^T\\ \tilde{B}_{w_i}^T & -\gamma^2 I & (*)^T & (*)^T\\ P+Y_j^TB_i^T & 0 & -\frac{1}{\beta}W & (*)^T\\ \tilde{C}_iP+\tilde{C}_iY_j^TB_i^T & 0 & 0 & -I\end{array}\right)<0.\tag{60}$$

Now, by employing the same technique used in the proof of Theorem 3.1, we obtain

$$\left(\begin{array}{cc}\left(\begin{array}{c} A_i^TE_{ij}^TQ+QE_{ij}A_i+\tilde{C}_i^T\tilde{C}_i\\ +QE_{ij}A_{d_i}S^{-1}A_{d_i}^TE_{ij}^TQ+\beta S\end{array}\right) & (*)^T\\ \tilde{B}_{w_i}^TE_{ij}^TQ & -\gamma^2 I\end{array}\right)<0\tag{61}$$

where $W=S^{-1}$ and $i,j=1,2,\ldots,r$. Since (61) is less than zero, and because $\bar{h}_n\ge 0$ and $\sum\limits_{n=1}^{l}\bar{h}_n=1$, then (58) becomes

$$\dot{V}(x(t))\le -\tilde{z}^T(t)\tilde{z}(t)+\gamma^2\sum_{i=1}^{l}\sum_{j=1}^{l}\sum_{m=1}^{l}\sum_{n=1}^{l}\bar{h}_i\bar{h}_j\bar{h}_m\bar{h}_n\left[\tilde{w}^T(t)\tilde{w}(t)\right].\tag{62}$$

Integrating both sides of (62); by referring that $V(x(0))=0$ and $V(x(T_f))\ge 0$ for all $T_f\ne 0$, we obtain

$$\int_0^{T_f}\tilde{z}^T(t)\tilde{z}(t)dt\le\gamma^2\left[\int_0^{T_f}\sum_{i=1}^{l}\sum_{j=1}^{l}\sum_{m=1}^{l}\sum_{n=1}^{l}\bar{h}_i\bar{h}_j\bar{h}_m\bar{h}_n\left[\tilde{w}^T(t)\tilde{w}(t)\right]dt\right]\tag{63}$$

and we then derive

$$\int_0^{T_f}z^T(t)z(t)dt\le\gamma^2\left[\int_0^{T_f}w^T(t)w(t)dt\right].\tag{64}$$

Hence, the inequality (9) holds. When $w(t)=0$, (63) becomes $\dot{V}(t)\le -z^T(t)z(t)\le 0$. Therefore, the system (14) is asymptotically stable, and (b) in Remark 3.2 is achieved. This completes the proof. □