International Journal of Innovative Computing, Information and Control
Volume **16**, Number **3**, June **2020**
 ISSN 1349-4198
pp. 783–798

AN ADAPTIVE DIRECT DATA DRIVEN CONTROL SCHEME FOR UNKNOWN PLANT

Jianhong Wang, Ricardo A. Ramirez-Mendoza
and Jorge de J Lozoya Santos

School of Engineering and Sciences
Tecnologico de Monterrey
Monterrey, Ave. Eugenio Garza Sada 2501, Monterrey, N.L. 64849, Mexico
wangjianhong@nuaa.edu.cn

Received October 2019; revised February 2020

Abstract. *This paper develops an adaptive direct data driven control scheme for unknown plant in one closed loop system, whose goal is to achieve plant-model perfect matching condition. To apply direct data driven control to designing forward controller without the model of plant, virtual input is constructed to derive one optimization problem, whose decision variables are the unknown controller parameters. Through using the idea of adaptation, one parameter adjustment loop is added as the outer loop in such a way the unknown controller parameters are changed with environment, according to our constructed parameter adjustment law. Furthermore Lyapunov's stability theory is also used to derive parameter adjustment law such that stability can be guaranteed for the whole adaptive system. Such an adaptive direct data driven control can not only design controller without the model of plant, but also adjust unknown controller parameters adaptively through one constructed parameter adjustment mechanism. Finally two simulation examples confirm our theoretical results.*
Keywords: Direct data driven control, Adaptive mechanism, Lyapunov stability, Parameter adjustment law

1. **Introduction.** Adaptation means to change a behavior to conform to new circumstances, and then an adaptive controller is thus a controller that can modify its behavior in response to changes in the dynamics of the process and the character of the disturbances. Intuitively an adaptive controller is a controller with adjustable parameters and a mechanism for adjusting the parameters. Moreover an adaptive control system can be thought of as having two loops. One loop is a normal feedback with the process and the controller. The other loop is the parameter adjustment loop. As adaptive systems have useful properties, which can be profitably used to design control systems with improved performance and functionality, so a control engineer should know about adaptive systems. In designing the feedback controller for every control system structure, we always assume that the mathematical model corresponding to the plant is known in prior, i.e., the process of designing the controller is based on the priori knowledge of the plant. However, this assumption does not hold in reality, i.e., the considered plant is unknown, and then the mathematical model of the plant may be identified by lots of system identification methods. Generally in order to design the feedback controller in the closed loop system, there exist two kinds of approaches based on the priori knowledge of the plant. These two approaches are divided into the model based control and direct data driven control. The main difference between these two approaches is that whether the model for plant is needed to be identified in prior. The model based control approach needs to construct the

DOI: 10.24507/ijicic.16.03.783

mathematical model of the plant by using mechanical modeling technique or system identification idea, and then this mathematical model is used to design the later controller. So the controller performance of this model based control approach depends on the accuracy of the identified plant closely. In order to alleviate this dependency, some scholars propose to apply the input-output measured data to designing the controller directly and avoid the tedious identification process of the plant. From a system identification idea point of view, the above second approach is called the direct data driven control approach. The system identification idea plays one important role in theory and engineering, because it not only provides a method to construct one accurate mathematical model for the plant, but also gives a source of ideas for designing controller directly for the closed loop system. Furthermore as the direct data driven control approach avoids the identification process with respect to the plant and simplifies the whole designing process about the controller, in theory or engineering field it is more widely considered to achieve the control performance than the classical model based control method. Many common direct data driven control approaches include virtual reference feedback tuning control, subspace predictive control and iterative feedback tuning control, etc.

The main advantage of direct data driven control is that the controller is designed directly by using only input-output measured data without identifying the plant, i.e., the model for plant is unknown. Because of some safety and production restrictions, the open loop system is not used widely in many industrial production processes. So in such situation, it is very urgent to design controller in closed loop system. However, the difficulty about the controller design in closed loop system is that the correlation is considered between the input and external disturbance, induced by the feedback loop. If considering the problem of designing controller in closed loop system, the controller is always designed on the basis of a known model for plant. However, the model for plant cannot be easily determined in the industrial process, as the cost of developing the model for plant is very high.

In order to avoid the identification process for unknown model for plant and to design the controller directly in closed loop system, direct data driven control scheme is studied deep here, where the controller is adjusted by one adaptive mechanism. More specifically, the unknown controller parameter is not a constant, but time-varying, and it is modified with the environment changes. Adaptive control can deal with external disturbance [1], which adjusts the parameter estimates of controller based on feedback signal. When plant is affected by external disturbance such that output prediction deviates away our desired trajectory, then the generated error signal is used to adjust the parameters of controller until the output prediction corresponds to the designed trajectory. Many parameter adaptation algorithms are proposed in [2], whatever in deterministic environment or stochastic environment. In [3], a new multi variable adaptive control (MRAC) scheme is designed for plants of arbitrary relative degree, which does not require a stringent symmetry assumption related with the plant high frequency gain matrix. An adaptive controller makes use of a closed loop reference model as an observer [4], and guarantees global stability or asymptotic output tracking. A reference model robust adaptive controller and a linear quadratic regulator are combined to obtain a high performance and robust control system [5]. An extremum seeking algorithm is proposed for reference tracking and stabilization of unstable discrete time systems without control directions [6]. [7] establishes a novel result for adaptive asymptotic tracking control of uncertain switched linear systems. One commonly used adaptive mechanism is named as model reference adaptive control [8], where direct and indirect model reference adaptive control strategies are proposed for multi variable piecewise affine systems. These piecewise affine systems constitute a popular tool to model hybrid systems or nonlinear systems. [9] proposes a novel combined model reference adaptive control for unknown multi input multi output systems with guaranteed parameter convergence. To control the time delay system, some novel modifications are given for a predictor based model reference adaptive controller [10], so that input delays are compensated in uncertain nonlinear system. When considering stability and convergence for adaptive control system, Mazenc construction is used to design a simple strict Lyapunov function in a rather intuitive manner [11]. Multiple model adaptive control is applied to improving the transient response of nonlinear system [12], by using several switched models. Generally the related literature on adaptive control is very high, and here we cannot enumerate all of them. The detailed interplay of industry, applications, technology, theory and research on adaptive control is discussed in [13]. In recent years, direct data driven control is widely studied in research field, for example, direct data driven control is applied for designing controllers that handle constraints without deriving the model for plant and directly from data [14]. An on line direct data driven control is studied to exploit some results from set membership theory and the theory of learning [15]; at the same time, a predictable closed loop behavior is guaranteed by making use of a batch of available measured data. Furthermore the authors propose one zonotope parameter identification algorithm to calculate a set that contains the unknown controller parameters consistent with the measured output and the given bound of external disturbance [16]. And after transforming the problem of identifying unknown controller parameters into two linear regression models, the authors apply scenario optimization to determining the number of scenarios [17]. Then we find that the maximum number of data points equals the number of optimization variables.

Based on our earlier works [11-14], this paper develops a new idea of adaptation for direct data driven control scheme by using only measured data without the model for plant to design controller parameters. To the best of our knowledge that in all published papers on direct data driven control scheme, the obtained controller parameters are time-invariant, i.e., parameter estimates are constants. However, this constant condition of controller parameters is not realistic in reality, as the environment is changing, so the controller parameters must be time-varying with the environment changes. However, can we describe this property of the controller parameters with environment changes? The answer is to apply the adaptation into adjusting the controller parameters. The key problem in this paper is to determine the adjustment mechanism, so that the parameter adjustment mechanism is obtained to change the controller parameters adaptively. The controller parameters are changed on the basis of adjustment mechanism from error signal, which is the difference between the output of the closed loop system and the output of the reference model. Based on our earlier works [11-14], and the main processes in direct data driven control, the problem of designing controller can be transformed into one optimization problem, whose decision variables are the unknown controller parameters, and then in summary, the new contributions of this paper are

1) development of an adjustment mechanism to adjust the unknown controller parameters, by using only measured data, which corresponds to the designed controller, not the model for plant;

2) development of a new parameter adaptation algorithm to adjust the unknown controller parameters in the constructed adjustment mechanism.

This paper is organized as follows. In Section 2, the structure of our considered closed loop system is presented. A short introduction of model reference control is used to design the unknown parameterized controller, and the deficiency of classical model reference control is pointed out. In Section 3, direct data driven control scheme is proposed to design the unknown parameterized controller by minimization of one optimization problem. Section 4 develops the idea of adaptation into direct data driven control to form out proposed adaptive direct data driven control scheme. Section 5 shows how Lyapunov's stability theory can be used to construct adaption algorithm for adjusting unknown controller parameters in adaptive system. Section 6 presents simulation results to show the desired system performance. Section 7 points out our future work, and Section 8 ends the paper with final conclusion.

2. Problem Formulation.

2.1. Closed loop system. Assume the plant is a linear time invariant discrete time process. It is denoted by a rational transfer function form $P(z)$, and $P(z)$ is unknown. Throughout the closed loop experimental process, only a sequence of input-output measured data corresponding to the plant $P(z)$ are collected. The input-output relation is described as follows.

$$y(t) = P(z)u(t) \tag{1}$$

where z is a time shift operator, i.e., $zu(t) = u(t-1)$, $P(z)$ is one transfer function of the plant, $u(t)$ is the measured input, and $y(t)$ is the measured output corresponding to the plant $P(z)$. Consider the following simple closed loop system in Figure 1, the input-output relations in the whole closed loop system are written as follows.

$$\begin{cases} y(t) = P(z)u(t) \\ u(t) = C_1(z,\theta)\varepsilon(t) = C_1(z,\theta)[r(t) - y(t)] \end{cases} \tag{2}$$

where $r(t)$ is the excited signal, $C_1(z,\theta)$ is one unknown controller, which is parameterized by one unknown parameter vector θ, i.e., controller $C_1(z,\theta)$ is parameterized as the following linear affine form.

$$\begin{aligned} C_1(z,\theta) &= \alpha^T(z)\theta \\ \alpha(z) &= [\alpha_1(z), \alpha_2(z), \ldots, \alpha_n(z)]^T \\ \theta &= [\theta_1, \theta_2, \ldots, \theta_n]^T \end{aligned} \tag{3}$$

where $\alpha(z)$ denotes one known basis function vector, and θ is one unknown parameter vector with dimension n. The choice of this basic function vector $\alpha(z)$ can be as orthogonal basis function, and from practical perspective, $\alpha(z)$ is always chosen as follows.

$$\alpha(z) = \begin{bmatrix} 1 & z & z^2 & \cdots & z^n \end{bmatrix}$$

The parameterized form for controllers $C_1(z,\theta)$ is the common PID controller, i.e., the common PID controller can be rewritten as our considered parameterized form in Equation (3).

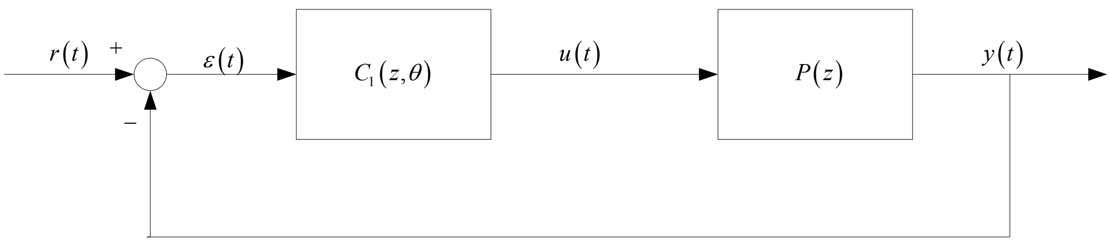

FIGURE 1. Closed loop system structure

Formulating Equation (2) again, the new input-output relations in terms of excited input $r(t)$ are given as follows.

$$y(t) = P(z)u(t) = P(z)[C_1(z,\theta)[r(t) - y(t)]]$$

$$= P(z)C_1(z,\theta)r(t) - P(z)C_1(z,\theta)y(t)$$
$$[1 + P(z)C_1(z,\theta)]y(t) = P(z)C_1(z,\theta)r(t)$$
$$y(t) = \frac{P(z)C_1(z,\theta)}{1 + P(z)C_1(z,\theta)}r(t) \tag{4}$$

2.2. **Classical model reference control.** In closed loop system with unknown controller, the transfer function from the excited signal $r(t)$ to measured output $y(t)$ is closed loop transfer function. From Equation (4), the control goal of classical model reference control is to tune the unknown parameter vector θ corresponding to unknown controller $C_1(z,\theta)$ in order to achieve the expected closed loop transfer function [18]. Given closed loop transfer function $M(z)$, we want to guarantee that closed loop transfer function approximates to its expected function $M(z)$. The expected closed loop transfer function $M(z)$ can be chosen in priori, based on the goal of the control designer. For example, if the goal of designing closed loop controllers is to achieve the zero tracking error, then the expected closed loop transfer function $M(z)$ can be chosen as 1.

The problem of tuning the unknown parameter vector θ is formulated as the following classical model reference control optimization problem.

$$\min_{\theta} J_{MR}(\theta) = \left\| \frac{P(z)C_1(z,\theta)}{1 + P(z)C_1(z,\theta)} - M(z) \right\|_2^2 \tag{5}$$

where $\|.\|_2^2$ is the common Euclidean norm.

In Equation (5), before solving this optimization problem with respect to unknown parameter vector θ, the priori knowledge about the plant $P(z)$ may be needed, for example, its transfer function form. So in this classical model reference control, as the plant $P(z)$ is unknown, firstly the identification strategy is needed to identify $P(z)$. To avoid the identification process of the plant $P(z)$, direct data driven control is proposed to directly identify unknown parameter vector θ in unknown controller $C_1(z,\theta)$ from the measured input-output data. $Z^N = \{u(t), y(t)\}_{t=1}^N$, where N is the number of data points.

3. **Direct Data Driven Control.** Given controller $\{C_1(z,\theta)\}$, as closed loop transfer function from $r(t)$ to $y(t)$ is $M(z)$, then we apply one arbitrary signal $r(t)$ to exciting the formal closed loop system (2), and the output of the closed loop system is described as that

$$y(t) = M(z)r(t)$$

Consider one special excited input $\bar{r}(t)$, the necessary condition about that the closed loop transfer function is $M(z)$ is that the two closed loop systems have the same output $y(t)$ under a given input [17]. During classical model reference control, this necessary condition holds in case of choosing suitable controller and excited signal. However, above description does not hold, due to unknown plant. The idea of direct data driven control means that virtual input $\bar{r}(t)$ needs to be constructed firstly, so we give the detailed process of constructing virtual input.

Suppose that a controller $\{C_1(z,\theta)\}$ results in a closed loop system, whose transfer function is $M(Z)$. Then if the closed loop system is fed by any reference signal $r(t)$, its output equals $M(Z)r(t)$. Hence, a necessary condition for the closed loop system to have the same transfer function as the reference model is that the output of the two systems for a given $\bar{r}(t)$. Classical model reference design methods try to impose such a necessary condition by first selecting a reference $\bar{r}(t)$ and then by choosing $\{C_1(z,\theta)\}$ such that the condition is satisfied. However, for a general selection of $\bar{r}(t)$, our task is difficult to accomplish if a model of the plant is not available. The basic idea of direct data driven control is to perform a wise selection of $\bar{r}(t)$.

Collecting input-output data $\{u(t), y(t)\}_{t=1}^{N}$ corresponding to the unknown plant $P(z)$, then for every measured output $y(t)$, define one virtual input $\bar{r}(t)$ such that

$$y(t) = M(z)\bar{r}(t) \tag{6}$$

This virtual input $\bar{r}(t)$ does not exist in reality, so it cannot be used to generate actual measured output $y(t)$. However, virtual input $\bar{r}(t)$ can be obtained by equation $y(t) = M(z)\bar{r}(t)$, i.e., $y(t)$ is the measured output of the closed loop system, when the excited signal $\bar{r}(t)$ is applied with no disturbance.

Due to the unknown plant $P(z)$, and when $P(z)$ is excited by $u(t)$, its output is $y(t)$, so we choose one suitable controller $\{C_1(z, \theta)\}$ to obtain one expected signal $u(t)$, if the closed loop system is excited by virtual input $\bar{r}(t)$ and $y(t)$ simultaneously. The construction of virtual input $\bar{r}(t)$ can be seen in Figure 2, where the tracking error $\varepsilon(t)$ is defined as

$$\varepsilon(t) = \bar{r}(t) - y(t) = \left(M^{-1}(z) - 1\right) y(t) \tag{7}$$

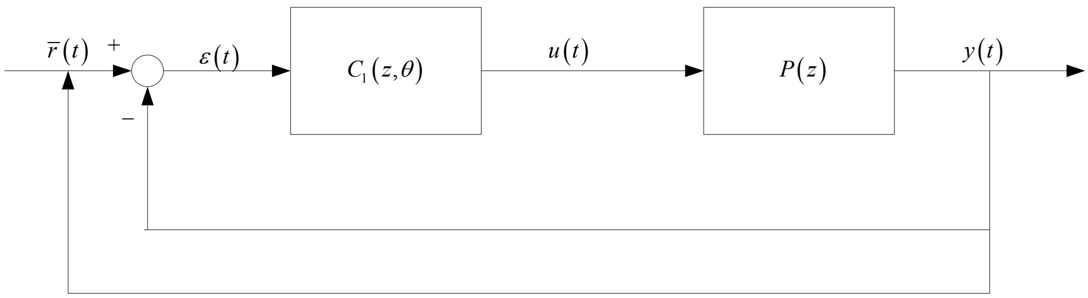

FIGURE 2. Construction of virtual input

From Figure 2 we see that when the closed loop system is excited by $(\bar{r}(t), y(t))$, the expression of $u(t)$ is got.

$$u(t) = C_1(z, \theta)\varepsilon(t) = C_1(z, \theta) \left(M^{-1}(z) - 1\right) y(t) \tag{8}$$

Using Equation (8), unknown parameter vector θ in controller $\{C_1(z, \theta)\}$ can be identified by solving the following optimization problem.

$$\min_{\theta} J_{VR}^{N}(\theta) = \frac{1}{N} \sum_{t=1}^{N} \left[u(t) - C_1(z, \theta) \left(M^{-1}(z) - 1\right) y(t)\right]^2 \tag{9}$$

where $M^{-1}(z)$ appears in above Equation (9). This inverse transfer function always exists in some special cases, for example, minimum phase system or linear causal system. However, if this inverse transfer function does not exist in reality, we can use their pseudo inverse forms in Equation (9).

Observing optimization problem (9) again, all variables are known except for that parameterized controller $\{C_1(z, \theta)\}$. More specifically, input-output data $\{u(t), y(t)\}_{t=1}^{N}$ can be collected by sensors, and the expected transfer function $M(z)$ is priori known. Roughly speaking, the plant $P(z)$ is not in optimization problem (9), which is the main contribution in direct data driven control. Furthermore optimization problem (9) embodies that the problem of designing one unknown controller can be transformed to identify one unknown parameter vector. This transformation will simplify the latter computational complexity about designing controller.

4. Adaptive Direct Data Driven Control. Before introducing adaptation into direct data driven control, measured data set $\left\{u(t), \varepsilon(t) = \bar{r}(t) - y(t) = \left(M^{-1}(z) - 1\right) y(t)\right\}_{t=1}^{N}$ is used to identify the unknown controller parameters θ. Through solving the optimization problem (9), parameter estimates of controller parameters θ are obtained by differentiation with respect to θ and by setting the derivative equal to zero. However, the obtained parameter estimates are fixed constants, and these constants keep invariant with environment changes. To describe the time variant property about the changing environment, one adjustment or adaptive mechanism is added in the considered closed loop system, so that the parameter estimates will be changed on the basis of adjustment mechanism. Based on the main process of direct data driven control, the adjustment mechanism must be added about measured data $\{u(t), \varepsilon(t)\}_{t=1}^{N}$, not data $\{y(t), M(z)r(t)\}_{t=1}^{N}$.

A block diagram of the adaptive direct data driven control scheme is shown in Figure 3. This adaptive system has one ordinary feedback loop composed of the plant and the controller and other feedback loop that changes the controller parameters. The parameters are changed on the basis of feedback from the error. The ordinary feedback loop is called the inner loop, and the parameter adjustment loop is called the outer loop.

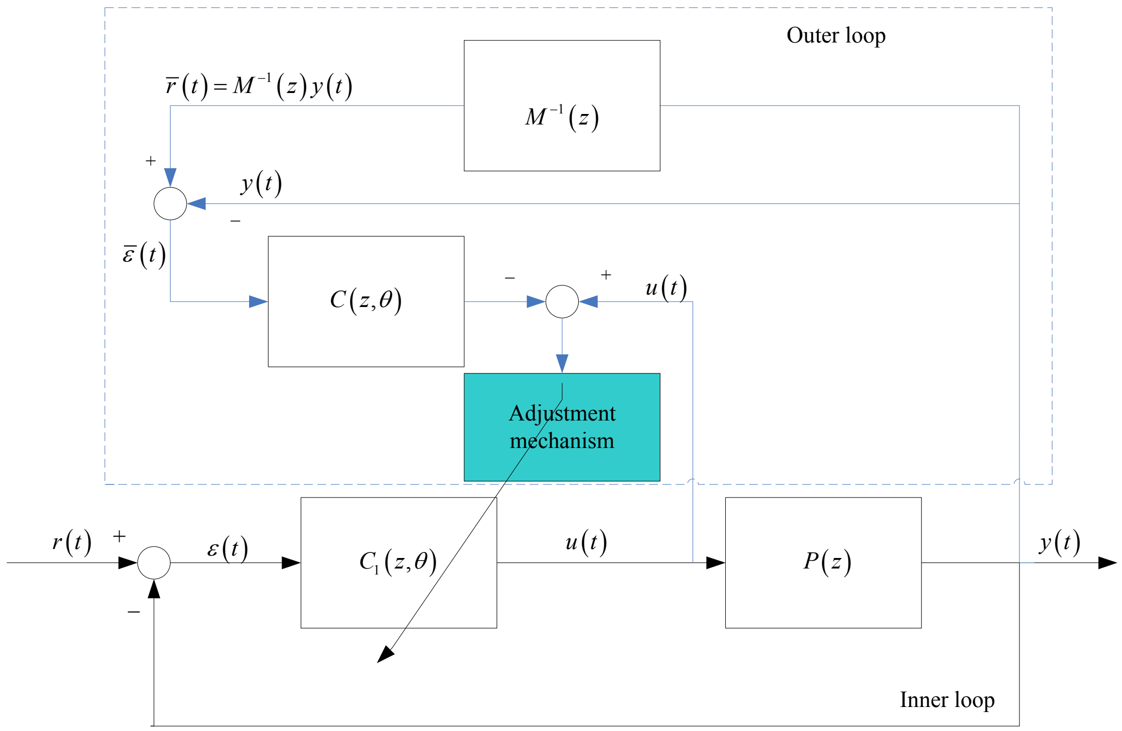

FIGURE 3. Block diagram of adaptive direct data driven control scheme

In Figure 3, $\bar{r}(t) = M^{-1}(z)y(t)$ is virtual input, $\bar{\varepsilon}(t) = \bar{r}(t) - y(t) = \left(M^{-1}(z) - 1\right) y(t)$ is virtual tracking error, and $C(z, \theta)$ is one virtual controller. When plant $P(z)$ changes, we identify the parameters in virtual controller $C(z, \theta)$, and the update controller parameters in true controller $C_1(z, \theta)$, such that the obtained transfer function of the closed loop system equals that desired or expected transfer function $M(z)$.

As PID controller widely used in engineering, here we use adaptive direct data driven control scheme to design PID controller. Let the given controller structure of PID controller be

$$u(t) = u(t-1) + K_p(e(t) - e(t-1)) + K_i e(t) + K_d(e(t) - 2e(t-1) + e(t-2)) \quad (10)$$

where in Equation (10), K_p, K_i, K_d are three controller parameters in PID controller, $u(t)$ is the input signal for unknown plant $P(z)$, and $e(t)$ is the tracking error for the whole closed loop system.

Reformulating the parameterized controller $C(z,\theta)$ in the adjustment mechanism as

$$\begin{aligned} C(z,\theta) &= K_p + \frac{K_i}{1-z^{-1}} + K_d\left(1-z^{-1}\right) \\ &= \begin{bmatrix} 1 & \frac{1}{1-z^{-1}} & 1-z^{-1} \end{bmatrix} \begin{bmatrix} K_p \\ K_i \\ K_d \end{bmatrix} = \alpha^T(z)\theta \end{aligned} \tag{11}$$

where

$$\alpha(z) = \begin{bmatrix} 1 & \frac{1}{1-z^{-1}} & 1-z^{-1} \end{bmatrix}^T; \quad \theta = \begin{bmatrix} K_p & K_i & K_d \end{bmatrix}^T$$

Then we have

$$u(t) = C(z,\theta)\bar{\varepsilon}(t) = \left(K_p + \frac{K_i}{1-z^{-1}} + K_d\left(1-z^{-1}\right)\right)\left(M^{-1}(z)-1\right)y(t) \tag{12}$$

Without loss of generality, expected transfer function $M(z)$ is chosen as a second model.

$$M(z) = \frac{cz^{-1} + dz^{-2}}{1 + az^{-1} + bz^{-2}} \tag{13}$$

Substituting Equation (13) into (12), we obtain

$$\begin{aligned} u(t) &= K_p\left(M^{-1}(z)-1\right)y(t) + \frac{K_i\left(M^{-1}(z)-1\right)}{1-z^{-1}}y(t) + K_d\left(1-z^{-1}\left(M^{-1}(z)-1\right)\right)y(t) \\ &= \begin{bmatrix} \varphi_p(t) & \varphi_i(t) & \varphi_d(t) \end{bmatrix}\theta = \varphi^T(t)\theta \end{aligned} \tag{14}$$

where regressor vector $\varphi(t)$ is defined as

$$\begin{aligned} \varphi(t) &= \begin{bmatrix} \varphi_p(t) & \varphi_i(t) & \varphi_d(t) \end{bmatrix}^T \\ \varphi_p(t) &= \left(M^{-1}(z)-1\right)y(t) = \frac{y(t+1) + (a-c)y(t) + (b-d)y(t-1)}{c + dz^{-1}} \\ \varphi_i(t) &= \frac{\left(M^{-1}(z)-1\right)}{1-z^{-1}}y(t) = \frac{y(t+1) + (a-c)y(t) + (b-d)y(t-1)}{c + (d-c)z^{-1} - dz^{-2}} \\ \varphi_d(t) &= \left(1-z^{-1}\left(M^{-1}(z)-1\right)\right)y(t) \\ &= \frac{y(t+1) + (a-c-1)y(t) + (b-d-a+c)y(t-1) + (d-b)y(t-2)}{c + dz^{-1}} \end{aligned} \tag{15}$$

As in adjustment mechanism, the goal of designing virtual controller $C(z,\theta)$ is to make the output of virtual controller $C(z,\theta)$ approach to the input $u(t)$ of plant $p(z)$, i.e., controller parameters in virtual controller $C(z,\theta)$ are identified through the following optimization problem.

$$\min_{\theta} J_1(\theta) = \left\|u(t) - \varphi^T(t)\theta\right\|_2^2 = \frac{1}{N}\sum_{t=1}^{N}\left[u(t) - \varphi^T(t)\theta\right]^2 \tag{16}$$

Due to the time varying property for plant $p(z)$, then controller parameters are also time varying. Controller parameters can be identified by recursive least squares with forgetting factor, where its recursive form is

$$\begin{cases} \hat{\theta}(t) = \hat{\theta}(t-1) + \dfrac{\varphi(t)}{\alpha(t)}\Big(u(t) - \varphi^T(t)\hat{\theta}(t-1)\Big) \\ \alpha(t) = \lambda\alpha(t-1) + \|\varphi(t)\|^2 \end{cases} \tag{17}$$

where in above recursive form, $\hat{\theta}(t)$ is controller parameters at time instant t, and $\hat{\theta}(t-1)$ means the previous controller parameters at time instant $t-1$. λ is one forgetting factor, and initial value $\alpha(0) = \alpha_0 > 0$. If $\lambda = 0$, then above recursive algorithm (17) reduced to projection algorithm. Further to adjust controller parameters in such way the loss function $J_1(\theta)$ is sufficiently small, it is reasonable to change the controller parameters in the direction of the negative gradient $J_1(\theta)$. That

$$\frac{d\theta}{dt} = -\gamma\frac{\partial J_1}{\partial\theta} = -\gamma e\frac{\partial e}{\partial\theta} \tag{18}$$

where parameter γ determines the adaptation rate, and

$$e(t) = u(t) - \varphi^T(t)\theta$$
$$J_1(\theta) = \|e(t)\|_2^2 = \frac{1}{N}\sum_{t=1}^{N} e^2(t)$$

The partial derivative $\frac{\partial e}{\partial\theta}$ is called the sensitivity derivative of the adaptive system.

Computing this partial derivative $\frac{\partial e}{\partial\theta}$ as

$$\frac{\partial e}{\partial\theta} = \varphi^T(t)$$

After substituting this partial derivative into Equation (18), it holds that

$$\frac{d\theta}{dt} = -\gamma e\frac{\partial e}{\partial\theta} = -\gamma e\varphi^T(t) \tag{19}$$

It means that

$$\begin{bmatrix} \dot{K}_p(t) \\ \dot{K}_i(t) \\ \dot{K}_d(t) \end{bmatrix} = -\gamma e \begin{bmatrix} \varphi_p(t) \\ \varphi_i(t) \\ \varphi_d(t) \end{bmatrix} \tag{20}$$

Equation (20) is called as the MIT rule in the parameter adjustment mechanism, and the MIT rule can be regarded as a gradient scheme to minimize the squared error.

Generally our proposed adaptive direct data driven control scheme is formulated as follows.

Step 1: Given our designed or expected closed loop transfer function $M(z)$;

Step 2: Collect measured data $\{u(t), y(t)\}_{t=1}^{N}$ to compute virtual input $M^{-1}(z)y(t)$;

Step 3: Choose one initial virtual controller $C(z,\theta)$ and one initial controller $C_1(z,\theta)$;

Step 4: Apply Equations (13), (14) and (15) to obtain regressor vector $\varphi(t)$;

Step 5: Choose recursive least squares with forgetting factor or MIT rule to identify parameter estimates $\hat{\theta}(t)$ in virtual controller $C(z,\theta)$;

Step 6: If $C(z,\theta) \neq C_1(z,\theta)$, then let $C_1(z,\theta) = C(z,\theta)$;

Step 7: Let $t = t+1$, return to step 3, until the above steps stop.

5. **Design Controller Using Lyapunov Theory.** The basic idea of adaptive direct data driven control is introduced in above Section 4. As we all know that stability is one important issue in all control theory, in this section we give how to use Lyapunov's theory to construct algorithm for adjusting controller parameters in adaptive system. To achieve this, we need to derive a differential equation for the tracking error $e(t) = u(t) - \varphi^T(t)\theta$. This differential equation contains the adjustment parameters. We try to construct a Lyapunov function and adaptive mechanism such that the tracking will go to zero. When using the Lyapunov theory for adaptive systems, partial derivative $\frac{dV}{dt}$ of one constructed Lyapunov function V must be negative semidefinite. Then the latter procedure is to determine the error equation and a Lyapunov function with a bounded second partial derivative.

Assume a true parameter vector θ_0 exists, such that the perfect matching can be achieved, i.e., $u(t) = \varphi^T(t)\theta_0$. Then the tracking error $e(t)$ is that

$$e(t) = u(t) - \varphi^T(t)\theta = \varphi^T(t)\theta_0 - \varphi^T(t)\theta = \varphi^T(t)(\theta_0 - \theta) \tag{21}$$

To construct one Lyapunov function, we firstly introduce a state space representation between the parameters θ and the tracking error $e(t)$ as

$$\begin{cases} \dfrac{dx}{dt} = Ax + B(\theta_0 - \theta) \\ e = Cx \end{cases} \tag{22}$$

where in Equation (22) variable t is neglected for notational clarity. From model control theory, it holds that

$$e(t) = \left[C(sI - A)^{-1}B\right](\theta_0 - \theta) = \varphi^T(t)(\theta_0 - \theta)$$

It means that

$$\left[C(sI - A)^{-1}B\right] = \varphi^T(t) = \begin{bmatrix} \varphi_p(t) & \varphi_i(t) & \varphi_d(t) \end{bmatrix}$$

If the homogeneous system $\frac{dx}{dt} = Ax$ is asymptotically stable, then there exist positive definite matrices P and Q such that

$$A^T P + PA = -Q$$

Then one Lyapunov function is constructed as follows.

$$V = \frac{1}{2}\left(\gamma x^T P x + (\theta_0 - \theta)^2\right) \tag{23}$$

Taking the partial derivation with respect to time, then it holds that

$$\frac{dV}{dt} = \frac{\gamma}{2}\left(\frac{dx^T}{dt}Px + x^T P \frac{dx}{dt}\right) - (\theta_0 - \theta)\frac{d\theta}{dt} \tag{24}$$

Substituting Equation (22) into (24), we obtain

$$\begin{aligned} \frac{dV}{dt} &= \frac{\gamma}{2}(\theta_0 - \theta)^T B^T P x + \frac{\gamma}{2}\left(x^T A^T P x + x^T P A x\right) + \frac{\gamma}{2} x^T P B(\theta_0 - \theta) - (\theta_0 - \theta)\frac{d\theta}{dt} \\ &= \frac{\gamma}{2} x^T\left(A^T P + PA\right)x + (\theta_0 - \theta)\left(-\frac{d\theta}{dt} + \frac{\gamma}{2}B^T P x + \frac{\gamma}{2}x^T P B\right) \\ &= -\frac{\gamma}{2}x^T Q x + (\theta_0 - \theta)\left(-\frac{d\theta}{dt} + \gamma B^T P x\right) \end{aligned} \tag{25}$$

If the parameter adjustment law is chosen as

$$\frac{d\theta}{dt} = \gamma B^T P x = \gamma B^T P(sI - A)^{-1}B(\theta_0 - \theta) \tag{26}$$

then the partial derivative of the constructed Lyapunov function will be negative as long as $x \neq 0$. The state vector x and the tracking error $e = Cx$ will go to zero as t goes to infinity. The parameter adjustment law (26) can be applied to replacing Equation (19) in the adaptive direct data driven control scheme.

6. **Simulation Example.** Here in this section, two simulation examples are used to prove the efficiency of our proposed theories about adaptive direct data driven control scheme without the plant.

1) Firstly consider the following linear discrete time system, whose model of the plant is that

$$P(z) = \frac{1 - 0.2z^{-1}}{1 - 0.6z^{-1}}$$

where true plant $P(z)$ is unknown, and the expected closed loop transfer function $M(z)$ is

$$M(z) = \frac{11.306(z - 0.53)}{z^2 - 0.706z + 0.32}$$

Applying direct data driven control to deal with the measured data, then the initial values for controller parameters are given as

$$\theta_0 = \begin{bmatrix} K_{p0} & K_{i0} & K_{d0} \end{bmatrix}^T = \begin{bmatrix} 0.4021 & 0.3504 & 0.2475 \end{bmatrix}^T$$

In simulation, sampled period is chosen as 0.5s. From the closed loop step response curve in Figure 4, we see that after initial parameter values of the controller are tuned by off-line direct data driven control, the closed loop step response starts to run without any substantial oscillation, and the overshot is very small. During the first 80 seconds, the initial parameter values of the controller keep running smoothly. After 80 seconds, some parameter values of the plant are changed as follows.

$$P(z) = \frac{1 - 1.8z^{-1}}{1 - 0.1z^{-1}}$$

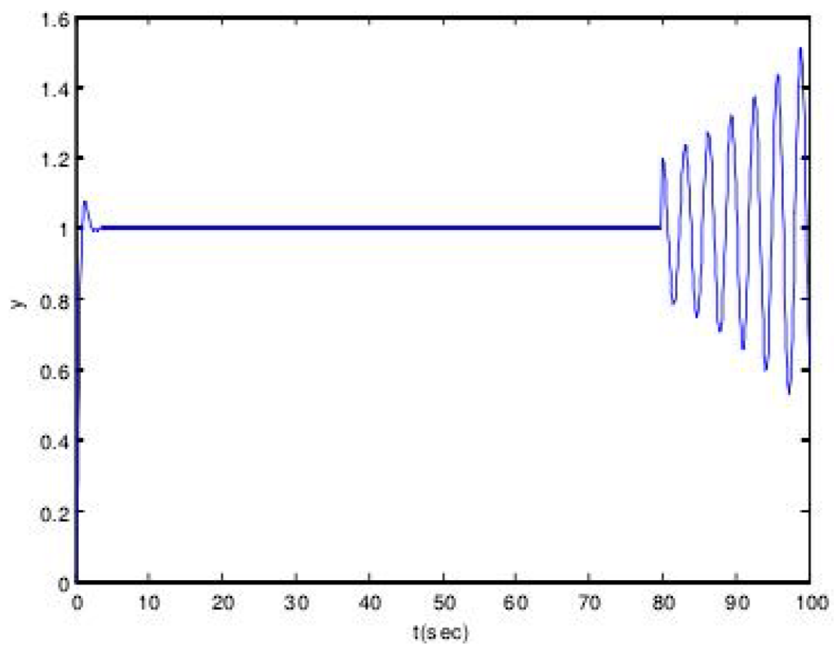

FIGURE 4. Step response curve for direct data driven control

Due to the insufficient stability of the PID controller, the considered closed loop system will diverge, and it means that the initial parameter values corresponding to direct data driven control cannot adapt to the time varying property. Closed loop step response obtained by our proposed adaptive direct data driven control is plotted in Figure 5, where at 80 seconds, the whole closed loop system fluctuates. However, after updating new measured data, the controller parameters are continuously corrected, so that the whole closed loop system will be stable in 4 seconds. Meanwhile, the simulation curves show when the forgetting factor is closed to the value of 1, the closed loop system's fluctuation is gentle, but the correction time is long. On the contrary, when the forgetting factor approaches to zero, the system starts to fluctuate greatly, and the correction process is relatively fast. Such above facts mean the choice of forgetting factor will affect the fluctuation size and the speed for correction.

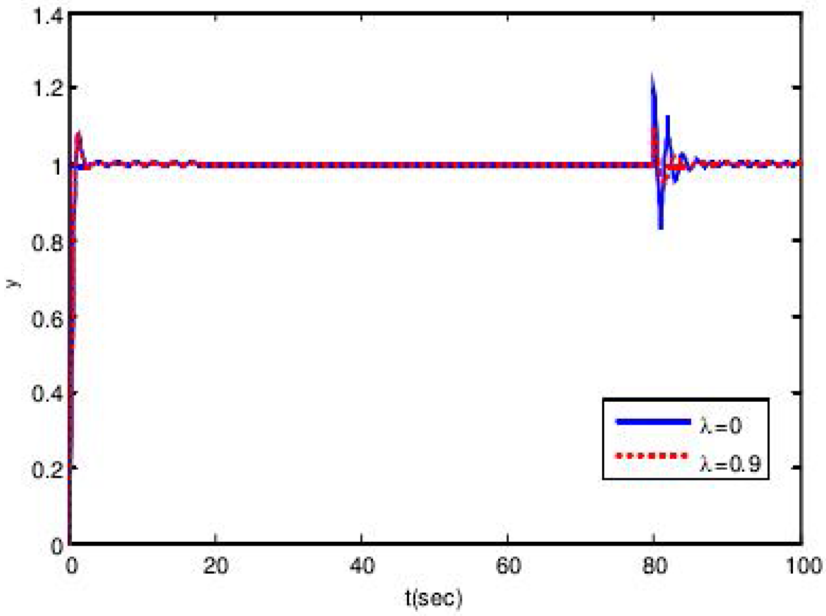

FIGURE 5. Step response curve for adaptive direct data driven control

2) Secondly consider one discrete time linear system, its transfer function form is described as follows.

$$P(z) = \frac{(z-1.2)(z-0.4)}{z(z-0.3)(z-0.8)}$$

One classical PID controller is used here in this second simulation.

$$C_1(z,\theta) = \alpha^T(z)\theta = \begin{bmatrix} \frac{z^2}{z^2-z} & \frac{z}{z^2-z} & \frac{1}{z^2-z} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

The true PID controller is given as

$$C_1(z,\theta) = \alpha^T(z)\theta = \begin{bmatrix} \frac{z^2}{z^2-z} & \frac{z}{z^2-z} & \frac{1}{z^2-z} \end{bmatrix} \begin{bmatrix} 0.86 \\ 0.2 \\ 0.1 \end{bmatrix}$$

The expected closed loop transfer function is chosen as

$$M(z) = \frac{z(z-1)\left(0.86z^4 - 1.1z^3 + 3.9z^2 + 0.8z + 0.48\right)}{z^7 - 3z^6 - 0.96z^5 - 0.72z^4 - 0.93z^3 + 3.9z^2 + 0.8z + 0.48}$$

The input-output measured data $\{u(t), y(t)\}_{t=1,2,\ldots,1000}$ are collected in the closed loop environment, and the number of data points is set 2000. To use the idea of virtual reference feedback tuning control in designing parameter vectors, the plant model $P(z)$ is excited by zero mean Gaussian white noise, which is plotted in Figure 6 with the number of data points being 1000, and the measured output data is seen in Figure 7. The adaptive parameter algorithm is used to solve the optimization problem (9). Before starting this iteration algorithm, the initial values of the unknown parameter vector are selected as

$$\theta = \begin{bmatrix} 0.75 \\ 0.25 \\ 0.15 \end{bmatrix}$$

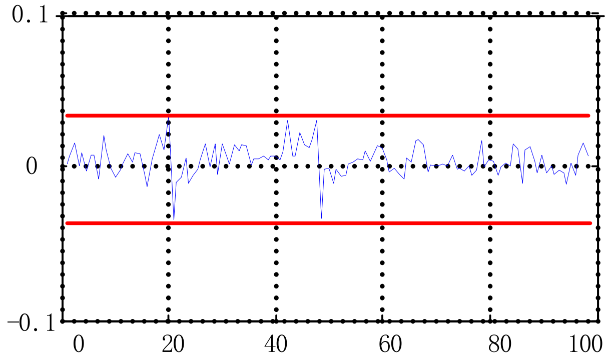

FIGURE 6. The input signal as a white noise

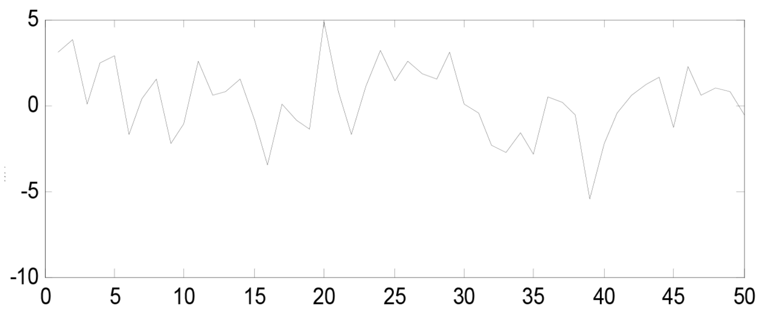

FIGURE 7. The measured output signal

As the considered adaptive parameter algorithm is also one iterative algorithm, then after 80 iterative steps we define the relative error for the obtained parameter estimators as $\frac{\|\hat{\theta}_N - \hat{\theta}_{N-1}\|}{\|\hat{\theta}_N\|}$. The tendency of cost function is shown in Figure 8, with the iterative steps increasing. From Figure 8, we see that cost function is decreased with the iterative steps, and after 80 iterative steps, the cost function will approach to zero value. It means the iterative parameter estimator $\hat{\theta}_{80}$ can be used as the final parameter estimator.

7. **Future Work.** As adaptive control and direct data driven control are two different and separate research fields, and this paper is the first one to bridge these two separable researches, the closed loop system studied in Figure 1 is a very simple system. In our earlier work [11-14], more complex closed loop system with two degrees of freedom controllers $(C_1(z, \theta), C_2(z, \eta))$ is studied in Figure 9, where two controllers are designed simultaneously by direct data driven control. So based on our results in this paper, we

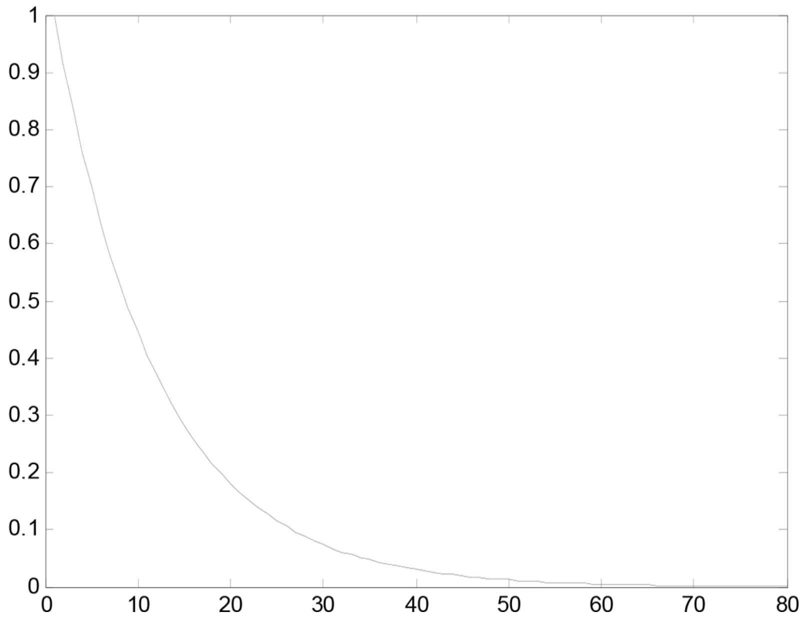

Figure 8. The decreased cost function

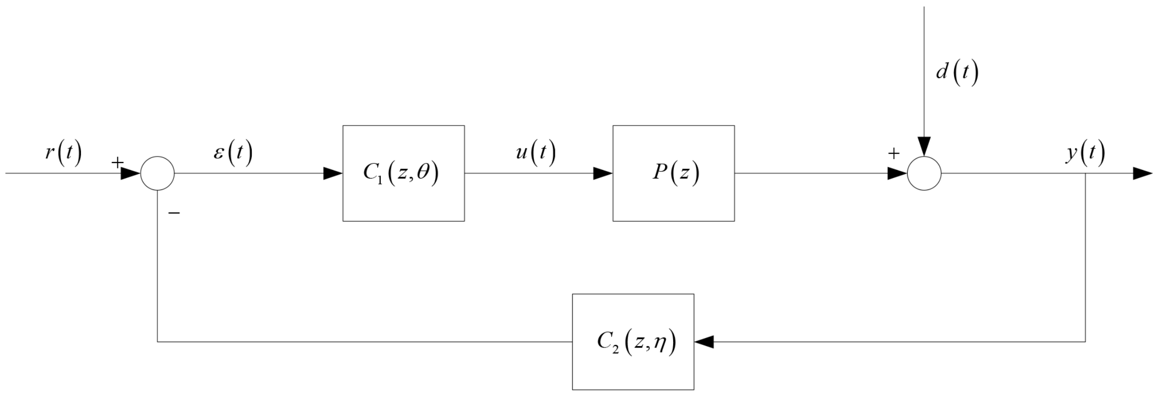

Figure 9. Closed loop system with two degrees of freedom controllers

will develop one adaptive direct data driven control for two degrees of freedom controllers in stochastic environment.

Due to two controllers in Figure 9, we need to determine where the adaptive mechanism is placed and whether centralized adjustment or distributed adjustment is applied to generating virtual input and virtual disturbance, for example, the centralized adjustment mechanism in Figure 10.

8. **Conclusion.** The paper connects adaptive control and direct data driven control to adjust the controller parameters adaptively. One adjustment mechanism is constructed by using measured data, and recursive least squares with forgetting factor or parameter adjustment law are proposed to achieve the adaptation. Furthermore Lyapunov's stability theory is also used to derive parameter adjustment law such that stability can be guaranteed. Such adaptive direct data driven control scheme expands the ability of the traditional adaptive control to deal with the changing environment, and can be applied to more complex closed loop system with two degrees of freedom controllers, which is our next idea.

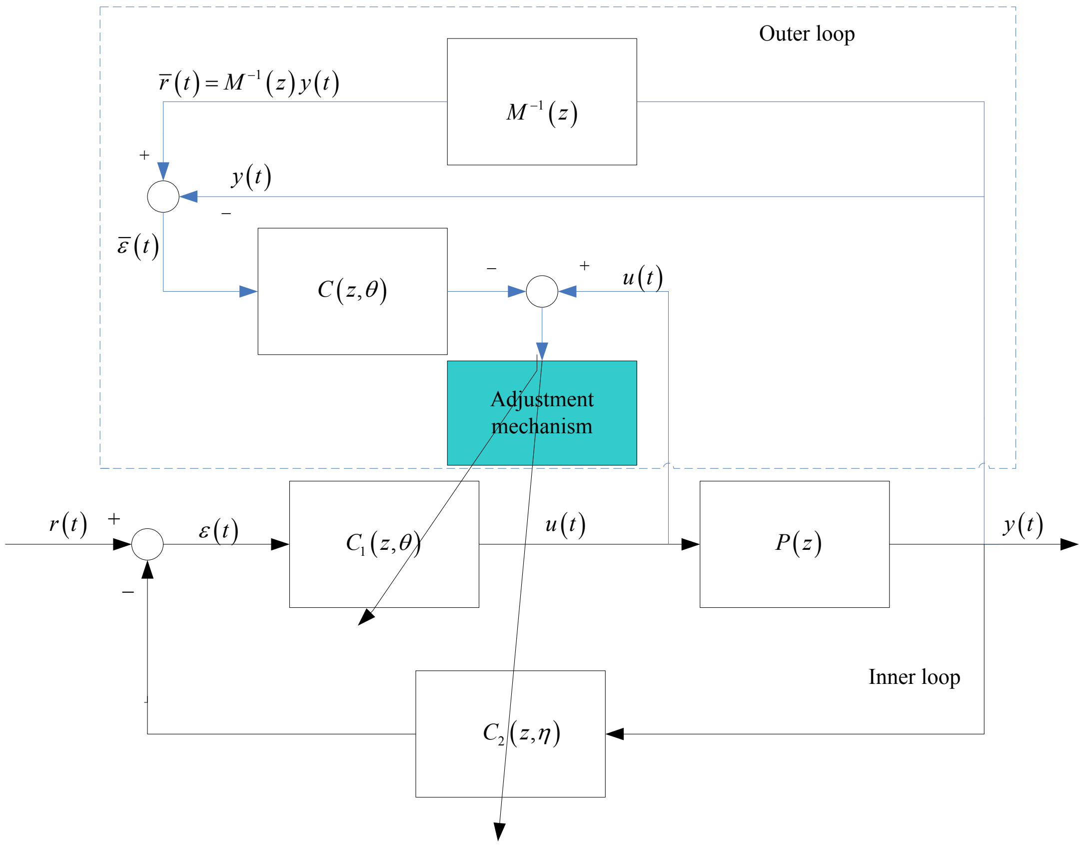

Figure 10. Centralized adjustment mechanism

Acknowledgment. This work is partially supported by the Grants from the National Science Foundation of China (No. 61364014) and (No. jxxjb18020). The author is grateful to Professor Eduardo F Camacho for his warm invitation in his control lab at the University of Seville, Seville, Spain. Thanks for his assistance and advice on zonotopes in guaranteed state estimation and model predictive control.

REFERENCES

[1] A. S. Bazanella, M. Gevers, L. Mišković and B. D. O. Anderson, Iterative minimization of H_2 control performance criterion, *Automatica*, vol.44, no.10, pp.2549-2559, 2008.
[2] A. Sala and A. Esparza, Extension to "virtual reference feedback tuning: A direct method for the design of feedback controllers", *Automatica*, vol.41, no.8, pp.1473-1476, 2005.
[3] G. Guardabassi, Virtual reference direct method: An off line approach to data based control system design, *IEEE Trans. Automatic Control*, vol.45, no.4, pp.954-960, 2002.
[4] J. M. Bravo, T. Alamo and E. F. Camacho, Bounded error identification of systems with time varying parameters, *IEEE Trans. Automatic Control*, vol.51, no.7, pp.1144-1150, 2006.
[5] J. M. Bravo, A. Suarez and M. Vasallo, Slide window bounded error time varying systems identification, *IEEE Trans. Automatic Control*, vol.61, no.9, pp.2282-2287, 2016.
[6] J. M. Bravo, T. Alamo and M. Vasallo, A general framework for predictions based on bounding techniques and local approximations, *IEEE Trans. Automatic Control*, vol.62, no.6, pp.430-446, 2017.
[7] M. Tanaskovic, L. Fagiano and C. Novara, Data driven control of nonlinear systems: An on line direct approach, *Automatica*, vol.75, no.12, pp.1-10, 2017.
[8] M. C. Campi, Virtual reference feedback tuning; A direct method for the design of feedback controllers, *Automatica*, vol.38, no.2, pp.1337-1346, 2002.
[9] M. C. Campi and A. Lechini, An approach of the virtual reference feedback tuning method to a benchmark problem, *European Journal of Control*, vol.9, no.5, pp.66-76, 2005.

[10] M. C. Campi, Direct nonlinear control design: The virtual reference feedback tuning approach, *IEEE Trans. Automatic Control*, vol.51, no.8, pp.14-27, 2006.
[11] J. Wang, Virtual reference feedback tuning control for constrained closed loop system, *Journal of Shanghai Jiaotong University*, vol.46, no.2, pp.1398-1405, 2012.
[12] J. Wang, Virtual reference feedback tuning control design for nonlinear closed loop system, *Electronic Optics and Control*, vol.20, no.1, pp.23-27, 2013.
[13] J. Wang, Virtual reference feedback tuning design in internal model control, *Journal of System Science and Mathematical Science*, vol.37, no.2, pp.355-369, 2017.
[14] J. Wang, Virtual reference feedback tuning control and iterative minimization identification, *Systems Engineering Theory and Practice*, vol.34, no.2, pp.1256-1266, 2014.
[15] S. Grammatico, X. Zhang and K. Margellos, A scenarion approach for non-convex control design, *IEEE Trans. Automatic Control*, vol.61, no.2, pp.334-344, 2016.
[16] A. Care, B. Csorji and M. C. Campi, Finite sample system identification: An overview and a new correlation method, *IEEE Control Systems Letters*, vol.2, no.1, pp.61-66, 2018.
[17] E. Weyer and M. C. Campi, Asymptotic properties of SPS confidence regions, *Automatica*, vol.82, no.3, pp.287-294, 2017.
[18] S. A. Mehta and D. M. Adhyaru, Review of solution methods for the fractional optimal control, *ICIC Express Letters*, vol.13, no.11, pp.995-1004, 2019.