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COST REDUCTION FUNCTION CONSIDERING STOCHASTIC RISKS IN THE PRODUCTION PROCESS

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Abstract. *In this paper, there are some risks assumed in small and medium-sized production systems: progress of process against demand, uncertainty of logistics, variation of process workers, and difficulty of synchronization with external suppliers. The first thing we have proposed for these challenges is a mathematical model of the work process flow. It was a model that utilized the heat conduction diffusion equation that handled several flowing processes in physics. In addition, fluctuations in demand and uncertainty in logistics were the introduction of stochastic models and the use of finance theory as stochastic phenomena. An important management theme is how to reduce production costs in consideration of these risks as a company. We propose a cost reduction function by analyzing these problems.*

Keywords: Learning curve theory, Phase-field, Throughput, Lead time, Potential, Production process

1. **Introduction.** In recent years, the number of skilled technicians at manufacturing sites has decreased, and design quality degradation has become a major problem. There is a tendency to reduce costs due to global cost competition. In order to overcome this problem, we must consider the cost reduction and make full use of mathematical sciences to maintain as high a quality as possible, and further improve the variation in production through data analysis related to manufacturing.

Several studies have addressed the problem of increasing the productivity of production processes used in the production industry [1, 2]. On the other hand, the previous research of a vertical take-off and landing rotorcraft was reported for power saving and long flight. The area and incident angle of the kite wing are changed in conjunction. This is a flexible kite wing attached multicopter with a completely new concept [3].

In a previous study, the research addressed the problem of reducing construction work and inventory in the steel industry. Specifically, they investigated the relationship between variations in the rate of construction and delivery rate. They have performed the analysis using the queuing model and applied log-normal distribution to modeling the system in the steel industry [4].

A previous study has also proposed a method that increases both production efficiency and production of a greater diversity of products for customer use. Their proposed approach results in shortened lead times and reduces the uncertainty in demand. Their

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method captures the stochastic demand of customers and produces solutions by solving a nonlinear stochastic programming problem [5]. Moreover, with respect to a mathematical modeling of production processes, the study has proposed the progressive closed loop approach, that is, an optimization method such as DSM (dependency structure matrix) and a probabilistic search function that provides an approximate process planning solution [6].

In summary, several studies have considered uncertainty and proposed practical approaches to shorten the lead time. The demand is treated as a stochastic variable and applies mathematical programming. To our knowledge, previous studies have not treated lead time as a stochastic variable. Because fluctuations in the supply chain and market demand and the changes in the production volume of suppliers are propagated to other suppliers, their effects are amplified. Therefore, because the amounts of stock are large, an increase or decrease of the suppliers stock is modeled using differential equation. This differential equation is said as Billwhip model, representing a stock congestion [7, 8].

The theory of constraints (TOC) describes the importance of avoiding bottlenecks in production processes [28]. When using production equipment, delays in one production step are propagated to the next. Hence, the use of production equipment may lead to delays. In this study, we apply a physical approach and regard each step as a continuous step. By applying this approach, we can mathematically analyze the delay of each step and obtain methods to address it.

In our previous study, we constructed a state in which the production density of each process corresponds to the physical propagation of heat [9]. Using this approach, we showed that a diffusion equation dominates the production process [10].

In other words, when minimizing the potential of the production field (stochastic field), the equation, which is defined by the production density function $S_i(x,t)$ and the boundary conditions, is described using the diffusion equation with advection to move in transportation speed ρ. The boundary condition means a closed system in the production field. The adiabatic state in thermodynamics represents the same state [10].

As previous research directly related to our research, according to Professor Kataoka of Toyohashi Sozo University as the research on cost reduction, the following research results had been proposed. In order to estimate the production cost, he had introduced the learning curve theory [11], which presented the mathematical equation as $y(x) = cp^{a(x)}$. The conditions being not considered in the above theory are as follows.

1) Uncertainty of logistics supplied to the production process.
2) Uncertainty in the production process of human skills.
3) Endogenous and exogenous disturbances to the lead time in the production process.
4) Non-uniformity of manufacturing specifications for processed products.

We clarified that the fluctuations in lead time were dependent on the state variable, which was a throughput deviation. The propagation of throughput deviation was restricted by Burgers equation of fluid dynamics. In our previous study, we reported that the normalized lead time data had an on-off intermittency [12]. To verify our analysis, we represented actual data that were obtained before/after the managing of processes using the cyclic production flow process [13]. Then, we have clarified the relation between lead time and production density by constructing two stochastic differential equations as a mathematical model. We have also clarified that production density is greatly affected by a fluctuation in lead time [14]. Moreover, we have reported an analysis of production processes using a lead time function. Two types of production demand are classified in a production business. One is custom-type production (asynchronous type), which has a stochastic element. The other is mass-produced production (synchronous type), which has almost no stochastic element. We have reported an optimal production allocation to maximize the rate of increase in cash flow by these two types of production requests (complex type). Our approach is to take advantage of the risk-sensitive control method, which is a powerful technique that takes robustness into account [15].

This study assumes product lead time as process delay. The reason is to make the mathematical modeling of production process easier to understand. Further, this study assumes that the specified control equipment is ordered by a customer and is classified into a number of production elements during the production process. The finished product is then delivered to the customer. Any process delay may largely impact the business production revenues, particularly those of small and medium-sized enterprises.

Finally, some risks have occurred in small and medium-sized production systems: progress of process against demand, uncertainty of logistics, variation of process workers, and difficulty of synchronization with external suppliers. We have consistently proposed production processes for these issues based on the thermal diffusion model. We have also proposed improvement measures for stochastic phenomena such as fluctuations in demand and supply chain delays. This paper proposes an improvement of cost reduction by applying a mathematical model based on past research results.

Therefore, we propose a cost reduction function considering various uncertainties in the production process in Kataoka's theory. The reason for the proposal is that, in the case of small-scale production, the power distribution characteristics of the production volume are not realistic when the influence of demand distribution and production varieties are mixed. Moreover, it is not realistic without considering various risks in the production system. This paper attempts a stochastic approach to the already reported production density diffusion equation based on the above-mentioned concept and based on the results reported so far [10]. The basis of this approach is that the fluctuation of throughput occurs due to the probability of the lead time function x [16, 17], and how the production density is constrained by it can be considered by the stochastic analysis.

To the best of our knowledge, this is the first study on the contribution of fluctuations to production processes.

2. **Learning Curve Theory.**

We outline the learning theory. According to Prof. Kataoka, the cost is the function of manpower and is defined as the function of x^n, ($n = 0, 1, 2, \ldots$) for the production x. If the unit production cost for the arbitrary production quantity x is $y(x)$, the following equation is obtained [11].

$$y(x) = cp^{a(x)} \tag{1}$$

$$a(x) = \frac{\ln x}{\ln 2} \tag{2}$$

where c and p ($0 < p < 1$) are the initial unit cost and the reduction rate respectively. p is also said to be a proficiency rate.

In the case of small-scale production, when the influence of demand distribution and production varieties are mixed, the power characteristic of the production volume is not realistic in the learning curve theory. In addition, various risks in the production system are not reflected.

According to the learning curve theory, the each parameter and function is defined as follows.

Definition 2.1.

$$p = \frac{y(2x)}{y(x)}, \quad 0 < p < 1 \tag{3}$$

There is also the following equation.

Definition 2.2.

$$y(x) = cx^{n(p)} \tag{4}$$

$$-n(p) = \frac{\ln p}{\ln 2} \tag{5}$$

where c is the first production time.

In addition, the logarithmic notation is based on the assumption of power production as described above.

We obtain Figures 1 and 2 by calculating based on Equations (3), (4) and (5). Figure 1 shows the number of production on the horizontal axis, and Figure 2 shows the learning rate on the horizontal axis. For example, according to Figure 1, when the production number changes to 1-5, the production effect is reduced by 40%. At this time, the proficiency rate is 0.8. What is characteristic in this theory is that immediately after moving from the initial production to the next production, a learning effect, that is, a production time reduction effect is seen. The feature of this theory is that the proficiency effect (the production time reduction effect) is immediately seen when moving from the initial production to the next production. According to this theory, the proficiency rate is constant according to 2^x $(x = 0, 1, 2, \ldots)$. Therefore, this theory is effective in the case of a product such as a relatively single processed product manufactured in large quantities.

This learning curve theory assumes a power characteristic of lot production and learning rate for simple lot production. In this theory, the theoretical model equation as the

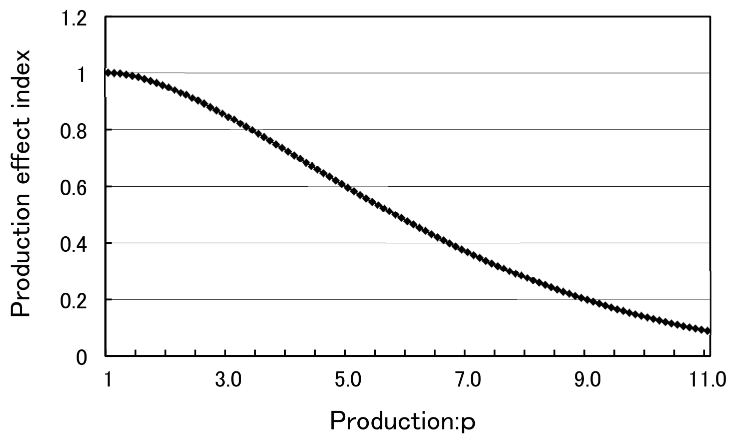

Figure 1. Number of production by learning theory

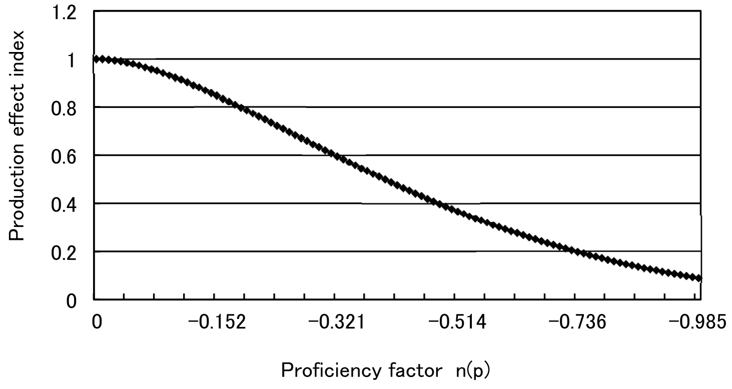

Figure 2. Learning rate by learning theory

exponential model was proposed assuming logarithmic linearity between the power values (x) and $y(x)$. The preconditions of this theory are as follows.

1) Reduction of production time by repeated production can be expected. As the condition, learning of the skill of the worker maturity, designing and process improvement activities. These conditions are well known in the mechanical frame assembly process.
2) The logarithmic linearity of the proficiency rate function and the production time is assumed.
3) The initial production processing time is assumed to be known.

3. **Distribution System and Diffusion Equation of the Production Process.** From Figure 3, we refer to the network capacity (i.e., a statically acceptable amount of production) in an interprocess network (a production field) as R. An interprocess network indicates a sequential flow from one process to the other after the completion of the current process. Here assuming that the production density function for the i-th equipment is $S_i(x,t)$, $S_i(x,t)$ is expressed by

$$[J(x,t)dt - J(x+dx,t)dt]R = [S_i(x,t+dt) - S_i(x,t)]Rdx \tag{6}$$

where J is the production flow [10, 19].

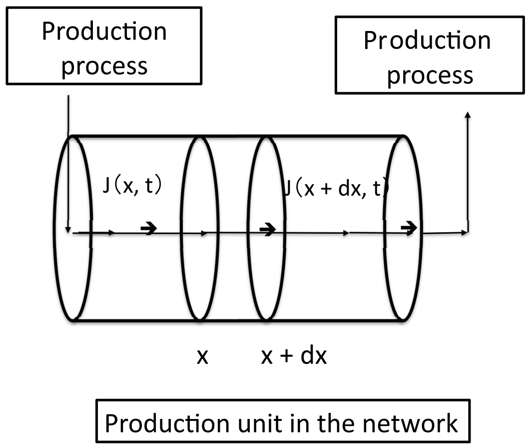

FIGURE 3. Network inter-process division of worker

We define production flow as the displacement of a production density function in the unit-production direction. The production density function is proportional to the cost necessary for production; thus, it can be considered as the production cost per unit production. Furthermore, as production leads to returns, the production density function can be considered as returns.

$$\frac{\partial S_i(x,t)}{\partial t} = D\frac{\partial^2 S_i(x,t)}{\partial x^2} \tag{7}$$

where D is the diffusion coefficient, t is the time variable, and x is the spatial variable.

This equation is equivalent to the diffusion equation derived from the minimization condition of free energy in a production field, indicating that the connections between processes can be treated as a diffusive propagation of products (refer to Figure 3) [10, 19].

A model of the production process, which is connected in one dimension, is described as follows. The process of production is indicated by the movement of production units from one process (node) to another. This production flow is equivalent to transmission rate, which is defined as the rate of data flow between connected nodes in communication engineering. Accordingly, we formulate the production model in a manner similar to heat propagation in physics. Thus, the production process is modeled mathematically using a continuous diffusion type of partial differential equation consisting of time and spatial variables [10].

Setting the network capacity (the available static production volume) to R in an inter-process network (production field, equivalent to a stochastic field), we obtain the following:

$$[J(x)dt - J(x+dx)dt]R = [S(t+dt) - S(t)]Rdx \tag{8}$$

where J is the production flow and S is the production density.

In the present model, the production flow indicates the displacement of production processes in the direction related to the production density. In other words, the production cost per production is as follows.

Definition 3.1. *Production cost per unit production*

$$J = -D\frac{\partial S}{\partial x} \tag{9}$$

where D is a diffusion coefficient.

From Equation (8), we obtain

$$-\frac{\partial J}{\partial x} = \frac{\partial S}{\partial t} \tag{10}$$

From Equations (9) and (10), we obtain

$$\frac{\partial S}{\partial t} = D\frac{\partial^2 S}{\partial x^2} \tag{11}$$

where $t \in [0, T]$, $x \in [0, L] \equiv \Omega$, $S(0, x) = S_0(x)$, $B_x S(t, x)|_{x=\partial\Omega}$.

Equation (11) is equivalent to the diffusion equation derived from the minimization condition of free energy in a production field [9, 10]. The connections between processes can be treated as a diffusive propagation of products (refer to Figure 3).

As shown in Figure 4, x represents the production elements that constitute a unit production and varies $x \to x'$ at $[t + dt]$. In other words, the unit production varies by exciting the external force and is the basis for revenue generation (an increase of potential energy). Therefore, in the transition $S_i(t, x) \to S_i\left(t', x'\right)$, the production cost, which is the cumulated external force, increases. The connections between production processes are referred to as "joints".

$$\frac{\partial S}{\partial t} + \nabla(v \cdot S) = \frac{1}{2}\Delta\left(D^2 S\right) + \lambda \tag{12}$$

where λ denotes a forced external force function and v denotes a production propagation speed. Here, λ is a forced increase function. We ignore λ from hereafter.

From Equation (12), a production density distribution varies according to increasing a production density.

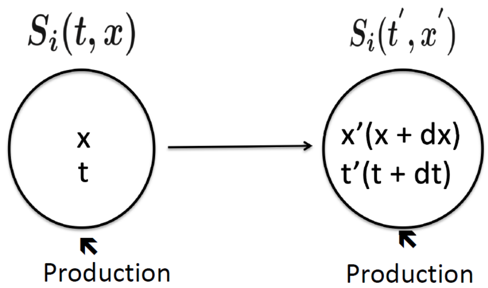

FIGURE 4. Unit of production by changing the excitation force

$S(t,x)$ satisfies a Fokker-Plank equation as follows [20, 21, 22, 23]:

$$\frac{\partial S(t,x)}{\partial t} = \frac{1}{2}\frac{\partial^2}{\partial x^2}\left\{D^2(t,x)S(t,x)\right\} - \frac{\partial}{\partial x}\{a(t,x)S(t,x)\} \tag{13}$$

where $x(t)$ satisfies Equation (14).

Definition 3.2. *Stochastic mathematical model at each stage*

$$dx(t) = \left\{a(t,x)dt + c(t,x)d\tilde{B}(t)\right\} + D(t,x)dB(t) \tag{14}$$

where $\tilde{B}$ and B denote an independent Brownian motion. c denotes a fluctuation term, which follows a stochastic differential equation.

The first term on the right-hand side of Equation (14) denotes the flow of the medium, and the second term represents the fluctuation of diffusion. Moreover, $a(t,\cdot)$ denotes an average lead time and $c(t,x)d\tilde{B}(t)$ denotes a fluctuation around processes [18, 24].

We report a stochastic approach for a production process based on the production density equation [10], i.e., a fluctuation is induced by a stochastic characteristic of a lead time function. In this case, we apply stochastic analysis to evaluating the manner in which the production density is constrained.

We assume that S defines as follows: S represents a production density with a fluctuation, and v also causes a fluctuation in throughput. As a result, a production is proportional to the slope of production density. Equation (13) is generally bound by Equation (14). According to Okazaki's analysis, we obtain as follows [23]:

$$\begin{aligned}\partial S(t,x) = &\left[\frac{1}{2}\frac{\partial^2}{\partial x^2}\left\{D^2(t,x) + c^2(t,x)\right\}S(t,x) - \frac{\partial}{\partial x}(a(t,x)S(t,x))\right]dt \\ &+ \frac{\partial}{\partial x}\{c(t,x)S(t,x)\}\partial\tilde{B}(t)\end{aligned} \tag{15}$$

where $D^2(t,x)+c^2(t,x)$ denotes a trend, $a(t,x)S(t,x)$ denotes a fluctuation of stages and $c(t,x)S(t,x)$ denotes also a fluctuation of lead time. $S(t,x)$ denotes a production density and is derived as follows [10]:

$$S\left(t, I_h^x\right) = \int_0^t P\left(\tau, x_0; t, I_h^x\right) S(\tau, x_0) d\tau \tag{16}$$

where $I_h^x \equiv [x, x+h]$ and $S\left(t, I_h^x\right)$ is the production density.

In our idea of production flow in Section 3, we describe the joint propagation model at multiple stages in the production process and the potential energy in the production field. Our research findings are as follows.

- The fluctuation in the lead time is caused by the propagation of the fluctuation of the state variables constrained by Burgers equation of fluid dynamics.
- A phenomenon similar to the occurrence in turbulent flow fields discussed in fluid dynamics is observed in production processes.
- The diffusion coefficient affected the fluctuation in turbulent spots in fluid mechanics.
- When the configuration parameters of the diffusion coefficient are considered as a trend coefficient and volatility, a production process can approach a synchronous process such as laminar flow in fluid dynamics by reducing the volatility of the production processes.

4. **Mathematical Model of Production System Considering Risks.** In the manufacturing industry, we manufacture jointly with external companies (supply chain risk). In this case, the requested process may not be completed as scheduled. Also, parts may not arrive due to delay in transportation (distribution risk). Work may be delayed due to lack of workers (manpower risk). These items are considered stochastic risks.

4.1. **Stochastic risks.** The following risks are assumed in the production system. We present a mathematical model that takes these risks into account [16, 17, 18]:

1) Stochastic volatility of process progress against demand
2) Stochastic volatility of logistics for production
3) Stochastic volatility of workers for process operations
4) Stochastic volatility for synchronization with external suppliers

Especially for work processes with many human factors, there is a problem of worker's progress or recognition of own process (process recognition) in the whole process. These issues affect the following throughput:

- Throughput in individual processes
- Throughput related to overall synchronization

Therefore, we describe the mathematical structure (stochastic differential equation) of throughput based on the example of the production system [16, 17, 18]. Furthermore, the following items are assumed mainly for manufacturing costs:

- Processed goods
- Price fluctuations of various parts used
- Price fluctuations of manufacturing consumables
- Price fluctuations in logistics costs

At this time, both of the processed products and the various parts used which depend on the physical distribution and the manpower work together with respect to the process. Synchronous working with external suppliers is also an important factor. It is very important in production systems to take account of stochastic volatilities with respect to the above items.

We have reported the following characteristics in the batch processing process (All of one skilled worker's manufacturing) and FPS method [12, 25, 26].

- Power characteristics
- Existing of fluctuation
- ON-OFF intermittency

These characteristics arise from the challenges mentioned above. We construct a portfolio by maximizing (minimizing) a utility that is reasonable for each field. We have reported the following power model [25].

$$dS = \mu S dt + \sigma S dW \tag{17}$$

where S, μ and σ are the rate of return, the trend (average) and the volatility respectively. W is the Wiener process.

4.2. **Potential model.** Furthermore, we proposed a mathematical model that is used in the following finances: that is, the Vasicek model, which is used in mathematical finance, was adopted to evaluate the production throughput of a production flow system. The production throughput was assumed to behave as an average regression. The production consisting of asynchronous and synchronous processes was evaluated theoretically using the average regression [27]. The following equation is derived as follows:

$$dS = \left(1 - \frac{D}{2M}\right) Sdt + \sigma SdW \tag{18}$$

where, regarding with $D/2M$, M and D are the actual data. M represents the elements of $[6 \times 9]$ (six workers and nine stages in the process). D is $\Delta X > K$ (now $K \geq 4$), which represents the number of elements for target throughput (WS in actual data). Then, D/M represents the error rate in Test-run1$\sim$Test-run3. $D/2M$ represents the average of D/M under normal distribution. Therefore, $\mu = 1 - (D/2M)$.

Here, we transform Equation (18) as follows:

$$\frac{dS}{S} = \left(1 - \frac{D}{2M} \pm \frac{P}{M}\right) dt + \sigma dW(t) \tag{19}$$

The fluctuations occur around the trend μ and ϵ is equivalent to P/M in Figure 5. Moreover, the displacement of the main distribution depends on the displacement of the potential in Figure 5.

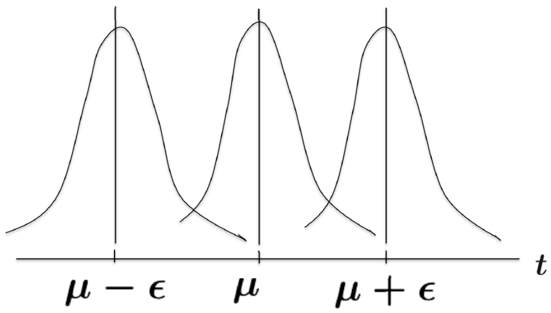

FIGURE 5. Probability distribution of S

The basis of our proposed solution is that the width of the product quality boundary region fluctuates with time. As shown in Figure 6, $\theta(x, t)$ is the function of the boundary width fluctuation (x) and time (t). Boundary width fluctuation refers to the width on the left and right, with θ as the centerline. Time is shown on the vertical axis in Figure 6. Figure 6 shows that the errors propagate along with the product. When noise-sensitive elements, which happen to be along the boundary line of the quality distribution, are activated, pronounced fluctuations can be caused by the noise. In these situations, the probability of an error increases (Figure 7), especially when products in the vicinity of the boundary contain unstable electronic components, and therefore have uncertain quality characteristics. For example, in this product, errors typically appear when parts susceptible to $1/f$ noise are used, and these result in a quality defect rate of about 3% when it is in line with the apparatus. However, these sensitive parts may be removed from the device, preventing the emergence of errors. While the influence of external environments is a critical consideration for manufacturing, it is still possible and beneficial to mathematically model and predict the appearance of product defects.

We defined the free energy of products that were within the product quality boundary region, and then analyzed these products based on the potential function in the boundary region, which constitutes to free energy operating in a stochastic region. We simulated measures that were meant to improve the quality on the premise that there is a quality probability distribution in the boundary area, whose width was assumed to change stochastically. We then introduced Ginzburg and Landau (GL) free energy theory in the quality boundary region and evaluated the resulting potential function. Based on this

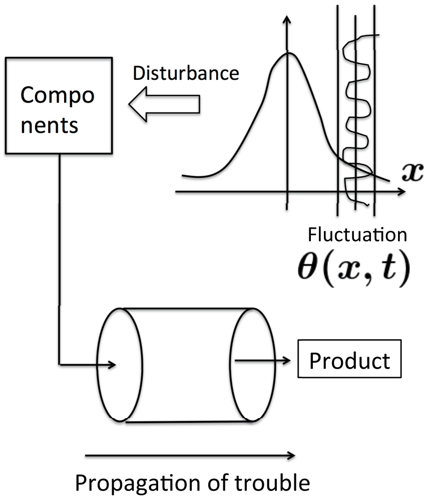

FIGURE 6. Trouble propagation and fluctuation

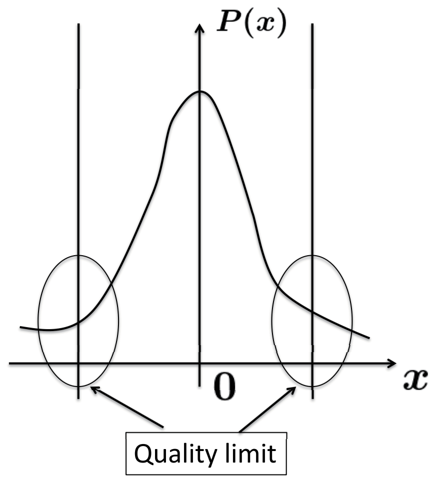

FIGURE 7. Quality probability distribution

concept, we proposed a set of corrections for circuits that are sensitive to electrical noise caused by the poor product quality, and then simulated their performance. Our first model of defect potential ignored the stochastic element. The general potential function was defined in terms of a periodic function and noise function.

Definition 4.1. *Potential function $W(\theta)$ for the boundary width θ*

$$W(\theta) = \int_0^x dx \left[\Delta\theta(x,t)\left\{a\nabla\theta - \frac{D}{2}(\Delta\theta(x,t))^2\right\}\right] + g(t)\theta(x,t) + r(t) \tag{20}$$

where D denotes a constant (> 0). $g(t)$ and $r(t)$ denote a cyclic function and noise function, respectively, and x is a variable belonging to a finite area, S, that has a smooth boundary in one-dimensional Euclidean space. The expression $g(t)\theta(x,t) + r(t)$ denotes an external stress, such as a fluctuation in temperature and humidity, which often causes defects in electronic components in Japan. In other words, this function represents potential with a width x ($0 \le x \le X$) for a product near the tolerance boundary. Figure 8 shows the range of fluctuation function $\theta(x)$. Figure 9 shows the domain of order parameter $\varphi^*(\theta,\xi,t)$. $\varphi^*(\theta,t) = 1$ and $\varphi^*(\theta,\xi,t) = 0$ describe stable and unstable conditions, respectively [21].

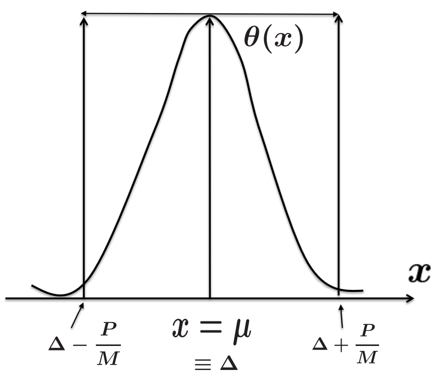

FIGURE 8. Range of fluctuation function $\theta(x)$

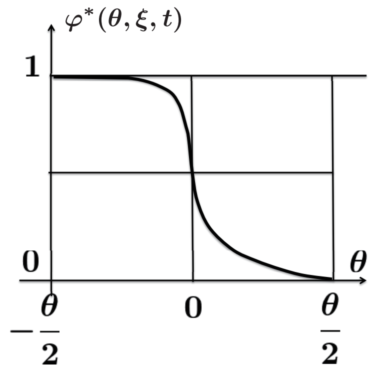

FIGURE 9. The domain of order parameter $\varphi^*(\theta,\xi,t)$

Definition 4.2. *Fluctuation function* $\varphi(\theta,\xi,t)$

$$\varphi(\theta,\xi,T)=h(\theta)f^S(\xi,T)+\{1-h(\theta)\}f^L(\xi,T)+W^{SL}\cdot g(\theta) \tag{21}$$

where $t=T$, $0\le\varphi(\theta,\xi,t)\le 1$, $h(0)=0$ *and* $h(1)=1$.

At this time, the free energy F in the system is as follows.

Definition 4.3. *Free energies* F *within boundaries function* θ

$$F=\int_{x\in S}\left\{\varphi(\theta,\xi,T)+\frac{1}{2}\epsilon^2|\nabla\theta|^2\right\}dx \tag{22}$$

where ϵ *denotes a positive constant,* $\nabla(\cdot)$ *denotes a* $\partial(\cdot)/\partial x$. $\varphi(\theta,\xi,T)$ *is defined as Equation (21).*

The production process is according to a thermal diffusion. $\nabla\theta$ represents the gradient energy density of the process in which the product is made. Free energy represents an indicator of stability and instability of products [21]. Generally, the free energy F does not increase with time, so the following equation holds in a closed system [30].

At this time, the following equation is obtained from the gradient principle.

$$\tau\frac{\partial\varphi}{\partial t}=-\frac{\delta F}{\delta\varphi}=\epsilon^2\nabla^2\varphi^2+\varphi(1-\varphi)\times\left(\varphi-\frac{1}{2}+f\right) \tag{23}$$

The special solution of Equation (23) is as follows [21]:

$$\varphi^*(\theta,\xi,t)=\frac{1}{2}\left\{1-\tanh\left(\frac{\theta-\nu t}{\sqrt{2}\epsilon}\right)\right\} \tag{24}$$

where $\nu=\left(\sqrt{2}\epsilon f\right)/\tau$.

Here, $\nu=0$. Then, we can obtain as follows:

$$\varphi^*(\theta,\xi,t)=\frac{1}{2}\left\{1-\tanh\left(\frac{\theta}{\sqrt{2}\epsilon}\right)\right\} \tag{25}$$

Figure 9 is shown by Equation (25). Therefore, $0\le\varphi(\theta,\xi,t)\le 1$ becomes the order parameter.

We define the order parameter $\varphi(\theta,\xi,t)$ as follows [26, 29]. The order parameter represents the degree of stable phase and unstable phase of the product.

Definition 4.4. *Potential function*

$$f(\varphi)=a\cdot h(\varphi)+b\cdot\{1-h(\varphi)\}+c\cdot g(\varphi) \tag{26}$$

where $h(0)=0$ *and* $h(1)=1$. *The function* $c\cdot g(\varphi)$ *has a maximal value as* $0<\varphi<1$ *and has zero as* $\varphi=0$ *and* $\varphi=1$. a, b *and* c *denote the normalized parameters for assumed noise intensity in the stable phase and normalized value for assumed noise intensity in the unstable phase respectively.*

Definition 4.5. $h(\varphi)$ *and* $g(\varphi)$

$$h(\varphi)\equiv\varphi^2(3-2\varphi) \tag{27}$$

$$g(\varphi)\equiv\varphi^2(1-\varphi)^2 \tag{28}$$

With respect to Equation (26), the potential function $f(\varphi)$ can be defined by using Equation (26) as the order variable of the fluctuation width function. It looks like Figure 9.

From Figure 8, we define the potential function as Equations (26)-(28) for the fluctuation width function $\theta(x)$, which is $-\frac{P}{M}\le\theta(x)\le+\frac{P}{M}$. Next, $\varphi(\theta,\xi,T)$ is obtained from the principle of the gradient method, and $0\le\varphi^*(\theta,\xi,T)\le 1.0$ is established from the result obtained from the special solution. $\varphi^*(\theta, \xi, T)$ becomes the order parameter (Refer to Figure 9). However, the free energy in the system is defined by Equation (22).

In our idea of stochastic risks in Section 4, our research findings are as follows:

- Based on our research results, we were able to observe and link the on-off intermittence in time with the fluctuations that we previously reported in 2014 [12].
- We derived the Burgers equation by recognizing the graph of the start time series (order time-series) of the lead time period in production processes.
- Based on our actual data, we presented the reductions in volatility, which led to an improved production throughput.

In Section 4, we describe some of risks, which are both of the processed products and the various parts used which depend on the physical distribution and the manpower work together with respect to the process. Synchronous working with external suppliers is also a risk factor. It is very important in production systems to take account of stochastic volatilities. With respect to the potential model, we utilize the Vasicek model, which is used in mathematical finance, and was adopted to evaluate the production throughput of a production flow system. The production throughput was assumed to behave as an average regression. The production consisting of asynchronous and synchronous processes is evaluated theoretically using the average regression [27].

5. New Cost Reduction Function.

Here, we propose a new cost function. From Equations (26)-(28), the parameters a, b and c are defined as follows.

Definition 5.1.

$$a = \frac{M}{M^*} \tag{29}$$

$$b = \frac{D}{M} \tag{30}$$

$$c = \frac{P}{M} \tag{31}$$

$$\frac{M}{M^*} \leq 1, \quad M \geq D, \quad M > P, \quad D > P \tag{32}$$

Regarding with Figure 10, the edge of fluctuation means the state in which the number of stages that fall within the allowable lead time and the number of stages that do not fall within the allowable lead time exist in the number of stages of all processes (here, 1 to 6 processes). M^*, M, D and P denote the maximum number of stages in the process, the number of partial stages in the process, the number of stages that fall within the acceptable evaluation criteria for the process and the number of stages that slightly deviate from the acceptable evaluation criteria in the process in Equations (29)-(31). Therefore, the edge of the fluctuation of $D = P$ of $[C]$ represents the state where the number of stages within the evaluation criterion and the number of stages slightly deviating from each other are almost balanced. That is, the phase transition to either A or B state is made by a slight change in the behavior pattern in the stage. At this time, $[A]$ indicates that the state of the process is relatively good, and $[B]$ indicates the opposite. $[C]$ represents the edge of fluctuation. Please refer to the references for "a slight change in the behavior pattern in the stage" in the above description [21].

Regarding with Figure 11, "Area-1" indicates a large-effect production area with high production efficiency and D and P are small. "Area-2" indicates an area with the small-quantity effect production area and D and P are large. D/M $(M > D)$ represents an ability ratio and P/M represents an uncertainty ratio.

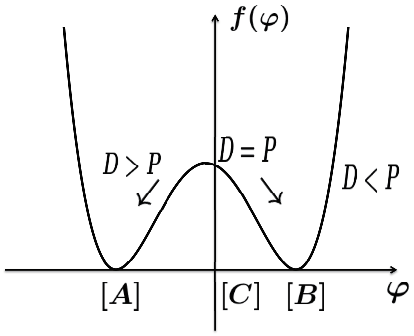

FIGURE 10. Edge of fluctuation from Equation (26)

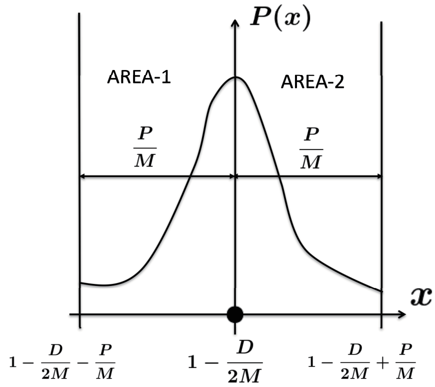

FIGURE 11. Standard normal distribution

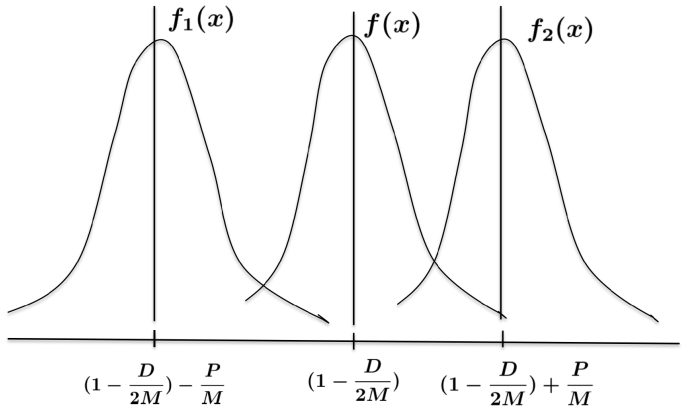

FIGURE 12. Main and transition distributions

Regarding with Figure 12, $f(x)$, $f_1(x)$ and $f_2(x)$ are the main distribution and the transition distribution, respectively, which all represent the normal distribution as follows:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{33}$$

$$f_1(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\left\{x-\left(\mu-\frac{P}{M}\right)\right\}^2}{2\sigma^2} \Big/ \left(\frac{D}{M}\right)\right) \tag{34}$$

$$f_2(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\left\{x-\left(\mu+\frac{P}{M}\right)\right\}^2}{2\sigma^2} \Big/ \left(1-\frac{D}{M}\right)\right) \tag{35}$$

where $\mu = 1 - \frac{D}{2M}$.

We present an evaluation of the production process model as follows. Our basic idea is under average regression.

When the transition from the main distribution to $f_1(x)$ is made according to the magnitude of D, $\exp(-\bullet)$ is normalized by (D/M) by the exponential function $\exp(-\bullet)$. In the case of $f_2(x)$, since the value of D is small, it is normalized by the value of $(1-D/M)$.

We define the stochastic occurrence as follows from Equations (33)-(35).

Definition 5.2. *Stochastic occurrence*

$$g(M,D,P;\sigma) = \left(\frac{D}{M}\right)\int_{\frac{1-\frac{D}{2M}-\frac{P}{M}}{\sigma}}^{\infty} \psi(z)dz + \left(1-\frac{D}{M}\right)\int_{\frac{1-\frac{D}{2M}-\frac{P}{M}}{\sigma}}^{\infty} \psi(z)dz \tag{36}$$

$$\psi(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right), \quad z = \frac{x-\mu}{\sigma} \tag{37}$$

It becomes effective when D increases large in the first term of Equation (36), and becomes effective when D decreases small in the second term of Equation (36). Depending on the magnitude of D, one of them occurs, and at this time, the reduction function is $(1-g)$. That is, g is the stochastic occurrence and represents the production efficiency, and $(1-g)$ represents the reduced rate as the opposite.

We define the cost reduction function $L(M, D, P; \sigma)$ as follows.

Definition 5.3.

$$L(M, D, P; \sigma) = 1 - g(M, D, P; \sigma) \tag{38}$$

$$g(M, D, P; \sigma) = \left(\frac{D}{M}\right) \Phi\left(\frac{1 - \frac{D}{2M} - \frac{P}{M}}{\sigma}\right) + \left(1 - \frac{D}{M}\right) \Phi\left(\frac{1 - \frac{D}{2M} - \frac{P}{M}}{\sigma}\right) \tag{39}$$

where $\Phi(\bullet)$ represents the cumulative normal distribution function.

Moreover, we present the cost reduction function by replacing Equation (39) as follows:

$$L(\eta) = \left(\frac{D}{M}\right) \Phi\left(\frac{1 - \frac{D}{2M} - \frac{P}{M}}{\sqrt{\eta}}\right) + \left(1 - \frac{D}{M}\right) \Phi\left(\frac{1 - \frac{D}{2M} - \frac{P}{M}}{\sqrt{\eta}}\right) \tag{40}$$

$$\sqrt{\eta} = \sigma \tag{41}$$

From Equation (40), we define the maximum production reduction ratio H as follows:

Definition 5.4.

$$H = \frac{L(\eta = 1.0)}{L(\eta = 0)} \tag{42}$$

At this time, we consider the volatility σ as the volatility of non-uniformity with respect to production or the ratio of the number of actual production lots to the maximum production number (maximum production number that can be produced). It represents fluctuations in production.

6. Numerical Simulation.

6.1. Production systems in the production equipment industry. From this chapter, we will describe the actual production process we are using. We refer to the production system in manufacturing equipment industry studied in this paper. This is not a special system but “Make-to-order system with version control”. Make-to-order system is a system which allows necessary manufacturing after taking orders from clients, resulting in “volatility” according to its delivery date and lead time. In addition, there is volatility in the lead time, depending on the content of the make-to-order products (production equipment).

However, effective utilization of the production forecast information on the orders may suppress certain amount of “variation”, but the complete suppression of variation will be difficult. In other words, “volatility” in monthly cash flow occurs and of course influences a rate of return in these companies. The production management system, suitable for the separate make-to-order system which is managed by numbers assigned to each product upon order, is called as “product number management system” and is widely used.

All productions are controlled with numbered products and instructions are given for each numbered product.

Thus, ordering design, logistics and suppliers are conducted for each manufacturer's serial numbers in most cases except for semifinished products (unit incorporated into the final product) and strategic stocks.

Therefore, careful management of the lead time or production date may not suppress "volatility" in manufacturing (production).

The company in this study is the "supplier" in Figure 13 and "factory" here. Companies are under the assumption that there are N (numbers of) suppliers; however, this study deals with one company because no data is published for the rest of the companies ($N-1$).

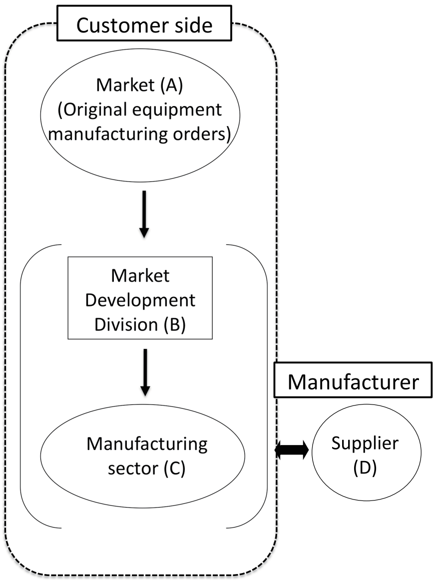

Figure 13. Business structure of company of research target

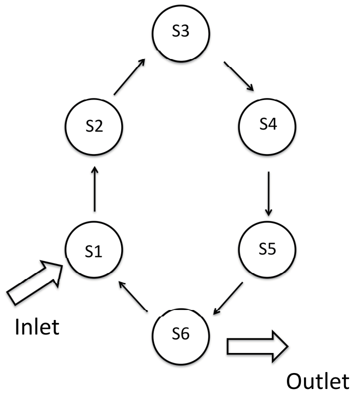

Figure 14. Production flow process

6.2. **Production flow process.** A manufacturing process that is termed as a production flow process is shown in Figure 14. The production flow process, which manufactures low volumes of a wide variety of products, is produced through several stages in the production process. In Figure 14, the processes consist of six production stages (S1-S6). In each step S1-S6 of the manufacturing process, materials are being produced.

The direction of the arrow represents the direction of the production flow. In this system, production materials are supplied from the inlet and the end product will be shipped from the outlet.

Assumption 6.1. *The manpower work in each process becomes non-linear because it varies depending on the ability of the worker.*

Assumption 6.2. *The production structure is a closed structure; that is, the production is driven by a cyclic system (production flow system). Production work is completed from S1 to S6 in Figure 14.*

Assumption 6.1 indicates that the determination of the production structure is considered a major factor, which includes the generation value of production or the throughput generation structure in a stochastic manufacturing process (hereafter called the manufacturing field). Because such a structure is at least dependent on the demand, it is considered to have a nonlinear structure.

Because the value of such a product depends on the throughput, its production structure is nonlinear. Therefore, Assumption 6.1 reflects the realistic production structure and is somewhat valid. Assumption 6.2 is completed in each step and flows from the next step until stage S6 is completed. Assumption 6.2 is reasonable because new production starts from S1.

Based on the control equipment, the product can be manufactured in one cycle. The production throughput required to maintain 6 pieces of equipment/day is as follows:

$$\frac{(60 \times 8 - 28)}{3} \times \frac{1}{6} \simeq 25 \text{ (min)} \tag{43}$$

where the throughput of the previous process is set as 20 (min). In Equation (43), "28" represents the throughput of the previous process plus the idle time for synchronization. "8" is the number of processes and the total number of all processes is "8" plus the previous process. "60" is given by 20 (min) $\times$ 3 (cycles).

One process throughput (20 min) in full synchronization is

$$T_s = 3 \times 120 + 40 = 400 \text{ (min)} \tag{44}$$

Therefore, a throughput reduction of about 10% can be achieved. However, the time between processes involves some asynchronous idle time.

Please refer to the Appendix. Test run results and Tables 4-9 are shown in it.

6.3. **Analysis of the test-run results.** Table 1 shows a comparison table of the working time for the production method of the Test-run1$\sim$Test-run3.

- (Test-run1): Each throughput in every process (S1-S6) is asynchronous, and its process throughput is asynchronous. Table 4 represents the production time (min) in each process. The volatilities of K3 and K8 increase due to the delay of K3 and K8 in Table 5. K3 and K8 of workers in Table 4 indicate the delay propagation of working time through S1-S6 stages. Table 5 represents the volatility in each process performed by workers. Table 4 represents the target time, and the theoretical throughput is given by $3 \times 199 + 2 \times 15 = 627$ (min).

  In addition, the total working time in stage S3 is 199 (min), which causes a bottleneck. Figure 20 is a graph illustrating the measurement data in Table 4, and it represents the total working time for each worker (K1-K9). The graph in Figure 21 represents the volatility data for each working time in Table 4.
- (Test-run2): Set to synchronously process the throughput.

  The target time in Table 6 is 500 (min), and the theoretical throughput (not including the synchronized idle time) is 400 (min). Table 7 represents the volatility data of each working process (S1-S6) for each worker (K1-K9).
- (Test-run3): Introducing a preprocess stage, the process throughput is performed synchronously with the reclassification of the process. The theoretical throughput (not including the synchronized idle time) is 400 (min) in Table 8. Table 9 represents the volatility data of each working process (S1-S6) for each worker (K1-K9).

  From this result, the idle time must be set at 100 (min). Based on the above results, the target theoretical throughput (T_s') is obtained using the "synchronization-with-preprocess" method. This goal is

$$\begin{aligned} T_s &\sim 20 \times 6 \text{ (First cycle)} + 17 \times 6 \text{ (Second cycle)} + 20 \times 6 \text{ (Third cycle)} \\ &\quad + 20 \text{ (Previous process)} + 8 \text{ (Idle-time)} \\ &= 370 \text{ (min)} \end{aligned} \tag{45}$$

  The full synchronous throughput in one stage (20 min) is

$$T_s' = 3 \times 120 + 40 = 400 \text{ (min)} \tag{46}$$

The throughput becomes about 10% reduction in result. Therefore, the "synchronization-with-preprocess" method is realistic in this paper, and it is recommended the "synchronization-with-preprocess" method in the flow production system [13].

Now, we manufacture one equipment at 3 cycle. For maintaining the throughput of 6 units/day, the production throughput is as follows.

In Table 8, the working times of the workers K4, K7 show shorter than others. However, the working time shows around target time.

Here, the preprocess represents the working until the process itself is entered. To eliminate the idle time after classification of the processes in advance, this preprocess was introduced. In Figure 15, for example, it represents the termination of the operation of step K5 during the preprocess. By making the corresponding step K5 to be the preprocess, there are eight remaining processes. When performing the 3 cycles in Figure 16, the first cycle is {K1, K2, K3}, the second cycle is {K4, K6, K7}, and the third cycle is {K8, K9}.

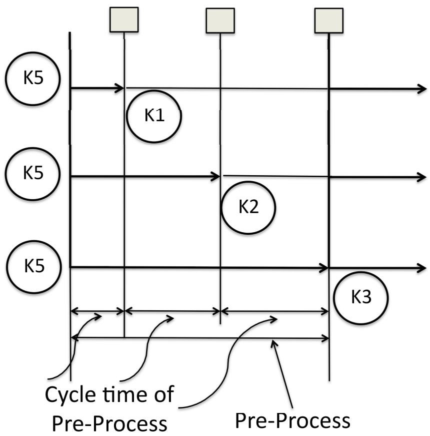

Figure 15. Previous process in production equipment

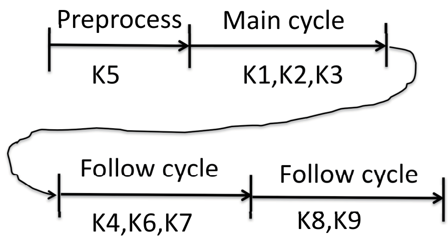

Figure 16. "Synchronization-with-preprocess" method in production equipment

After completion of the third cycle, the workers start production of the next product. That is, the first production process starts the first cycle. By adopting the preprocess cycle, the third cycle is adopted in a parallel process.

Here, the preprocess is adopted in Test-run3 only.

Table 1. Correspondence between the table labels and the test run number

	Table number	Production process	Working time	STD
Test-run1	Table 4	Asynchronous process	627 (min)	0.29
Test-run2	Table 6	Synchronous process	500 (min)	0.06
Test-run3	Table 8	Synchronous-with-preprocess	470 (min)	0.03

6.4. **Analysis of the cost reduction function in Figures 17-19.** Table 2 shows values obtained by applying the parameter values (M, D, P), the capacity ratio (D/M), and the uncertainty ratio (P/M) (Test-run1) used in Figures 17-19.

In Test-run1 (Figure 17), the variance of the workers is large and the convergence of the cost reduction function is slow. Therefore, it is understood that it is difficult to reduce costs. Test-run2 (Figure 18) and Test-run3 (Figure 19) are basically a synchronous production method that maximizes productivity. In particular, Test-run3 is a modified production system. Therefore, it can be seen that the cost reduction function converges quickly and that cost reduction can be expected. From the numerical results, Figure 19 has the highest cost reduction rate. The reason is that Test-run3 is the most productive (please see Section 6.3).

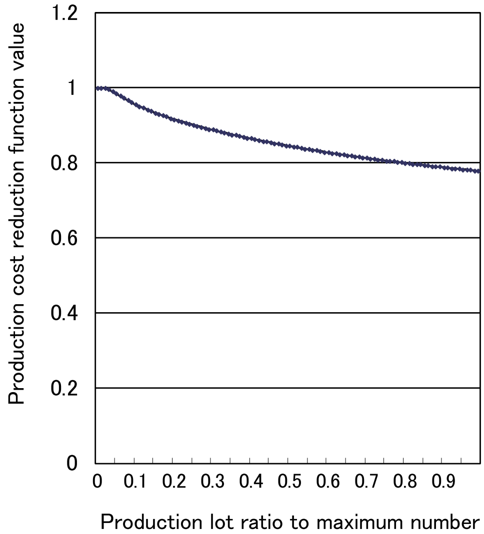

Figure 17. Cost reduction function in production system (Test-run1)

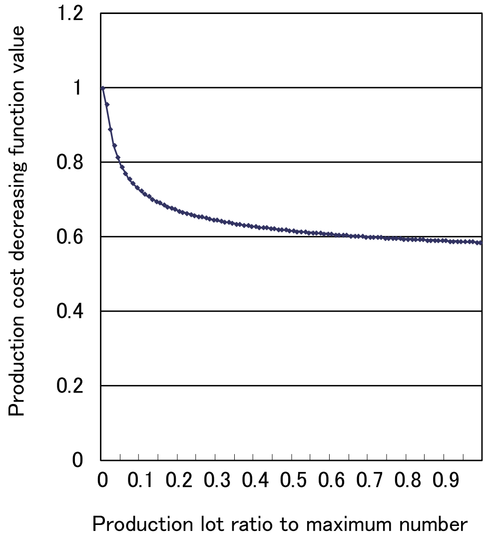

Figure 18. Cost reduction function in production system (Test-run2)

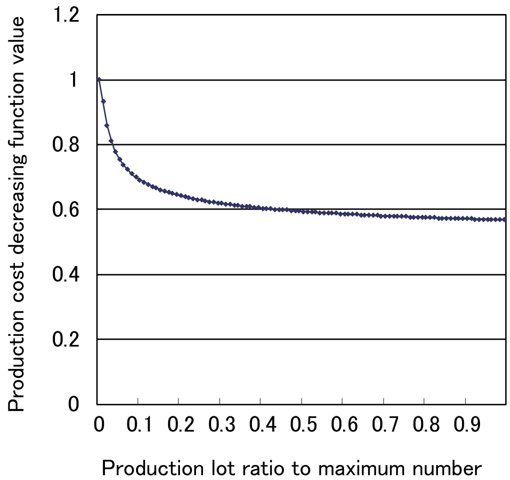

Figure 19. Cost reduction function in production system (Test-run3)

Table 2. Set values of parameters M, D, P, trend and volatility in Figures 17-19

	M	D	P	Ability rate (D/M)	Reduction rate (P/M)	Trend	Volatility
Figure 17 (Test-run1)	54	27	27	$(0.5)_1$	$(0.5)_1$	$(0.5)_1$	$(0.5)_1$
Figure 18 (Test-run2)	54	50	20	$(0.93)_2$	$(0.37)_2$	$(0.93)_2$	$(0.37)_2$
Figure 19 (Test-run3)	54	52	20	$(0.96)_3$	$(0.37)_3$	$(0.96)_3$	$(0.37)_3$

where $(\bullet)_1$, $(\bullet)_2$ and $(\bullet)_3$ are Tes-trun1, Test-run2 and Test-run3 respectively. From Equation (42), we obtain Table 3 under the parameters of Table 2. From the ability rate $(D/M)_1 < (D/M)_2 < (D/M)_3$, Test-run3 is the best production method. Similarly, from the reduction rate $(P/M)_1 > (P/M)_2 > (P/M)_3$, Test-run3 is the best production method.

Table 3. Inhomogeneous volatility value in Figures 17-19

	Inhomogeneous volatility value
Figure 17 (Test-run1)	0.7794 (0.7794/1.0)
Figure 18 (Test-run2)	0.5852 (0.5852/1.0)
Figure 19 (Test-run3)	0.5686 (0.5686/1.0)

7. **Conclusion.** We have proposed the new cost reduction function that considers the following:

- Uncertainty of logistics supplied to processing the uncertainties in the process of human skills
- Endogenous treatment for the lead time in the machining the external disturbances
- Consideration of non-uniformity of specifications for processed products

By evaluating the maximum production reduction ratio utilizing this cost reduction function, it is possible to increase the production efficiency.

REFERENCES

[1] M. E. Mundel, *Improving Productivity and Effectness*, Prentice-Hall, NZ, 1983.

[2] A. Neely, M. Gregorey and K. Platts, Performance measurement system design, *International Journal of Operations & Production Management*, vol.15, no.4, pp.5-16, 1995.

[3] K. Hayama and H. Irie, Trial production of kite wing attached multicopter for power saving and long flight, *ICIC Express Letters, Part B: Applications*, vol.10, no.5, pp.405-412, 2019.

[4] K. Nishioka, Y. Mizutani, H. Ueno et al., Toward the integrated optimization of steel plate production process –A proposal for production control by multi-scale hierarchical modeling–, *Synthesiology*, vol.5, no.2, pp.98-112, 2012.

[5] N. Ueno, M. Kawasaki, H. Okuhira and T. Kataoka, Mass customization production planning system for muli-process, *Journal of the Faculty of Management and Information Systems, Prefectural University of Hiroshima*, no.1, pp.183-192, 2009 (in Japanese).

[6] U. Asgher, R. Ahmad and S. I. Butt, Mathematical modeling of manufacturing process plan, optimization analysis with stochastic and DSM modeling techniques, *Advanced Materials Research*, vols.816-817, pp.1174-1180, 2013.

[7] H. L. Lee, V. Padmanabhan and S. Whang, The bullwhip effect in supply chains, *Sloan Management Review*, pp.93-102, 1997.

[8] H. Kondo and K. Nisinari, Modeling stock congestion in production management, *Reports of RIAM Symposium, Mathematics and Physics in Nonlinear Waves*, no.20, ME-S7, pp.146-149, 2008 (in Japanese).

[9] H. Tasaki, *Thermodynamics – A Contemporary Perspective (New Physics Series)*, Baifukan, Co., LTD., 2000.

[10] K. Shirai and Y. Amano, Production density diffusion equation and production, *IEEJ Trans. Electronics, Information and Systems*, vol.132-C, no.6, pp.983-990, 2012.

[11] S. Kataoka, The use of learning curve theory in estimating production cost, *The Bulletin of Toyohashi Junior College*, no.12, pp.147-161, 1995.

[12] K. Shirai and Y. Amano, On-off intermittency management for production process improvement, *International Journal of Innovative Computing, Information and Control*, vol.11, no.3, pp.815-831, 2015.

[13] K. Shirai, Y. Amano and S. Omatu, Improving throughput by considering the production process, *International Journal of Innovative Computing, Information and Control*, vol.9, no.12, pp.4917-4930, 2013.

[14] K. Shirai and Y. Amano, Investigation of the relation between production density and lead-time via stochastic analysis, *International Journal of Innovative Computing, Information and Control*, vol.13, no.4, pp.1117-1133, 2017.

[15] K. Shirai and Y. Amano, Determination of allocation rate of production projects utilizing risk-sensitive control theory, *International Journal of Innovative Computing, Information and Control*, vol.13, no.3, pp.847-871, 2017.

[16] K. Shirai and Y. Amano, A study on mathematical analysis of manufacturing lead time –Application for deadline scheduling in manufacturing system–, *IEEJ Trans. Electronics, Information and Systems*, vol.132-C, no.12, pp.1973-1981, 2012.

[17] K. Shirai and Y. Amano, Analysis of production processes using a lead-time function, *International Journal of Innovative Computing, Information and Control*, vol.12, no.1, pp.125-138, 2016.

[18] K. Shirai and Y. Amano, Model of production system with time delay using stochastic bilinear equation, *Asian Journal of Management Science and Applications*, vol.1, no.1, pp.1-15, 2015.

[19] K. Shirai and Y. Amano, Application of an autonomous distributed system to the production process, *International Journal of Innovative Computing, Information and Control*, vol.10, no.4, pp.1247-1265, 2014.

[20] K. Shirai and Y. Amano, Synchronization analysis of a production process utilizing stochastic resonance, *International Journal of Innovative Computing, Information and Control*, vol.12, no.3, pp.899-914, 2016.

[21] K. Shirai and Y. Amano, Synchronization analysis of the production process utilizing the phase-field model, *International Journal of Innovative Computing, Information and Control*, vol.12, no.5, pp.1597-1613, 2016.

[22] K. Shirai and Y. Amano, Mathematical modeling and potential function of a production system considering the stochastic resonance, *International Journal of Innovative Computing, Information and Control*, vol.12, no.6, pp.1761-1776, 2016.

[23] S. Okazaki, Generalization of Fokker-Planck equations by means of projection operator technique –A study of stochastic processes subject to additive noises–, *RISM (Repository of the Institute of Statistical Mathematics)*, vol.38, no.1, pp.1-18, 1990.

[24] K. Shirai, Y. Amano and S. Omatu, Propagation of working-time delay in production, *International Journal of Innovative Computing, Information and Control*, vol.10, no.1, pp.169-182, 2014.

[25] K. Shirai, Y. Amano, S. Omatu and E. Chikayama, Power-law distribution of rate-of-return deviation and evaluation of cash flow in a control equipment manufacturing company, *International Journal of Innovative Computing, Information and Control*, vol.9, no.3, pp.1095-1112, 2013.

[26] K. Shirai and Y. Amano, Self-similarity of fluctuations for throughput deviations within a production process, *International Journal of Innovative Computing, Information and Control*, vol.10, no.3, pp.1001-1016, 2014.

[27] K. Shirai and Y. Amano, Production throughput evaluation using the Vasicek model; *International Journal of Innovative Computing, Information and Control*, vol.11, no.1, pp.1-17, 2015.

[28] S. J. Baderstone and V. J. Mabin, A review goldratt's theory of constraints (TOC) – Lessons from the international literature, *Operations Research Society of New Zealand 33rd Annual Conference*, University of Auckland, New Zealand, 1998.

[29] K. Shirai, Y. Amano and S. Omatu, Consideration of phase transition mechanisms during production in manufacturing processes, *International Journal of Innovative Computing, Information and Control*, vol.9, no.9, pp.3611-3626, 2013.

[30] R. Yamamoto, T. Nakaturu, K. Miyajima and M. Ishikawa, On the mathematical modeling of order-disorder transition by stochastic partial differential equations, *Yamaguchi Univ. Engineering Faculty Research Report*, vol.50, no.1, pp.45-51, 1999.

Appendix. Analysis of Actual Data in the Production Flow System. The results are as follows. Here, the trend coefficient, which is the actual number of pieces of equipment/the target number of equipment, represents a factor that indicates the degree of the number of pieces of production equipment.

Test-run1: 4.4 (pieces of equipment)/6 (pieces of equipment) = 0.73,
Test-run2: 5.5 (pieces of equipment)/6 (pieces of equipment) = 0.92,
Test-run3: 5.7 (pieces of equipment)/6 (pieces of equipment) = 0.95.

Volatility data represent the average value of each test-run.

Table 4. Total production time at each stage for each worker (Asynchronous)

	WS	S1	S2	S3	S4	S5	S6
K1	15	20	20	25	20	20	20
K2	20	22	21	22	21	19	20
K3	10	20	26	25	22	22	26
K4	20	17	15	19	18	16	18
K5	15	15	20	18	16	15	15
K6	15	15	15	15	15	15	15
K7	15	20	20	30	20	21	20
K8	20	29	33	30	29	32	33
K9	15	14	14	15	14	14	14
Total	145	172	184	199	175	174	181

Table 5. Volatility of Table 4

K1	1.67	1.67	3.33	1.67	1.67	1.67
K2	2.33	2	2.33	2	1.33	1.67
K3	1.67	3.67	3.33	2.33	2.33	3.67
K4	0.67	0	1.33	1	0.33	1
K5	0	1.67	1	0.33	0	0
K6	0	0	0	0	0	0
K7	1.67	1.67	5	1.67	2	1.67
K8	4.67	6	5	4.67	5.67	6
K9	0.33	0.33	0	0.33	0.33	0.33

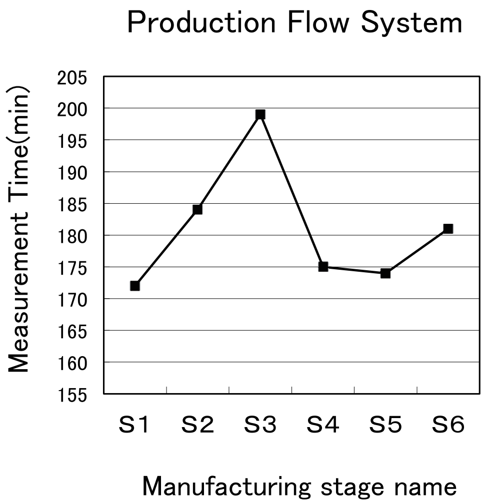

Figure 20. Total production time of each stage by each worker

Figure 21. STD data of each worker at each stage

Table 6. Total production time at each stage by each worker (Synchronous)

	WS	S1	S2	S3	S4	S5	S6
K1	20	18	19	18	18	18	18
K2	20	18	18	18	18	18	18
K3	20	21	21	21	21	21	21
K4	*16	13	11	11	13	13	13
K5	*16	16	16	17	17	16	16
K6	*16	18	18	18	18	18	18
K7	20	14	14	13	14	14	13
K8	20	22	22	22	22	22	22
K9	20	20	20	20	20	20	20
Total	168	165	164	163	166	165	164

Table 7. Volatility of Table 6

K1	0.67	0.33	0.67	0.67	0.67	0.67
K2	0.67	0.67	0.67	0.67	0.67	0.67
K3	0.33	0.33	0.33	0.33	0.33	0.33
K4	1	1.67	1.67	1	1	1
K5	0	0	0.33	0.33	0	0
K6	0.67	0.67	0.67	0.67	0.67	0.67
K7	2	2	2.33	2	2	2.33
K8	0.67	0.67	0.67	0.67	0.67	0.67
K9	1.67	1.67	1.67	1.67	1.67	1.67

Table 8. Total production time at each stage for each worker (synchronous-with-pre-process), K5 (*): Preprocess

	WS	S1	S2	S3	S4	S5	S6
K1	20	18	19	18	18	18	18
K2	20	18	18	18	18	18	18
K3	20	21	21	21	21	21	21
K4	16	13	11	11	13	13	13
K5	16	*	*	*	*	*	*
K6	16	18	18	18	18	18	18
K7	16	14	14	13	14	14	13
K8	20	22	22	22	22	22	22
K9	20	20	20	20	20	20	20
Total	148	144	143	141	144	144	143

Table 9. Volatility of Table 8 K5: Previous process

K1	0.67	0.33	0.67	0.67	0.67	0.67
K2	0.67	0.67	0.67	0.67	0.67	0.67
K3	0.33	0.33	0.33	0.33	0.33	0.33
K4	1	1.67	1.67	1	1	1
K5	*	*	*	*	*	*
K6	0.67	0.67	0.67	0.67	0.67	0.67
K7	0.67	0.67	1	0.67	0.67	1
K8	0.67	0.67	0.67	0.67	0.67	0.67
K9	0	0	0	0	0	0