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ADAPTIVE FINITE-TIME SYNERGETIC CONTROL DESIGN FOR POWER SYSTEMS WITH STATIC VAR COMPENSATOR

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Abstract. *In this paper, an adaptive finite-time synergetic control for an electric power system including Static Var Compensator (SVC) is proposed. Despite unknown parameters, the developed controller is able to enhance transient stability and guarantee that all trajectories of the whole closed-loop system are driven to the desired manifold in a finite time. In this work, there are two unknown parameters of particular interest, that is, a damping coefficient and a mechanical input power. The dynamic characteristics of the proposed control are evaluated using a Single-Machine Infinite Bus (SMIB) power system implemented in MATLAB environment. The simulation results are presented to verify the effectiveness and feasibility of the developed control scheme once compared with that of an adaptive synergetic control technique. Further, they illustrate that in spite of unknown parameters, the proposed scheme offers better transient performances in terms of faster damping oscillations, a shorter settling time, and so on.*
Keywords: Transient stability, Generator excitation, SVC, Synergetic control theory, Adaptive control, Finite-time stabilization

1. **Introduction.** It is well-known that the size and complexity of power systems are continuously increasing, leading to the trend of power grid development. Nevertheless, once power systems are further connected until the whole power systems become a large-scale power system. Thus, maintaining power system stability along with frequency and voltage regulation for large-scale system is of greater importance. Further, under several contingencies, large-scale power systems, which have highly nonlinear characteristics, may encounter lots of unstable factors, thereby resulting in power system instability and decreasing the stability margin of power systems. As a result, there are recently attempts to find out an effective control technique to deal with the adverse effect of nonlinear behaviors of power systems and enhance power system stability once disturbances occur. Recently, the design of an advanced nonlinear control strategy has received a variety of research to not only enhance power system stability margins, but also increase power flow transfer capability in transmission line systems.

According to the recent literature, a promising and efficient methodology to improve power system stability and increase power transfer capability is a coordination of generator excitation control and Flexible AC Transmission Systems (FACTS) devices. FACTS

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devices [1, 2] are of great importance because they are used to augment the power flow and voltage controllability along with the power system stability. Additionally, they include SVC, STATCOM, TCSC, SSSC, TCPAR, UPFC, etc. These FACTS are always used to enhance power flow and voltage stability, to damp out inter-area and system oscillations, to augment reactive power control, and to enhance steady-state and dynamic stability of interconnected and long-distance transmission systems. In a family of these FACTS devices, Static Var Compensator (SVC) is of particular interest in this work because it is extensively utilized in power systems. In particalar, both active and reactive power can be both injected and absorbed from SVC control algorithm for grid transfer capability augmentation, smooth and rapid reactive power compensation, and damping oscillations [3-5].

There are recently a variety of researches focusing on the coordination of generator excitation and SVC controller for power systems via the advanced nonlinear control theory [6-13]. In [6], an incorporation of generator excitation control and SVC control for power systems was designed by using a means of the exact linearization method. Using a combination of feedback linearization design and control of differential and algebraic systems, Ruan and Wang [7] proposed a nonlinear control capable of improving power system stability and voltage stability for power systems including SVC. An adaptive controller based on a combined backstepping design and sliding mode scheme was reported in [9] for mitigating the undesired effects of external disturbances on the system output and achieving strong robustness properties in spite of system parameter variations. With the help of a coordination of adaptive Immersion and Invariance (I&I) method and $\mathcal{L}_2$ control design, an adaptive control law [10] has been presented to improve the system transient performances and the convergence speed of the system states despite having the non-parametric uncertainties. Kanchanaharuthai [11] proposed a nonlinear control control by means of I&I design methodology to accomplish power system stability, frequency and voltage regulation in spite of having both large and small perturbations. This method confirms superior performances over both coodinated passivation control and feedback linearizing control. In [12], a combined excitaion and SVC controller design has been presented by using a backstepping-like control to achieve both the system stability improvement and better dynamic properties than a conventional backstepping control. More recently, an improved adaptive backstepping sliding mode control [13] for power system with SVC was developed. This technique included the results of error compensation and $\mathcal{K}$-class function used to obtain better performances than a conventaional adaptive backstepping sliding mode controller, thereby leading to further improved performances of the overall closed-loop system stability.

This work continues this line of investigation and improves further transient performance of the system. Consequently, this paper concentrates on the design of an adaptive finite-time control scheme based on synergetic control theory for power systems with SVC. The synergetic control method was first developed by Kolesnikov et al. [14, 15]. There are recently lots of successful applications of synergetic control approach to many practical engineering systems as reported in [16-25]. Therefore, the main contributions of this paper can be listed as follows. (i) Based on synergetic control theory, an adaptive finite-time control scheme for power systems including SVC is developed for driving all trajectories to the equilibrium in finite time. (ii) With the help of Lyapunov stability, the stability analysis for the controlled power system is proven. (iii) Despite the presence of unknown parameters, the proposed controller is successfully applied to achieving improvement of transient stability and regulation of both frequency and terminal voltage. (iv) In comparison with an adaptive synergetic control method, the developed control method provides better dynamic performances and faster ability of suppressing power oscillation.

The remainder of this paper is organized as follows. A dynamic model of an SMIB power system including SVC is briefly mentioned in Section 2. Adaptive finite-time synergetic control design and stability analysis are stated in Section 3 while simulation results are stated in Section 4. Finally, a conclusion is given in Section 5.

2. **Power System Model with SVC.** According to the result presented in [11], an SMIB power system with generator excitation control of a synchronous generator and SVC control can be expressed as

$$\begin{cases} \dot{\delta} = \omega - \omega_s \\ \dot{\omega} = \dfrac{1}{M}(P_m - P_e - P_{svc} - D(\omega - \omega_s)) \\ \dot{E} = -\dfrac{X_{d\Sigma}}{X'_{d\Sigma}T'_0}E + \dfrac{X_d - X'_d}{X'_{d\Sigma}T'_0}V_\infty \cos\delta + \dfrac{u_f}{T'_0} \end{cases} \tag{1}$$

where δ is the power angle of the generator, ω denotes the relative speed of the generator, $D \geq 0$ is a damping constant, P_e is the electrical power delivered by the generator to the voltage at the infinite bus V_∞, ω_s is the synchronous machine speed, $\omega_s = 2\pi f$, H represents the per unit inertial constant, f is the system frequency and $M = 2H/\omega_s$. $X'_{d\Sigma} = X'_d + X_T + X_L$ is the reactance consisting of the direct axis transient reactance of SG, the reactance of the transformer, and the reactance of the transmission line X_L. Similarly, $X_{d\Sigma} = X_d + X_T + X_L$ is identical to $X'_{d\Sigma}$ except that X_d denotes the direct axis reactance of SG. T'_0 is the d-axis transient short-circuit time constant. $X_1 = X'_d + X_T$, $X_2 = X_L$, u_f is the field voltage control input. P_m is the mechanical input power to be assumed constant throughout this paper.

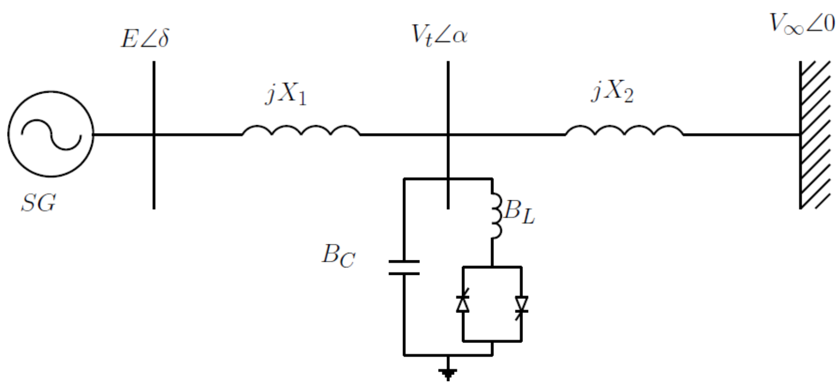

FIGURE 1. Network with Fixed Capacitor and Thyristor Controlled Reactor (FC-TCR) SVC

Although the static var compensator has a lot of different models, in this paper we focus on a kind of SVC consisting of Fixed Capacitor and Thyristor Controlled Reactor (FC-TCR). For this kind of SVC, the current (I_L) of the reactor thyristor relies on a firing angle α that is controlled via a control law presented in this work. Also, the current can be expressed in the following equation:

$$I_L(\alpha) = \frac{V_t(2(\pi - \alpha) + \sin 2\alpha)}{\pi\omega L} \tag{2}$$

where $\pi/2 \leq \alpha \leq \pi$. Let us define $\sigma = 2(\pi - \alpha)$ as the conduction angle. Thus, we have the reactor thyristor current as follows:

$$I_L(\sigma) = \frac{V_t(\sigma - \sin\sigma)}{\pi\omega L} \tag{3}$$

Subsequently, it is straightforward to compute the susceptance of reactor thyristor as

$$B_L(\sigma) = \frac{I_L(\sigma)}{V_t} = \frac{\sigma - \sin\sigma}{\pi\omega L} \tag{4}$$

Also, the equivalent SVC susceptance becomes the summation of the fixed capacitor susceptance (B_c) and the thyristor controlled variable reactor suspectance (B_L) that can be expressed as

$$B_{SVC} = B_c + B_L(\sigma) \tag{5}$$

In this work, the dynamics of SVC is assumed as the following first-order system:

$$\dot{B}_{SVC} = \frac{1}{T_r}(B_{SVC} - B_{SVC0} + u_r) \tag{6}$$

where B_{SVC0} is the initial value of the inductor in SVC (pu.), u_r is the SVC control input to be designed, and T_r is an SVC time constant.

For convenience, let us introduce the vector of the state variable as $x = [x_1, x_2, x_3, x_4]^T = [\delta, \omega - \omega_s, P_e, P_{svc}]^T$, where

$$\begin{aligned} P_e &= \frac{EV_\infty \sin\delta}{X'_{d\Sigma}} \\ P_{svc} &= \frac{(X'_d + X_T)\, X_L(B_{SVC} - B_c)}{X'_{d\Sigma} - (X'_d + X_T)\, X_L(B_{SVC} - B_c)} P_e. \end{aligned}$$

Then, taking derivative of such state variables with respect to time, we obtain the dynamic model of power systems including SVC written in an affine nonlinear system as follows:

$$\dot{x} = f(x) + g(x)u(x) \tag{7}$$

with

$$f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{1}{M}(P_m - Dx_2 - x_3 - x_4) \\ (-a + x_2 \cot x_1)x_3 + \frac{bV_\infty \sin 2x_1}{X'_{d\Sigma}} \\ \tilde{M}(x_1, x_3, x_4) f_3(x) + \tilde{N}(x_1, x_3, x_4)(B_L - B_{L0}) \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix},$$

$$g(x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ g_{31}(x) & 0 \\ g_{41}(x) & g_{42}(x) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{V_\infty \sin x_1}{X'_{d\Sigma}} & 0 \\ \tilde{M}(x_1, x_3, x_4) g_{31}(x) & -\tilde{N}(x_1, x_3, x_4) \end{bmatrix}, \quad u(x) = \begin{bmatrix} \frac{u_f}{T'_0} \\ \frac{u_r}{T_r} \end{bmatrix}$$

where

$$\begin{aligned} \tilde{M}(x_1, x_3, x_4) &= \left(\frac{X'_{d\Sigma}}{X'_{d\Sigma} - (X'_d + X_T)X_L(B_{SVC} - B_c)} - 1 \right), \\ \tilde{N}(x_1, x_3, x_4) &= -\frac{x_3(X'_d + X_T)X_L X'_{d\Sigma}}{(X'_{d\Sigma} - (X'_d + X_T)X_L(B_{SVC} - B_c))^2}, \\ B_{SVC} &= \frac{1}{(X'_d + X_T)X_L}\left(X'_{d\Sigma} - \frac{x_3 X'_{d\Sigma}}{x_3 + x_4} \right) + B_c, \quad a = \frac{X_{d\Sigma}}{X'_{d\Sigma} T'_0}, \quad b = \frac{X_{d\Sigma} - X'_{d\Sigma}}{X'_{d\Sigma} T'_0} V_\infty. \end{aligned}$$

Further, the bus (terminal) voltage V_t, as shown in Figure 1, can be directly computed from (7); thus, it is easy to obtain the following expression

$$V_t = \left(1 + \frac{x_4}{x_3}\right) \cdot \frac{\sqrt{\left(\frac{x_3 X'_{d\Sigma} X_2}{V_\infty \sin x_1}\right)^2 + (V_\infty X_1) + 2X_1 X_2 X'_{d\Sigma} x_3 \cot x_1}}{X'_{d\Sigma}}.$$

Additionally, the region of operation is defined in the set $\mathcal{D} = \left\{x \in \mathcal{S} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \middle| \, 0 < x_1 < \frac{\pi}{2}\right\}$. The open loop operating equilibrium is denoted by $x_e = [x_{1e},\ x_{2e},\ x_{3e},\ x_{4e}]^T = [\delta_e,\ 0,\ P_m,\ 0]^T$.

In this paper, an adaptive controller based on finite-time synergetic control design is proposed for the system stability improvement of power systems with SVC in the presence of two unknown parameters: a damping coefficient and a mechanical input power [26]. Let us define $\theta = [\theta_1, \theta_2]^T = [-D, P_m]^T$ as the vector of unknown constant parameters of interest. As a result, the system (7) can be rewritten as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{1}{M}(\theta_2 + \theta_1 x_2 - x_3 - x_4) \\ \dot{x}_3 = f_3(x) + g_{31}(x)\dfrac{u_f}{T'_0} \\ \dot{x}_4 = f_4(x) + g_{41}(x)\dfrac{u_f}{T'_0} + g_{42}(x)\dfrac{u_r}{T_r} \end{cases} \tag{8}$$

Therefore, the aim of this work is to design an adaptive (state) feedback control via the adaptive finite-time control strategy to enhance transient stability for the system (8) in the presence of unknown constant parameters θ.

$$\begin{cases} u = \phi\left(x, \hat{\theta}\right) \\ \dot{\hat{\theta}} = \varpi\left(x, \hat{\theta}\right) \end{cases} \tag{9}$$

such that all trajectories of the overall closed-loop system converge to the desired equilibrium point (x_e, θ) in a finite time where $\hat{\theta}$ is the estimate of $\theta = [\theta_1, \theta_2]^T$.

Before turning to the design procedure in the next section, the following technical lemmas are used to design the proposed control scheme.

Lemma 2.1. [27] *Suppose that a continuous, positve-definite function $V(t)$ satisfies the following inequality*

$$\dot{V}(t) \leq -\alpha V^\eta, \quad \forall t \geq t_0, \quad V(t_0) \geq 0 \tag{10}$$

where $\alpha > 0$, $0 < \eta < 1$ are two constants. Then, for all any initial time t_0, $V(t)$ satisfies the following inequalitiy:

$$V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - \alpha(1-\eta)(t - t_0), \quad t_0 < t < t_1 \tag{11}$$

and

$$V \equiv 0, \quad \forall t \geq t_1 \tag{12}$$

with t_1 computed by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{\alpha(1-\eta)} \tag{13}$$

Lemma 2.2. *Suppose $a_1, a_2, \ldots, a_n$ and $0 < q < 2$ are all real numbers, then the following inequality holds:*

$$(|a_1|^q + |a_2|^q + \cdots + |a_n|^q) \leq \left(a_1^2 + a_2^2 + \cdots + a_n^2\right)^{\frac{q}{2}} \tag{14}$$

Lemma 2.3. (*Jensen's inequality*)*: If $0 < a_1 < a_2$, then $\left(\sum_{i=1}^{n} x_i^{a_2}\right)^{\frac{1}{a_2}} \leq \left(\sum_{i=1}^{n} x_i^{a_1}\right)^{\frac{1}{a_1}}$ for $x_i \geq 0$, $i = 1, 2, \ldots, n$.*

Problem statement: The control objective of this paper is to solve the problem of the stabilization of the system (8) with unknown parameters θ, which can be formulated as follows: with the help of synergetic control theory, we design an adaptive finite-time control law $u(x)$ and parameter estimation $\hat{\theta}$ as follows:

$$u = \phi\left(x, \hat{\theta}\right), \quad \dot{\hat{\theta}} = \varphi\left(x, u, \hat{\theta}\right) \tag{15}$$

such that all trajectories of the overall closed-loop system (8) and (15) converge to the desired equilibrium in finite time, where $\hat{\theta}$ is the estimate of θ.

3. Design of Adaptive Finite-Time Synergetic Controller.

3.1. Finite-time synergetic control method.

As developed in [19], a finite-time synergetic design is an invariant-manifold-based control method capable of steering the state trajectories of the closed-loop system to the desired manifold in a finite time. This technique is further extended from a synergetic control design as proposed in [14, 15, 20] for systems. Also, it is successfully applied for a variety of practical applications.

Let us consider an n-dimensional nonlinear dynamic equation[1] in the state space form as

$$\dot{x}(t) = f(x, u, t) \tag{16}$$

where $x \in \mathbb{R}^n$ denotes the system state variable vector, $u \in \mathbb{R}^m$ is the control input vector to be designed, t is time, and $x_e \in \mathbb{R}^n$ denotes an assignable equilibrium point to be stabilized, respectively. The design procedure of the finite-time synergetic control design requires the following three steps.

- **Step 1:** Define a macro-variable as $\varphi(x)$ where $\varphi(x)$ is a function of the system states. With the help of finite-time synergetic control strategy, a stabilizing control law to steer the state trajectories into the desired manifold $\mathcal{M}$ relies on this macro-variable.
- **Step 2:** Design a control law which is able to drive all system states onto the specified manifold $\mathcal{M}$ smoothly at finite time and then remain on this manifold thereafter. Also, the desired control law needs to satisfy an evolution constraint as follows

$$T\dot{\varphi}(x)^{\frac{p}{r}} + \varphi(x) = 0, \quad T > 0 \tag{17}$$

  where T is a design parameter. p and r are positive odd numbers satisfying the condition $1 < p/r < 2$. This constraint indicates that the both macro-variable φ and its derivative $\dot{\varphi}$ will be driven to zero in finite time. Also, it can be observed that the parameters in the constraint above directly affect the convergence rate of the system trajectories approaching the manifold $\mathcal{M}$ at a finite time.

[1]It is assumed that throughout this paper all functions and mappings are $\mathbb{C}^\infty$.

- **Step 3:** Differentiating the selected macro-variable $\varphi(x)$ with respect to the system variable x, taking account of the chain rule of differentiation, and then plugging (16) back in (17), it is straightforward to obtain the following constraint:

$$\frac{\partial\varphi(x)}{\partial x}f(x,u,t)+\left(T^{-1}\varphi(x)\right)^{\frac{r}{p}}=0 \tag{18}$$

For the constraint above, the desired control can be found from selecting a suitable macro-variable φ and the design parameter T together with two positive odd numbers p, r. Thus, after directly solving the constraint (18), the desired controller is expressed as

$$u(x)=\phi(x,\varphi(x),T,t,p,r) \tag{19}$$

It can be observed that from solving the constraint (18), an analytical controller can not only drive the system trajectories to the desired manifold $\mathcal{M}$ in a finite time, but also accomplish the desired control performances and specifications.

3.2. **Adaptive finite-time control design.** The objective of this subsection is to drive the state trajectories of the power systems with SVC in spite of unknown parameters to approach the specified manifold $\mathcal{M}$ in a given finite time and remain on it forever. Nevertheless, to tackle the effect of unknown parameters, parameter update rule is defined, and finite-time stability is guaranteed as well. From (8), an error coordinate variable is defined as

$$e=\begin{bmatrix}e_1\\e_2\\e_3\\e_4\end{bmatrix}=\begin{bmatrix}x_1-x_{1e}\\x_2-x_{2e}\\x_3-x_{3e}\\x_4-x_{4e}\end{bmatrix}=\begin{bmatrix}x_1-\delta_e\\x_2\\x_3-P_m\\x_4\end{bmatrix} \tag{20}$$

Thus, the proposed control law based on finite time synergetic control scheme above is provided by

$$\begin{cases}\dfrac{u_f\left(x,\hat{\theta}\right)}{T_0'}=-\dfrac{1}{\beta_1 g_{31}(x)}\left[\beta_1 f_3(x)+x_2+\dot{\hat{x}}_2-\left(-\dfrac{1}{T_1}\varphi_1\right)^{\frac{r_1}{p_1}}\right]\\ \dfrac{u_r\left(x,\hat{\theta}\right)}{T_r}=-\dfrac{1}{\beta_2 g_{42}(x)}\left[\beta_2\left(f_4(x)+g_{41}(x)\dfrac{u_f}{T_f}\right)+x_2+\dot{\hat{x}}_2-\left(-\dfrac{1}{T_2}\varphi_2\right)^{\frac{r_2}{p_2}}\right]\end{cases} \tag{21}$$

where $\dot{\hat{x}}_2=\frac{1}{M}\left(\hat{\theta}_2+\hat{\theta}_1x_2-x_3-x_4\right)$. $\hat{\theta}$ is an estimation of the unknown parameter θ. In addition, the suitable parameter update laws for unknown parameters of the controlled system can be selected as

$$\begin{cases}\dot{\hat{\theta}}_1=\gamma_1\left[k_1\tilde{\theta}_1^{\frac{r_1}{p_1}}+\dfrac{x_2}{M}(\varphi_1+\varphi_2)\right]\\ \dot{\hat{\theta}}_2=\gamma_2\left[k_2\tilde{\theta}_2^{\frac{r_2}{p_2}}+\dfrac{1}{M}(\varphi_1+\varphi_2)\right]\end{cases} \tag{22}$$

where γ_j, k_j $(j=1,2)$ are positive design parameters. φ_i, $(i=1,2)$ are the macro-variables that need to be selected.

In what follows, the finite-time stability of the overall closed-loop power systems with SVC in the presence of unknown parameters is proved.

Theorem 3.1. *Consider the power systems with SVC in the presence of the unknown parameters. If the developed control law is chosen as (21) together with (22), the state trajectories of the system will converge to the specified manifold $\mathcal{M}$ in a given finite time and remain on it forever.*

Proof: The proof of the result can be done by the following three steps.

Step 1: Consider the following macro-variables as follows:

$$\begin{cases} \varphi_1 = s_1(e) = e_1 + e_2 + \beta_1 e_3 \\ \varphi_2 = s_2(e) = e_1 + e_2 + \beta_2 e_4 \end{cases} \tag{23}$$

where β_1, β_2 denote positive design parameters.

Step 2: The system trajectories of the system will be steered and forced to approach to the desired manifold $\varphi = \dot{\varphi} = 0$ in finite time, and then they remain on it thereafter. Therefore, in the dynamics of the evolution, each macro-variable (23) is given by

$$T_i \dot{\varphi}_i^{\frac{p_i}{r_i}} + \varphi_i = 0, \quad T_i > 0, \quad i = 1, 2 \tag{24}$$

where T_i are the pre-specified controller parameters indicating the converging speed of the closed-loop dynamics to the desired manifold $\varphi(x) = 0$. p_i and r_i are positive odd number satisfying the condition $1 < p_i/r_i < 2$.

Thus, the time derivative of the error variables (23) along the system trajectory (8) becomes

$$\dot{\varphi}_i = \left(-\frac{1}{T}\varphi\right)^{\frac{r_i}{p_i}} \tag{25}$$

$$\begin{bmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial s_1}{\partial e_1}\dot{e}_1 + \frac{\partial s_1}{\partial e_2}\dot{e}_2 + \frac{\partial s_1}{\partial e_3}\dot{e}_3 \\ \frac{\partial s_2}{\partial e_1}\dot{e}_1 + \frac{\partial s_2}{\partial e_2}\dot{e}_2 + \frac{\partial s_2}{\partial e_4}\dot{e}_4 \end{bmatrix} = \begin{bmatrix} \dot{e}_1 + \dot{e}_2 + \beta_1 \dot{e}_3 \\ \dot{e}_1 + \dot{e}_2 + \beta_2 \dot{e}_4 \end{bmatrix} = \begin{bmatrix} \left(-\frac{1}{T_1}\varphi_1\right)^{\frac{r_1}{p_1}} \\ \left(-\frac{1}{T_2}\varphi_2\right)^{\frac{r_2}{p_2}} \end{bmatrix} \tag{26}$$

where $\frac{\partial s_1}{\partial e_1} = \frac{\partial s_1}{\partial e_2} = \frac{\partial s_2}{\partial e_1} = \frac{\partial s_2}{\partial e_2} = 1$, $\frac{\partial s_1}{\partial e_3} = \beta_1$, $\frac{\partial s_2}{\partial e_4} = \beta_2$. From (26), it is necessary to employ the dynamics of error variables; thus, one has

$$\begin{cases} \dot{e}_1 = x_2 \\ \dot{e}_2 = \frac{1}{M}(\theta_2 + \theta_1 x_2 - x_3 - x_4) \\ \dot{e}_3 = f_3(x) + g_{31}(x)\frac{u_f}{T_0'} = -\frac{1}{\beta_1}\left(\dot{e}_1 + \dot{e}_2 - \left(-\frac{1}{T_1}\varphi_1(x)\right)^{\frac{r_1}{p_1}}\right) \\ \dot{e}_4 = f_4(x) + g_{41}(x)\frac{u_f}{T_0'} + g_{42}(x)\frac{u_r}{T_r} = -\frac{1}{\beta_2}\left(\dot{e}_1 + \dot{e}_2 - \left(-\frac{1}{T_2}\varphi_2(x)\right)^{\frac{r_2}{p_2}}\right) \end{cases} \tag{27}$$

Step 3: In order to design the desired finite-time synergetic controller, Lyapunov stability strategy will be used to ensure the finite-time stability of the overall closed-loop dynamics including unknown parameters. Since there are two unknown parameters in the system (8), for simplicity, let us define the estimation error $\tilde{\theta} = \theta - \hat{\theta}$. Thus, consider the following Lyapunov function candidate:

$$V\left(\varphi, \hat{\theta}, t\right) = \frac{1}{2}\left(\varphi_1^2 + \varphi_2^2 + \frac{1}{\gamma_1}\tilde{\theta}_1^2 + \frac{1}{\gamma_2}\tilde{\theta}_2^2\right) \tag{28}$$

with constant adaptation gains $\gamma_1 > 0$, $\gamma_2 > 0$ and $\dot{\tilde{\theta}}_i = -\dot{\hat{\theta}}_i$, $(i = 1, 2)$. The time derivative of the Lyapunov function can be stated as

$$\dot{V}(t) = \varphi_1\dot{\varphi}_1 + \varphi_2\dot{\varphi}_2 - \frac{1}{\gamma_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 - \frac{1}{\gamma_2}\tilde{\theta}_2\dot{\hat{\theta}}_2 \tag{29}$$

After differentiating (28) and substituting (26) and (27) into (29), we have

$$\dot{V}(t) = \varphi_1\left[\underbrace{\dot{e}_1 + \dot{e}_2 + \beta_1\dot{e}_3(t)}_{\left(-\frac{1}{T_1}\varphi_1\right)^{\frac{r_1}{p_1}}}\right] + \varphi_2\left[\underbrace{\dot{e}_1 + \dot{e}_2 + \beta_2\dot{e}_4(t)}_{\left(-\frac{1}{T_2}\varphi_2\right)^{\frac{r_2}{p_2}}}\right] - \frac{1}{\gamma_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 - \frac{1}{\gamma_2}\tilde{\theta}_2\dot{\hat{\theta}}_2 \tag{30}$$

Based on Lyapunov stability theory, $\dot{V}(t)$ must be a negative definite function. Consequently, an appropriate control law is given in (21). Then, substituting the control law $u\left(x, \hat{\theta}\right)$ into (30), we have

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^{2}\left(-\frac{1}{T_i}\varphi_i\right)^{\frac{r_i}{p_i}}\varphi_i + \tilde{\theta}_1\underbrace{\left(\frac{x_2}{M}(\varphi_1+\varphi_2) - \frac{1}{\gamma_1}\dot{\hat{\theta}}_1\right)}_{-k_1\tilde{\theta}_1^{\frac{r_1}{p_1}}} + \tilde{\theta}_2\underbrace{\left(\frac{1}{M}(\varphi_1+\varphi_2) - \frac{1}{\gamma_2}\dot{\hat{\theta}}_2\right)}_{-k_2\tilde{\theta}_2^{\frac{r_2}{p_2}}} \\ &= -\frac{1}{T_1}\varphi_1^{\frac{r_1+p_1}{p_1}} - \frac{1}{T_2}\varphi_2^{\frac{r_2+p_2}{p_2}} - k_1\tilde{\theta}_1^{\frac{r_1+p_1}{p_1}} - k_2\tilde{\theta}_2^{\frac{r_2+p_2}{p_2}}\end{aligned} \tag{31}$$

In accordance with Lemma 2.2, it is easy to obtain

$$\begin{aligned}\dot{V}(t) \le &-\frac{1}{T_1}2^{\frac{r_1+p_1}{2p_1}}\left(\frac{1}{2}\varphi_1^2\right)^{\frac{r_1+p_1}{2p_1}} - \frac{1}{T_2}2^{\frac{r_2+p_2}{2p_2}}\left(\frac{1}{2}\varphi_2^2\right)^{\frac{r_2+p_2}{2p_2}} - k_1(2\gamma_1)^{\frac{r_1+p_1}{2p_1}}\left(\frac{1}{2\gamma_1}\tilde{\theta}_1^2\right)^{\frac{p_1+r_1}{2p_1}} \\ &- k_2(2\gamma_2)^{\frac{r_2+p_2}{2p_2}}\left(\frac{1}{2\gamma_2}\tilde{\theta}_2^2\right)^{\frac{r_2+p_2}{2p_2}}\end{aligned} \tag{32}$$

Using Lemma 2.3 and choosing $c = \min\{c_1, c_2, c_3, c_4\} > 0$ and $\eta = \max\{\eta_1, \eta_2\}$ where $c_1 = \frac{2^{\frac{r_1+p_1}{2p_1}}}{T_1}$, $c_2 = \frac{2^{\frac{r_2+p_2}{2p_2}}}{T_2}$, $c_3 = k_1(2\gamma_1)^{\frac{r_1+p_1}{2p_1}}$, $c_4 = k_2(2\gamma_2)^{\frac{r_2+p_2}{2p_2}}$, $\eta_1 = \frac{r_1+p_1}{2p_1}$, $\eta_2 = \frac{r_2+p_2}{2p_2}$, we have

$$\begin{aligned}\dot{V}(t) &= -c_1\left(\frac{1}{2}\varphi_1^2\right)^{\eta_1} - c_2\left(\frac{1}{2}\varphi_2^2\right)^{\eta_2} - c_3\left(\frac{1}{2\gamma_1}\tilde{\theta}_1^2\right)^{\eta_1} - c_4\left(\frac{1}{2\gamma_1}\tilde{\theta}_1^2\right)^{\eta_2} \\ &\le -c\left[\left(\frac{1}{2}\varphi_1^2\right)^{\eta} + \left(\frac{1}{2}\varphi_2^2\right)^{\eta} + \left(\frac{1}{2\gamma_1}\tilde{\theta}_1^2\right)^{\eta} + \left(\frac{1}{2\gamma_1}\tilde{\theta}_1^2\right)^{\eta}\right] \\ &\le -c\left[\left(\frac{1}{2}\varphi_1^2\right) + \left(\frac{1}{2}\varphi_2^2\right) + \left(\frac{1}{2\gamma_1}\tilde{\theta}_1^2\right) + \left(\frac{1}{2\gamma_1}\tilde{\theta}_1^2\right)\right]^{\eta}\end{aligned} \tag{33}$$

Thus, the expression above becomes

$$\dot{V} \le -cV^{\eta} \tag{34}$$

For simplicity if we select $\eta = \max\{\eta_1, \eta_2\} = \frac{r+p}{2p}$ and straightforwardly solving (34), we obtain

$$\int_{t_0}^{t} V^{-\frac{r+p}{2p}}\dot{V}(t)dt \;\; \le \;\; -c(t-t_0) \Rightarrow \frac{2p}{p-r}V^{\frac{p-r}{2p}}(t) \le \frac{2p}{p-r}V^{\frac{p-r}{2p}}(t_0) - c(t-t_0) \tag{35}$$

Therefore, it is obvious from (35) that $\varphi_i \equiv 0$ and $\tilde{\theta}_i \equiv 0$ for all $t \geq t_1$ where $t_1 = t_0 + \frac{2p}{c(p-r)} V^{\frac{p-r}{2p}}(t_0) = t_0 + \frac{V^{\left(1-\frac{r+p}{2p}\right)}(t_0)}{c\left(1-\frac{r+p}{2p}\right)} = t_0 + \frac{V^{1-\eta}(t_0)}{c(1-\eta)}$, $\eta = \frac{r+p}{2p}$, as given in Lemma 2.1.

From (35), it is easy to know that φ_i, $\tilde{\theta}_i$ $(i = 1, 2)$ converge to zero in finite time, and the closed-loop system is finite-time stable. Because $\tilde{\theta}_i \to 0$, $\hat{\theta}_i$ goes to the real values θ_i in finite time as well. Likewise, φ_i is a function of error signals. Thus, by the definition of φ_i in (23) after φ_i settle down to zero, e_k $(k = 1, 2, 3, 4)$ eventually converge to zero in finite-time. In addition, it can be observed that the convergence speed of the selected macro-variables (φ_i) depends upon a suitable selection of design parameters c, p, r, T_i, γ_i, k_i $(i = 1, 2)$ in spite of unknown parameters. This completes the proof.

4. **Simulation Results.** In this section, the simulations are given to exhibit the efficiency and applicability of the proposed controller evaluated on SMIB power system including SVC as shown in Figure 2. The performance of the proposed control scheme is verified in MATLAB environment under the effect of severe disturbance. For the simulations, there is a case used to evaluate the performance of the developed controller. We assume that there is a symmetrical three-phase fault occurring at the point P as shown in Figure 2. For this case, there are five stages of interest as follows. Firstly, all state variables are at pre-fault steady state. The fault occurs at $t = 0.5$ sec, Afterward, the fault is cleared by opening the breaker at $t = 0.7$ sec. The transmission line can be restored at $t = 2.0$ sec. Eventually, the system returns to a post-fault state.

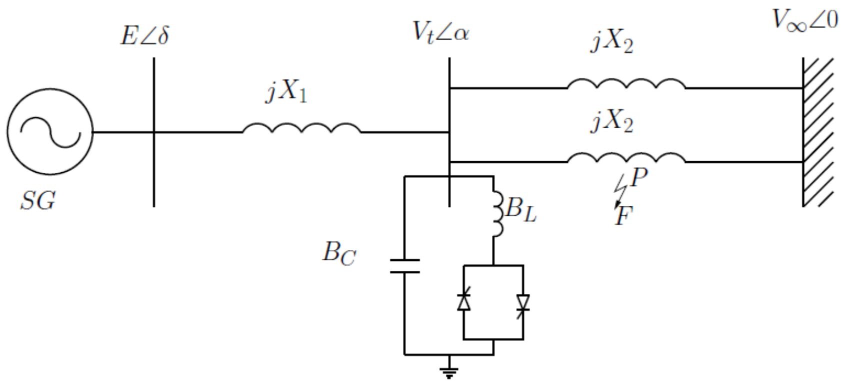

Figure 2. Network with TCR-FC SVC

The physical parameters (pu.), the controller parameters, and initial parameters used for this power system model are as follows:

- The actual parameters of synchronous generators, SVC, and transmission line: $\omega_s = 2\pi f$ rad/s, $D = 0.2$, $H = 5$, $f = 60$ Hz, $T_0' = 4$, $V_\infty = 1\angle 0°$, $X_d = 1.1$, $X_d' = 0.2$, $X_T = 0.1$, $T_r = 1$, $X_2 = X_L = 0.2$, $P_m = 1$,
- The designed parameters of the develped controller are $\beta_1 = \beta_2 = 50$, $T_1 = 0.005$, $T_2 = 1$; $p = p_1 = p_2 = 7$, $r = r_1 = r_2 = 5$, $\gamma_1 = \gamma_2 = 5$,
- Initial parameters $\delta_e = 0.4964$ rad, $\omega_e = \omega_s$, $P_{ee} = 1$ pu., $P_{svce} = 0$ pu., $\hat{\theta}_{10} = -0.5$, $\hat{\theta}_{20} = 0$.

The time domain simulations are carried out to investigate the system stability enhancement and the dynamic performance of the designed controller and the parameter adaptive law, as given in (21) and (37). The performance of the proposed controller (adaptive finite-time synergetic controller) is compared with that of the adaptive synergetic controller $(p_i = r_i)$, as given in (36).

$$\begin{cases} \dfrac{u_f\left(x,\hat{\theta}\right)}{T_0'} = -\dfrac{1}{\beta_1 g_{31}(x)}\left[\beta_1 f_3(x) + x_2 + \dot{\hat{x}}_2 + \dfrac{1}{T_1}\varphi_1\right] \\ \dfrac{u_r\left(x,\hat{\theta}\right)}{T_r} = -\dfrac{1}{\beta_2 g_{42}(x)}\left[\beta_2\left(f_4(x) + g_{41}(x)\dfrac{u_f}{T_f}\right) + x_2 + \dot{\hat{x}}_2 + \dfrac{1}{T_2}\varphi_2\right] \end{cases} \tag{36}$$

where $\dot{\hat{x}}_2$ is identical to the dynamics of the proposed control. Also, the parameter update laws for unknown parameters of the controlled system can be selected as

$$\begin{cases} \dot{\hat{\theta}}_1 = \gamma_1\left[k_1\tilde{\theta}_1 + \dfrac{x_2}{M}(\varphi_1+\varphi_2)\right] \\ \dot{\hat{\theta}}_2 = \gamma_2\left[k_2\tilde{\theta}_2 + \dfrac{1}{M}(\varphi_1+\varphi_2)\right] \end{cases} \tag{37}$$

where γ_j, k_j $(j = 1, 2)$ are positive design parameters. The controller parameters of this scheme are set as $\beta_1 = \beta_2 = 50$, $T_1 = 0.005$, $T_2 = 1$, $\gamma_1 = \gamma_2 = 5$.

The simulation results are illustrated in Figures 3-5. From Figure 3, it can be seen that under the proposed method and the adaptive synergetic method, time histories of power angle, frequency, and terminal voltage converge to the specified equilibrium point in a finite time with different convergence rates. It is obvious that time histories of the developed control law are faster a lot despite unknown parameters. From different methods, Figure 4 exhibits comparative plots of the parameter estimates for unknown parameters with different rates of convergence to real values. It is also seen that the parameter estimate of the developed scheme rapidly returns to the real value of the damping coefficient constant and mechanical power input almost without any oscillations. In contrast, the parameter estimate of the adaptive synergetic control still exhibits considerable oscillations.

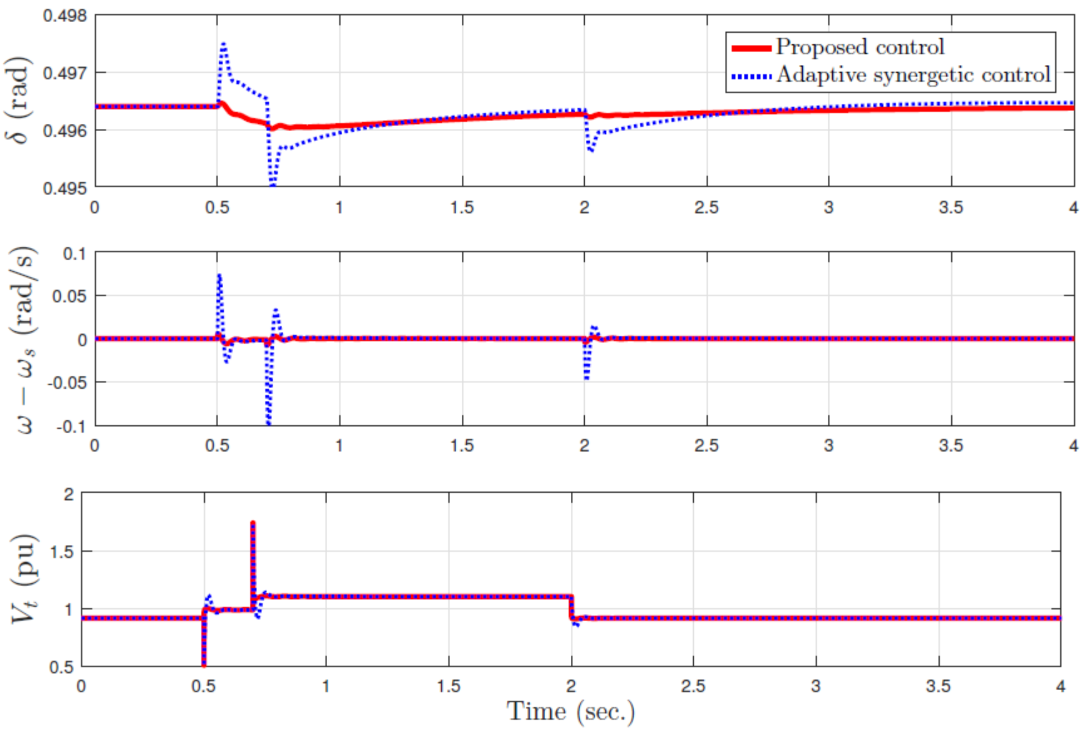

FIGURE 3. Controller performances – Power angles (δ) (rad.), frequency ($\omega - \omega_s$) rad/s. and terminal voltage (V_t) pu. (Solid: Proposed control, Dotted: Adaptive synergetic controll)

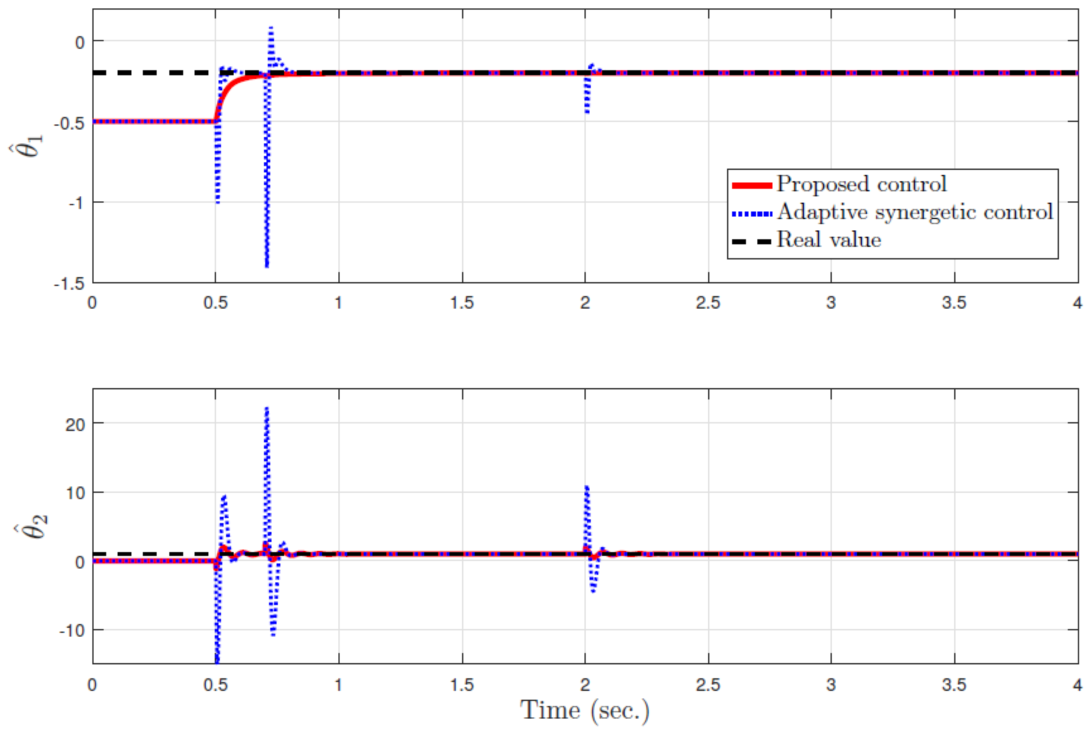

FIGURE 4. Unknown constant parameters – Damping coefficient estimate $\left(\hat{\theta}_1\right)$ and mechanical input power estimate $\left(\hat{\theta}_2\right)$ (Solid: Proposed control, Dotted: Adaptive synergetic controll, Dashed: Real value)

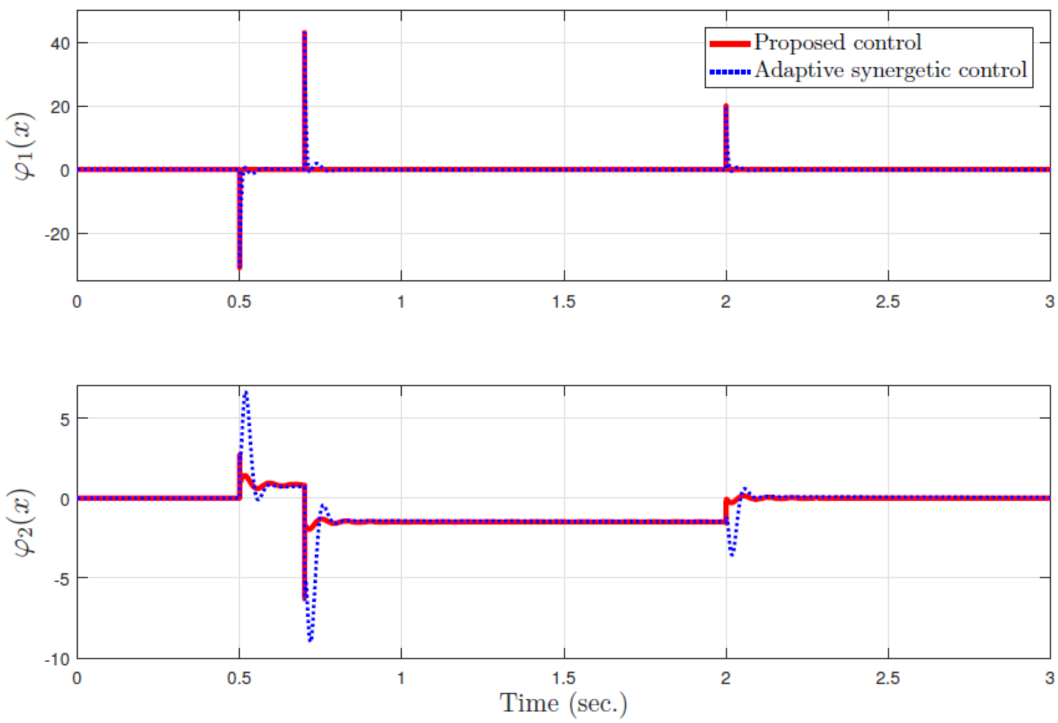

FIGURE 5. The selected macro-variable $\varphi(x) = [\varphi_1(x), \varphi_2(x)]^T$

This clearly indicates the proposed controller offers superior dynamic performances over the adaptive synergetic controller. It can be observed from Figure 5 that time responses of the selected macro-variable that can converge toward the desired manifold $\varphi(x) = 0$ with different convergence rates as shown in Figures 3 and 4. From these figures, the p-resented design offers smaller overshoot magnitude of oscillation, shorter rising time, and very shorter settling time as compared with adaptive synergetic control. This illustrates improved transient performances despite having unknown parameters in the system.

In summary, the simulation results earlier confirm that the developed strategy not only improves transient stability along with voltage and frequency regulations, but also provides an opportunity to improve dynamic properties in spite of having unknown parameters. Also, all signals of the proposed control are driven to the desired equilibrium point in finite time while those of the adaptive synergetic control asymptotically approach to the specified equilibrium point.

5. **Conclusion.** With the help of adaptive finite-time synergetic control strategy, a nonlinear adaptive controller has been proposed in this paper to improve transient stability and voltage regulation for SMIB power system including SVC. Under a large disturbance, the simulations have illustrated that the proposed control is capable of steering all trajectories (power angle, terminal voltage, and frequency) to the desired equilibrium point in finite time despite unknown parameters. In accordance with the presented design procedure and the simulations, the developed strategy offers superior dynamic performance over adaptive synergetic control strategy. Moreover, although there are inevitably unknown parameters in the system, the comparative results confirm the efficacy of the designed controller that can rapidly damp out power oscillations, further improve transient stability, and effectively regulate both terminal voltage and frequency of the overall system. Future study will be devoted to extension of this scheme to a large-scale power system in the presence of external disturbances, unknown nonlinearities and unmodeled dynamics.

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