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LOW-SENSITIVITY CONTROL WITH ROBUST STABILITY USING DOUBLE-FEEDBACK CONTROL FOR MIMO TIME-DELAY SYSTEMS

Daisuke Koyama, Kotaro Hashikura, Md Abdus Samad Kamal and Kou Yamada

Graduate School of Science and Technology
Gunma University
1-5-1 Tenjincho, Kiryu, Gunma 376-8515, Japan
{ t171b041; k-hashikura; maskamal; yamada }@gunma-u.ac.jp

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Abstract. *In this paper, we consider the problem of a two-degree-of-freedom control system using double-feedback control with low sensitivity and robust stability for multiple-input/multiple-output time-delay systems having a varying number of unstable poles. A control system is desired to achieve low sensitivity such as reducing the effect of the uncertainty for the output. In general, it is difficult for a control system with robust stability to reduce low-sensitivity characteristics. Yamada shows a robust stability condition that can achieve low-sensitivity characteristics for a plant having a varying number of poles. Yu et al. expand the results of Yamada and propose a design method for two-degree-of-freedom control systems for single-input/single-output minimum-phase systems using double-feedback control with robust stability to reduce the effect of the uncertainty for the output to be much smaller than that of the conventional two-degree-of-freedom control system. However, a design method for a double-feedback control system for multiple-input/multiple-output time-delay systems having a varying number of unstable poles has not been considered. In this paper, we expand the results of Yu et al. and propose a design method for a two-degree-of-freedom control system using double-feedback control with low sensitivity and robust stability to reduce the effect of the uncertainty for the output to be smaller than that of a conventional two-degree-of-freedom control system for multiple-input/multiple-output time-delay systems having a varying number of unstable poles.*

Keywords: Multivariable system, Time-delay system, Low-sensitivity control, Sensitivity function, Robust stability

1. **Introduction.** In this paper, we consider a design method for a two-degree-of-freedom control system using double-feedback control with low sensitivity and robust stability for multiple-input/multiple-output time-delay systems. The problem of stabilizing the control system for uncertainty in a plant is called the robust stabilization problem, and has been considered by several researchers [1, 2, 3, 4, 5, 6, 7]. The robust stabilization problem was first formulated by Doyle and Stein [1]. In [1], Doyle and Stein show the robust stability conditions for multiplicative uncertainty and additive uncertainty. Chen and Desoer give a complete proof of the problem described by Doyle and Stein [2]. Kimura solves the robust stabilization problem for single-input/single-output systems [8]. The result by Kimura is expanded for multiple-input/multiple-output systems by Vidyasagar and Kimura [9].

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According to previous studies [1, 2, 3, 4, 5, 6, 7], a complementary sensitivity function needs to yield a small value to maintain stability for a large uncertainty. Making the complementary sensitivity function small reduces the performance of control systems, such as disturbance attenuation. To design a control system having a high performing disturbance attenuation property and so on, we need to make the sensitivity function yield a small value. However, we cannot reduce the sensitivity and complementary sensitivity functions simultaneously because the sum of these functions is equal to 1. Therefore, it is difficult to make a control system with both low sensitivity and robust stability. However, low-sensitivity control does not always make the control system unstable for systems with uncertainty. Maeda and Vidyasagar consider this an infinite gain margin problem [10, 11]. Nogami et al. clarify the condition that the high-gain controller does not make the system unstable and propose a design method [12]. Doyle et al. show the condition that a low-sensitivity control system leads to robust stability for some class of uncertainty from a specific viewpoint; a class of uncertainty exists such that the low-sensitivity control ensures robust stability [13]. Although the class of uncertainty proposed by Doyle et al. is suitable for a high-performance robust control system design, this class of uncertainty cannot be applied to a system with a varying number of existing right-half plane applications such that the number of unstable poles changes. Because the small gain theorem exists, it is difficult to solve this problem to obtain the robust stability condition for a system having an uncertain number of poles in the closed right-half plane. Yamada considers this problem using some class of uncertainty [14]. Although Verma also considers this problem similarly to Yamada, the class of uncertainty considered by Yamada is different from that considered by Verma. In [15], Hoshikawa et al. clarify the robust stability condition that achieves low sensitivity for multiple-input/multiple-output minimum-phase time-delay systems with a varying number of poles in the closed right-half plane from the same perspective as [14].

It is important to consider the control system structure so the control system has low-sensitivity characteristics. It is well known that the internal model control (IMC) structure is effective for low sensitivity [16]. However, the IMC structure cannot be applied to systems having unstable poles. Zhou and Ren consider this problem and propose a generalized internal model control (GIMC) structure [17]. Several researchers have used the GIMC structure in their studies [18, 19, 20, 21]. In [18, 19, 20], the GIMC structure is applied to fault-tolerant control for mechatronic systems [18, 19]. Okajima et al. propose a compensator structure, which minimizes the error between the plant and nominal plant [22]. By contrast, Yu et al. expand the result of [14] and propose a new control structure called double-feedback control for single-input/single-output minimum-phase systems [23]. In [23], the design method of the two-degree-of-freedom control system using double-feedback control with robust stability can reduce the effect of the uncertainty on the output more than single-loop feedback two-degree-of-freedom control systems. However, to our knowledge, no paper has considered low-sensitivity control using double-feedback control for multiple-input/multiple-output time-delay systems having a varying number of unstable poles. Some applications of time-delay systems have been reported, such as the networked control system and cooperative control [24]. However, designing a control system with robust stability for time-delay systems is difficult because the stability margin is lost by the time delay.

In this paper, we expand the result of [14, 15, 23] and consider a design method for a two-degree-of-freedom control system for multiple-input/multiple-output time-delay systems having a varying number of unstable poles to reduce the effect of uncertainty for the output. Such a system can be built using a low-sensitivity controller. This paper is organized as follows. In Section 2, we explain the two-degree-of-freedom control system using double-feedback control and its associated problems. The proposed control system using double-feedback control for a multiple-input/multiple-output time-delay system can achieve both low sensitivity and robust stability. In addition, the control system proposed in this paper makes the effect of the uncertainty for the output smaller than that of a single-loop feedback two-degree-of-freedom control system. Hoshikawa et al. consider a similar robust stabilization problem [15]. However, to our knowledge, no paper has considered a control system with robust stability such that the effect of the uncertainty for the output is much smaller than that of a single-loop feedback two-degree-of-freedom control system from the perspective of the control structure. In Section 3, we clarify the robust stability condition of the single-loop feedback two-degree-of-freedom control system. In Section 4, we clarify the robust stability condition of the two-degree-of-freedom control system using double-feedback control. Then, we clarify that the robust stability condition of the double-feedback control system is related to minimizing a sensitivity function. In Section 5, we compare the influence of the uncertainty for the two-degree-of-freedom control system using double-feedback control and that of the single-loop feedback control system. From the result of the comparison of the influence of the uncertainty for the output, we clarify that the two-degree-of-freedom control system using double-feedback control reduces the effect of the uncertainty for the output less than the single-loop feedback two-degree-of-freedom control system. Finally, conclusions are given in Section 6.

Notations

R	the set of real numbers.
$R^{m\times p}$	the set of real number matrices.
C	the set of complex numbers.
$R(s)$	the set of real relational functions with s.
$R^m(s)$	the set of m real relational vectors with s.
$R^{m\times p}(s)$	the set of $m \times p$ real relational matrices with s.
RH_∞	the set of stable proper real relational functions.
$RH_\infty^{m\times p}$	the set of $m \times p$ stable proper real relational matrices.
$\left[\begin{array}{c\|c} A & B \\ \hline C & D \end{array}\right]$	the state-space realization of $C(sI - A)^{-1}B + D$
$\\| \cdot \\|_\infty$	H_∞ norm of $\cdot$.
$\bar{\sigma}(\cdot)$	a maximum singular value of $\cdot$.

2. **Problem Formulation.** In this section, we explain the two-degree-of-freedom control system using double-feedback control for a multiple-input/multiple-output time-delay system and the problem considered in this paper. The two-degree-of-freedom control system using double-feedback control includes a single-loop feedback two-degree-of-freedom control system. To explain the two-degree-of-freedom control system using double-feedback control, we first need to explain the single-loop feedback two-degree-of-freedom control system and provide the results regarding its control system.

Consider a single-loop feedback two-degree-of-freedom control system in Figure 1. Here, $G_0(s)e^{-sT}$ is a plant of the multiple-input/multiple-output time-delay systems. $T > 0$ is a time-delay, and $G_0(s)e^{-sT} \in R^{m\times p}(s)$ is assumed to be a strictly proper plant and to have no zero in the closed right-half plane. $F_0(s) \in R^{m\times p}(s)$ is also assumed to be strictly proper and have no zero in the closed right-half plane. $C_1(s) \in R^{p\times m}(s)$ is a feedback controller, $F_1(s)e^{-sT}$ and $Q_1(s) \in RH_\infty^{p\times p}$ is a feed-forward controller. $F_1(s) \in RH_\infty^{m\times p}$ has no zero in

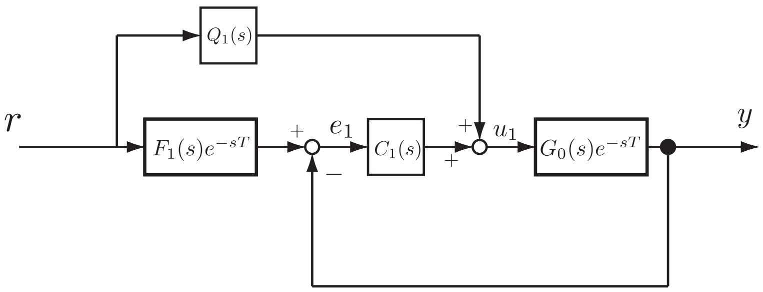

Figure 1. The single-loop feedback two-degree-of-freedom control system

the closed right-half plane. In addition, $Q_1(s)$ satisfies $F_0(s)Q_1(s) = F_1(s)$. $r \in R^p(s)$ is a reference input, $e_1 \in R^m(s)$ is an error of the single-loop feedback two-degree-of-freedom control system in Figure 1, $u_1 \in R^p(s)$ is the control input, and $y \in R^m(s)$ is a controlled output. The plant $G_0(s)$ is assumed to be stabilizable, detectable, and $p \leq m$. The state-space description of $G_0(s)$ is denoted by

$$G_0(s) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \in R^{m\times p}(s), \tag{1}$$

where $A \in R^{n\times n}$, $B \in R^{n\times p}$, $C \in R^{m\times n}$, and $D \in R^{m\times p}$. A nominal plant of $G_0(s)e^{-sT}$ is denoted by $F_0(s)e^{-sT}$. The state-space description of $F_0(s)$ is written as

$$F_0(s) = \left[\begin{array}{c|c} A_m & B_m \\ \hline C_m & D_m \end{array}\right] \in R^{m\times p}(s), \tag{2}$$

where $A_m \in R^{n_m\times n_m}$, $B_m \in R^{n_m\times p}$, $C_m \in R^{m\times n_m}$, and $D_m \in R^{m\times p}$. From the assumption that $G_0(s)$ and $F_0(s)$ have no zero in the closed right-half plane, there exists no $s_0 \in C$ in the closed right-half plane satisfying

$$\text{rank} \begin{bmatrix} A - s_0 I & B \\ C & D \end{bmatrix} < n + p, \tag{3}$$

and no $\bar{s}_0 \in C$ in the closed right-half plane satisfied

$$\text{rank} \begin{bmatrix} A_m - \bar{s}_0 I & B_m \\ C_m & D_m \end{bmatrix} < n_m + p. \tag{4}$$

In general, the nominal plant $G_0(s)e^{-sT}$ is not equal to the nominal plant $F_0(s)e^{-sT}$. That is, an error between $G_0(s)e^{-sT}$ and $F_0(s)e^{-sT}$ exists. Using $F_0(s)e^{-sT}$, $G_0(s)e^{-sT}$ is assumed to be written by the form in

$$G_0(s)e^{-sT} = (I + \Delta(s))F_0(s)e^{-sT}, \tag{5}$$

where $\Delta(s)$ is an uncertainty. Without loss of generality, $I + \Delta(s)$ is assumed to be of normal full rank. To design a single-loop feedback two-degree-of-feedback control system with robust stability in Figure 1 to reduce the effect of the uncertainty $\Delta(s)$ for the output y, we adopt the following set of $G_0(s)e^{-sT}$.

Definition 2.1. *The plant $G_0(s)e^{-sT}$ defined by $G_0(s)e^{-sT} = (I + \Delta(s))F_0(s)e^{-sT}$ is called an element of set $\Omega\left(F_0(s)e^{-sT}, W(s)\right)$ if the following expressions are valid.*

1) The relative degree of $G_0(s)$ is equal to that of $F_0(s)$.
2) $\Delta(s)$ satisfies

$$\bar{\sigma}\left\{(I + \Delta(j\omega))^{-1}\Delta(j\omega)\right\} = \bar{\sigma}\left\{\Delta(j\omega)(I + \Delta(j\omega))^{-1}\right\} < |W(j\omega)| \ \forall\omega \in R \cup \{\infty\}, \tag{6}$$

where $W(s)$ is a stable rational function.

The first problem is to consider a design method for a single-loop feedback two-degree-of-freedom control system in Figure 1 to reduce the effect of the uncertainty $\Delta(s)$ for the output y.

To reduce the influence of the uncertainty for the output less than that of the single-feedback two-degree-of-freedom control system in Figure 1, we consider the control system shown in Figure 2. Here, the block diagram $G_1(s)e^{-sT}$ is shown in Figure 3. From Figure 2, Figure 1, and Figure 3, the control structure in Figure 2 includes the single-loop feedback two-degree-of-freedom control system shown in Figure 1. This is the reason why the control system in Figure 2 is called the double-feedback control system. Here, $C_2(s) \in R(s)^{m\times p}$ is the feedback controller, and $F_2(s)e^{-sT}$ and $Q_2(s) \in RH_\infty^{p\times p}$ are the feed-forward controllers. $F_2(s) \in RH_\infty^{m\times p}$ has no zero in the closed right-half plane. In addition, $Q_2(s)$ satisfies $F_1(s)Q_2(s) = F_2(s)$. $r \in R^p(s)$ is a reference input, $e_2 \in R^m(s)$ is an error of the double-feedback control system in Figure 2, $u_2 \in R^p(s)$ is a control input, and $y \in R^m(s)$ is a controlled output.

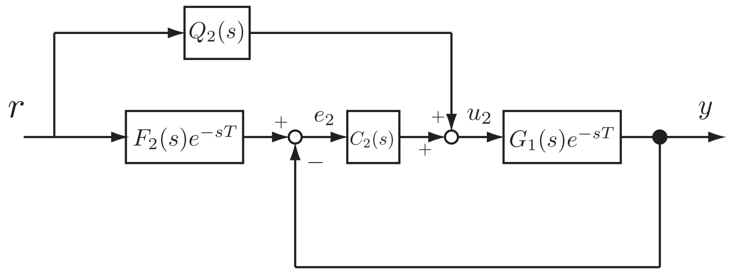

Figure 2. The double-feedback control system

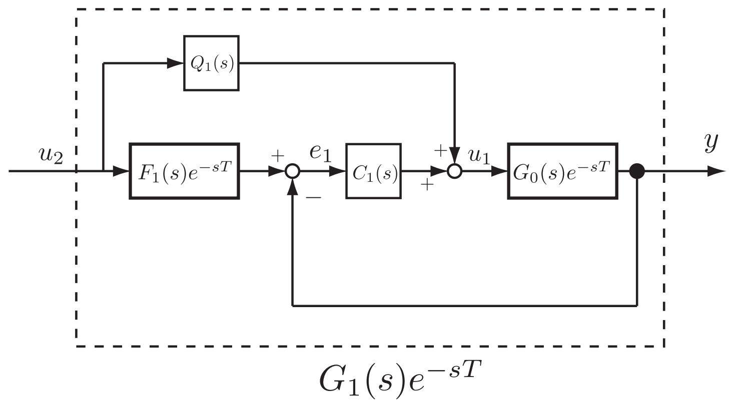

Figure 3. The block diagram of $G_1(s)e^{-sT}$

The second problem is to consider the robust stability condition of the two-degree-of-freedom control system in Figure 2 to reduce the effect of the uncertainty $\Delta(s)$ for the output y. The third problem is to consider a design method for the two-degree-of-freedom control system in Figure 2 such that the effect of the uncertainty $\Delta(s)$ for the output y is reduced compared with that of the two-degree-of-freedom control system in Figure 1.

3. **A Robust Stability Condition of the Single-Loop Feedback Two-Degree-of-Freedom Control System.** In this section, we propose a single-loop feedback two-degree-of-freedom control system in Figure 1 for multiple-input/multiple-output time-delay systems to reduce the effect of the uncertainty $\Delta(s)$ for the output y.

To clarify the effect of the uncertainty $\Delta(s)$ for the output y in Figure 1, the transfer matrix from r to y in Figure 1 is written as

$$y = (I + H_1(s))F_1(s)e^{-sT}r, \tag{7}$$

where $F_1(s)e^{-sT}$ is the transfer matrix from r to y in the case of $\bar{\sigma}\{\Delta(j\omega)\} = 0\ \forall\omega \in R$ and $H_1(s)$ is the transfer matrix written as

$$H_1(s) = \left\{I - S_1(s)\Delta(s)(I + \Delta(s))^{-1}\right\}^{-1} S_1(s)\Delta(s)(I + \Delta(s))^{-1}. \tag{8}$$

Here, $S_1(s)$ is the sensitivity function in Figure 1 written by

$$S_1(s) = \left(I + F_0(s)C_1(s)e^{-sT}\right)^{-1}. \tag{9}$$

From (7), (9), and (8), to reduce the effect of $\Delta(s)$ for y in Figure 1, the feedback controller is designed to minimize $\|S_1(s)W(s)\|_\infty$ because the upper bound of $\Delta(s)(I + \Delta(s))^{-1}$ is $W(s)$ from (6).

To maintain the internal stability condition, the single-feedback two-degree-of-freedom control system in Figure 1 must be well-posed; therefore, the controller must be proper. When the controller $C_1(s)$ is proper, the sensitivity function has the following property:

$$\lim_{\omega\to\infty} \bar{\sigma}\{S_1(j\omega)\} = \lim_{\omega\to\infty} \left\{\left(I + F_0(j\omega)C_1(j\omega)e^{-j\omega T}\right)^{-1}\right\} = 1, \tag{10}$$

because of the assumption that $F_0(s)$ is strictly proper. To satisfy (6) and (10), the following relation is required:

$$\lim_{\omega\to\infty} \bar{\sigma}\left\{(I + \Delta(j\omega))^{-1}\Delta(j\omega)\right\} < \lim_{\omega\to\infty} |W(j\omega)| \le 1. \tag{11}$$

From (11), we obtain the following theorem.

Theorem 3.1. *A necessary condition that $\Delta(s)$ satisfies (6) and (11) is that $I + \Delta(s)$ is biproper. That is, when $I + \Delta(s)$ is denoted by*

$$I + \Delta(s) = \left[\begin{array}{c|c} A_d & B_d \\ \hline C_d & D_d \end{array}\right], \tag{12}$$

the necessary condition that $\Delta(s)$ satisfies (6) and (11) is

$$\mathrm{rank} D_d = p. \tag{13}$$

Proof: The proof of Theorem 3.1 is obtained by showing that if $F_0(s)$ is not biproper, that is, (13) is not satisfied, then (11) is not satisfied. For simplicity, let $\bar{\Delta}(s) = I + \Delta(s)$, then

$$(I + \Delta(s))^{-1}\Delta(s) = \bar{\Delta}^{-1}(s)\left(\bar{\Delta}(s) - I\right) = I - \bar{\Delta}^{-1}(s). \tag{14}$$

If $I + \Delta(s)$ is not biproper, but proper, then $\bar{\Delta}^{-1}(s)$ is not proper. This means that $I - \bar{\Delta}^{-1}(s)$ is also improper. Thus, we have

$$\lim_{\omega\to\infty} \bar{\sigma}\left\{(I + \Delta(j\omega))^{-1}\Delta(j\omega)\right\} = \infty. \tag{15}$$

Therefore, (11) is not satisfied.

Conversely, if $I + \Delta(s)$ is improper, then $\bar{\Delta}^{-1}(s)$ is not biproper, but proper. This means that $I - \bar{\Delta}^{-1}(s)$ is proper,

$$\mathrm{rank}\left[\lim_{\omega\to\infty}\left\{\bar{\Delta}^{-1}(s)\right\}\right] < p,$$

and at least one of the eigenvalues of

$$\lim_{\omega\to\infty}\left\{I - \bar{\Delta}^{-1}(j\omega)\right\} \tag{16}$$

is equal to 1. Thus, we have

$$\lim_{\omega\to\infty} \bar{\sigma}\left\{(I+\Delta(j\omega))^{-1}\Delta(j\omega)\right\} \geq 1. \tag{17}$$

This does not satisfy (11), thereby completing the proof of Theorem 3.1. □

When $I+\Delta(s)$ is biproper, we have the following theorem.

Theorem 3.2. *Assume that $F_0(s)$ and $G_0(s)$ is the strictly proper minimum-phase system. If $I+\Delta(s)$ is biproper, then the following expressions hold.*

1) The number of zeros in the closed right-half plane of $I+\Delta(s)$ is equal to the number of poles in the closed right-half plane of $F_0(s)$.

2) The number of poles in the closed right-half plane of $I+\Delta(s)$ is equal to that of $G_0(s)$.

To prove Theorem 3.2, the following lemmas are required.

Lemma 3.1. *Let $\bar{G}(s) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$, where $A \in R^{n\times n}$. If*

$$\text{rank}\bar{G}(s) = p, \tag{18}$$

then

$$\text{rank}\begin{bmatrix} A-sI & B \\ C & D \end{bmatrix} = n+p. \tag{19}$$

The matrix $\begin{bmatrix} A-sI & B \\ C & D \end{bmatrix}$ is called the system matrix of $\bar{G}(s)$.

Lemma 3.2. *The zeros of the system consist of the following four elements.*

1) All transmission zeros of the system.

2) All uncontrollable and unobservable poles of the system.

3) One or all uncontrollable and observable poles of the system.

4) One or all controllable and unobservable poles of the system.

Theorem 3.2 is shown by using the presented lemmas.

Proof: *The proof of Theorem 3.2.* To show the proof of Theorem 3.2, it is sufficient to show only the following two expressions.

1) Zeros in the closed right-half plane of $I+\Delta(s)$ consist of poles in the closed right-half plane of the nominal plant $F_0(s)$. That is, without loss of generality, when $I+\Delta(s)$ is assumed to have no poles and some zeros in the closed right-half plane, the results show that zeros in the closed right-half plane of $I+\Delta(s)$ are poles in the closed right-half plane of $F_0(s)$.
2) Poles in the closed right-half plane of $I+\Delta(s)$ consist of poles in the closed right-half plane of the nominal plant $G_0(s)$. That is, without loss of generality, when $I+\Delta(s)$ is assumed to have some poles and no zeros in the closed right-half plane, the results show that poles in the closed right-half plane of $I+\Delta(s)$ are poles in the closed right-half plane of $G_0(s)$.

At first, the results show that zeros in the closed right-half plane of $I+\Delta(s)$ are poles in the closed right-half plane of $F_0(s)$. Without loss of generality, $I+\Delta(s)$ is assumed to have no poles in the closed right-half plane and only some zeros in the closed right-half plane. From Theorem 3.1, $I+\Delta(s)$ is denoted as

$$I+\Delta(s) = \left[\begin{array}{c|c} A_d & B_d \\ \hline C_d & D_d \end{array}\right], \tag{20}$$

where $A_d \in R^{n_d \times n_d}$, $B_d \in R^{n_d \times p}$, $C_d \in R^{p \times n_d}$, and $D_d \in R^{p \times p}$ are nonsingular. Therefore, the state-space description of $(I + \Delta(s))F_0(s)$ is written by the form in

$$(I + \Delta(s))F_0(s) = \left[\begin{array}{cc|c} A_d & B_dC_m & B_dD_m \\ 0 & A_m & B_m \\ \hline C_d & D_dC_m & D_dC_m \end{array}\right]. \tag{21}$$

Let s_0 be a right-half plane zero of $I + \Delta(s)$. Then, we have

$$\text{rank}\begin{bmatrix} A_d - s_0I & B_d \\ C_d & D_d \end{bmatrix} < n_d + p. \tag{22}$$

From the presented equation, there exists $[\ \xi_1 \quad \xi_2\ ] \neq 0$ satisfying

$$[\ \xi_1 \quad \xi_2\ ]\begin{bmatrix} A_d - s_0I & B_d \\ C_d & D_d \end{bmatrix} = 0. \tag{23}$$

From (21) and (23), for the system matrix of $(I + \Delta(s))F_0(s)$, we have

$$\begin{bmatrix} \xi_1 & 0 & \xi_2 \end{bmatrix}\begin{bmatrix} A_d - s_0I & B_dC_m & B_dD_m \\ 0 & A_m - s_0I & B_m \\ C_d & D_dC_m & D_dC_m \end{bmatrix} = 0.$$

This equation implies that s_0 is also a zero of $(I + \Delta(s))G_m(s)$. From Lemma 3.2 and the assumption that $G_0(s)$ has no zeros in the closed right-half plane, s_0 is either an uncontrollable pole of $(I + \Delta(s))F_0(s)$ or an unobservable pole of $(I + \Delta(s))F_0(s)$. Thus, s_0 is either a pole in the closed right-half plane of $I + \Delta(s)$ or $F_0(s)$. To assume that $I + \Delta(s)$ and G_0 have no poles in the closed right-half plane, s_0 is a pole of $F_0(s)$. The presented discussion shows that zeros in the closed right-half plane of $I + \Delta(s)$ are equivalent to poles in the closed right-half of $F_0(s)$.

Next, it is shown that poles in the closed right-half plane of $I + \Delta(s)$ consist of poles in the closed right-half plane of $G_0(s)$. Without loss of generality, $I + \Delta(s)$ is assumed to have no zeros in the closed right-half plane and some poles in the closed right-half plane. Because D_d is nonsingular, the state-space description of $F_0(s)$ is rewritten as

$$\begin{aligned} F_0(s) &= (I + \Delta(s))^{-1}G_0(s) \\ &= \left[\begin{array}{c|c} A_d - B_dD_d^{-1}C_d & B_dD_d^{-1} \\ \hline -D_d^{-1}C_d & D_d^{-1} \end{array}\right]\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \\ &= \left[\begin{array}{cc|c} A_d - B_dD_d^{-1}C_d & B_dD_d^{-1}C & B_dD_d^{-1}D \\ 0 & A & B \\ \hline -D_d^{-1}C_d & D_d^{-1}C & D_d^{-1}D \end{array}\right]. \end{aligned} \tag{24}$$

Let $\bar{s}_0$ be a pole in the closed right-half plane of $I + \Delta(s)$. From

$$\begin{aligned} &\text{rank}\begin{bmatrix} \left(A_d - B_dD_d^{-1}C_d\right) - \bar{s}_0 & B_dD_d^{-1} \\ -D_d^{-1}C_d & D_d^{-1} \end{bmatrix} \\ &= \text{rank}\left\{\begin{bmatrix} I & B_d \\ 0 & I \end{bmatrix}\begin{bmatrix} A_d - \bar{s}_0I & 0 \\ -D_d^{-1}C_d & D_d^{-1} \end{bmatrix}\right\} \\ &= \text{rank}\begin{bmatrix} A_d - \bar{s}_0I & 0 \\ -D_d^{-1}C_d & D_d^{-1} \end{bmatrix} \\ &< n_d + p, \end{aligned}$$

$\bar{s}_0$ is also a zero of $(I+\Delta(s))^{-1}$. Thus, there exists $\left[\begin{array}{cc}\bar{\xi}_1 & \bar{\xi}_2\end{array}\right]$ satisfying

$$\left[\begin{array}{cc}\bar{\xi}_1 & \bar{\xi}_2\end{array}\right]\left[\begin{array}{cc}A_d-\bar{s}_0 I & 0\\ -D_d^{-1}C_d & D_d^{-1}\end{array}\right]=0.$$

From this equation and (24), for the system matrix of $(I+\Delta(s))^{-1}G_0(s)$, we have

$$\left[\begin{array}{ccc}\bar{\xi}_1 & 0 & \bar{\xi}_2\end{array}\right]\left[\begin{array}{ccc}\left(A_d-B_dD_d^{-1}C_d\right)-\bar{s}_0 & B_dD_d^{-1}C & B_dD_d^{-1}D\\ 0 & A-\bar{s}_0I & B\\ -D_d^{-1}C_d & D_d^{-1}C & D_d^{-1}D\end{array}\right]=0.$$

This implies that $\bar{s}_0$ is also a zero of $(I+\Delta(s))^{-1}G_0(s)$. From Lemma 3.2 and the assumption that $F_0(s)$ has no zeros in the closed right-half plane, $\bar{s}_0$ is either an uncontrollable pole of $(I+\Delta(s))^{-1}G_0(s)$ or an unobservable pole of $(I+\Delta(s))^{-1}G_0(s)$. To assume that $F_0(s)$ has no zeros in the closed right-half plane, $\bar{s}_0$ is either a zero of $I+\Delta(s)$ or a pole of $G_0(s)$. From the assumption that $I+\Delta(s)$ has no zeros in the closed right-half plane, $\bar{s}_0$ is a pole of $G_0(s)$. From the presented discussion, poles in the closed right-half plane of $I+\Delta(s)$ are those of $G_0(s)$.

We have completely proven the proof of Theorem 3.2. □

Then, the robust stability condition of the single-loop feedback two-degree-of-freedom control system in Figure 1 for $G_0(s)e^{-sT}\in\Omega\left(F_0(s)e^{-sT},W(s)\right)$ is summarized as the following theorem.

Theorem 3.3. *Assume that the controller $C_1(s)$ stabilizes $F_0(s)e^{-sT}$ and that $F_1(s)\in RH_\infty^{m\times p}$ is a transfer matrix such that $F_0(s)Q_1(s)=F_1(s)$ satisfies $Q_2(s)\in RH_\infty^{p\times p}$. The single-feedback two-degree-of-freedom control system in Figure 1 is stable for $\Omega\big(F_0(s)e^{-sT}$, $W(s)\big)$ if and only if the H_∞ norm of $S_1(s)W(s)$ satisfies*

$$\|S_1(s)W(s)\|_\infty<1. \tag{25}$$

The proof of Theorem 3.3 requires the following lemmas.

Lemma 3.3 (According to [25]). *Suppose $M\in RH_\infty$ and $\gamma<\|M\|_\infty$. Then, there exists a σ_0 (>0) such that for any given $\sigma\in[0,\sigma_0]$, there exists a $\Delta(s)\in RH_\infty$ with $\|\Delta(s)\|_\infty<1/\gamma$ such that* $\det(I-M(s)\Delta(s))$ *has a zero on the axis $Re(s)=\sigma$.*

Lemma 3.4. *Let $W(s)$ satisfy (11). $F_0(s)$ is assumed to have no zeros in the closed right-half plane and the p_m-th number of poles in the closed right-half plane. $G_0(s)$ is also assumed to have no zeros in the closed right-half plane and the p_0-th number of poles in the closed right-half plane. Then, the Nyquist plot of* $\det(I+\Delta(s))$ *encircles the origin $(0,0)$ p_0-p_m times in the counter-clockwise direction.*

Proof: *The proof of Lemma 3.4.* From the assumption that $W(s)$ satisfies (11), $I+\Delta(s)$ is biproper. From Theorem 3.2, the number of zeros in the closed right-half plane of $I+\Delta(s)$ is equal to the number of poles in the closed right-half plane of $F_0(s)$, and the number of poles in the closed right-half plane of $I+\Delta(s)$ is equal to the number of poles in the closed right-half plane of $G_0(s)$. From the assumption that $G_0(s)$ and $F_0(s)$ have no zeros in the closed right-half plane, $I+\Delta(s)$ has p_m number of zeros and p_0 number of poles in the closed right-half plane. According to the argument principle, the Nyquist plot of $\det(I+\Delta(s))$ encircles the origin $(0,0)$ $p-p_m$ times in the counter-clockwise direction. □

The proof of Theorem 3.3 is proven by using Lemma 3.4.

Proof: *The proof of Theorem 3.3.* The characteristic matrix of the single-loop feedback two-degree-of-freedom control system in Figure 1 is given by $I+C_1(s)G_0(s)e^{-sT}$. If the

Nyquist plot of $\det\left(I + C_1(s)G_0(s)e^{-sT}\right)$ for $-\infty < \omega < \infty$ encircles the origin $(0,0)$ $p_0 + p_c$ times in the counter-clockwise direction, then the system in Figure 1 is robustly stable. Here, p_{c1} means the number of poles in the closed right-half plane of $C_1(s)$ and p_0 means the number of poles in the closed right-half plane of $F_0(s)$. The determinant of the characteristic polynomial is written as

$$\begin{aligned}
&\det\left(I + F_0(s)C_1(s)e^{-sT}\right) \\
&= \det\left\{I + (I + \Delta(s))F_0(s)C_1(s)e^{-sT}\right\} \\
&= \det\left[\left\{I + \Delta(s)F_0(s)C_1(s)e^{-sT}\left(I + F_0(s)C_1(s)e^{-sT}\right)^{-1}\right\}\left(I + F_0(s)C_1(s)e^{-sT}\right)\right] \\
&= \det(I + \Delta(s))\det\left\{I - (I + \Delta(s))^{-1}\Delta(s)S_1(s)\right\}\det\left(I + F_0(s)C_1(s)e^{-sT}\right). \qquad (26)
\end{aligned}$$

From the assumption that $C_1(s)$ stabilizes $F_0(s)e^{-sT}$, the Nyquist plot of $\det\big(I + F_0(s)C_1(s)e^{-sT}\big)$ encircles the origin $(0,0)$ $p_m + p_{c1}$ times in the counter-clockwise direction. Here, p_m is the number of poles in the closed right-half plane of $F_0(s)$. Therefore, if the Nyquist plot of

$$\det(I + \Delta(s))\det\left\{I - (I + \Delta(s))^{-1}\Delta(s)S_1(s)\right\}$$

forall $\Delta(s) \in \Omega\left(F_0(s)e^{-sT}, W(s)\right)$ encircles the origin $(0,0)$ $p_0 - p_m$ times in the counter-clockwise direction, then the single-feedback two-degree-of-freedom control system in Figure 1 is robustly stable. From Lemma 3.4, the Nyquist plot of $\det(I + \Delta(s))$ encircles the origin $(0,0)$ $p_0 - p_m$ times in the counter-clockwise direction. Therefore, the necessary and sufficient condition that the single-loop feedback two-degree-of-freedom control system in Figure 1 is robustly stable is that the Nyquist plot of

$$\det\left\{I - (I + \Delta(s))^{-1}\Delta(s)S_1(s)\right\}$$

does not encircle the origin $(0,0)$ any time.

The remaining problem is to prove that the presented condition is equivalent to (25). We adopt the same procedure in [13] to prove this.

The sufficient part of the proof is as follows. Assume that $\|S_1(s)W(s)\|_\infty < 1$. It is clear that the Nyquist plot of

$$\det\left\{I - (I + \Delta(s))^{-1}\Delta(s)S_1(s)\right\}$$

does not encircle the origin $(0,0)$ even if we select any $\Delta(s) \in \Omega\left(F_0(s)e^{-sT}, W(s)\right)$.

The necessary part is as follows. From Lemma 3.3, if (25) does not hold, then $(I + \Delta(s))^{-1}\Delta(s) \in RH_\infty$ with

$$\left\|(I + \Delta(s))^{-1}\Delta(s)/W(s)\right\|_\infty < 1$$

to let the Nyquist plot of

$$\det\left\{I - (I + \Delta(s))^{-1}\Delta(s)S_1(s)\right\}$$

cross at the origin $(0,0)$.

From the presented discussion, Theorem 3.3 is proven. □

Theorem 3.3 means that minimizing $\|S_1(s)W(s)\|_\infty$ ensures making the double-feedback control system in Figure 2 robustly stable for $G_0(s)e^{-sT} \in \Omega\left(F_0(s)e^{-sT}, W(s)\right)$. That is, the double-feedback control system can concurrently achieve robust stability and reduce the effect of $\Delta(s)$ for y by satisfying Theorem 4.1.

4. **A Robust Stability Condition of the Double-Feedback Control System.** In this section, we clarify the robust stability condition for the double-feedback control system in Figure 2.

To reduce the effect of uncertainty $\Delta(s)$ for the output y in the double-feedback control system Figure 2, the transfer matrix from the reference input r to the output y in Figure 2 is written as

$$y = (I + H_2(s))F_2(s)e^{-sT}r, \tag{27}$$

where $F_2(s)e^{-sT}$ is the transfer function from r to y in Figure 2 in the case of $\bar{\sigma}\{\Delta(j\omega)\} = 0$ $\forall\omega \in R$ and $H_2(s)$ is the transfer matrix written as

$$H_2(s) = \left\{I - S(s)\Delta(s)(I + \Delta(s))^{-1}\right\}^{-1} S(s)\Delta(s)(I + \Delta(s))^{-1}. \tag{28}$$

Here, $S(s)$ is the sensitivity function in Figure 2 written as

$$S(s) = S_2(s)S_1(s), \tag{29}$$

where $S_2(s)$ is a transfer matrix written as

$$S_2(s) = \left(I + F_1(s)C_2(s)e^{-sT}\right)^{-1}. \tag{30}$$

From (28) and (30), the double-feedback control system can reduce the effect of the uncertainty $\Delta(s)$ for the output y by minimizing $\|S(s)W(s)\|_\infty$. When $C_2(s)$ tends to increase, (27) is closer to

$$y = F_2(s)e^{-sT}r. \tag{31}$$

Thus, there exists the possibility that the effect of $\Delta(s)$ for y in Figure 2 is smaller than that in Figure 1 because it is related to not only the controller $C_1(s)$ but also the controller $C_2(s)$.

The robust stability condition for $G_0(s)e^{-sT} \in \Omega\left(F_0(s)e^{-sT}, W(s)\right)$ of the double-feedback control system in Figure 2 is summarized as the following theorem.

Theorem 4.1. *Assume that the conditions in Theorem 3.3 hold. In addition, the results show that $C_2(s)$ stabilizes $F_1(s)e^{-sT}$, $F_2(s) \in RH_\infty^{m\times p}$ is a transfer matrix satisfying $F_1(s)Q_2(s) = F_2(s)$, and $Q_2(s) \in RH_\infty^{p\times p}$. The double-feedback control system in Figure 2 is stable for $G_0(s)e^{-sT} \in \Omega\left(F_0(s)e^{-sT}, W(s)\right)$ if and only if the H_∞ norm of $S(s)W(s)$ is satisfied*

$$\|S(s)W(s)\|_\infty = \|S_2(s)S_1(s)W(s)\|_\infty < 1. \tag{32}$$

The proof of Theorem 4.1 is proven using Theorem 3.3, Lemma 3.3, and Lemma 3.4.

Proof: *The proof of Theorem 4.1.* The characteristic matrix of the double-feedback control system in Figure 2 is given by

$$I + G_0(s)e^{-sT}\left\{\left(C_1(s)F_0(s)e^{-sT} + Q_1(s)\right)C_2(s) + C_1(s)\right\}.$$

If the Nyquist plot of

$$\det\left\{I + G_0(s)e^{-sT}\left\{\left(C_1(s)F_0(s)e^{-sT} + Q_1(s)\right)C_2(s) + C_1(s)\right\}\right\}$$

encircles the origin $(0,0)$ $p_0 + p_{c1} + p_{c2}$ times in the counter-clockwise direction, then the double-feedback control system in Figure 2 is robustly stable. Here, p_{c1} means the number of poles in the closed right-half plane of $C_1(s)$, p_{c2} means the number of poles in the closed right-half plane of $C_2(s)$, and p_0 means the number of poles in the closed right-half plane of $F_0(s)$. The determinant of a characteristic polynomial is written as

$$\begin{aligned}
&\det\left\{I + G_0(s)e^{-sT}\left\{\left(C_1(s)F_0(s)e^{-sT} + Q_1(s)\right)C_2(s) + C_1(s)\right\}\right\} \\
&= \det\left\{I + (I + \Delta(s))F_0(s)e^{-sT}\left\{(C_1(s)F_0(s)e^{-sT} + Q_1(s))C_2(s) + C_1(s)\right\}\right\} \\
&= \det\left\{(I + \Delta(s))\right\}\det\left\{I - (I + \Delta(s))^{-1}\Delta(s)S_2(s)S_1(s)\right\} \\
&\quad \det\left\{I + F_0(s)C_1(s)e^{-sT}\right\}\det\left\{I + F_1(s)C_2(s)e^{-sT}\right\}.
\end{aligned} \tag{33}$$

From the assumption that $C_1(s)$ stabilizes $F_0(s)e^{-sT}$, the Nyquist plot of

$$\det\left\{I + F_0(s)C_1(s)e^{-sT}\right\}$$

encircles the origin $(0, 0)$ $p_m + p_{c1}$ times in the counter-clockwise direction, where p_m is the number of poles in the closed right-half plane of $F_0(s)$. In addition, from the assumption that $C_2(s)$ stabilizes $F_1(s)e^{-sT}$ and $F_1(s)e^{-sT} \in RH_\infty$, the Nyquist plot of

$$\det\left\{I + F_1(s)C_2(s)e^{-sT}\right\}$$

encircles the origin $(0, 0)$ p_{c2} times in the counter-clockwise direction. Therefore, if the Nyquist plot of

$$\det\left\{(I + \Delta(s))\right\}\det\left\{I - (I + \Delta(s))^{-1}\Delta(s)S_2(s)S_1(s)\right\}$$

for $-\infty < \omega < \infty$ encircles the origin $(0, 0)$ $p_m - p_0$ times in the counter-clockwise direction, then the double-feedback control system in Figure 2 is stable for $G_0(s)e^{-sT} \in \Omega\left(F_0(s)e^{-sT}, W(s)\right)$. From Lemma 3.4, the Nyquist plot of $\det\left\{(I + \Delta(s))\right\}$ encircles the origin $(0, 0)$ $p_m - p_0$ times in the counter-clockwise direction. Therefore, the necessary and sufficient condition for the double-feedback control system in Figure 2 for $G_0(s)e^{-sT} \in \Omega\left(F_0(s)e^{-sT}, W(s)\right)$ is that the Nyquist plot of

$$\det\left\{I - (I + \Delta(s))^{-1}\Delta(s)S_2(s)S_1(s)\right\}$$

does not encircle the origin any time. Thus, the remaining problem is to prove that the presented condition is equivalent to (32). We adopt the same procedure in [13] to prove this.

The sufficient part of the proof is as follows. Assume that $\|S_2(s)S_1(s)W(s)\|_\infty < 1$. Thus, the Nyquist plot of

$$\det\left\{I - (I + \Delta(s))^{-1}\Delta(s)S_2(s)S_1(s)\right\}$$

does not encircle the origin $(0, 0)$, even if we select any $G_0(s)e^{-sT} \in \Omega\left(F_0(s)e^{-sT}, W(s)\right)$.

In addition, from Lemma 3.3, if (32) does not hold, then $(I + \Delta(s))^{-1}\Delta(s) \in RH_\infty$ with

$$\left\|(I + \Delta(s))^{-1}\Delta(s)/W(s)\right\|_\infty < 1$$

to let the Nyquist plot of

$$\det\left\{I - (I + \Delta(s))^{-1}\Delta(s)S_2(s)S_1(s)\right\}$$

cross at the origin $(0, 0)$.

From the presented discussion, Theorem 4.1 is proven. □

Theorem 4.1 means that minimizing $\|S(s)W(s)\|_\infty$ ensures making the double-feedback control system in Figure 2 robustly stable for $G_0(s)e^{-sT} \in \Omega\left(F_0(s)e^{-sT}, W(s)\right)$. That is, the double-feedback control system with robust stability can reduce the effect of $\Delta(s)$ for y. In the next section, we consider the design problem of the two-degree-of-freedom control system in Figure 2 such that the effect of the uncertainty $\Delta(s)$ for the output y is reduced compared with the two-degree-of-freedom control system in Figure 1 by comparing the effects of the uncertainty $\Delta(s)$ for y in Figure 2 and that in Figure 1.

5. **Comparison of the Effects of the Uncertainty $\Delta(s)$.**

In this section, we compare the effects of the uncertainty $\Delta(s)$ for the output y between the single-loop feedback two-degree-of-freedom control system in Figure 1 and the double-feedback control system in Figure 2.

To compare the effects of the uncertainty $\Delta(s)$ for the output y between the single-loop feedback two-degree-of-freedom control system in Figure 1 and the double-feedback control system in Figure 2, $F_1(s) = F_2(s)$ is needed. If the maximum singular value of

$H_2(s)$ is less than that of $H_1(s)$, then the effect of $\Delta(s)$ for y in Figure 2 is less than that in Figure 1.

From (8) and (28), the relationship between $H_1(s)$ and $H_2(s)$ is written as

$$H_2(s) = (I - K(s))H_1(s), \tag{34}$$

where $K(s)$ is given by

$$\begin{aligned} K(s) &= \left\{I - S(s)\Delta(s)(I + \Delta(s))^{-1}\right\}^{-1}(I - S_2(s)) \\ &= \left\{I - S_2(s)S_1(s)\Delta(s)(I + \Delta(s))^{-1}\right\}^{-1}(I - S_2(s)). \end{aligned} \tag{35}$$

For the frequency range ω_0 to satisfy $\bar{\sigma}\{S(j\omega_0)\} < 1$, $\bar{\sigma}\{S_2(j\omega_0)\} < 1$, and $\bar{\sigma}\{S_1(j\omega_0)\} < 1$, then we have

$$\bar{\sigma}\{H_2(j\omega_0)\} \leq \bar{\sigma}\{S_2(j\omega_0)\}\bar{\sigma}\{H_1(j\omega_0)\} < \bar{\sigma}\{H_1(j\omega_0)\}. \tag{36}$$

That is, for the frequency range ω_0 to satisfy $\bar{\sigma}\{S(j\omega_0)\} < 1$, $\bar{\sigma}\{S_2(j\omega_0)\} < 1$, and $\bar{\sigma}\{S_1(j\omega_0)\} < 1$, the effect of $\Delta(s)$ for y in Figure 2 is less than that in Figure 1. For the frequency range ω_1 to satisfy $\bar{\sigma}\{S(j\omega_1)\} = \bar{\sigma}\{S_1(j\omega_1)\} = \bar{\sigma}\{S_2(j\omega_1)\} = 1$, then we have

$$\begin{aligned} \bar{\sigma}\{H_2(j\omega_1)\} &= \left[\bar{\sigma}\left\{\Delta(j\omega_1)(I + \Delta(j\omega_1))^{-1}\right\} - 1\right]^{-1} \\ &\quad \times \left[\bar{\sigma}\left\{\Delta(j\omega_1)(I + \Delta(j\omega_1))^{-1}\right\} - 1\right]\bar{\sigma}\{H_1(j\omega_1)\} \\ &= \bar{\sigma}\{H_1(j\omega_1)\}. \end{aligned} \tag{37}$$

That is, for the frequency range ω_1 to satisfy $\bar{\sigma}\{S(j\omega_1)\} = \bar{\sigma}\{S_1(j\omega_1)\} = \bar{\sigma}\{S_2(j\omega_1)\} = 1$, the effect of $\Delta(s)$ for y in Figure 2 is equal to that in Figure 1.

From the presented discussion, if the maximum singular value of $S(s)$, $S_1(s)$, and $S_2(s)$ is less than 1, then the effect of $\Delta(s)$ for y in Figure 2 is less than that in Figure 1.

6. **Conclusion.** In this paper, we expanded the results of [14, 15, 23] and considered a design method for a two-degree-of-freedom control system with low sensitivity and robust stability for multiple-input/multiple-output time-delay systems having a varying number of unstable poles to reduce the effect of the uncertainty for the output. We can design a two-degree-of-freedom control system such that the effect of the uncertainty for the output is less than that of the conventional two-degree-of-freedom control system by using double-feedback control. Due to space limitations, we will show numerical examples and a design procedure in another article. In addition, we will expand these results and examine the control system using multiplex feedback control to design low-sensitivity control and clarify the limitations of the control system using multiplex feedback control.

REFERENCES

[1] J. C. Doyle and G. Stein, Multivariable feedback design: Concepts for a classical modern synthesis, *IEEE Trans. Automatic Control*, vol.26, no.1, pp.4-16, 1981.

[2] M. J. Chen and C. A. Desoer, Necessary and sufficient condition for robust stability of linear distributed feedback systems, *International Journal of Control*, vol.35, pp.255-267, 1982.

[3] J. C. Doyle, J. E. Wall and G. Stein, Performance and robustness analysis for structured uncertainty, *Proc. of the 21st IEEE CDC*, pp.629-636, 1982.

[4] M. S. Verma, J. W. Helton and E. A. Jonckheere, Robust stabilization of a family of plants with varying number of right half plane poles, *Proc. of 1986 American Control Conference*, pp.1827-1832, 1986.

[5] K. Glover and J. C. Doyle, State-space formulae for all stabilizing controllers that satisfy an H_∞ norm bound and relations to risk sensitivity, *Systems and Control Letters*, vol.11, pp.167-172, 1988.

[6] J. C. Doyle, K. Glover, P. P. Khargonekar and B. A. Francis, State-space solution to standard H_2 and H_∞ control problems, *IEEE Trans. Automatic Control*, vol.34, no.8, pp.831-847, 1989.

[7] A. P. Kishore and J. B. Pearson, Uniform stability and performance in H_∞, *Proc. of the 31st IEEE CDC*, pp.1991-1996, 1992.

[8] H. Kimura, Robust stabilizability for a class of transfer function, *IEEE Trans. Automatic Control*, vol.29, no.9, pp.788-793, 1984.
[9] M. Vidyasagar and H. Kimura, Robust controllers for uncertain linear multivariable system, *Auctomatica*, vol.22, no.1, pp.85-94, 1986.
[10] H. Maeda and M. Vidyasagar, Infinite gain margin problem in multivariable feedback systems, *Automatica*, vol.22, no.1, pp.131-133, 1986.
[11] H. Maeda and M. Vidyasagar, Design of multivariable feedback systems with infinite gain margin and decoupling, *Systems and Control Letters*, vol.6, pp.127-130, 1985.
[12] H. Nogami, H. Maeda, M. Vidyadagar and S. Kodama, Design of high gain feedback system with robust stability, *Transactions of the Society of Instrument and Control Engineers*, vol.22, pp.1014-1020, 1986 (in Japanese).
[13] J. C. Doyle, B. A. Francis and A. Tannenbaum, *Feedback Control Theory*, Macmillan Publishing, 1992.
[14] K. Yamada, Robust stabilization for the plant with varying number of unstable poles and low sensitivity characteristics, *Proc. of 1998 American Control Conference*, pp.2050-2054, 1998.
[15] T. Hoshikawa, K. Yamada, T. Hagiwara, Y. Ando, I. Murakami and Y. Tatsumi, Achievement of low sensitivity characteristics and robust stability condition for multiple-input/multiple-output minimum-phase time-delay systems, *ICIC Express Letters*, vol.6, no.3, pp.693-698, 2012.
[16] M. Morari and E. Zafiriou, *Robust Process Control*, Prentice Hall, 1989.
[17] K. Zhou and Z. Ren, A new controller architecture for high performance, robust, and fault-tolerant control, *IEEE Trans. Automatic Control*, vol.46, no.10, pp.1613-1618, 2001.
[18] T. Namerilawa and H. Maruyama, High performance robust control of magnetic suspension systems using GIMC structure, *Transactions of the Society of Instrument and Control Engineers*, vol.42, no.11, pp.1181-1187, 2006 (in Japanese).
[19] T. Namerikawa and J. Miyakawa, GIMC structure considering communication delay and its application to mechatronic system, *Proc. of 2007 American Control Conference*, pp.1532-1537, 2007.
[20] W. Guo, A. Qiu and C. Wen, Active actuator fault tolerant control based on generalized internal model control and performance compensation, *IEEE Access*, vol.7, pp.175514-175521, 2019.
[21] K. Xiong, L. Ma, N. Qin, L. Peng, H. Liu and B. Su, Generalized internal model control strategy for vehicular single-phase PWM rectifiers under parametric uncertainties, *2019 Chinese Control Conference (CCC)*, Guangzhou, China, pp.2353-2358, 2019.
[22] H. Okajima, H. Umei, N. Matsunaga and T. Asai, A design method of compensator to minimize model error, *SICE Journal of Control, Measurement, and System Integration*, vol.6, no.4, pp.267-275, 2013.
[23] X. Yu, J. Li, D. Koyama, S. Shiomi, T. Suzuki and K. Yamada, Low sensitivity control for minimum-phase systems using double feedback control, *ICIC Express Letters, Part B: Applications*, vol.8, no.1, pp.35-42, 2017.
[24] T. Insperger, T. Ersal and G. Orosz, *Time Delay Systems: Theory, Numerics, Applications, and Experiments*, Springer, 2017.
[25] K. Zhou, J. C. Doyle and K. Glover, *Robust and Optimal Control*, Prentice Hall, 1995.