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IMPROVED ROBUST ABSOLUTE STABILITY OF TIME-DELAYED LUR'E SYSTEMS

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Abstract. *In this paper, we focus on the problem of the absolute and robust absolute stability for time-delayed Lur'e systems. Both the cases with time-invariant and time-varying nonlinearities are considered. By using two improved relaxed integral inequality lemmas, new delay-dependent absolute and robust absolute stability criteria are proposed via Lyapunov-Krasovskii functional (LKF) approach. The stability conditions can be expressed as convex linear matrix inequality (LMI) framework, which can be solved by using standard LMI convex optimization solvers. The criteria proposed in this paper are less conservative than some previous ones. Finally, some numerical examples are presented to show the effectiveness of the proposed approach.*

Keywords: Lur'e nonlinear system, Lyapunov-Krasovskii functional, Robust absolute stability, Time-varying delay

1. **Introduction.** Time-delay is often attributed as the major source of poor performance and instability. Lots of researchers have focused on the stability problems of time-delayed systems. Thereby, effective methods have been developed to derive less conservative delay-dependent stability criteria for time-delayed systems [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15], such as, free-weighting matrix method [1, 16], piecewise analysis method [1, 4, 17], reciprocally convex method [5], Wirtinger-based integral inequality approach [6, 14], combined convex technique [7], integral inequality approach and delay decomposed approach [2, 3, 18, 19, 20, 21], free-matrix-based double integral inequality [12, 13].

Most systems are nonlinear in practical engineering. It is well known that many of certain nonlinear systems can be modeled as Lur'e system, which consists of a feedback connection of a linear dynamical system and a nonlinearity satisfying the sector bounded condition [22]. It plays an important role in many practical projects, such as the Chua's circuit, the Lorenz system, spacecraft control, communications, machinery design and other fields. Meanwhile, the stability problems for Lur'e systems have been considered extensively [23, 24, 25, 26, 27, 28, 29], where lots of significant robust stability criteria had been given. In [30, 31], Han et al. investigate the absolute stability of Lur'e systems with constant delay and time-varying delay, respectively. In [32], to obtain a less conservative condition for Lur'e systems with time-varying delays, improved free-matrix-weighting (IFMW) approach is used to estimate single integral term with time-varying

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delay information appearing in the derivative of LKF. Delay decomposed approach is fully used in [24], where the time delay interval is decomposed to n ($n \geq 2$) equal subintervals. However, the dimension of LMIs increases sharply with the increase of the decomposed interval, which makes the computational cost higher. Recently, an improved free-matrix-based inequality (FMBI) is derived in [33], and less conservative delay-dependent stability criteria for linear time-delay system are obtained via the FMBI approach. Xiao et al. in [25] also improve the robust absolute stability criteria of Lur'e systems with time-varying delay based on the FMBI. Nevertheless, many slack matrices bring heavy computation complexity, and it is a bit difficult to judge how to introduce slack matrices reasonably [34].

Although, as mentioned above, several significant robust absolute stability criteria for uncertain Lur'e systems with time-varying delays have been given, the following ideas of reducing the conservation of the proposed stability criteria should be addressed. (i) An extended double-integral inequalities method, proposed in [35, 36], has been used to reduce conservation of a stability criterion (see Lemma 2.2 below). Recently, in [9], two relaxed integral inequalities are obtained via the combination of the Wirtinger-based inequality and the reciprocally convex lemma, without requiring any extra slack matrix (see Lemma 2.1 below). This further reduces the conservation of the proposed stability criteria of linear systems with time-varying delays. It is natural whether the extended double-integral inequality method and the improved relaxed integral inequality can be used simultaneously to further reduce the conservation of stability criteria for time-delayed Lur'e systems. (ii) In [25], many slack matrices bring heavy computation complexity via the FMBI approach. Is it possible to overcome the difficulties without increasing conservation? (iii) Another new LKF with more information of the state-based vectors, such as $x(t)$, $x(t-h(t))$, $x(t-h)$ and $\dot{x}(t)$, may reduce the conservation of the proposed stability criteria.

Based on the analysis above, it is an interesting and still challenging problem to address the above issues, which offers motivation to derive less conservative and more efficient calculative stability criteria for the time-delayed Lur'e system.

In this paper, we contribute to the absolute and robust absolute stability criteria for a class of uncertain Lur'e system with time-varying delays and sector bounded nonlinearities. Both the cases with time-invariant and time-varying nonlinearities are considered. By constructing a modified LKF based on the single-integral or double-integral, some improved delay-dependent absolute and robust absolute stability criteria are derived in terms of LMIs. Our criteria are less conservative than some ones recently proposed. The contribution in reducing conservation of the proposed stability criteria relies on the meanwhile using of the extended double-integral inequalities method and the improved relaxed integral inequalities proposed in [9, 36]. Finally, some detailed illustrative examples are provided to show that the proposed approach improves existing methods and gives better results for stability than those reported earlier.

Notation: The notation $P > 0$ (< 0) means that matrix P is positive (negative) definite. I denotes an identity matrix with appropriate dimensions. $*$ denotes the symmetric terms in a block matrix and diag$\{\cdots\}$ denotes a block-diagonal matrix. e_i ($i = 1, \ldots, m$) are block entry matrices. For example, $e_2^T = \begin{bmatrix} 0 & I & \underbrace{0 \cdots 0}_{m-2} \end{bmatrix}$. $F[h(t)]$ denotes F is the function of $h(t)$. $\mathrm{Sym}\{B\} = B + B^T$.

2. **Problem Statement and Preliminaries.** Consider the following Lur'e system with time-varying delays and a time-varying nonlinearity:

$$\begin{cases} \dot{x}(t) = [A + \Delta A(t)]x(t) + [A_1 + \Delta A_1(t)]x(t-h(t)) + [B + \Delta B(t)]w(t), \\ z(t) = Mx(t) + Nx(t-h(t)), \\ w(t) = -\varphi(t, z(t)), \\ x(s) = \psi(s),\ s \in [-h, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $z(t) \in \mathbb{R}^m$ and $w(t) \in \mathbb{R}^m$ are the state, input and output vectors of the system, respectively. A, A_1, B, M and N are real constant matrices with appropriate dimensions; $\psi(s)$ is an $\mathbb{R}^n$-valued continuous initial functional specified on $[-h, 0]$ with known a positive scalar h. $\varphi(t, z(t)) \in \mathbb{R}^m$ is the nonlinear functional in the feedback path. The time-varying delay $h(t)$ is continuous-time functional and satisfies the following conditions:

$$0 \le h(t) \le h, \quad \mu_1 \le \dot{h}(t) \le \mu_2, \quad \forall t \ge 0, \tag{2}$$

where h, μ_1 and μ_2 are constants.

The nonlinear functional $\varphi(t, z(t))$ in the feedback path is continuous in t, globally Lipschitz in $z(t)$, and satisfies

$$[\varphi(t, z(t)) - K_1 z(t)]^T [\varphi(t, z(t)) - K_2 z(t)] \le 0 \tag{3}$$

for $\forall t \ge 0$, $\varphi(t, 0) = 0$, where K_1 and K_2 are real matrices and $K = K_2 - K_1$ is a symmetric positive definite matrix. It is customarily said that the nonlinear function, $\varphi(t, z(t))$, belongs to the sector $[K_1, K_2]$.

$\Delta A(t)$, $\Delta A_1(t)$ and $\Delta B(t)$ denote real-valued matrix functions representing parameter uncertainties, which are assumed to satisfy

$$[\Delta A(t) \quad \Delta B(t) \quad \Delta A_1(t)] = DF(t) [E_a \quad E_b \quad E_{a1}], \tag{4}$$

where D, E_a, E_b and E_{a1} are known constant matrices with appropriate dimensions, and $F(t)$ is an unknown matrix with Lebesgue-measurable elements and satisfies

$$F^T(t)F(t) \le I, \quad \forall t \ge 0. \tag{5}$$

Definition 2.1. (Robust Absolute Stability) *The uncertain Lur'e system described by* (1) *is said to be robust absolutely stable in the sector* $[K_1, K_2]$*, if the system is asymptotically stable for any nonlinear function* $\varphi(t, z(t))$ *satisfying* (3) *and all admissible uncertainties.*

Lemma 2.1. [9] *For a block symmetric matrix* $\tilde{R} = \operatorname{diag}\{R, 3R\}$ *with* $R > 0$, $0 < h(t) < h$ *and any matrix* M*, then, the following inequality holds for all continuously differentiable function* $x(t)$*:*

$$h \int_{t-h}^{t} \dot{x}^T(s) R \dot{x}(s) ds \ge \alpha^T(t) [E_1 \ E_2] \left(\begin{bmatrix} \tilde{R} & M \\ * & \tilde{R} \end{bmatrix} + \begin{bmatrix} \frac{h-h(t)}{h} T_1 & 0 \\ 0 & \frac{h(t)}{h} T_2 \end{bmatrix} \right) \begin{bmatrix} E_1^T \\ E_2^T \end{bmatrix} \alpha(t),$$

where

$$\alpha^T(t) = \left[x^T(t) \ \ x(t-h(t)) \ \ x(t-h) \ \ \int_{t-h(t)}^{t} \frac{x(s)}{h(t)} ds \ \ \int_{t-h}^{t-h(t)} \frac{x(s)}{h-h(t)} ds \right],$$

$$E_1 = [e_1 - e_2 \ e_1 + e_2 - 2e_4], \ E_2 = [e_2 - e_3 \ e_2 + e_3 - 2e_5],$$

$$T_1 = \tilde{R} - M\tilde{R}^{-1}M^T, \ T_2 = \tilde{R} - M^T \tilde{R}^{-1} M.$$

Lemma 2.2. *For any matrix $R > 0$, scalars $h \geq 0$, and any continuous function x in $R \to R^n$, the following inequality holds*

$$\frac{h^2}{2}\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)R\dot{x}(s)dsd\theta \geq \psi^T\phi\psi$$

with $\psi^T = \left[x^T(t) \ \int_{t-h}^{t} x^T(s)ds \ \int_{-h}^{0}\int_{t+\theta}^{t} x^T(s)ds\right]$, $\phi = \frac{1}{h^2}\begin{bmatrix} \frac{5h^4}{2}R & 2h^3R & -9h^2R \\ * & 7h^2R & -18hR \\ * & * & 54R \end{bmatrix}$.

Proof: It is easy to obtain the following inequality from Lemma 4 of [36]

$$\begin{aligned}\frac{h^2}{2}\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)R\dot{x}(s)dsd\theta \geq& \left[\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)dsd\theta\right]R\left[\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}(s)dsd\theta\right]\\ &+\frac{24}{h^2}\Delta^T R\Delta\end{aligned}$$

with $\Delta = \frac{1}{2}\int_{-h}^{0}\int_{-h}^{\theta}\int_{t+r}^{t}\dot{x}(s)dsdrd\theta - \int_{-h}^{0}\int_{t+\theta}^{t}\int_{t+\theta}^{r}\dot{x}(s)dsdrd\theta.$

Then, based on the formula above, the following inequality holds:

$$\begin{aligned}\frac{h^2}{2}\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)R\dot{x}(s)dsd\theta \geq& \left[\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)dsd\theta\right]R\left[\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}(s)dsd\theta\right]\\ &+\frac{24}{h^2}\left[\frac{h^2}{4}x(t)+\frac{h}{2}\int_{t-h}^{t}x(s)ds-\frac{3}{2}\int_{-h}^{0}\int_{t+\theta}^{t}x(s)dsd\theta\right]^T R\\ &\times\left[\frac{h^2}{4}x(t)+\frac{h}{2}\int_{t-h}^{t}x(s)ds-\frac{3}{2}\int_{-h}^{0}\int_{t+\theta}^{t}x(s)dsd\theta\right]\\ =& \ \psi^T\phi\psi.\end{aligned}$$

This completes the proof.

Lemma 2.3. [37] *Given matrices Γ, Ξ and $\Omega = \Omega^T$, the following inequality $\Omega + \Gamma F(\sigma)\Xi + \Xi^T F^T(\sigma)\Gamma^T < 0$ holds for any $F(\sigma)$ satisfying $F^T(\sigma)F(\sigma) \leq I$, if and only if there exists a scalar $\varepsilon > 0$ such that $\Omega + \varepsilon^{-1}\Gamma\Gamma^T + \varepsilon\Xi^T\Xi < 0$.*

Remark 2.1. *Recently, the FWM-based method has been used to reduce conservatism of a stability criterion for time-delay Lur'e system in* [25]. *However, many slack matrices bring heavy computation complexity. Therefore, Lemma 2.1 is used in this paper to overcome the difficulties without increasing conservatism. This can make the calculation convenient and fast. Moreover, Lemmas 2.1 and 2.2 are used simultaneously to further reduce the conservation of stability criteria for time-delay Lur'e system.*

3. Main Results.

In this section, we will give some absolute stability criteria and robust absolute stability criteria for time-delay Lur'e system.

3.1. Absolute stability criteria for time-delay Lur'e system.

In this subsection, first, by applying the loop transformation [20], we get that the normal form of system (1) in the sector $[K_1, K_2]$ is equal to that of the following system:

$$\begin{cases}\dot{x}(t) = \bar{A}x(t) + \bar{A}_1 x(t-h(t)) + Bw(t),\\ z(t) = Mx(t) + Nx(t-h(t)),\\ w(t) = -\varphi(t, z(t)),\\ x(s) = \psi(s),\ s \in [-h, 0]\end{cases} \tag{6}$$

in the sector $[0, K]$. Here, $\bar{A} = A - BK_1M$ and $\bar{A}_1 = A_1 - BK_1N$.

Theorem 3.1. *The system* (6) *satisfying the conditions* (2)-(3) *is absolutely stable for given values of $h \geq 0$, μ_1 and μ_2, if there exist real positive definite matrices $P = [P_{ab}]_{5n\times 5n}$, Q, W, R, Z, $G = [G_{ab}]_{2n\times 2n}$, positive semi-definite diagonal matrices $S = \mathrm{diag}\{s_1, s_2, \ldots, s_m\}$ and any matrices U_1, U_2 and $T = [T_{ab}]_{2n\times 2n}$ with appropriate dimensions such that the following LMIs hold for $\dot{h}(t) \in \{\mu_1, \mu_2\}$:*

$$\begin{bmatrix} \Sigma\big|_{h(t)=0} - \Theta & E_1^T T \\ * & -\tilde{R} \end{bmatrix} < 0, \tag{7}$$

$$\begin{bmatrix} \Sigma\big|_{h(t)=h} - \Theta & E_2^T T^T \\ * & -\tilde{R} \end{bmatrix} < 0, \tag{8}$$

where

$$\begin{aligned}
\Sigma =& \mathrm{Sym}\left\{\Pi_1[h(t)] P \Gamma^T[h(t)]\right\} - [E_1\ E_2] \begin{bmatrix} h_1[h(t)]\tilde{R} & T \\ * & h_2[h(t)]\tilde{R} \end{bmatrix} [E_1\ E_2]^T \\
&+ \beta\Phi\beta^T + e_1\left(h^2 W\right)e_1^T + e_2\left(1 - \dot{h}(t)\right)Q e_2^T - e_3 Q e_3^T + e_4\left(h^2 R + \frac{h^4}{4}Z\right)e_4^T \\
&+ [e_1\ e_4]\, G\, [e_1\ e_4]^T - \left(1 - \dot{h}(t)\right)[e_2\ e_5]\, G\, [e_2\ e_5]^T - h h(t) e_7 W e_7^T \\
&- h(h - h(t)) e_8 W e_8^T + \Pi_2\Pi_3, \\
\Pi_1[h(t)] =& [e_1\ e_2\ h(t)e_7\ (h - h(t))e_8\ e_9], \\
\Gamma[h(t)] =& \left[e_4\ \left(1 - \dot{h}(t)\right)e_5\ e_1 - \left(1 - \dot{h}(t)\right)e_2\ \left(1 - \dot{h}(t)\right)e_2 - e_3\ h e_1 - h(t)e_7 - (h - h(t))e_8\right], \\
h_1[h(t)] =& \frac{2h - h(t)}{h},\ h_2[h(t)] = \frac{h + h(t)}{h}, \\
\Pi_2 =& [e_1 U_1 + e_4 U_2],\ \Pi_3 = \left[\bar{A}e_1^T + \bar{A}_1 e_2^T - e_4^T + B e_6^T\right], \\
\Theta =& \mathrm{Sym}\left\{e_6 S\left(e_6^T + KM e_1^T + KN e_2^T\right)\right\},\ \tilde{R} = \mathrm{diag}\{R,\ 3R\}, \\
E_1 =& [e_1 - e_2\ \ e_1 + e_2 - 2e_7],\ \ E_2 = [e_2 - e_3\ \ e_2 + e_3 - 2e_8], \\
\beta =& [e_1\ \ h(t)e_7 + (h - h(t))e_8\ \ e_9], \\
\Phi =& \begin{bmatrix} -\frac{5h^2}{2}Z & -2hZ & 9Z \\ * & -7Z & \frac{18}{h}Z \\ * & * & -\frac{54}{h^2}Z \end{bmatrix}.
\end{aligned}$$

Proof: The proof of Theorem 3.1 is shown in Appendix A.

Remark 3.1. *To further reduce the conservatism of a stability criterion* [25], *LKF* (6) *with a triple-integral term is chosen via an improved double-integral inequality technology of Lemma 2.2 application in this paper. It is worthy pointing out that the proposed LKF* (6) *reduces to the LKF (11) of* [25] *only with a double-integral term when choosing $Z = 0$ and $W = 0$. Therefore, the absolute stability criterion obtained via the LKF (11) of* [25] *is summarized as follows.*

Remark 3.2. *Theorem 3.1 gives the absolute stability criterion of the time-delayed Lur'e system* (1). *By solving the LMIs in Theorem 3.1, we can easily obtain the maximum allowable delay upper bounds to guarantee the absolute stability of the time-delayed Lur'e system* (1). *Thus, the influence of time delays on system performance can be further understood.*

Corollary 3.1. *The system* (6) *satisfying the conditions* (2)-(3) *is absolutely stable for given values of $h \geq 0$, μ_1 and μ_2, if there exist real positive definite matrices $P =$*

$[P_{ab}]_{4n\times 4n}$, Q, R, $G=[G_{ab}]_{2n\times 2n}$, *positive semi-definite diagonal matrices* $S=\mathrm{diag}\{s_1,s_2,\ldots,s_m\}$ *and any matrices* U_1, U_2 *and* $T=[T_{ab}]_{2n\times 2n}$ *with appropriate dimensions such that the following* ***LMI****s hold for* $\dot{h}(t)\in\{\mu_1,\mu_2\}$*:*

$$\begin{bmatrix} \tilde{\Sigma}\big|_{h(t)=0}-\tilde{\Theta} & \tilde{E}_1^T T \\ * & -\tilde{R} \end{bmatrix}<0, \tag{9}$$

$$\begin{bmatrix} \tilde{\Sigma}\big|_{h(t)=h}-\tilde{\Theta} & \tilde{E}_2^T T^T \\ * & -\tilde{R} \end{bmatrix}<0, \tag{10}$$

where

$$\begin{aligned}
\tilde{\Sigma} =& \mathrm{Sym}\left\{\tilde{\Pi}_1[h(t)]P\tilde{\Gamma}^T[h(t)]\right\}-\left[\tilde{E}_1\ \tilde{E}_2\right]\begin{bmatrix} h_1[h(t)]\tilde{R} & T \\ * & h_2[h(t)]\tilde{R}\end{bmatrix}\left[\tilde{E}_1\ \tilde{E}_2\right]^T \\
&+\tilde{e}_2\left(1-\dot{h}(t)\right)Q\tilde{e}_2-\tilde{e}_3Q\tilde{e}_3^T+\tilde{e}_4\left(h^2R\right)\tilde{e}_4^T \\
&+[\tilde{e}_1\ \tilde{e}_4]\,G\,[\tilde{e}_1\ \tilde{e}_4]^T-\left(1-\dot{h}(t)\right)[\tilde{e}_2\ \tilde{e}_5]\,G\,[\tilde{e}_2\ \tilde{e}_5]^T \\
&+\tilde{\Pi}_2\tilde{\Pi}_3, \\
\tilde{\Pi}_1[h(t)] =& \left[\tilde{e}_1\ \tilde{e}_2\ h(t)\tilde{e}_7\ (h-h(t))\tilde{e}_8\right], \\
\tilde{\Gamma}[h(t)] =& \left[\tilde{e}_4\ \left(1-\dot{h}(t)\right)\tilde{e}_5\ \tilde{e}_1-\left(1-\dot{h}(t)\right)\tilde{e}_2\ \left(1-\dot{h}(t)\right)\tilde{e}_2-\tilde{e}_3\right], \\
h_1[h(t)] =& \frac{2h-h(t)}{h},\ h_2[h(t)]=\frac{h+h(t)}{h}, \\
\tilde{\Pi}_2 =& \left[\tilde{e}_1U_1+\tilde{e}_4U2\right],\ \Pi_3=\left[\bar{A}\tilde{e}_1^T+\bar{A}_1\tilde{e}_2^T-\tilde{e}_4^T+B\tilde{e}_6^T\right], \\
\tilde{\Theta} =& \mathrm{Sym}\left\{\tilde{e}_6S\left(\tilde{e}_6^T+KM\tilde{e}_1^T+KN\tilde{e}_2^T\right)\right\},\ \tilde{R}=\mathrm{diag}\{R,\ 3R\}, \\
\tilde{E}_1 =& \left[\tilde{e}_1-\tilde{e}_2\quad \tilde{e}_1+\tilde{e}_2-2\tilde{e}_7\right],\ \tilde{E}_2=\left[\tilde{e}_2-\tilde{e}_3\quad \tilde{e}_2+\tilde{e}_3-2\tilde{e}_8\right].
\end{aligned}$$

Remark 3.3. *To reduce the conservatism of sufficient stability conditions,* $V_2(x(t))$ *of LKF (9) can be rewritten as*

$$\begin{aligned}
\tilde{V}_2(x(t)) =& \int_{t-h(t)}^{t}\begin{bmatrix} x(s) \\ \dot{x}(s) \\ \int_s^t \dot{x}(u)du\end{bmatrix}^T G\begin{bmatrix} x(s) \\ \dot{x}(s) \\ \int_s^t \dot{x}(u)du\end{bmatrix}ds \\
&+\int_{t-h}^{t-h(t)}\begin{bmatrix} x^T(s) \\ \int_s^t \dot{x}(u)du\end{bmatrix}^T L\begin{bmatrix} x^T(s) \\ \int_s^t \dot{x}(u)du\end{bmatrix}ds.
\end{aligned}$$

Therefore, we can obtain a new LKF with replacing $V_2(x(t))$ *of LKF (9) with the equation above. By calculating their time derivatives, we have*

$$\begin{aligned}
\dot{\tilde{V}}_2(x(t)) =& \begin{bmatrix} x(t) \\ \dot{x}(t) \\ 0\end{bmatrix}^T G\begin{bmatrix} x(t) \\ \dot{x}(t) \\ 0\end{bmatrix}-\left(1-\dot{h}(t)\right)\begin{bmatrix} x(t-h(t)) \\ \dot{x}(t-h(t)) \\ \int_{t-h(t)}^t \dot{x}(u)du\end{bmatrix}^T G\begin{bmatrix} x(t-h(t)) \\ \dot{x}(t-h(t)) \\ \int_{t-h(t)}^t \dot{x}(u)du\end{bmatrix} \\
&+\left(1-\dot{h}(t)\right)\begin{bmatrix} x(t-h(t)) \\ \int_{t-h(t)}^t \dot{x}(u)du\end{bmatrix}^T L\begin{bmatrix} x(t-h(t)) \\ \int_{t-h(t)}^t \dot{x}(u)du\end{bmatrix} \\
&-\begin{bmatrix} x(t-h) \\ \int_{t-h}^t \dot{x}(u)du\end{bmatrix}^T L\begin{bmatrix} x(t-h) \\ \int_{t-h}^t \dot{x}(u)du\end{bmatrix}+2\int_{t-h(t)}^t\begin{bmatrix} x(s) \\ \dot{x}(s) \\ \int_s^t \dot{x}(u)du\end{bmatrix}^T G\begin{bmatrix} 0 \\ 0 \\ \dot{x}(t)\end{bmatrix}
\end{aligned}$$

$$+2\int_{t-h}^{t-h(t)}\begin{bmatrix} x(s) \\ \int_s^t \dot{x}(u)du \end{bmatrix}^T L \begin{bmatrix} 0 \\ \dot{x}(t) \end{bmatrix}$$

where $G = [G_{ab}]_{3n\times 3n} > 0$ *and* $L = [L_{ab}]_{2n\times 2n} > 0$.

In $\dot{\tilde{V}}_2(x(t))$, *more information of the state-based vectors, such as* $x(t)$, $x(t-h(t))$, $x(t-h)$ *and* $\dot{x}(t)$, *are considered, which may further reduce the conservatism of stability conditions in Theorem 3.1. However, such improvement may come at the cost of calculative complexity due to the introduction of matrix* G *and* L. *Naturally, the absolute stability criterion obtained via the LKF (9) with* $\tilde{V}_2(x(t))$ *is summarized as follows.*

Corollary 3.2. *The system* (6) *satisfying the conditions* (2)-(3) *is absolutely stable for given values of* $h \geq 0$, μ_1 *and* μ_2, *if there exist real positive definite matrices* $P = [P_{ab}]_{5n\times 5n}$, W, R, Z, $\bar{G} = [\bar{G}_{ab}]_{3n\times 3n}$, $L = [L_{ab}]_{2n\times 2n}$, *positive semi-definite diagonal matrices* $S = \mathrm{diag}\{s_1, s_2, \ldots, s_m\}$ *and any matrices* U_1, U_2 *and* $T = [T_{ab}]_{2n\times 2n}$ *with appropriate dimensions such that the following LMIs hold for* $\dot{h}(t) \in \{\mu_1, \mu_2\}$:

$$\begin{bmatrix} \bar{\Sigma}\big|_{h(t)=0} - \Theta & E_1^T T \\ * & -\tilde{R} \end{bmatrix} < 0, \tag{11}$$

$$\begin{bmatrix} \bar{\Sigma}\big|_{h(t)=h} - \Theta & E_2^T T^T \\ * & -\tilde{R} \end{bmatrix} < 0, \tag{12}$$

where

$$\begin{aligned}
\bar{\Sigma} =& \mathrm{Sym}\left\{\Pi_1[h(t)]P\Gamma^T[h(t)]\right\} - [E_1\ E_2]\begin{bmatrix} h_1[h(t)]\tilde{R} & T \\ * & h_2[h(t)]\tilde{R} \end{bmatrix}[E_1\ E_2]^T + \beta\Phi\beta^T \\
&+ e_1(h^2W)e_1^T + e_4\left(h^2R + \frac{h^4}{4}Z\right)e_4^T - hh(t)e_7We_7^T - h(h-h(t))e_8We_8^T + \Pi_2\Pi_3 \\
&+ [e_1\ e_4\ 0]\bar{G}[e_1\ e_4\ 0]^T - \left(1-\dot{h}(t)\right)[e_2\ e_5\ e_1-e_2]\bar{G}[e_2\ e_5\ e_1-e_2]^T \\
&+ \left(1-\dot{h}(t)\right)[e_2\ e_1-e_2]L[e_2\ e_1-e_2]^T - [e_3\ e_1-e_3]L[e_3\ e_1-e_3]^T \\
&+ \mathrm{Sym}\big\{[h(t)e_7\ e_1-e_2\ h(t)(e_1-e_7)]\bar{G}[0\ 0\ e_4]^T \\
&+ [(h-h(t))e_8\ (h-h(t))(e_1-e_8)]K[0\ e_4]^T\big\}.
\end{aligned}$$

Remark 3.4. *When the nonlinear function in systems* (1) *and* (6) *is time-invariant decentralized, the stability problem for the class of Lur'e systems with time-varying delay has been considered in* [7, 36, 38, 39, 40, 41, 42, 43]. *In the following discussion of this paper, we consider the class of uncertain Lur'e systems with time-varying delays described by*

$$\begin{cases} \dot{x}(t) = [A+\Delta A(t)]x(t) + [A_1+\Delta A_1(t)]x(t-h(t)) + [B+\Delta B(t)]f(z(t)), \\ z(t) = Mx(t), \\ x(s) = \psi(s),\ s\in[-h,0], \end{cases} \tag{13}$$

where the nonlinear feedback part $f(z(t))$ *belongs to the set of nonlinear functions with bounded sector constrains, that is,* $f(z(t))$ *satisfies the following condition* [7, 36, 38, 39, 40, 41, 43]:

$$\begin{aligned}\phi =:& \big\{f:\mathbb{R}^m \mapsto \mathbb{R}^m : f(z) = [f_1(z_1)\ f_2(z_2)\ \cdots\ f_m(z_m)]^T, \\ & \alpha_i z_i^2 \leq z_i f_i(z_i) \leq \beta_i z_i^2 \ for\ z_i \neq 0,\ \alpha_i, \beta_i \geq 0,\ i=1,2,\ldots,m\big\},\end{aligned} \tag{14}$$

where α_i *and* β_i *are given constants. Here, for simplicity, let us define* $K_1 = \mathrm{diag}\{\alpha_1, \alpha_2, \ldots, \alpha_m\}$ *and* $K_2 = \mathrm{diag}\{\beta_1, \beta_2, \ldots, \beta_m\}$.

For a nominal case of time-delayed Lur'e system (13), we have the following delay-dependent absolute stability results.

Theorem 3.2. *The nominal case of system* (13) *satisfying the conditions* (2) *and* (14) *is absolutely stable for given values of $h \geq 0$, μ_1, μ_2, diagonal matrices K_1 and K_2 if there exist real positive definite matrices $P = [P_{ab}]_{5n\times 5n}$, W, R, Z, $\bar{G} = [\bar{G}_{ab}]_{3n\times 3n}$, $L = [L_{ab}]_{2n\times 2n}$, positive semi-definite diagonal matrices $S_1 = \mathrm{diag}\{s_{11}, s_{12}, \ldots, s_{1m}\}$, $S_2 = \mathrm{diag}\{s_{21}, s_{22}, \ldots, s_{2m}\}$, $\Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_m\}$ and any matrices U_1, U_2 and $T = [T_{ab}]_{2n\times 2n}$ with appropriate dimensions such that the following LMIs hold for $\dot{h}(t) \in \{\mu_1, \mu_2\}$:*

$$\begin{bmatrix} \bar{\Sigma}\big|_{h(t)=0} + \bar{\Theta} & E_1^T T \\ * & -\tilde{R} \end{bmatrix} < 0, \tag{15}$$

$$\begin{bmatrix} \bar{\Sigma}\big|_{h(t)=h} + \bar{\Theta} & E_2^T T^T \\ * & -\tilde{R} \end{bmatrix} < 0, \tag{16}$$

where

$$\begin{aligned} \bar{\Theta} =& \mathrm{Sym}\left\{e_1\left[M^T(K_2S_1 - K_1S_2)M\right]e_4^T + e_4M^T(S_2 - S_1)e_6^T\right\} \\ &- e_1M^T\left[K_1\Lambda K_2 + K_2\Lambda K_1\right]Me_1^T + 2e_1\left[M^T(K_2 + K_1)\Lambda\right]e_6^T - 2e_6\Lambda e_6^T. \end{aligned}$$

Proof: The proof of Theorem 3.2 is shown in Appendix B.

Remark 3.5. *Actually, with the help of delay decomposition technology,*

$$\int_{t-\frac{h}{2}}^{t} f(s)ds,\quad \int_{t-h}^{t-\frac{h}{2}} f(s)ds,\quad \int_{-\frac{h}{2}}^{0}\int_{t+\theta}^{t} g(s)dsd\theta,\quad \int_{-h}^{-\frac{h}{2}}\int_{t+\theta}^{t} g(s)dsd\theta$$

can be scaled properly with Lemmas 2.1 and 2.2 at each subintervals in estimating an upper bound of $\dot{V}(x(t))$. These can give another improved feasible region for delay-dependent stability criteria, which may be an interesting topic in our future work.

3.2. **Robust absolute stability criteria for time-delay Lur'e system.** In this subsection, we extend the obtained absolute stability conditions above to robust absolute stability problem for uncertain Lur'e systems (1) and (13) with time-varying parameter uncertainties satisfying (4) and (5).

Theorem 3.3. *The uncertain Lur'e system* (1) *satisfying the conditions* (2)-(5) *is robust absolutely stable for given values of $h \geq 0$, μ_1 and μ_2, if there exist real positive definite matrices $P = [P_{ab}]_{5n\times 5n}$, Q, W, R, Z, $G = [G_{ab}]_{2n\times 2n}$, positive semi-definite diagonal matrices $S = \mathrm{diag}\{s_1, s_2, \ldots, s_m\}$ and any matrices U_1, U_2, $T = [T_{ab}]_{2n\times 2n}$ with appropriate dimensions and a scalar $\varepsilon > 0$ such that the following **LMI**s hold for $\dot{h}(t) \in \{\mu_1, \mu_2\}$:*

$$\begin{bmatrix} \Sigma\big|_{h(t)=0} - \Theta + \varepsilon\Delta_2^T\Delta_2 & E_1^T T & \Delta_1^T \\ * & -\tilde{R} & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0, \tag{17}$$

$$\begin{bmatrix} \Sigma\big|_{h(t)=h} - \Theta + \varepsilon\Delta_2^T\Delta_2 & E_2^T T^T & \Delta_1^T \\ * & -\tilde{R} & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0, \tag{18}$$

where $\Delta_1 = [e_1U_1D + e_4U_2D]^T$, $\Delta_2 = \left[e_1E_a^T + e_2E_{a1}^T + e_6E_b^T\right]^T$.

Proof: If we replace $\bar{A}$, $\bar{A}_1$ and $\bar{B}$ in **LMI**s (7) and (8) with $\bar{A}+DF(t)E_a$, $\bar{A}_1+DF(t)E_{a1}$, $\bar{B}+DF(t)E_b$, respectively, **LMI**s in (7) and (8) can be rewritten as

$$\begin{aligned}&\Sigma[h(t)]-\frac{h-h(t)}{h}E_1^TT^T\tilde{R}^{-1}TE_1-\frac{h(t)}{h}E_2^TT\tilde{R}^{-1}T^TE_2\\&+\Delta_1F(t)\Delta_2+\Delta_2^TF^T(t)\Delta_1^T<0.\end{aligned}\tag{19}$$

According to Theorem 3.1, it is obvious that the uncertain system (1) is robust absolutely stable for all admissible uncertainties, if LMI in (19) holds for $\dot{h}(t)\in\{\mu_1,\mu_2\}$. It follows from Lemma 2.3 that LMI in (19) holds if and only if there exist positive scalars $\varepsilon>0$, such that the following matrix inequality holds:

$$\Sigma[h(t)]-\frac{h-h(t)}{h}E_1^TT^T\tilde{R}^{-1}TE_1-\frac{h(t)}{h}E_2^TT\tilde{R}^{-1}T^TE_2+\varepsilon^{-1}\Delta_1\Delta_1^T+\varepsilon\Delta_2^T\Delta_2<0,\tag{20}$$

which is equivalent to **LMI**s in (17) and (18), respectively, by **Schur complement equivalence**. From Definition 2.1, this completes the proof.

Nextly, when the LKF satisfies Remark 3.1, the following corollary is obtained:

Corollary 3.3. *The uncertain Lur'e system* (1) *satisfying the conditions* (2)-(5) *is absolutely stable for given values of $h\geq 0$, μ_1 and μ_2, if there exist real positive definite matrices $P=[P_{ab}]_{4n\times 4n}$, Q, R, $G=[G_{ab}]_{2n\times 2n}$, positive semi-definite diagonal matrices $S=\mathrm{diag}\{s_1,s_2,\ldots,s_m\}$ and any matrices U_1, U_2, $T=[T_{ab}]_{2n\times 2n}$ with appropriate dimensions and a scalar $\varepsilon>0$ such that the following* ***LMI****s hold for $\dot{h}(t)\in\{\mu_1,\mu_2\}$:*

$$\begin{bmatrix}\tilde{\Sigma}\big|_{h(t)=0}-\tilde{\Theta}+\varepsilon\tilde{\Delta}_2^T\tilde{\Delta}_2 & \tilde{E}_1^TT & \tilde{\Delta}_1^T\\ * & -\tilde{R} & 0\\ * & * & -\varepsilon I\end{bmatrix}<0,\tag{21}$$

$$\begin{bmatrix}\tilde{\Sigma}\big|_{h(t)=h}-\tilde{\Theta}+\varepsilon\tilde{\Delta}_2^T\tilde{\Delta}_2 & \tilde{E}_2^TT^T & \tilde{\Delta}_1^T\\ * & -\tilde{R} & 0\\ * & * & -\varepsilon I\end{bmatrix}<0,\tag{22}$$

where $\tilde{\Delta}_1=[\tilde{e}_1U_1D+\tilde{e}_4U_2D]^T$, $\tilde{\Delta}_2=\left[\tilde{e}_1E_a^T+\tilde{e}_2E_{a1}^T+\tilde{e}_6E_b^T\right]^T$.

Nextly, when the LKF satisfies Remark 3.3, the following corollary is obtained:

Corollary 3.4. *The system* (1) *satisfying the conditions* (2)-(5) *is absolutely stable for given values of $h\geq 0$, μ_1 and μ_2, if there exist real positive definite matrices $P=[P_{ab}]_{5n\times 5n}$, W, R, Z, $\bar{G}=[\bar{G}_{ab}]_{3n\times 3n}$, $L=[L_{ab}]_{2n\times 2n}$, positive semi-definite diagonal matrices $S=\mathrm{diag}\{s_1,s_2,\ldots,s_m\}$, any matrices U_1, U_2 and $T=[T_{ab}]_{2n\times 2n}$ with appropriate dimensions and a scalar $\varepsilon>0$ such that the following LMIs hold for $\dot{h}(t)\in\{\mu_1,\mu_2\}$:*

$$\begin{bmatrix}\bar{\Sigma}\big|_{h(t)=0}-\Theta+\varepsilon\Delta_2^T\Delta_2 & E_1^TT & \Delta_1^T\\ * & -\tilde{R} & 0\\ * & * & -\varepsilon I\end{bmatrix}<0,\tag{23}$$

$$\begin{bmatrix}\bar{\Sigma}\big|_{h(t)=h}-\Theta+\varepsilon\Delta_2^T\Delta_2 & E_2^TT^T & \Delta_1^T\\ * & -\tilde{R} & 0\\ * & * & -\varepsilon I\end{bmatrix}<0.\tag{24}$$

When the nonlinear function in systems (1) and (6) is time-invariant decentralized, the the following corollary is obtained:

Corollary 3.5. *The uncertain system* (13) *satisfying the conditions* (2), (4), (5) *and* (14) *is robust absolutely stable for given values of $h\geq 0$, μ_1, μ_2, diagonal matrices K_1 and*

K_2 *if there exist real positive definite matrices* $P = [P_{ab}]_{5n\times 5n}$, W, R, Z, $\bar{G} = [\bar{G}_{ab}]_{3n\times 3n}$, $L = [L_{ab}]_{2n\times 2n}$, *positive semi-definite diagonal matrices* $S_1 = \mathrm{diag}\{s_{11}, s_{12}, \ldots, s_{1m}\}$, $S_2 = \mathrm{diag}\{s_{21}, s_{22}, \ldots, s_{2m}\}$, $\Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_m\}$, *any matrices* U_1, U_2 *and* $T = [T_{ab}]_{2n\times 2n}$ *with appropriate dimensions and a scalar* $\varepsilon > 0$ *such that the following LMIs hold for* $\dot{h}(t) \in \{\mu_1, \mu_2\}$:

$$\begin{bmatrix} \bar{\Sigma}\big|_{h(t)=0} + \bar{\Theta} + \varepsilon \Delta_2^T \Delta_2 & E_1^T T & \Delta_1^T \\ * & -\tilde{R} & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0, \tag{25}$$

$$\begin{bmatrix} \bar{\Sigma}\big|_{h(t)=h} + \bar{\Theta} + \varepsilon \Delta_2^T \Delta_2 & E_2^T T^T & \Delta_1^T \\ * & -\tilde{R} & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0. \tag{26}$$

4. **Numerical Example.** In this section, we give examples to show the effectiveness of the criteria proposed in this paper. The conservatism of the criteria is checked based on the calculated maximal admissible delay upper bounds (MADUPs). Moreover, the index of the number of decision variables (NoV) is applied to showing the complexity of criteria.

Remark 4.1. *In order to illustrate the effectiveness of the stability criteria proposed in this paper, some numerical examples commonly used in many recent studies, such as* [7, 18, 24, 25, 30, 32, 36, 38, 39, 42, 43, 44, 45, 46]*, are chosen. Therefore, by comparing our results with others in the recent literature, the advancement of our stability criteria are shown conveniently based on the lager MADUPs than some recent literature.*

Example 4.1. [24, 25, 30, 32, 47]. *Consider the uncertain Lur'e system* (1) *with the system parameters described as:*

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},\ A_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},\ B = \begin{bmatrix} -0.2 \\ -0.3 \end{bmatrix},$$
$$M = \begin{bmatrix} 0.3 & 0.1 \end{bmatrix},\ N = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix},\ K_1 = 0.2,\ K_2 = 0.5,$$
$$D = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\ E_a = E_{a1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ E_b = 0.$$

TABLE 1. **MADUPs** h for different μ (Example 4.1)

Methods\\μ	0.0	0.3	0.6	0.9	NoVs
[30]	3.3057	2.0787	1.4195	0.9228	10
[32]	3.3057	2.2262	1.7409	1.4682	24
[24] ($m = 2$)	4.1076	2.4660	1.8787	1.7190	58
[24] ($m = 3$)	4.2664	2.5164	1.9147	1.7923	91
[25]	4.3332	2.6873	2.2021	1.9897	105
[47]	4.3956	2.9358	2.4721	2.2356	539
Corollary 3.3	4.3332	2.6972	2.1946	1.9987	78
Theorem 3.3	4.5922	2.7084	2.2136	2.0059	103
Corollary 3.4	4.6208	3.0948	2.5765	2.4055	120

In Table 1, the maximum allowed delay bounds MADUPs h of the Lur'e system (1) for different μ by using our results and methods in [24, 25, 30, 32, 47] are compared. From the tables, it is found that our results give better upper bounds on the delay h for robust absolute stability of the Lur'e system (1) than some recent results. Moreover, the NoV of Theorem 2 in [25] is $23.5n^2 + 4.5n + m$, while the NoVs of Corollary 3.3

and Theorem 3.3 in this paper are $17n^2 + 4n + m$ and $22.5n^2 + 5.5n + m$, respectively. The MADUPs of Corollary 3.3 are much the same as those of Theorem 2 [25] and the MADUPs of Theorem 3.3 are lager than those of Theorem 2 [25]. However, the NoVs of Corollary 3.3 and Theorem 3.3 are $6.5n^2+0.5n$ and n^2-n less than that of Theorem 2 [25], respectively, which means that the former improves the results but does not require extra decision variables. This advantage grows with the increase of n. Here $-\mu_1 = \mu_2 = \mu$.

The NoV of Corollary 3.4 is $26.5n^2 + 6.5n + m$, which is $4n^2 + n$ more than that of Theorem 3.3 due to the introduction of $\tilde{V}_2(x(t))$. However, the MADUPs of Corollary 3.4 are lager than those of Theorem 3.3, which means that the former further improves the results.

To confirm the obtained results ($h = 4.6208$), the simulation result is shown in Figure 1 which shows that the state responses of the Lur'e system (1) with $\varphi(t, z(t)) = 0.3tanh(z(t))$ and $h(t) = 4.6208$ converge to zero under the random initial state.

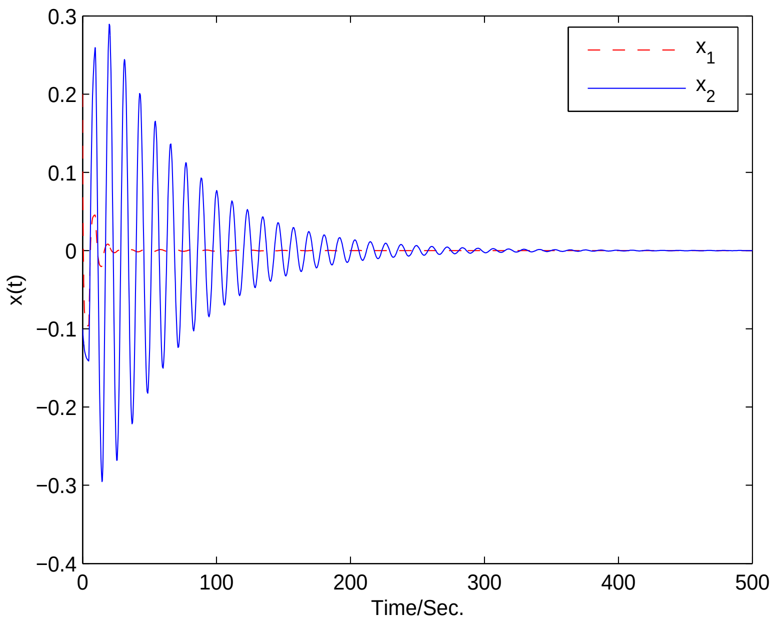

FIGURE 1. The state responses of system (1)

Example 4.2. [7, 38, 39, 42]. *Consider the uncertain Lur'e system* (13) *with the system parameters described as:*

$$A = \begin{bmatrix} -1.2 & 0 \\ 0.8 & -1 \end{bmatrix}, \; A_1 = \begin{bmatrix} -1 & 0.6 \\ -0.6 & -1 \end{bmatrix}, \; B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix},$$
$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; D = \begin{bmatrix} \theta & 0 \\ 0 & \theta \end{bmatrix}, \; E_a = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$
$$E_b = E_{a1} = \begin{bmatrix} 0.03 & 0 \\ 0 & 0.03 \end{bmatrix}, \; K_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \; K_2 = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}.$$

In Table 2, the MADUPs h of the Lur'e system (13) for $\mu_1 = 0$ and different μ_2 by using Corollary 3.5 in this paper and methods in [7, 38, 39, 42] are compared. From the table, it is found that Corollary 3.5 gives better upper bounds on the delay h for robust absolute stability of the Lur'e system (13) than some recent results. To confirm the obtained results ($h = 5.259$), the simulation result is shown in Figure 2 which shows that the state

TABLE 2. **MADUPs** with fixed $\mu_1 = 0$ (Example 4.2)

μ_2	Methods\ θ	0	0.2	0.4	0.6	0.8	1	NoVs
0	[39]	1.113	1.062	1.014	0.967	0.921	0.887	60
	[38]	3.325	3.128	2.849	2.780	2.651	2.522	153
	[7]	3.355	3.172	2.912	2.876	2.734	2.614	287
	[42]	4.372	3.840	3.456	3.160	2.921	2.723	86
	Corollary 3.5	5.259	4.515	3.955	3.513	3.158	2.867	122
0.1	[39]	1.026	0.984	0.940	0.898	0.857	0.818	60
	[38]	3.160	2.899	2.840	2.702	2.575	2.460	153
	[7]	3.224	3.046	2.900	2.804	2.603	2.554	287
	[42]	3.616	3.295	3.039	2.828	2.649	2.491	86
	Corollary 3.5	4.379	3.867	3.468	3.149	2.888	2.671	122

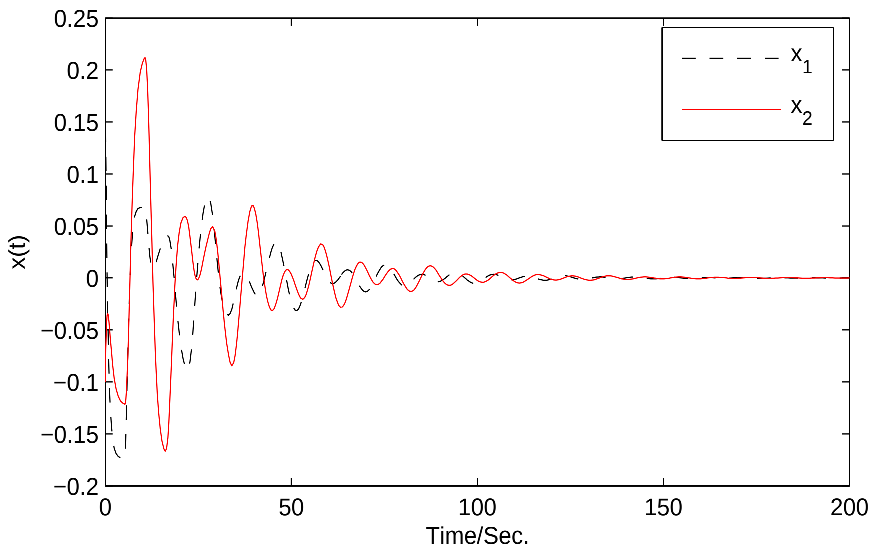

FIGURE 2. The state responses of system (13)

responses of the Lur'e system (13) with $f(z(t)) = 0.3tanh(z(t))$ and $h(t) = 5.259$ converge to zero under the random initial state.

Example 4.3. [18, 36, 43, 44, 45, 46]. *Consider the uncertain system* (13) *with following parameters:*

$$A = \begin{pmatrix} -2 & 0 \\ 0 & -0.9 \end{pmatrix},\ A_1 = \begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix},\ B = \begin{pmatrix} -0.2 \\ -0.3 \end{pmatrix},\ M = \begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix},$$

$$D = E_a = E_{a1} = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix},\ E_b = \begin{pmatrix} 0.1 \\ 0 \end{pmatrix},\ F(t) = \begin{pmatrix} \sin(\omega t) & 0 \\ 0 & \sin(\omega t) \end{pmatrix}.$$

Let $\dot{h}(t) = 0$ and $K_1 = 0$ for the example. By applying the criteria in [18, 44] and Corollary 3.5 in this study, the MADUPs h for $K_2 = 0.5$ are computed as 4.7062, 4.7846 and 6.3195, respectively. It is showed that our criteria are less conservative than ones in [18, 44].

The MADUPs h for different values of K_2 are listed in Table 3 along with some existing results given in the references [36, 41, 43, 45, 46, 48]. We can find that the proposed stability criterion is less conservative than the previous ones. The simulation result is shown in Figure 3 which shows that the state responses of the neutral-type Lur'e system

TABLE 3. **MADUPs** h for different K_2 (Example 4.3)

Methods\\K_2	0.5	100	NoVs
[48]	4.3196	4.1792	48
[41]	5.1366	4.9572	72
[46]	5.1666	4.9731	74
[45]	5.3486	5.1828	65
[43]	5.4352	5.2517	92
[36]	5.5513	5.3557	93
Corollary 3.5	6.3195	6.1992	122

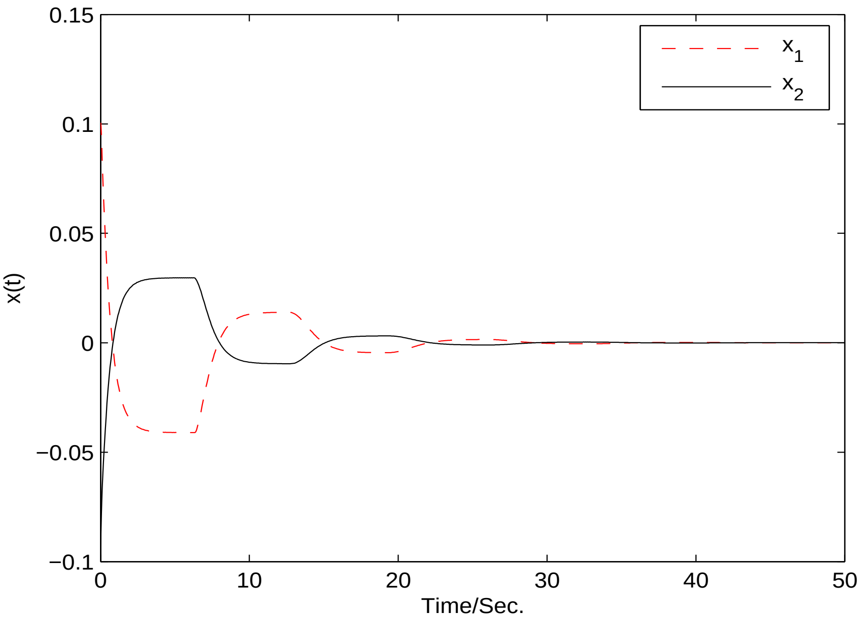

FIGURE 3. The state responses of system (13)

(13) with $f(\sigma(t)) = 0.5tanh(\sigma(t))$ and $h(t) = 6.3195$ converge to zero under the random initial state.

5. **Conclusion.** In this paper, some new absolute and robust absolute stability criteria are proposed for the uncertain Lur'e systems with time-varying delays and sector bounded nonlinearities via a modified LKF. Some effective techniques, such as improved relaxed integral inequality method, extended double-integral inequalities lemma, are applied to reduce the conservation of the proposed criteria from some existing results. Finally, some numerical examples are used to illustrate the effectiveness of the proposed approaches.

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Appendix A. The proof of Theorem 3.1 is as follows. For the sake of simplicity on matrix representation, the notations of several symbols and matrices are defined as

$$\xi^T(t) = \Big[x^T(t) \;\; x^T(t-h(t)) \;\; x^T(t-h) \;\; \dot{x}(t) \;\; \dot{x}^T(t-h(t)) \;\; w^T(t)$$
$$\frac{1}{h(t)}\int_{t-h(t)}^{t} x^T(s)ds \;\; \frac{1}{h-h(t)}\int_{t-h}^{t-h(t)} x^T(s)ds \;\; \int_{-h}^{0}\int_{t+\theta}^{t} x^T(s)dsd\theta\Big],$$
$$\zeta^T(t) = \left[x^T(t) \;\; x^T(t-h(t)) \;\; \int_{t-h(t)}^{t} x^T(s)ds \;\; \int_{t-h}^{t-h(t)} x^T(s)ds \;\; \int_{-h}^{0}\int_{t+\theta}^{t} x^T(s)dsd\theta\right].$$

Proof: Construct an **LKF** candidate as

$$V(x(t)) = \sum_{i=1}^{4} V_i(x(t)) \tag{27}$$

with

$$V_1(x(t)) = \zeta^T(t)P\zeta(t),$$
$$V_2(x(t)) = \int_{t-h(t)}^{t} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T G \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds + \int_{t-h}^{t-h(t)} x^T(s)Qx(s)ds,$$
$$V_3(x(t)) = h\int_{-h}^{0}\int_{t+\theta}^{t} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} W & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds,$$
$$V_4(x(t)) = \frac{h^2}{2}\int_{-h}^{0}\int_{\theta}^{0}\int_{t+r}^{t} \dot{x}^T(s)Z\dot{x}(s)dsdrd\theta.$$

The time derivative of $V(x(t))$ with respect to time along the trajectory of (6) is as follows:

$$\dot{V}_1(x(t)) = \xi^T(t)\mathrm{Sym}\left\{\mathit{\Pi}_1[h(t)]P\mathit{\Gamma}^T[h(t)]\right\}\xi(t),$$
$$\dot{V}_2(x(t)) = \left(1-\dot{h}(t)\right)x^T(t-h(t))Qx(t-h(t)) - x^T(t-h)Qx(t-h)$$
$$+ \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T G \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} - \left(1-\dot{h}(t)\right)\begin{bmatrix} x(t-h(t)) \\ \dot{x}(t-h(t)) \end{bmatrix}^T G \begin{bmatrix} x(t-h(t)) \\ \dot{x}(t-h(t)) \end{bmatrix},$$
$$\dot{V}_3(x(t)) = h^2x^T(t)Wx(t) + h^2\dot{x}^T(t)R\dot{x}(t) - h\int_{t-h}^{t} x^T(s)Wx(s)ds - h\int_{t-h}^{t}\dot{x}^T(s)R\dot{x}(s)ds,$$
$$\dot{V}_4(x(t)) = \frac{h^4}{4}\dot{x}^T(t)Z\dot{x}(t) - \frac{h^2}{2}\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Z\dot{x}(s)ds.$$

It is easy to get the following inequalities from **Jensen inequality** [49]:

$$-h\int_{t-h}^{t} x^T(s)Wx(s)ds = -h\int_{t-h(t)}^{t} x^T(s)Wx(s)ds - h\int_{t-h}^{t-h(t)} x^T(s)Wx(s)ds$$
$$\le -h\xi^T(t)\left(h(t)e_7We_7^T + (h-h(t))e_8We_8^T\right)\xi(t). \tag{28}$$

Using Lemmas 2.1 and 2.2, we have

$$-h\int_{t-h}^{t} \dot{x}^T(s)R\dot{x}(s)ds \le -\xi^T(t)\left[E_1\ E_2\right]\begin{bmatrix} \frac{2h-h(t)}{h}\tilde{R} & T \\ * & \frac{h+h(t)}{h}\tilde{R}\end{bmatrix}\left[E_1\ E_2\right]^T\xi(t)$$
$$+\xi^T(t)\left\{\frac{h-h(t)}{h}E_1^T T\tilde{R}^{-1}T^T E_1 + \frac{h(t)}{h}E_2^T T^T\tilde{R}^{-1}TE_2\right\}\xi(t), \tag{29}$$

$$-\frac{h^2}{2}\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Z\dot{x}(s)dsd\theta \le \xi^T(t)\beta\Phi\beta^T\xi(t). \tag{30}$$

For any appropriately dimensioned matrices U_1 and U_2, it is true that

$$0 = \left[x(t)U_1 + \dot{x}(t)U_2\right]\left[\bar{A}x(t) + \bar{A}_1x(t-h(t)) + Bw(t) - \dot{x}(t)\right]. \tag{31}$$

The nonlinear function $w(t)$ in the feedback path satisfying the sector conditions (3), for any $S = \mathrm{diag}\{s_1, s_2, \ldots, s_m\} \ge 0$, it follows from (3) and (6) that

$$0 \le -2w^T(t)Sw(t) - 2w^T(t)SK\left[Mx(t) + Nx(t-h(t))\right]$$
$$= \xi^T(t)\mathrm{Sym}\left\{-e_6S\left[e_6^T + KMe_1^T + KNe_2^T\right]\right\}\xi(t). \tag{32}$$

Then, it follows from inequalities (28)-(32) that

$$\dot{V}(x(t)) \le \sum_{i=1}^{4}\dot{V}_i(x(t)) + \left[x(t)U_1 + \dot{x}(t)U_2\right]\left[\bar{A}x(t) + \bar{A}_1x(t-h(t)) + Bw(t) - \dot{x}(t)\right]$$
$$-2w^T(t)Sw(t) - 2w^T(t)SK\left[Mx(t) + Nx(t-h(t))\right]$$
$$\le \xi^T(t)\left[\Sigma[h(t)] + \frac{h-h(t)}{h}E_1^T T\tilde{R}^{-1}T^T E_1 + \frac{h(t)}{h}E_2^T T^T\tilde{R}^{-1}TE_2\right]\xi(t). \tag{33}$$

Therefore, LMIs (7) and (8) hold for $\dot{h}(t) \in \{\mu_1, \mu_2\}$, which together with **Schur complement equivalence** imply that $\dot{V}(x(t)) < 0$.

Hence, it follows from the Lyapunov stability theory that the nominal system (6) is absolutely stable for any nonlinear function $\varphi(t, z(t))$ satisfying (3). From Definition 2.1, this completes the proof.

Appendix B. The proof of Theorem 3.2 is as follows.

Proof: Construct an **LKF** candidate as

$$\bar{V}(x(t)) = V_1(x(t)) + \tilde{V}_2(x(t)) + V_3(x(t)) + V_4(x(t))$$
$$+ 2\sum_{i=1}^{m}\int_0^{z_i(t)}\left[s_{1i}(f_i(s) - \alpha_is) + s_{2i}(\beta_is - f_i(s))\right]ds. \tag{34}$$

The time derivative of $2\sum_{i=1}^{m}\int_0^{z_i(t)}\left[s_{1i}(f_i(s) - \alpha_is) + s_{2i}(\beta_is - f_i(s))\right]ds$ is as follows:

$$2\left[f(z(t)) - K_1Mx(t)\right]^T S_1M\dot{x}(t) + 2\left[K_2Mx(t) - f(z(t))\right]^T S_2M\dot{x}(t). \tag{35}$$

According to the sector condition (14), for any $\Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_m\} \geq 0$, it follows from (14) that

$$0 \leq 2\left[f(z(t)) - K_1 M x(t)\right]^T \Lambda \left[K_2 M x(t) - f(z(t))\right]. \tag{36}$$

Then, it follows from inequalities (28)-(31) and (35)-(36) that

$$\dot{V}(x(t)) \leq \hat{\xi}^T(t) \left[\bar{\Sigma}[h(t)] + \bar{\Theta} + \frac{h - h(t)}{h} E_1^T T \tilde{R}^{-1} T^T E_1 + \frac{h(t)}{h} E_2^T T^T \tilde{R}^{-1} T E_2\right] \hat{\xi}(t), \tag{37}$$

where

$$\hat{\xi}^T(t) = \Big[x^T(t) \;\; x^T(t-h(t)) \;\; x^T(t-h) \;\; \dot{x}(t) \;\; \dot{x}^T(t-h(t)) \;\; f^T(z(t)) \\ \frac{1}{h(t)} \int_{t-h(t)}^{t} x^T(s) ds \;\; \frac{1}{h-h(t)} \int_{t-h}^{t-h(t)} x^T(s) ds \;\; \int_{-h}^{0} \int_{t+\theta}^{t} x^T(s) ds d\theta\Big].$$

Therefore, LMIs (15) and (16) hold for $\dot{h}(t) \in \{\mu_1, \mu_2\}$, which together with **Schur complement equivalence** imply that $\dot{V}(x(t)) < 0$.