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ADAPTIVE DYNAMIC SURFACE SATURATED CONTROL OF FLEXIBLE HYPERSONIC VEHICLE

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Abstract. *This paper investigates the flight longitudinal tracking control problem of flexible hypersonic vehicle with the external disturbance, model parameter uncertainties and input saturation. First, velocity and altitude subsystems are established using decomposing the longitudinal dynamics of flexible hypersonic vehicle, where the flexible effects, external disturbance and model parameter uncertainties are regarded as unknown bounded lumped disturbance. Second, the robust saturated controllers are designed for velocity and altitude subsystems by using the back-stepping method, adaptive tracking differentiator and tangent function with approximate saturation function, respectively. Finally, Lyapunov stability theory and simulations are adopted to verify the effectiveness of the control strategy.*

Keywords: Flexible hypersonic vehicle, Backstepping control, Tracking control, Input saturation, Finite time stable

1. **Introduction.** The hypersonic vehicle has characteristics of fast time-varying, strong non-linearity, strong coupling and strong uncertainty, etc. [1,2]. Additionally, the usually adopted slender configuration and light structure design have led to the flexible deformation of the structure of hypersonic vehicle, which have caused the induced angle of attack to affect the controlling input variable of the vehicle and magnified the probability of the actuator's generating saturation [3]. Therefore, the robust saturated control method of flexible hypersonic vehicle has possessed important significance.

Domestic and overseas scholars have conducted related studies on the tracking control of the hypersonic vehicle and achieved rich research achievements [4,5]. In [6], the adaptive dynamic surface tracking controller was provided aiming at hypersonic vehicle with aerodynamic parameter uncertainties, which has realized the fast and stable tracking of velocity and altitude signal. In [7], the anti-saturation adaptive back-stepping control strategy was proposed for the velocity and altitude subsystem based on compensating signals and auxiliary system. In [8], the adaptive fast terminal tracking controller was designed for hypersonic vehicle, where the non-homogeneous disturbance observer was introduced to estimate disturbances. An obstacle avoidance control algorithm was designed for the unmanned underwater vehicle by using the convolution neural network in [9]. In order to deal with the flexible effect of hypersonic vehicle, the robust tracking controller was designed for flexible hypersonic vehicle by combining super-twisting sliding mode theory and adaptive control technique in [10,11]. Based on neural networks, back-stepping method and tracking differentiator, the dynamic surface controller was proposed for the hypersonic flight vehicle in [12-14]. Due to the complex flight conditions, it is easy for

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the actuator to have saturation phenomenon, which can weaken system control performance, and even may lead to system instability. Therefore, the input saturation must be considered in designed controller. In [15], the saturated control strategy for hypersonic vehicle was on basis of the mean-value theorem and sliding model control. In [16], the adaptive anti-saturation tracking controller was constructed for hypersonic vehicle by using the Nussbaum function, dynamic surface control and nonlinear observer. In [17], the dynamic surface tracking controller was designed by utilizing finite-time compensator to deal with the problem of the input saturation. In [18], the robust anti-saturation tracking controller was put forward for the flexible hypersonic vehicle based on the neural networks theory. The neutral networks back-stepping saturated tracking controller was designed for flexible hypersonic vehicle, which can satisfy the requirement of control accuracy and input constraint in [19,20].

To further solve the control problem of the flexible hypersonic vehicle with external disturbance, model parameter uncertainties and input saturation, the robust saturated control scheme is designed based on the back-stepping method and hyperbolic tangent function. Compared with the existing works, the main contributions of the thesis are shown as follows:

1) Compared with [8], the input saturation and flexible effects are taken into account in this paper, which makes the designed controllers be more practical;

2) Compared with [13], the robust saturated control scheme can guarantee the states of the system converge to zero in finite time;

3) An adaptive tracking differentiator filter is developed in conjunction with the back-stepping control that eliminates "explosion of terms" in the virtual controls.

This paper is organized as follows. The flexible hypersonic vehicle dynamic model is established in the following section. In Section 3, the tracking saturated controllers are designed for velocity and altitude subsystems, and then, the corresponding stability proofs are given as well. Numerical simulations are presented in Section 4. The paper is closed with some concluding remarks in Section 5.

Nomenclature

m – Mass (kg)
g – Acceleration of gravity (m/s^2)
I_{yy} – Moment of inertia (kg·m^2)
L – Lift (N)
D – Drag (N)
T – Thrust (N)
M – Pitching (N.m)
$C_{T,\phi}$ – The coefficient of thrust relative to throttle setting
N_i – Generalized force
$\bar{q}$ – Dynamic pressure (kg/s^2)
s – Reference area (m^2)
ϕ – Throttle setting
C_M – Moment coefficient
C_{N_i} – Elastic coefficient
ρ – Density of air
δ_e – Elevator deflection
δ_c – Canard angle
z_T – The arm of thrust (m)
$\bar{c}$ – Mean aerodynamic chord (m)
C_T – Thrust coefficient
C_D – Drag coefficient
C_L – Lift coefficient
M_∞ – Free-stream Mach number

2. Preliminaries.

2.1. Problem formulation.

The flexible hypersonic vehicle is described as follows [4].

$$\dot{h} = V \sin(\theta - \alpha)$$
$$\dot{V} = \frac{T\cos\alpha - D}{m} - g\sin(\theta - \alpha)$$

$$\begin{aligned}
\dot{\alpha} &= -\frac{L + T\sin\alpha}{mV} + q + \left(\frac{g}{V} - \frac{V}{r}\right)\cos(\theta - \alpha) \\
\dot{\theta} &= q \\
\dot{q} &= \frac{M}{I_{yy}} \\
\ddot{\eta}_1 &= -2\zeta_1\omega_1\dot{\eta}_1 - \omega_1^2\eta_1 + N_1 \\
\ddot{\eta}_2 &= -2\zeta_2\omega_2\dot{\eta}_2 - \omega_2^2\eta_2 + N_2
\end{aligned} \tag{1}$$

where h, V, α, θ and q are altitude, velocity, angle of attack, pitch angle and pitch rate of flexible hypersonic vehicle, respectively. η_1, $\dot{\eta}_1$, η_2 and $\dot{\eta}_2$ represent flexible modes, ω_i and ζ_i $(i = 1, 2)$ are corresponding natural frequencies and damping coefficients, respectively. m, g and I_{yy} are mass, acceleration of gravity and moment of inertia, respectively. Other related variables in this model can be seen in nomenclature.

T, D, L, M and N_i mean thrust, drag, lift, pitching moment and generalized force, respectively. And the expressions are described as

$$\begin{aligned}
&T = \bar{q}\left[\phi C_{T,\phi}(\alpha, \Delta\tau_1, M_\infty) + C_T(\alpha, \Delta\tau_1, M_\infty, A_d)\right] \\
&D = \bar{q}sC_D(\alpha, \delta_e, \delta_c, \Delta\tau_1, \Delta\tau_2),\ L = \bar{q}sC_L(\alpha, \delta_e, \delta_c, \Delta\tau_1, \Delta\tau_2) \\
&M_y = z_T T + \bar{q}s\bar{c}C_M(\alpha, \delta_e, \delta_c, \Delta\tau_1, \Delta\tau_2),\ N_i = \bar{q}C_{N_i}(\alpha, \delta_e, \delta_c, \Delta\tau_1, \Delta\tau_2),\ i = 1, 2
\end{aligned} \tag{2}$$

where $C_{T,\phi}(\alpha, \Delta\tau_1, M_\infty)$, $C_T(\alpha, \Delta\tau_1, M_\infty, A_d)$, $C_D(\alpha, \delta_e, \delta_c, \Delta\tau_1, \Delta\tau_2)$, $C_L(\alpha, \delta_e, \delta_c, \Delta\tau_1, \Delta\tau_2)$, $C_M(\alpha, \delta_e, \delta_c, \Delta\tau_1, \Delta\tau_2)$ and $C_{N_i}(\alpha, \delta_e, \delta_c, \Delta\tau_1, \Delta\tau_2)$ are the nonlinear functions of M_∞, A_d, α, $\Delta\tau_1$ and $\Delta\tau_2$; the expressions are described as follows

$$\begin{aligned}
C_{T,\phi} &= C_{T,\phi}^{\alpha}\alpha + C_{T,\phi}^{\alpha M_{-\infty}^{-2}}\alpha M_{-\infty}^{-2} + C_{T,\phi}^{\alpha\tau_1}\alpha\tau_1 + C_{T,\phi}^{M_\infty^{-2}}M_\infty^{-2} + C_{T,\phi}^{\Delta\tau_1^2}\Delta\tau_1^2 \\
C_T &= C_T^{A_d}A_d + C_T^{\alpha}\alpha + C_T^{M_\infty^{-2}}M_\infty^{-2} + C_T^{\Delta\tau_1}\Delta\tau_1 + C_T^0 \\
C_D &= C_D^{(\alpha+\Delta\tau_1)^2}(\alpha + \Delta\tau_1)^2 + C_D^{(\alpha+\Delta\tau_1)}(\alpha + \Delta\tau_1) + C_D^{\delta_e^2}\delta_e^2 + C_D^{\delta_e}\delta_e \\
&\quad + C_D^{\delta_c^2}\delta_c^2 + C_D^{\delta_c}\delta_c + C_D^{\alpha\delta_e}\alpha\delta_e + C_D^{\alpha\delta_c}\alpha\delta_c + C_D^{\Delta\tau_2}\Delta\tau_2 + C_D^0 \\
C_L &= C_L^{\alpha}\alpha + C_L^{\delta_e}\delta_e + C_L^{\delta_c}\delta_c + C_L^{\Delta\tau_1}\Delta\tau_1 + C_L^{\Delta\tau_2}\Delta\tau_2 + C_L^0 \\
C_M &= C_M^{\alpha}\alpha + C_M^{\delta_e}\delta_e + C_M^{\delta_c}\delta_c + C_M^{\Delta\tau_1}\Delta\tau_1 + C_M^{\Delta\tau_2}\Delta\tau_2 + C_M^0 \\
C_{N_i} &= C_{N_i}^{\alpha}\alpha + C_{N_i}^{\delta_e}\delta_e + C_{N_i}^{\delta_c}\delta_c + C_{N_i}^{\Delta\tau_1}\Delta\tau_1 + C_{N_i}^{\Delta\tau_2}\Delta\tau_2 + C_{N_i}^0
\end{aligned} \tag{3}$$

where $M_\infty = V/M_0$, M_0 is the sound velocity. τ_1 and τ_2 are the deflection angles of the forebody angle and afterbody angle, respectively. Other parameter variables are seen in [4].

2.2. **Control-oriented model.** The aerodynamic parameter uncertainties are considered, and the aerodynamic force, aerodynamic torque and the uncertainty parts of generalized forces are obtained as follows

$$\begin{aligned}
&\Delta T = \bar{q}(\phi\Delta C_{T,\phi} + \Delta C_T),\ \Delta D \approx \bar{q}s\Delta C_D,\ \Delta L \approx \bar{q}s\Delta C_L \\
&\Delta M_y = z_T\Delta T + \bar{q}s\bar{c}\Delta C_M,\ \Delta N_i = \bar{q}\Delta C_{N_i},\ i = 1, 2
\end{aligned}$$

where $\Delta C_{T,\phi}$, ΔC_T, ΔC_D, ΔC_L, ΔC_M and ΔC_{N_i} are described as follows

$$\begin{aligned}
\Delta C_{T,\phi} &= \Delta C_{T,\phi}^{\alpha}\alpha + \Delta C_{T,\phi}^{\alpha M_{-\infty}^{-2}}\alpha M_{-\infty}^{-2} + \Delta C_{T,\phi}^{M_\infty^{-2}}M_\infty^{-2} + \Delta C_{T,\phi}^0 + \Delta C_{T,\phi}^{\alpha\tau_1}\alpha\tau_1 \\
&\quad + \Delta C_{T,\phi}^{\Delta\tau_1^2}\Delta\tau_1^2 + \Delta C_{T,\phi}^{\Delta\tau_1}\Delta\tau_1 \\
\Delta C_T &= \Delta C_T^{A_d}A_d + \Delta C_T^{\alpha}\alpha + \Delta C_T^{M_\infty^{-2}}M_\infty^{-2} + \Delta C_T^{\Delta\tau_1}\Delta\tau_1 + \Delta C_T^0 \\
\Delta C_D &= \Delta C_D^{(\alpha+\Delta\tau_1)^2}(\alpha + \Delta\tau_1)^2 + \Delta C_D^{(\alpha+\Delta\tau_1)}(\alpha + \Delta\tau_1) + \Delta C_D^{\delta_e^2}\delta_e^2 + \Delta C_D^{\delta_e}\delta_e
\end{aligned}$$

$$+\Delta C_D^{\delta_c^2}\delta_c^2+\Delta C_D^{\delta_c}\delta_c+\Delta C_D^{\alpha\delta_e}\alpha\delta_e+\Delta C_D^{\alpha\delta_c}\alpha\delta_c+\Delta C_D^{\Delta\tau_2}\Delta\tau_2+\Delta C_D^0$$
$$\Delta C_L=\Delta C_L^{\alpha}\alpha+\Delta C_L^{\delta_e}\delta_e+\Delta C_L^{\delta_c}\delta_c+\Delta C_L^{\Delta\tau_1}\Delta\tau_1+\Delta C_L^{\Delta\tau_2}\Delta\tau_2+\Delta C_L^0$$
$$\Delta C_M=\Delta C_M^{\alpha}\alpha+\Delta C_M^{\delta_e}\delta_e+\Delta C_M^{\delta_c}\delta_c+\Delta C_M^{\Delta\tau_1}\Delta\tau_1+\Delta C_M^{\Delta\tau_2}\Delta\tau_2+\Delta C_M^0$$
$$\Delta C_{N_i}=\Delta C_{N_i}^{\alpha}\alpha+\Delta C_{N_i}^{\delta_e}\delta_e+\Delta C_{N_i}^{\delta_c}\delta_c+\Delta C_{N_i}^{\Delta\tau_1}\Delta\tau_1+\Delta C_{N_i}^{\Delta\tau_2}\Delta\tau_2+\Delta C_{N_i}^0$$

Considering the flexible modes as an unknown bounded disturbance of rigid body system, the control model is established as the following

$$\begin{aligned}\dot V&=f_V+g_V\phi+d_V\\ \dot h&=V\sin\gamma\\ \dot\gamma&=f_\gamma+g_\gamma\alpha+d_\gamma\\ \dot\alpha&=f_\alpha+g_\alpha q+d_\alpha\\ \dot q&=f_q+g_q\delta_e+d_q\end{aligned}\tag{4}$$

where d_V, d_γ, d_α and d_q are unknown bounded disturbance, including the aerodynamic parameter uncertainty, flexible modes, gas-push coupling and the coupling terms between the control inputs. The expressions of each function are defined as

$$f_V=\frac{1}{m}\left(\bar qC_{T,1}\cos\alpha-\bar qSC_D^0\right)-g\sin\gamma,\ g_V=\frac{1}{m}\bar qC_{T,\phi,1}\cos(\alpha)$$
$$C_{T,1}=C_T^{A_d}A_d+C_T^{\alpha}\alpha+C_T^{M_\infty^{-2}}M_\infty^{-2}+C_T^0$$
$$C_{T,\phi,1}=C_{T,\phi}^{\alpha}\alpha+C_{T,\phi}^{\alpha M_\infty^{-2}}\alpha M_\infty^{-2}+C_{T,\phi}^{M_\infty^{-2}}M_\infty^{-2}+C_{T,\phi}^0$$
$$d_V=\frac{1}{m}\bar q\Delta T\cos\alpha$$
$$-\frac{1}{m}\bar qS\left[\begin{array}{l}\left(C_D^{(\alpha+\Delta\tau_1)^2}+\Delta C_D^{(\alpha+\Delta\tau_1)^2}\right)(\alpha+\Delta\tau_1)^2+\left(C_D^{(\alpha+\Delta\tau_1)}+\Delta C_D^{(\alpha+\Delta\tau_1)}\right)(\alpha+\Delta\tau_1)\\ +\left(C_D^{\Delta\tau_2}+\Delta C_D^{\Delta\tau_2}\right)\Delta\tau_2+\left(C_D^{\delta_e^2}+\Delta C_D^{\delta_e^2}\right)\delta_e^2+\left(C_D^{\delta_e}+\Delta C_D^{\delta_e}\right)\delta_e\\ +\left(C_D^{\alpha\delta_e}+\Delta C_D^{\alpha\delta_e}\right)\alpha\delta_e+\left(C_D^{\delta_c^2}+\Delta C_D^{\delta_c^2}\right)\delta_c^2+C_D^{\delta_c}\delta_c+\left(C_D^{\alpha\delta_c}+\Delta C_D^{\alpha\delta_e}\right)\alpha\delta_c\\ +\left(C_D^{\alpha\delta_e}+\Delta C_D^{\alpha\delta_e}\right)\alpha\delta_e+\left(C_D^{\alpha\delta_c}+\Delta C_D^{\alpha\delta_e}\right)\alpha\delta_c+\Delta C_D^0\end{array}\right]$$
$$f_\gamma=\frac{1}{mV}\left[\bar qS\left(C_L^{\alpha}\alpha+C_L^0\right)+T_1\sin\alpha-mg\cos\gamma\right],\ g_\gamma=\frac{1}{mV}\bar qSC_L^{\delta_c}$$
$$d_\gamma=\frac{1}{mV}\bar qS\left[\left(C_L^{\Delta\tau_1}+\Delta C_L^{\Delta\tau_1}\right)\Delta\tau_1+\left(C_L^{\Delta\tau_2}+\Delta C_L^{\Delta\tau_2}\right)\Delta\tau_2+\left(C_L^{\delta_2}+\Delta C_L^{\delta_2}\right)\delta_e\right]$$
$$+\frac{1}{mV}\left[\bar qS\Delta C_L^{\alpha}\alpha+\Delta T\sin a+\bar qS\Delta C_L^0+\bar qS\Delta C_L^{\delta_c}\delta_c\right]$$
$$T_1=\bar q\left(C_{T,\phi,1}\phi+C_{T,1}\right),\ C_{T,1}=C_T^{A_d}A_d+C_T^{\alpha}\alpha+C_L^{M_\infty^{-2}}M_\infty^{-2}+C_T^0$$
$$C_{T,\phi,1}=C_{T,\phi}^{\alpha}\alpha+C_{T,\phi}^{\alpha M_\infty^{-2}}\alpha M_\infty^{-2}+C_{T,\phi}^{M_\infty^{-2}}M_\infty^{-2}+C_{T,\phi}^0$$
$$f_q=\frac{1}{I_{yy}}\left[z_TT_1+\bar qS\bar c\left(C_M^{\alpha}\alpha+C_M^0\right)\right]$$
$$g_q=\frac{1}{I_{yy}}\bar qS\bar c\delta_e,\ d_q=\frac{1}{I_{yy}}\left[z_T\Delta T+\bar qS\bar c\left(\Delta C_M+C_M^{\Delta\tau_1}\Delta\tau_1\right)+\bar qS\bar c\left(C_M^{\Delta\tau_2}\Delta\tau_2+C_M^{\delta_c}\delta_c\right)\right]$$

Considering the input saturation, then (4) can be rewritten as

$$\begin{aligned}\dot V&=f_V+g_V\mathrm{sat}(\phi)+d_V\\ \dot h&=V\sin\gamma\\ \dot\gamma&=f_\gamma+g_\gamma\alpha+d_\gamma\end{aligned}\tag{5}$$

$$\dot{\alpha} = f_\alpha + g_\alpha q + d_\alpha$$
$$\dot{q} = f_q + g_q \text{sat}(\delta_e) + d_q$$

Control Objective: Aiming at considering flexible hypersonic vehicle subject to input saturation, the tracking saturated controllers are designed for velocity and altitude subsystems, under which the velocity V and altitude h can track the expected reference signals V_d and h_d, respectively. Meanwhile, the flight path angle, angle of attack, pitch rate and flexible effects can be kept in a certain variation range.

2.3. **Related lemmas and assumptions.**

Lemma 2.1. [11] *Considering the system* $\dot{\xi} = f(\xi)$, $f(0) = 0$, $\xi(0) = \xi_0$, $\xi \in \mathrm{R}^n$, *suppose that there exists a continuous positive definite function* $V_f(t)$, *and that* $\dot{V}_f(t) \le -\alpha_1 V_f(t) - \alpha_2 V_f(t)^\eta$, $\forall t > t_0$, *where* $\alpha_1 > 0$, $\alpha_2 > 0$ *and* $0 < \eta < 1$. *Then, the system state converges to the equilibrium point in finite time, where the settling time can be estimated by*

$$T_r \le \frac{1}{\alpha_1(1-p)} \ln \frac{\alpha_1 V_f(t_0)^{1-p} + \alpha_2}{\alpha_2} \tag{6}$$

Lemma 2.2. [5] *For* $n+1$ *order tracking differentiator (7), if the input signal* α_r *contains a bounded noise* $|\chi_1 - \alpha_r| \le \kappa$, *there exist positive constants* ν_i *and* $\bar{r}_i$ *that make Inequality (8) hold*

$$\begin{cases} \dot{\chi}_1 = -r_1 |\chi_1 - \alpha_r|^{\frac{n}{n+1}} \text{sign}(\chi_1 - \alpha_r) + \chi_2 \\ \dot{\chi}_i = -r_i |\chi_i - \dot{\chi}_{i-1}|^{\frac{n+1-i}{n+2-i}} \text{sign}(\chi_i - \dot{\chi}_{i-1}) + \chi_{i+1} \\ \quad \cdots \\ \dot{\chi}_n = -r_n |\chi_n - \dot{\chi}_{n-1}|^{\frac{1}{2}} \text{sign}(\chi_n - \dot{\chi}_{n-1}) + \chi_{n+1} \\ \dot{\chi}_{n+1} = -r_n \text{sign}(\chi_{n+1} - \dot{\chi}_n) \end{cases} \tag{7}$$

$$\begin{gathered} |\chi_i - \alpha_{ri}| \le \nu_i \kappa^{\frac{n+2-i}{n+1}}, \ i = 1, 2, \ldots, n \\ \left|\upsilon_j - \alpha_{r(j+1)}\right| \le \bar{r}_j \kappa^{\frac{n+1-j}{n+1}}, \ j = 1, 2, \ldots, n-1 \end{gathered} \tag{8}$$

where r_i $(i = 1, 2, \ldots, n+1)$ *is positive constant, and* $\alpha_{r(j+1)}$ *represents* j *order differential of* α_r.

3. **Main Results.** Regarding the flexible effects, external disturbance and model parameter uncertainty as the kind of disturbance with unknown upper bound, the back-stepping saturated tracking controllers are designed for velocity and altitude subsystems combining back-stepping method, hyperbolic tangent function and adaptive tracking differentiator, respectively. The detailed processes of the design are given as follows.

3.1. **Design for velocity subsystem.** The tracking error of velocity is defined as

$$z_V = V - V_d \tag{9}$$

where V_d is reference signal.

Compute the first order derivative of z_V

$$\begin{aligned} \dot{z}_V &= \dot{V} - \dot{V}_d \\ &= f_V + g_V \text{sat}(\phi) + d_V - \dot{V}_d \end{aligned} \tag{10}$$

The hyperbolic tangent function is introduced to deal with input saturation as follows

$$\text{sat}(\phi) = h(\phi) = \phi + \Delta\phi \tag{11}$$

$$h(\phi) = \phi_{\max} \tanh(\phi/\phi_{\max}) \tag{12}$$

where $\phi_{\max}$ is the positive constant and $\tanh\left(\phi/\phi_{\max}\right) = \dfrac{e^{\phi/\phi_{\max}} - e^{-\phi/\phi_{\max}}}{e^{\phi/\phi_{\max}} + e^{-\phi/\phi_{\max}}}$. Then (10) can be rewritten as

$$\begin{aligned} \dot{z}_V &= f_V + g_V h(\phi) + d_V(t) - \dot{V}_d \\ &= f_V + g_V \phi + g_V \Delta\phi + d_V(t) - \dot{V}_d \end{aligned} \tag{13}$$

Assumption 3.1. *The disturbances $g_V \Delta\phi + d_V(t)$ in (13) are assumed to be bounded, and satisfy the inequality $|g_V \Delta\phi + d_V(t)| \le \varepsilon_V$, where ε_V is a positive constant.*

Remark 3.1. *The disturbances $g_V \Delta\phi + d_V(t)$ include two uncertainties. As for $d_V(t)$, from [19], it can be seen that it consists of bounded terms produced by aerodynamic parameter uncertainty, flexible modes, gas-push coupling and the coupling terms between the control inputs, so it is bounded. The hyperbolic tangent function is a bounded function and according to [15], it can be obtained that $\Delta\phi$ is bounded, so $g_V \Delta\phi$ is also bounded. Therefore, Assumption 3.1 is rational.*

According to (13), based on back-stepping control and adaptive control method, the anti-saturation tracking controller is designed as

$$\phi = \frac{1}{g_V}\left(-f_V - k_{V1} z_V - k_{V2} |z_V|^{\gamma} \operatorname{sign}(z_V) - \delta_V \hat{\varepsilon}_V \operatorname{sign}(z_V) + \dot{V}_d\right) \tag{14}$$

$$\dot{\hat{\varepsilon}}_V = \delta_V |z_V|, \ (\hat{\varepsilon}_V(0) > 0) \tag{15}$$

where k_{V1} and k_{V2} are positive constants, $\delta_V > 1$, $0 < \gamma < 1$.

Theorem 3.1. *Considering model (5) with Assumption 3.1, the states of the system under the designed controller (14) are bounded. Furthermore, the tracking error variable z_V converges to zero in finite time.*

Proof: The proof is provided in Appendix A.

Remark 3.2. *In Theorem 3.1, according to (50), it can be seen the variables z_V and ε_V are bounded. Therefore, we put forward the next Lyapunov function V_2 which can guarantee that z_V converges to zero in finite time.*

Remark 3.3. *Due to the complex external environment, upper bound of flexible effects, external disturbance and model parameter uncertainties of the system are generally unknown, the adaptive control (15) is adopted to estimate the upper bound of the disturbance which is not required to be known in advance.*

3.2. **Design for altitude subsystem.** Based on the altitude reference signal, the flight path angle is chosen as follows

$$\gamma_c = \left[-k_{h1}(h - h_d) - k_{h2} |h - h_d|^{\gamma} \operatorname{sign}(h - h_d) + h_d\right] / V_d \tag{16}$$

where k_{h1} and k_{h2} are positive constants, and h_d is an altitude reference signal.

Step 1: The altitude tracking error is defined as

$$z_h = h - h_d \tag{17}$$

Compute the first order derivative of (17)

$$\dot{z}_h = V \sin\gamma - \dot{h}_d \tag{18}$$

As the flight path angle is very small, $\sin\gamma \approx \gamma$, then (18) can be derived

$$\begin{aligned} \dot{z}_h &= z_V \gamma + V\gamma - \dot{h}_d \\ &= z_V \gamma + V\gamma_d + V z_\gamma - \dot{h}_d \end{aligned} \tag{19}$$

Substituting (16) into (19), it can be rewritten as

$$\dot{z}_h = -k_{h1}V_d z_h - k_{h2}V_d |z_h|^{\gamma} \operatorname{sign}(z_h) + z_V\gamma + V_d z_\gamma \tag{20}$$

where z_γ is the flight path angle tracking error variable.

Choose Lyapunov function as

$$V_h = \frac{1}{2}z_h^2 \tag{21}$$

The time derivative of (21) yields

$$\begin{aligned} \dot{V}_h &= z_h\dot{z}_h \\ &= z_h\left(-k_h V_d z_h - k_{h2}V_d |z_h|^{\gamma} \operatorname{sign}(z_h) + z_V\gamma + V_d z_\gamma\right) \\ &= -k_h V_d z_h^2 - k_{h2} z_h V_d |z_h|^{\gamma} \operatorname{sign}(z_h) + z_h z_V\gamma + V_d z_h z_\gamma \end{aligned} \tag{22}$$

To avoid the derivative of virtual controller, the second-order adaptive tracking differentiator (23) is introduced as follows

$$\begin{cases} \dot{\chi}_{1\gamma} = -r_1 |\chi_{1\gamma} - \gamma_c|^{\frac{1}{2}} \operatorname{sign}(\chi_{1\gamma} - \gamma_c) + \chi_{2\gamma} \\ \dot{\chi}_{2\gamma} = -r_2 \operatorname{sign}(\chi_{2\gamma} - \dot{\chi}_{1\gamma}) \end{cases} \tag{23}$$

where r_1 and r_2 are positive constants.

Applying Lemma 2.2, Inequality (24) can be obtained

$$|\chi_{1\gamma} - \gamma_c| \le l_{\gamma 1},\ |\dot{\chi}_{1\gamma} - \dot{\gamma}_c| \le l_{\gamma 2} \tag{24}$$

where $l_{\gamma1}$ and $l_{\gamma2}$ are positive constants.

Step 2: The flight path angle tracking error z_γ is defined as

$$z_\gamma = \gamma - \gamma_c \tag{25}$$

The time derivative of z_γ can be written as

$$\dot{z}_\gamma = \dot{\gamma} - \dot{\gamma}_c = f_\gamma + g_\gamma\alpha + d_\gamma - \dot{\gamma}_c \tag{26}$$

Define the virtual control α_c as

$$\begin{aligned} \alpha_c = -\frac{1}{g_\gamma}\Big(& f_\gamma + k_{\gamma1}z_\gamma + k_{\gamma2}\operatorname{sig}(z_\gamma)^{\gamma} + \delta_{\gamma1}\hat{\varepsilon}_\gamma \operatorname{sign}(z_\gamma) \\ & + \delta_{\gamma2}\hat{l}_{\gamma2}\operatorname{sign}(z_\gamma) + \frac{1}{z_\gamma} z_h z_V \gamma + V_d z_h - \dot{\chi}_{1\gamma}\Big) \end{aligned} \tag{27}$$

$$\dot{\hat{\varepsilon}}_\gamma = \delta_{\gamma1}|z_\gamma|,\ (\hat{\varepsilon}_\gamma(0) > 0) \tag{28}$$

$$\dot{\hat{l}}_{\gamma2} = \delta_{\gamma2}|z_\gamma|,\ \left(\hat{l}_{\gamma2}(0) > 0\right) \tag{29}$$

where $k_{\gamma1}$ and $k_{\gamma2}$ are positive constants, $\delta_{\gamma1} > 1$, $\delta_{\gamma2} > 1$, $0 < \gamma < 1$.

To avoid the derivative of virtual controller, the second-order adaptive tracking differentiator (30) is adopted as the following

$$\begin{cases} \dot{\chi}_{1\alpha} = -r_1 |\chi_{1\alpha} - \alpha_c|^{\frac{1}{2}} \operatorname{sign}(\chi_{1\alpha} - \alpha_c) + \chi_{2\alpha} \\ \dot{\chi}_{2\alpha} = -r_2 \operatorname{sign}(\chi_{2\alpha} - \dot{\chi}_{1\alpha}) \end{cases} \tag{30}$$

where r_1 and r_2 are positive constants.

Applying Lemma 2.2, Inequality (31) can be obtained

$$|\chi_{1\alpha} - \alpha_c| \le l_{\alpha1},\ |\dot{\chi}_{1\alpha} - \dot{\alpha}_c| \le l_{\alpha2} \tag{31}$$

where $l_{\alpha1}$ and $l_{\alpha2}$ are positive constants.

Choose Lyapunov function V_γ as

$$V_\gamma = \frac{1}{2}z_\gamma^2 + \tilde{\varepsilon}_\gamma^2 + \tilde{l}_{\gamma2}^2 \tag{32}$$

where $\tilde{\varepsilon}_\gamma = \varepsilon_\gamma - \hat{\varepsilon}_\gamma$, $\tilde{l}_{\gamma 2} = l_{\gamma 2} - \hat{l}_{\gamma 2}$.

Applying (27)-(28), the derivative of (32) can be written as

$$\begin{aligned}
\dot{V}_\gamma &= z_\gamma \dot{z}_\gamma - \tilde{\varepsilon}_\gamma \dot{\hat{\varepsilon}}_\gamma - \tilde{l}_{\gamma 2} \dot{\hat{l}}_{\gamma 2} \\
&= z_\gamma \left(f_\gamma + g_\gamma \left(z_\alpha + \alpha_c \right) + d_\gamma - \dot{\gamma}_d \right) - \tilde{\varepsilon}_\gamma \dot{\hat{\varepsilon}}_\gamma - \tilde{l}_{\gamma 2} \dot{\hat{l}}_{\gamma 2} \\
&= -k_{\gamma 1} z_\gamma^2 - k_{\gamma 2} z_\gamma \mathrm{sig}\left(z_\gamma \right)^\gamma + z_\gamma g_\gamma z_\alpha + z_\gamma \left(l_{\gamma 2} - \delta_{\gamma 2} \hat{l}_{\gamma 2} \mathrm{sign}\left(z_\gamma \right) \right) \\
&\quad + z_\gamma \left(d_\gamma - \delta_{\gamma 1} \hat{\varepsilon}_\gamma \mathrm{sign}\left(z_\gamma \right) \right) - \left(\varepsilon_\gamma - \hat{\varepsilon}_\gamma \right) \delta_{\gamma 1} \left| z_\gamma \right| - \left(l_{\gamma 2} - \hat{l}_{\gamma 2} \right) \delta_{\gamma 2} \left| z_{\alpha_\gamma} \right| \\
&\quad - V_d z_h z_\gamma - z_h z_V \gamma \\
&\le -k_{\gamma 1} z_\gamma^2 - k_{\gamma 2} z_\gamma \mathrm{sig}\left(z_\gamma \right)^\gamma + \left| z_\gamma \right| \varepsilon_\gamma - \varepsilon_\gamma \delta_{\gamma 1} \left| z_\gamma \right| + \left| z_\gamma \right| l_{\gamma 2} - l_{\gamma 2} \delta_{\gamma 2} \left| z_\gamma \right| \\
&\quad + z_\gamma g_\gamma z_\alpha - z_h z_V \gamma - V_d z_h z_\gamma
\end{aligned} \tag{33}$$

Step 3: The angle of attack tracking error is defined as $z_\alpha = \alpha - \alpha_c$ and the time derivative of z_α can be written as

$$\dot{z}_\alpha = \dot{\alpha} - \dot{\alpha}_c = f_\alpha + g_\alpha \left(z_q + q_c \right) + d_\alpha - \dot{\alpha}_c \tag{34}$$

Define the virtual control q_c as follows

$$\begin{aligned}
q_c = -\frac{1}{g_\alpha} \big(& f_\alpha + k_{\alpha 1} z_\alpha + k_{\alpha 2} \mathrm{sig}\left(z_\alpha \right)^\gamma + \delta_{\alpha 1} \hat{\varepsilon}_\alpha \mathrm{sign}\left(z_\alpha \right) \\
& + \delta_{\alpha 2} \hat{l}_{\alpha 2} \mathrm{sign}\left(z_\alpha \right) + z_\gamma g_\gamma - \dot{\chi}_{1\alpha} \big)
\end{aligned} \tag{35}$$

$$\dot{\hat{\varepsilon}}_\alpha = \delta_{\alpha 1} \left| z_\alpha \right|, \ \left(\hat{\varepsilon}_\alpha, (0) > 0 \right) \tag{36}$$

$$\dot{\hat{l}}_{\alpha 2} = \delta_{\alpha 2} \left| z_\alpha \right|, \ \left(\hat{l}_{\alpha 2}, (0) > 0 \right) \tag{37}$$

where $k_{\alpha 1}$ and $k_{\alpha 2}$ are positive constants, $\delta_{\alpha 1} > 1$, $\delta_{\alpha 2} > 1$, $0 < \gamma < 1$.

Choose Lyapunov function V_α as

$$V_\alpha = \frac{1}{2} z_\alpha^2 + \tilde{\varepsilon}_\alpha^2 + \tilde{l}_{\alpha 2}^2 \tag{38}$$

where $\tilde{\varepsilon}_\alpha = \varepsilon_\alpha - \hat{\varepsilon}_\alpha$, $\tilde{l}_{\alpha 2} = l_{\alpha 2} - \hat{l}_{\alpha 2}$.

Computing the derivative of V_α and inserting (35), it yields

$$\begin{aligned}
\dot{V}_\alpha &= z_\alpha \left(f_\alpha + g_\alpha \left(z_q + q_c \right) + d_\alpha - \dot{\alpha}_c \right) - \tilde{\varepsilon}_\alpha \dot{\hat{\varepsilon}}_\alpha - \tilde{l}_{\alpha 2} \dot{\hat{l}}_{\alpha 2} \\
&= -k_{\alpha 1} z_\alpha^2 - k_{\alpha 2} z_\alpha \mathrm{sig}\left(z_\alpha \right)^\gamma + z_\alpha g_\alpha z_q + z_\alpha \left(l_{a2} - \delta_{\alpha 2} \hat{l}_{\alpha 2} \mathrm{sign}\left(z_\alpha \right) \right) \\
&\quad + z_\alpha \left(d_\alpha - \delta_{\alpha 1} \hat{\varepsilon}_\alpha \mathrm{sign}\left(z_\alpha \right) \right) - \left(\varepsilon_\alpha - \hat{\varepsilon}_\alpha \right) \delta_{\alpha 1} \left| z_\alpha \right| \\
&\quad - \left(l_{\alpha 2} - \hat{l}_{\alpha 2} \right) \delta_{\alpha 2} \left| z_\alpha \right| - z_\gamma g_\gamma z_\alpha \\
&\le -k_{\alpha 1} z_\gamma^2 - k_{\alpha 2} z_\alpha \mathrm{sig}\left(z_\alpha \right)^\gamma + \left| z_\alpha \right| \varepsilon_\alpha - \varepsilon_\alpha \delta_{\alpha 1} \left| z_\alpha \right| + \left| z_\alpha \right| l_{\alpha 2} - l_{\alpha 2} \delta_{\alpha 2} \left| z_\alpha \right| \\
&\quad + z_\alpha g_\alpha z_q - z_\gamma g_\gamma z_\alpha
\end{aligned} \tag{39}$$

To avoid the derivative of the virtual controller, the second-order adaptive tracking differentiator is given as

$$\begin{cases} \dot{\chi}_{1q} = -r_1 \left| \chi_{1q} - q_c \right|^{\frac{1}{2}} \mathrm{sign}\left(\chi_{1q} - q_c \right) + \chi_{2q} \\ \dot{\chi}_{2q} = -r_2 \mathrm{sign}\left(\chi_{2q} - \dot{\chi}_{1q} \right) \end{cases} \tag{40}$$

where r_1 and r_2 are positive constants.

Applying Lemma 2.2, Inequality (41) can be obtained

$$\left| \chi_{1q} - q_c \right| \le l_{q1}, \ \left| \dot{\chi}_{1q} - \dot{q}_c \right| \le l_{q2} \tag{41}$$

where l_{q1} and l_{q2} are positive constants.

Step 4: The pitch rate tracking error is defined as $z_q = q - q_c$, and the time derivative of z_q can be written as

$$\dot{z}_q = \dot{q} - \dot{q}_c = f_q + g_q \text{sat}(\delta_e) + d_q - \dot{q}_c \tag{42}$$

The hyperbolic tangent function is introduced to deal with input saturation as follows

$$\text{sat}(\delta_e) = h(\delta_e) = \delta_e + \Delta\delta_e \tag{43}$$

$$h(\delta_e) = \delta_{\text{emax}} \tanh(\delta_e/\delta_{\text{emax}}) \tag{44}$$

where δ_{emax} is positive constant and $\tanh(\delta_e/\delta_{\text{emax}}) = \frac{e^{\delta_e/\delta_{\text{emax}}} - e^{-\delta_e/\delta_{\text{emax}}}}{e^{\delta_e/\delta_{\text{emax}}} + e^{-\delta_e/\delta_{\text{emax}}}}$. Then (10) can be rewritten as

$$\begin{aligned} \dot{z}_q &= f_q + g_q h(\delta_e) + d_q - \dot{q}_d \\ &= f_q + g_q \delta_e + g_q \Delta\delta_e + d_q - \dot{q}_d \end{aligned} \tag{45}$$

Assumption 3.2. *The disturbances $g_q\Delta\delta_e + d_q$ in (45) are assumed to be bounded, and satisfy the inequality $|g_q\Delta\delta_e + d_q| \le \varepsilon_h$, where ε_h is a positive constant.*

Based on (45), the anti-saturation tracking controller δ_e is designed as

$$\begin{aligned} \delta_e = &-\frac{1}{g_q}\Big(f_q + k_{q1} z_q + k_{q2} |z_q|^\gamma \operatorname{sign}(z_q) + \delta_{q1}\hat{\varepsilon}_q \operatorname{sign}(z_q) \\ &+ \delta_{q2}\hat{l}_{q2} \operatorname{sign}(z_q) + z_\alpha g_\alpha - \dot{\chi}_{1q}\Big) \end{aligned} \tag{46}$$

$$\dot{\hat{\varepsilon}}_q = \delta_{q1}|z_q|, \ (\hat{\varepsilon}_q(0) > 0) \tag{47}$$

$$\dot{\hat{l}}_{q2} = \delta_{q2}|z_q|, \ \left(\hat{l}_{q2}(0) > 0\right) \tag{48}$$

where k_{q1} and k_{q2} are positive constants, $\delta_{q1} > 1$, $\delta_{q2} > 1$, $0 < \gamma < 1$.

Theorem 3.2. *Considering model (5) with Assumption 3.2, the states of the system under the designed controller (46) and adaptive laws (47)-(48) are bounded; the tracking error variables z_h, z_γ, z_α and z_q converge to zero in finite time.*

Proof: The proof is provided in Appendix B.

Remark 3.4. *In this paper, with the aid of the adaptive tracking differentiator, the problem of repetitive differentiation of the virtual controllers is eliminated. And tangent function is introduced to solve input saturation problem.*

4. **Numerical Examples.** In order to verify the effectiveness of the control strategy designed in this paper, the nonlinear motion (1) and aerodynamic model (3) of the flexible hypersonic vehicle are taken as the simulation objects, and the parameters of the hypersonic vehicle referring to [4] are shown in Table 1.

TABLE 1. The parameters of flexible hypersonic vehicle

Symbol	Value	Symbol	Value	Symbol	Value
h	33528 m	η_1	1.5122	m	446.4469 kg/m
V	4590.3 m/s	$\dot{\eta}_1$	0	ζ_1	0.02
α	0.02645 rad	η_2	1.2144	ω_1	6 rad/s
θ	0.02645 rad	$\dot{\eta}_2$	0	ζ_2	0.02
q	0 rad/s			ω_2	19.6 rad/s

The reference velocity and altitude of the flexible hypersonic vehicle are given as $V_d = 4670.3$ m/s and $h_{d1} = 34028$ m, respectively. In order to test the robustness to the proposed control strategy, the simulation is divided into two cases as follows

Case 1: Simulation without aerodynamic uncertainties

Case 2: Simulation with aerodynamic uncertainties as

$$\begin{cases} C_L = C_L^* \left(1 + U_{ft} + U_V\right) \\ C_D = C_D^* \left(1 + U_{fD} + U_V\right) \\ C_T = C_T^* \left(1 + U_{fT} + U_V\right) \\ C_M = C_M^* \left(1 + U_{fM} + U_V\right) \end{cases}$$

where C^*, U_f and U_V denote the nominal value, fixed parameter uncertainties, and time-varying part, respectively. The uncertainties are considered as: $U_{ft} = -20\%$, $U_{fD} = 20\%$, $U_{fT} = -20\%$, $U_{fM} = 20\%$ and $U_V = 0.2\sin(0.1t)$.

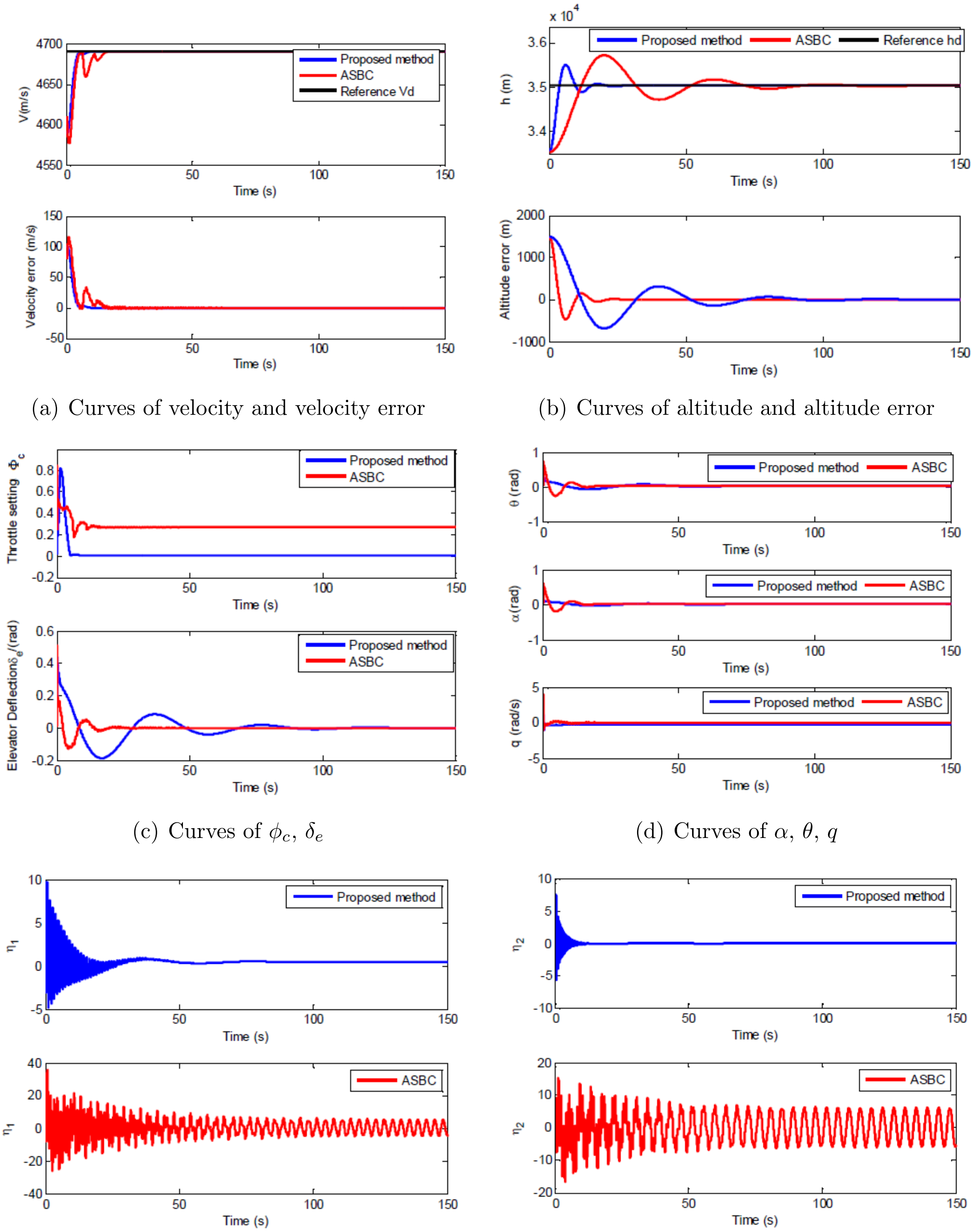
4.1. **The simulation analysis without aerodynamic uncertainties.** For performance comparison, the adaptive saturated back-stepping controller (ASBC) in [7] is also simulated under the same conditions. We select parameters of the controller as $k_{V1} = 1.2$, $k_{V2} = 0.25$, $\delta_V = 1.15$, $\gamma = 0.65$, $k_{h1} = 0.8$, $k_{h2} = 0.35$, $k_{\gamma1} = 1.05$, $k_{\gamma2} = 0.4$, $k_{\alpha1} = 1.15$, $k_{\alpha2} = 0.15$, $k_{q1} = 0.85$, $k_{q2} = 0.4$, $r_1 = 0.05$, $r_2 = 0.05$, $\delta_{\gamma1} = \delta_{\alpha1} = \delta_{q1} = 1.25$, and $\delta_{\gamma2} = \delta_{\alpha2} = \delta_{q2} = 1.1$. And the simulation result is shown in Figure 1.

Figure 1(a) and Figure 1(b) give the tracking error curves of velocity and altitude respectively; it can be known that the proposed method and the ASBC can guarantee the velocity error and altitude error converge to any small neighborhood and satisfy the requirement of control accuracy for Case 1. However, under the proposed method the convergence speed of the velocity error and altitude error is much faster and the curve of the velocity error and altitude error is smoother than that under the ASBC. Figure 1(c) gives the curves of control input, from which it can be known the control input under both the two controllers are within the reasonable bounds. In addition, it is obvious that the proposed method does not bring about the smooth compared with ASBC. From Figure 1(d), it can be noticeable that the flight path angle, angle of attack, pitch rate can tend to be stable values within short period of time. From the curves of flexible modal given in Figure 1(e) and Figure 1(f), it demonstrates that the proposed method can ensure that the flexible modal has been kept in the bounded range during the whole control process. However, the flexible modal is not considered into ASBC, which makes the flexible mode of hypersonic vehicle tend to be unstable in the control process.

4.2. **The simulation analysis with aerodynamic uncertainties.** For Case 2, its control parameters and gains are the same as those of Case 1. The simulation results are shown in Figure 2.

Figures 2(a)-2(f) show the curves of velocity, altitude, control input, other state variables and flexible modal of the hypersonic vehicle under the designed controller. From the simulation results, it can be concluded that when the aerodynamic uncertainties are considered, proposed control strategy possesses the stronger robustness. From the tracking curves in Figures 2(a)-2(b), it can be seen that they still satisfy the requirement of the control precision compared with Case 1. Form Figure 2(c), it can be seen that the control input amplitude value is slightly larger than that in Case 1. As shown in Figure 2(d), it can be seen that the other states of the system tend to be steady-state values in short time. It can be clearly observed from Figures 2(e)-2(f) that the flexible modal approaches to a steady value after a period of time.

5. **Conclusions.** In this paper, the tracking saturated control problem of flexible hypersonic vehicle with external disturbance, model parameter uncertainties and input saturation has been studied. The conclusions are as the following four points.



(a) Curves of velocity and velocity error

(b) Curves of altitude and altitude error

(c) Curves of ϕ_c, δ_e

(d) Curves of α, θ, q

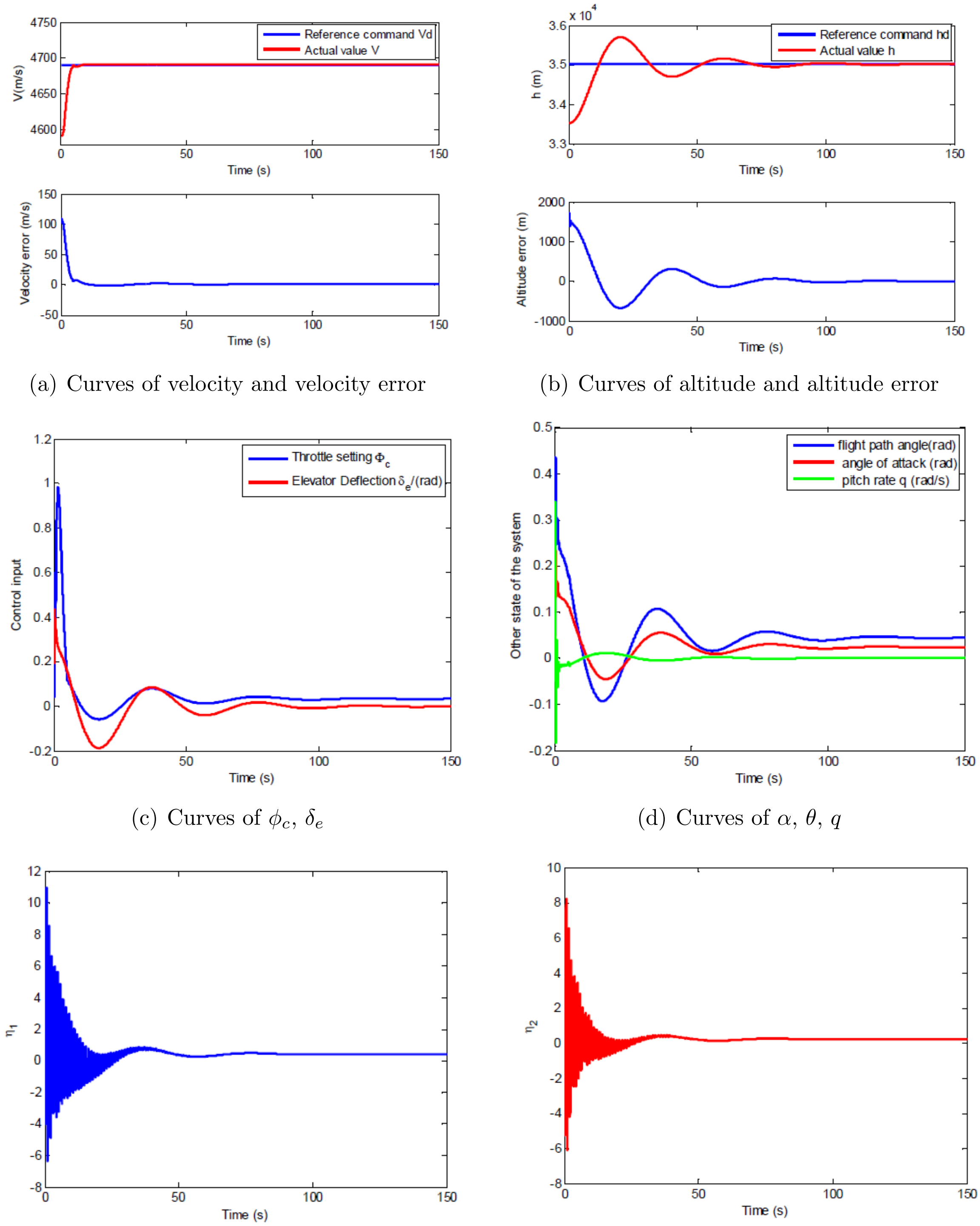
(e) Curves of flexible mode η_1

(f) Curves of flexible mode η_2

FIGURE 1. Simulation results under designed controller for Case 1

1) The adaptive back-stepping saturated tracking controllers are designed for velocity and altitude subsystems. And the adaptive tracking differentiator is used to avoid the differential of the virtual control signals.

2) Under the designed control scheme, the states of system are bounded respectively, and the tracking errors can converge to zero in finite time.



(a) Curves of velocity and velocity error (b) Curves of altitude and altitude error

(c) Curves of ϕ_c, δ_e (d) Curves of α, θ, q

(e) Curves of flexible mode η_1 (f) Curves of flexible mode η_2

FIGURE 2. Simulation results under designed controller for Case 2

3) The simulation results demonstrate the robustness of controllers and good tracking performance of the desired reference signals.

4) In the future, the problem of actuator failures and state constraints should be considered in the design of flexible hypersonic control system. Based on the existing nonlinear control theory, the different advanced control methods should be combined with each other to improve the performance of the control algorithm and solve the control problem.

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Appendix A. Proof of Theorem 3.1.

Proof: The Lyapunov function is chosen as the following

$$V_1 = \frac{1}{2}z_V^2 + \frac{1}{2}\tilde{\varepsilon}_V^2 \tag{49}$$

where $\tilde{\varepsilon}_V = \varepsilon_V - \hat{\varepsilon}_V$.

Computing the first order derivative of V_1, one obtains

$$\begin{aligned} V_1 &= z_V\left(f_V + g_V\phi + g_V\Delta\phi + d_V(t) - \dot{V}_d\right) - \tilde{\varepsilon}_V\dot{\hat{\varepsilon}}_V \\ &= -k_{V1}z_V^2 - z_V k_{V2}|z_V|^\gamma \operatorname{sign}(z_V) + z_V(\varepsilon_V - \delta_V\hat{\varepsilon}_V\operatorname{sign}(z_V)) \\ &\quad - (\varepsilon_V - \hat{\varepsilon}_V)\delta_V|z_V| \\ &\le -k_{V1}z_V^2 - z_V k_{V2}|z_V|^\gamma \operatorname{sign}(z_V) + (1-\delta_V)\varepsilon_V|z_V| \\ &\le -k_{V1}z_V^2 - z_V k_{V2}|z_V|^\gamma \operatorname{sign}(z_V) \\ &\le 0 \end{aligned} \tag{50}$$

From (50), it can be seen that $\dot{V}_1 \le 0$, so V_1 is not increasing, which means that z_V and ε_V are bounded.

Choose the Lyapunov function as

$$V_2 = \frac{1}{2}z_V^2 \tag{51}$$

Applying (14) and (15), the derivative of (51) can be written as

$$\begin{aligned} \dot{V}_2 &= z_V\dot{z}_V \\ &= z_V\left(-k_{V1}z_V^2 - z_V k_{V2}|z_V|^\gamma \operatorname{sign}(z_V) + \varepsilon_V - \delta_V\hat{\varepsilon}_V\operatorname{sign}(z_V)\right) \\ &\le -k_{V1}z_V^2 - z_V k_{V2}|z_V|^\gamma \operatorname{sign}(z_V) + (\varepsilon_V - \delta_V\hat{\varepsilon}_V)|z_V| \end{aligned} \tag{52}$$

On the basis of $\hat{\varepsilon}_V(0) > 0$ and $\dot{\hat{\varepsilon}}_V = \delta_V|z_V| \ge 0$, it can be obtained that $\hat{\varepsilon}_V(t) > \hat{\varepsilon}_V(0)$. $\hat{\varepsilon}_V(0)$ is chosen big enough, and δ_V satisfies $\delta_V \ge \dfrac{\sqrt{z_V^2(0) + \hat{\varepsilon}_V^2(0)}}{\hat{\varepsilon}_V(0)} + 1$, the inequality should be derived

$$\begin{aligned} \varepsilon_V - \delta_V\hat{\varepsilon}_V &\le \varepsilon_V - \sqrt{z_V^2(0) + \hat{\varepsilon}_V^2(0)} - \hat{\varepsilon}_V(0) \\ &\le \sqrt{\hat{\varepsilon}_V^2(0)} - \sqrt{z_V^2(0) + \hat{\varepsilon}_V^2(0)} \\ &\le 0 \end{aligned} \tag{53}$$

According to (53), then (52) can be further simplified as

$$\begin{aligned} \dot{V}_2 &\le -k_{V1}z_V^2 - z_V k_{V2}|z_V|^\gamma \operatorname{sign}(z_V) + (\varepsilon_V - \delta_V\hat{\varepsilon}_V)|z_V| \\ &\le -k_{V1}z_V^2 - z_V k_{V2}|z_V|^\gamma \operatorname{sign}(z_V) \\ &\le -2k_{V1}V_2 - (\sqrt{2})^{\frac{1+\gamma}{2}}k_{V2}V_2^{\frac{1+\gamma}{2}} \end{aligned} \tag{54}$$

According to Lemma 2.1, it can be concluded that the tracking error variable z_V converges to zero in finite time.

Appendix B. Proof of Theorem 3.2.

Proof: Choose the Lyapunov function candidate as

$$V_3 = V_h + V_\gamma + V_\alpha + \frac{1}{2}z_q^2 + \tilde{\varepsilon}_q^2 + \tilde{l}_{q2}^2 \tag{55}$$

where $\tilde{\varepsilon}_q = \varepsilon_q - \hat{\varepsilon}_q$, $\tilde{l}_{q2} = l_{q2} - \hat{l}_{q2}$.

Applying (22), (33) and (39), the time derivative of (55) can be written as

$$\begin{aligned}\dot{V}_3 &= \dot{V}_h + \dot{V}_\gamma + \dot{V}_\alpha + f_q + g_q\delta_e + g_q\Delta\delta_e + d_q - \dot{q}_d - \tilde{\varepsilon}_q\dot{\hat{\varepsilon}}_q \\ &\le \dot{V}_h + \dot{V}_\gamma + \dot{V}_\alpha + \varepsilon_q\left|z_q\right| - \delta_{q1}\left|z_q\right|\varepsilon_q + l_{q2}\left|z_q\right| - \delta_{q2}\left|z_q\right| l_{q2} - z_\alpha g_\alpha z_q \\ &\le -k_{h1}V_d z_h^2 - k_{\gamma1}z_\gamma^2 - k_{\alpha1}z_\gamma^2 - k_{q1}z_q^2 - k_{h2}z_h V_d\left|z_h\right|^\gamma \operatorname{sign}(z_h) - k_{\alpha2}z_\alpha \operatorname{sig}(z_\alpha)^\gamma \\ &\quad - k_{\gamma2}z_\gamma\operatorname{sig}(z_\gamma)^\gamma - k_{q2}z_q\left|z_q\right|^\gamma\operatorname{sign}(z_q) + \left|z_\gamma\right|\varepsilon_\gamma(1-\delta_{\gamma1}) + \left|z_\gamma\right| l_{\gamma2}(1-\delta_{\gamma2}) \\ &\quad + \left|z_\alpha\right|\varepsilon_\alpha(1-\delta_\alpha) + \left|z_\alpha\right| l_{\alpha2}(1-\delta_{\alpha2}) + \varepsilon_q\left|z_q\right|(1-\delta_q) + \left|z_q\right| l_{q2}(1-\delta_{q2}) \\ &\le 0\end{aligned} \tag{56}$$

From (56), it can be seen the variables z_h, z_γ, z_α, z_q, $\tilde{\varepsilon}_\gamma$, $\tilde{\varepsilon}_\alpha$, $\tilde{\varepsilon}_q$, $\tilde{l}_{\gamma2}$, $\tilde{l}_{\alpha2}$ and $\tilde{l}_{q2}$ are bounded.

Choose the Lyapunov function candidate as

$$V_4 = \frac{1}{2}z_h^2 + \frac{1}{2}z_\gamma^2 + \frac{1}{2}z_\alpha^2 + \frac{1}{2}z_q^2 \tag{57}$$

Computing the derivative of V_1 and applying (22), (33) and (39), it yields

$$\begin{aligned}\dot{V}_4 &= z_h\dot{z}_h + z_\gamma\dot{z}_\gamma + z_\alpha\dot{z}_\alpha + z_q\dot{z}_q \\ &\le -k_{h1}V_d z_h^2 - k_{\gamma1}z_\gamma^2 - k_{\alpha1}z_\gamma^2 - k_{q1}z_q^2 - k_{h2}z_h V_d\left|z_h\right|^\gamma\operatorname{sign}(z_h) - k_{\alpha2}z_\alpha\operatorname{sig}(z_\alpha)^\gamma \\ &\quad - k_{\gamma2}z_\gamma\operatorname{sig}(z_\gamma)^\gamma - k_{q2}z_q\left|z_q\right|^\gamma\operatorname{sign}(z_q) + \left|z_\gamma\right|(\varepsilon_\gamma - \delta_{\gamma1}\hat{\varepsilon}_\gamma) \\ &\quad + \left|z_\gamma\right|\left(l_{\gamma2} - \delta_{\gamma2}\hat{l}_{\gamma2}\right) + \left|z_\alpha\right|(\varepsilon_\alpha - \delta_\alpha\hat{\varepsilon}_\alpha) + \left|z_\alpha\right|\left(l_{\alpha2} - \delta_{\alpha2}\hat{l}_{\alpha2}\right) \\ &\quad + \left|z_q\right|(\varepsilon_q - \delta_q\hat{\varepsilon}_q) + \left|z_q\right|\left(l_{q2} - \delta_{q2}\hat{l}_{q2}\right)\end{aligned} \tag{58}$$

As $\hat{\varepsilon}_i(0) > 0$ $(i = \gamma, \alpha, q)$, $\hat{l}_{i2}(0) > 0$ $(i = \gamma, \alpha, q)$ and $\dot{\hat{\varepsilon}}_i = \delta_{i1}\left|z_i\right| \ge 0$ $(i = \gamma, \alpha, q)$, $\dot{\hat{l}}_{i2} = \delta_{i2}\left|z_i\right| \ge 0$ $(i = \gamma, \alpha, q)$, it can be obtained that $\hat{\varepsilon}_i(t) > \hat{\varepsilon}_i(0)$ and $\hat{l}_{i2}(t) > \hat{l}_{i2}(0)$. $\hat{\varepsilon}_\gamma(0)$ $(i = \gamma, \alpha, q)$ and $\hat{l}_{i2}(0)$ $(i = \gamma, \alpha, q)$ are chosen big enough, and δ_{i1} $(i = \gamma, \alpha, q)$, δ_{i2} $(i = \gamma, \alpha, q)$ satisfy $\delta_{i1} \ge \frac{\sqrt{z_i^2(0) + \hat{\varepsilon}_i^2(0)}}{\hat{\varepsilon}_i(0)} + 1$ $(i = \gamma, \alpha, q)$, and $\delta_{i2} \ge \frac{\sqrt{z_i^2(0) + \hat{l}_{i2}^2(0)}}{\hat{l}_{i2}(0)} + 1$ $(i = \gamma, \alpha, q)$, the inequalitiy can be got as

$$\begin{aligned}\varepsilon_i - \delta_{i1}\hat{\varepsilon}_i &\le \varepsilon_i - \sqrt{z_i^2(0) + \hat{\varepsilon}_i^2(0)} - \hat{\varepsilon}_i(0) \\ &\le \sqrt{\hat{\varepsilon}_i^2(0)} - \sqrt{z_i^2(0) + \hat{\varepsilon}_i^2(0)} \\ &\le 0 \ (i = \gamma, \alpha, q)\end{aligned} \tag{59}$$

$$l_{i2} - \delta_{i2}\hat{l}_{i2} \le l_{i2} - \sqrt{z_i^2(0) + \hat{l}_{i2}^2(0)} - \hat{l}_{i2}(0) \le 0 \ (i = \gamma, \alpha, q) \tag{60}$$

According to (59) and (60), then (58) can be further simplified as

$$\begin{aligned}\dot{V}_4 &\le -k_{h1}V_d z_h^2 - k_{\gamma1}z_\gamma^2 - k_{\alpha1}z_\gamma^2 - k_{q1}z_q^2 - k_{h2}z_h V_d\left|z_h\right|^\gamma\operatorname{sign}(z_h) \\ &\quad - k_{\alpha2}z_\alpha\operatorname{sig}(z_\alpha)^\gamma - k_{\gamma2}z_\gamma\operatorname{sig}(z_\gamma)^\gamma - k_{q2}z_q\left|z_q\right|^\gamma\operatorname{sign}(z_q) \\ &\le -2\min(k_{h1}V_d, k_{\gamma1}, k_{\alpha1}, k_{q1})V_4 - \left(\sqrt{2}\right)^{\frac{1+\gamma}{2}}\min(k_{h2}V_d, k_{\alpha2}, k_{\gamma2}, k_{q2})V_4^{\frac{1+\gamma}{2}}\end{aligned} \tag{61}$$

From Lemma 2.1 and (61), it can be obtained that tracking error variables z_h, z_γ, z_α and z_q converge to zero in finite time.