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INTEGRATED FAULT DIAGNOSIS AND FAULT-TOLERANT CONTROL OF NONLINEAR NETWORK CONTROL SYSTEMS

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Abstract. *A new integrated fault diagnosis and fault tolerant control algorithm of nonlinear networked control systems is proposed in this paper. A Laplace transform strategy is used to analyze the random time delay problem. Besides, a new fault diagnosis algorithm based on learning observer is designed. Then, based on the fault diagnosis information, a new state feedback fault-tolerant controller is designed, which can ensure that the output of the post-fault system can still track the given target. Finally, a single-link robot is taken as an example for the numerical simulation. The simulation results show the effectiveness of the proposed method.*
Keywords: Laplace transform, Nonlinear networked control systems, Time delay, Learning observer, State feedback control

1. **Introduction.** With the development of science and technology, the safety and stability of actual industrial equipment are paid more attention. Based on this, many scholars began to conduct in-depth research on fault diagnosis and fault-tolerant control. In the actual industrial system, the system may cause malfunctions, and once fault occurs, it will cause huge losses and even disasters. Therefore, the research on fault diagnosis is not only of great theoretical significance, but also of high practical significance. In the past three decades, fault diagnosis and fault-tolerant control has been the research focus of many scholars [1-3]. The basic concept of a networked control system was put forward in the 1980s. In the last 20 years, because the networked control system has the advantages of low cost, easy maintenance and high system flexibility, the domestic and foreign scholars have studied it deeply in all aspects [4-7].

The typical networked control-inducted problem in detail is introduced in [4]. In [5], a reduced-order memoryless state observer for networked control systems with long delays is presented. The reduced-order observer is used for fault diagnosis. In [6], a T-S fuzzy model is used to deal with the delay problem caused by the network in a networked control system. An observer-based fault detection filter and a reference residual model are introduced to construct a novel model for the continuous-time networked control system in [7]. It can effectively solve the problems of network-induced delay, such as the delay of the controller to the actuator and data loss. In [9], the design of event-triggered control for networked control systems with dynamic quantization and faults is studied. Some classical fault diagnosis algorithms based on linear networked control are given in the literature.

Although some effective solutions to the problem of delay and packet loss in networked control systems are proposed in the above literature, they are almost aimed at linear

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systems. As we all know, nonlinearity is universal in practical industrial control systems. However, considering modeling nonlinearity is difficult, the design of observer and controller is also more difficult. In addition, there are few methods to deal with the nonlinearity. In [8], a fault diagnosis method based on a robust observer is proposed, in which uncertain nonlinear systems are studied and a direction for fault diagnosis of non-linear networked control systems is provided. In [10-12], a robust sliding mode observer is designed to diagnose the fault of a nonlinear control system. In [14,15], a fault diagnosis algorithm based on a learning observer is proposed and fault reconstruction is considered. However, this method has a limit scope of application, certain equivalence condition between fault coefficient and output coefficient needs to be satisfied, and many systems do not meet this limitation. In addition, the gain generated by this method is too large, which is easy to cause fluctuations and even does not meet the linear matrix inequality condition. Besides, input delay is not considered in the literature.

Fault-tolerant control can be divided into passive fault-tolerant control and active fault-tolerant control. Passive fault-tolerant control appears earlier, is simple in design, and has strong robustness [16,17]. However, passive fault-tolerant control can only be used for a certain type of fault. Because of the disadvantages of passive fault-tolerant control, the theory of active fault-tolerant control develops rapidly. A fault-tolerant control scheme based on impulsive system techniques is proposed for a class of nonlinear networked control systems in [18]. Cooperative fault-tolerant controllers are designed in [19], in which the non-faulty subsystems must follow local trajectories to compensate the deviant behavior of the fault subsystem. In [20], the problem of fault-tolerant control for a class of networked control systems with delay, packet dropout, packet disordering and nonlinear modeling uncertainties is studied. In [21], based on the estimation of system state and fault signal, a fault-tolerant controller is proposed to compensate the influence of fault.

Based on the above analysis, a new integrated fault diagnosis and fault-tolerant control algorithm is proposed. A new augmented fault diagnosis based on the learning observer design method for nonlinear networked control systems is presented in this paper. In this paper not only the time delay problem in the networked control system is considered, but also the nonlinear problem in the process of signal transmission is considered. Compared with the method proposed in [14,15], the constraints are relaxed and parameters are more applicable. Based on the estimation of system state and fault signal, a state feedback fault-tolerant controller is designed to compensate the influence of fault and maintain the system performance in this paper.

The rest of this paper is organized as follows. Section 2 presents the problem description. The fault detection method is given in Section 3. In Section 4, the design of fault-tolerant control (FTC) is proposed. Section 5 gives the simulation results, which is followed by some concluding remarks in Section 6.

Notation: The notation used in this paper is standard. X^T and X^{-1} are the transpose and the inverse of matrix X, respectively. The star symbol $*$ in a symmetric matrix denotes the transposed block in the symmetric position. A real symmetric matrix $P > (\geq)$ 0 denotes P being a positive definite (positive semi-definite) matrix, and $A > (\geq) B$ means $A - B > (\geq) 0$. I is used to denote an identity matrix with proper dimension. For a vector $s(t)$, its Euclidean norm is defined by $\|s(t)\|^2 = s^T(t)s(t)$.

2. Statistic Information and System Model Description.

The following nonlinear networked control system satisfying Lipschitz condition is considered as:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + g(t, x(t)) + Bu(t-\tau) + Ef(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input vector, $f(t) \in R^l$ is the fault vector, $y(t) \in R^p$ is the system output vector, $g(t, x(t))$ is a non-linear term satisfying the Lipschitz condition, τ is the random delay from the controller to the actuator, (A, C) is observable, and A, B, C, E are known system parameter matrices with appropriate dimensions.

It is assumed that the probability density function τ of random delay block $\gamma(\tau)$ exists and its distribution function can be written as follows:

$$F(\tau) = \int_0^{\tau} \gamma(\tau) d\tau \tag{2}$$

By using Laplace transform, the description of random delay block in frequency domain can be obtained as follows:

$$F_T(s) = \int_0^{\infty} e^{-s\tau} \gamma(\tau) d\tau \tag{3}$$

It is assumed that the random time delay τ from the controller is exponentially distributed and the equivalent transformation is carried out for the networked control system which obeys the random delay of exponential distribution.

$$F_T(s) = \int_0^{\infty} \lambda e^{-\lambda\tau} e^{-s\tau} \gamma(\tau) d\tau = \frac{\lambda}{\lambda + s} \tag{4}$$

Define the augmented variable as follows

$$z(t) = E(u(t - \tau)) \tag{5}$$

Since the expected output response of the random delay block equals the response of the corresponding Laplace transform $F_T(s)$ under the same step size signal, it can be obtained that

$$E(u(t - \tau)) = L^{-1} \left\{ \frac{\lambda}{\lambda + s} u(s) \right\} \tag{6}$$

From (5) and (6), it can be obtained that

$$z(s) = \frac{\lambda}{\lambda + s} u(s) \tag{7}$$

After the Laplace inverse transformation, the upper form can also be written as

$$\dot{z}(s) = -\lambda z(s) + \lambda u(s) \tag{8}$$

System (1) can be written as the following equivalent system

$$\begin{aligned} \dot{\bar{x}}(t) &= \bar{A}\bar{x}(t) + \bar{g}(t, x(t)) + \bar{B}u(t) + \bar{E}f(t) \\ Y(t) &= \bar{C}\bar{x}(t) \end{aligned} \tag{9}$$

where $\bar{x}(t) = \begin{bmatrix} x^T(t) & z(t) \end{bmatrix}^T$, $\bar{A} = \begin{bmatrix} A & B \\ 0 & -\lambda \end{bmatrix}$, $\bar{B} = \begin{bmatrix} 0 \\ \lambda \end{bmatrix}$, $\bar{E} = \begin{bmatrix} E \\ 0 \end{bmatrix}$, $\bar{g}(t, x(t)) = \begin{bmatrix} g(t, x(t)) \\ 0 \end{bmatrix}$, $\bar{C} = \begin{bmatrix} C & 0 \end{bmatrix}$.

3. **Fault Diagnosis via a Learning Observer.**

In order to effectively diagnose the fault, based on the equivalent principle, the following form of fault diagnosis learning observer is designed as

$$\begin{aligned} \dot{\hat{\bar{x}}}(t) &= \bar{A}\hat{\bar{x}}(t) + \bar{g}\left(t, \hat{x}(t)\right) + \bar{B}u(t) + \bar{E}Z(t) + \bar{L}\varepsilon(t) \\ Z(t) &= K_1 Z(t - \tau_1) + K_2 \varepsilon(t - \tau_1) \\ \dot{\hat{f}}(t) &= WZ(t) \end{aligned} \tag{10}$$

where $\dot{\hat{f}}(t) \in R^l$ denotes the first derivative of fault estimation, $\varepsilon(t)$ represents the residual, and $Z(t)$ is the input of the learning observer. The characteristic of the learning observer in this paper is that its input can be obtained from the residual signal and input signal of the previous moment, and can be continuously updated and adjusted. $\bar{L}$, K_1, K_2 and W are the gain matrices of the appropriate dimension to be determined.

The residual can be expressed as follows

$$\varepsilon(t) = Y(t) - \hat{Y}(t) = \bar{C}\left(\bar{x}(t) - \hat{\bar{x}}(t)\right) = \bar{C}e_x(t) \tag{11}$$

$e_x(t)$ denotes the state observation error and can be expressed as follows

$$e_x(t) = \hat{x}(t) - \hat{\bar{x}}(t) \tag{12}$$

The dynamic system of the observation error can be obtained by Formulae (10), (11) and (12) as follows

$$\dot{e}_x(t) = \dot{\bar{x}}(t) - \dot{\hat{\bar{x}}}(t) = \left(\bar{A} - \bar{L}\bar{C}\right)e_x(t) + \bar{g}(t, x(t)) - \bar{g}\left(t, \hat{x}(t)\right) + \bar{E}f(t) - \bar{E}Z(t) \tag{13}$$

Assumption 3.1. *Suppose $\bar{g}(t, x(t))$ meets the Lipschitz conditions, $\|\bar{g}(t, x(t)) - \bar{g}(t, \hat{x}(t))\| \leq \gamma \|x(t) - \hat{x}(t)\|$, where γ is a given small positive constant.*

Lemma 3.1. *For any suitable dimensional matrices, the following inequality is established.*

$$2X^T \cdot Y \leq X^T \cdot X + Y^T \cdot Y, \quad \forall X, Y \in R^n \tag{14}$$

Lemma 3.2. *For the learning observer as defined in (10), the following formula holds*

$$\begin{aligned} Z^T(t)Z(t) &= Z^T(t-\tau)K_1^TK_1Z(t-\tau) + Z^T(t-\tau)K_1^TK_2\bar{C}e_x(t-\tau) \\ &\quad + e_x^T(t-\tau)(K_2C)^TK_1Z_1(t-\tau) + e_x^T(t-\tau)\left(K_2\bar{C}\right)^T\left(K_2\bar{C}\right)e_x(t-\tau) \\ &\leq 2Z^T(t-\tau)K_1^TK_1Z(t-\tau) + 2e_x^T(t-\tau)\left(K_2\bar{C}\right)^T\left(K_2\bar{C}\right)e_x(t-\tau) \end{aligned} \tag{15}$$

Combining with Lemma 3.1 and Lemma 3.2 the following formula can be obtained as

$$2Z^T(t)Z(t) \leq 4Z^T(t-\tau)K_1^TK_1Z(t-\tau) + 4e_x^T(t-\tau)\left(K_2\bar{C}\right)^T\left(K_2\bar{C}\right)e_x(t-\tau) \tag{16}$$

Theorem 3.1. *When Assumption 3.1 is established, there are positive definite matrices $P \in R^{n+1}$, $R \in R^{n+1}$, and $K_1 \in R^{r\times r}$, $K_2 \in R^{r\times r}$, $Y \in R^{r\times r}$ to make the following conditions be satisfied:*

$$P\bar{A} + \bar{A}^TP - Y\bar{C} - \bar{C}^TY^T + R + 2\gamma P + P\bar{E}\bar{E}^TP = -Q \tag{17}$$

$$0 < (4 + 2\sigma)K_1^TK_1 \leq I \tag{18}$$

$$0 < (4 + 2\sigma)\left(K_2\bar{C}\right)^T\left(K_2\bar{C}\right) \leq R \tag{19}$$

where $Y = P * L$.

Proof: Select the Lyapunov function as follows:

$$V_1(t) = e_x^T(t)P_1e_x(t) + \int_{t-\tau}^{t} e_x^T(s)Re_x(s)ds + \int_{t-\tau}^{t} Z^T(s)Z(s)ds \tag{20}$$

The first-order derivative of $V_1(t)$ is obtained as follows:

$$\begin{aligned} \dot{V}_1(t) &= e_x^T(t)\left[P\left(\bar{A} - L\bar{C}\right) + \left(\bar{A} - L\bar{C}\right)^TP + 2P\gamma\right]e_x(t) \\ &\quad + 2e_x^T(t)PEf(t) - 2e_x^T(t)PEZ(t) + e_x^T(t)Re_x(t) \\ &\quad - e_x^T(t-\tau)Re_x(t-\tau) + Z^T(t)Z(t) - Z^T(t-\tau)Z(t-\tau) \end{aligned} \tag{21}$$

From Lemma 3.1, it can be obtained that

$$2\left\|e_x(t)PE\right\|\left\|Z(t)\right\| \leq e_x^T(t)PE^TPe_x(t) + Z^T(t)Z(t) \tag{22}$$

It is assumed that the fault is bounded, $\|f\| \leq K_f$, where K_f is the upper bound of the fault. Then from (13) and (16), it can be obtained that

$$\begin{aligned}\dot{V}_1(t) \leq\ & e_x^T(t)\left[P\left(\bar{A}-L\bar{C}\right)+\left(\bar{A}-L\bar{C}\right)^TP+R+2P\gamma\right]e_x(t)\\ & +2K_f\left\|P\bar{E}\right\|\left\|e_x(t)\right\|+e_x^T(t)P\bar{E}\bar{E}^TPe_x(t)\\ & +\sigma Z^T(t)Z(t)-\sigma Z^T(t)Z(t)+2Z^T(t)Z(t)\\ & -e_x^T(t-\tau)Re_x(t-\tau)-Z^T(t-\tau)Z(t-\tau)\end{aligned} \tag{23}$$

where σ is a positive constant.

Substituting (16) into (23), the upper form can also be written as:

$$\begin{aligned}\dot{V}_1(t) \leq\ & e_x^T(t)\left[P\left(\bar{A}-L\bar{C}\right)+\left(\bar{A}-L\bar{C}\right)^TP+R+2P\gamma+P\bar{E}\bar{E}^TP\right]e_x(t)\\ & +2K_f\left\|P\bar{E}\right\|\left\|e_x(t)\right\|+(4+2\sigma)Z_1^T(t-\tau)K_1^TK_1Z_1(t-\tau)\\ & +(4+2\sigma)e_x^T(t-\tau)(K_2C)^T\left(K_2C\right)e_x(t-\tau)-e_x^T(t-\tau)Re_x(t-\tau)\\ & -\sigma Z^T(t)Z(t)-Z^T(t-\tau)Z(t-\tau)\\ =\ & e_x^T(t)\left[P\left(\bar{A}-L\bar{C}\right)+\left(\bar{A}-L\bar{C}\right)^TP+R+2P\gamma+P\bar{E}\bar{E}^TP\right]e_x(t)\\ & +2K_f\left\|P_1\bar{E}\right\|\left\|e_x(t)\right\|+Z^T(t-\tau)\left((4+2\sigma)K_1^TK_1-I\right)Z(t-\tau)\\ & +e_x^T(t-\tau)\left((4+2\sigma)\left(K_2\bar{C}\right)^T\left(K_2\bar{C}\right)-R\right)e_x(t-\tau)-\sigma Z^T(t)Z(t)\end{aligned} \tag{24}$$

Equation (17) can be written as the following inequality:

$$\begin{bmatrix} P\left(\bar{A}-L\bar{C}\right)+\left(\bar{A}-L\bar{C}\right)^TP+R+2P\gamma & P\bar{E} \\ * & -I\end{bmatrix} < 0 \tag{25}$$

Substituting Equations (18), (19) and (25) into Equation (24), the following equality can be obtained as:

$$\dot{V}_1(t) \leq -\lambda_{\min}\left(Q_1\right)\left\|e_x(t)\right\|^2 + 2K_f\left\|P\bar{E}\right\|\left\|e_x(t)\right\| \tag{26}$$

When $\lambda_{\min}(Q_1) > \frac{2K_f\|PE\|\|e_z(t)\|}{\|e_x(t)\|^2}$, $\dot{V}_1(t) \leq 0$, where

$$Q_1 = \begin{bmatrix} P\left(\bar{A}-L\bar{C}\right)+\left(\bar{A}-L\bar{C}\right)^TP+R+2P\gamma & P\bar{E} \\ * & -I\end{bmatrix}.$$

The proof of stability is complete.

Remark 3.1. *Because the state estimation information and fault estimation information have been obtained in fault diagnosis phase, the fault-tolerant controller can be designed based on the information of fault and state estimation in the fault-tolerant control phase.*

4. **Fault-Tolerant Controller Design Based on State Feedback.** The aim of fault-tolerant control is to satisfy the requirements on the closed-loop system despite the presence of fault. Since the fault estimation information has been obtained during the fault diagnosis phase, then based on the obtained online fault estimation information, the following fault-tolerant controller is designed to ensure the stability of the post-fault system.

Assumption 4.1. *rank* $\left(\bar{B}, E\right) = $ *rank* $\left(\bar{B}\right)$.

There is a matrix $\bar{B}^* \in R^{m\times n}$ such that

$$(I_n - B * B^*)\, E = 0 \tag{27}$$

It is assumed that the output error vector of networked control system is $V(t)$ and the reference output is $V_d(t)$. Then the state feedback controller can be designed as follows:

$$\begin{aligned} V(t) &= Y(t) - Y_d(t) \\ u(t) &= -L_2\hat{\bar{x}}(t) - L_3\hat{f}(t) - \bar{B}^*\bar{g}\left(t, \hat{x}(t)\right) + DV(t) \end{aligned} \tag{28}$$

where $L_2 \in R^{m\times n}$ is state feedback matrix, and D is a known dimensional matrix.

For system (9), if no fault occurs, the actual output should be able to follow the expected output.

When no fault occurs, $f(t) = 0$, substituting Equation (28) into Equation (9), the following equality can be obtained as

$$\begin{aligned} \dot{\bar{x}}(t) &= \bar{A}\bar{x}(t) + \bar{g}(t, x(t)) + \bar{B}\left(-L_2\hat{\bar{x}}(t) - L_3\hat{f}(t) - \bar{B}^*\bar{g}\left(t, \hat{\bar{x}}(t)\right) + DV(t)\right) \\ &= \left(\bar{A} - \bar{B}L_2\right)\bar{x}(t) + \bar{B}L_2 e_x(t) + \bar{g}(t, x(t)) - \bar{g}\left(t, \hat{\bar{x}}(t)\right) + \bar{B}DV(t) - \bar{B}L_3\hat{f} \end{aligned} \tag{29}$$

From Assumption 3.1, it can be formulated that

$$\begin{aligned} \dot{\bar{x}}(t) &\le \left(\bar{A} - \bar{B}L_2\right)\bar{x}(t) + \left(\bar{B}L_2 + \gamma\right)e_x(t) + \bar{B}DV(t) - \bar{B}L_3\hat{f} \\ &= \left(\bar{A} - \bar{B}L_2\right)\bar{x}(t) + \left[\begin{array}{cc} \bar{B}L_2 + \gamma - \bar{B}L_3 & \bar{B}D \end{array}\right]\zeta(t) \end{aligned} \tag{30}$$

where $\zeta(t) = \left[\begin{array}{ccc} e_x^T & \hat{\bar{f}}(t) & V(t) \end{array}\right]^T$.

Lemma 4.1. *According to the bounded real Lemma in [22], for a given scalar $\tau > 0$, if there exists a positive definite matrix $P \in R^{n\times n}$ and matrices L_2, L_3 making the following inequality be satisfied:*

$$\left[\begin{array}{ccccc} \Omega_{11} & PBL_2 + \gamma P & PBL_3 & PBD & I_n \\ * & -\tau I_n & 0 & 0 & 0 \\ * & * & -\tau I_r & 0 & 0 \\ * & * & * & -\tau I_d & 0 \\ * & * & * & * & -\tau I_n \end{array}\right] < 0 \tag{31}$$

where $\Omega_{11} = P\left(\bar{A} - \bar{B}L_2\right) + \left(\bar{A} - \bar{B}L_2\right)^T P$, the state feedback fault-tolerant controller is robust and stable.

When fault occurs, the state equation of the system is expressed as follows:

$$\begin{aligned} \dot{\bar{x}}(t) &= \bar{A}\bar{x}(t) + \bar{g}(t, x(t)) + \bar{B}\left(-L_2\hat{\bar{x}}(t) - L_3\hat{f}(t) - \bar{B}^*\bar{g}(t, \hat{\bar{x}}(t)) + DV(t)\right) + \bar{E}f(t) \\ &= \left(\bar{A} - \bar{B}L_2\right)\bar{x}(t) + \bar{B}L_2 e_x(t) + \bar{g}(t, x(t)) - \bar{g}(t, \hat{\bar{x}}(t)) + \bar{B}DV(t) + \bar{E}f(t) \\ &\quad - \bar{B}L_3\hat{f} \\ &\le \left(\bar{A} - \bar{B}L_2\right)\bar{x}(t) + \left(\bar{B}L_2 + \gamma\right)e_x(t) + \bar{B}DV(t) + \bar{E}f(t) - \bar{B}L_3\hat{f} \\ &= \left(\bar{A} - \bar{B}L_2\right)\bar{x}(t) + \left[\begin{array}{cccc} \bar{B}L_2 + \gamma & \bar{E} & -\bar{B}L_3 & \bar{B}D \end{array}\right]\zeta(t) \end{aligned} \tag{32}$$

where $\zeta(t) = \left[\begin{array}{cccc} e_x^T(t) & \bar{f}(t) & \hat{\bar{f}}(t) & V(t) \end{array}\right]^T$.

The proof process is similar to that of the no fault condition. The LMI is solved as follows

$$\begin{bmatrix} \Omega_{11} & PBL_2+\gamma P & P\bar{E} & -P\bar{B}L_3 & P\bar{B}D & I_n \\ * & -\tau I_n & 0 & 0 & 0 & 0 \\ * & * & -\tau I_r & 0 & 0 & 0 \\ * & * & * & -\tau I_r & 0 & 0 \\ * & * & * & * & -\tau I_d & 0 \\ * & * & * & * & * & \tau I_n \end{bmatrix} < 0 \tag{33}$$

where $\Omega_{11} = P\left(\bar{A} - \bar{B}L_2\right) + \left(\bar{A} - \bar{B}L_2\right)^T P$.

In Inequality (33), since P, L_2 and L_3 are unknown, the inequality is likely to have no solution. For ease of calculation, Inequality (33) is respectively left multiplied and right multiplied $X = P^{-1}$ to obtain the following inequality

$$\begin{bmatrix} \Pi_{11} & BL_2X+\gamma X & \bar{E}X & -\bar{B}L_3X & \bar{B}DX & I_n \\ * & -\tau I_n & 0 & 0 & 0 & 0 \\ * & * & -\tau I_r & 0 & 0 & 0 \\ * & * & * & -\tau I_r & 0 & 0 \\ * & * & * & * & -\tau I_d & 0 \\ * & * & * & * & * & \tau I_n \end{bmatrix} < 0 \tag{34}$$

where $\Pi_{11} = \left(\bar{A} - \bar{B}L_2\right)X + X^T\left(\bar{A} - \bar{B}L_2\right)^T$.

5. A Simulation Example.

In this paper, a single link manipulator is considered, whose rotating joint is driven by a DC motor [23]. The status is the angular position, motor and link speed $x^T = (x_1, x_2, x_3, x_4)^T = (\theta_m, \omega_m, \theta_1, \omega_1)$, and the state space model is expressed as follows:

$$\begin{aligned} \dot{\theta}_m &= \omega_m \\ \dot{\omega}_m &= \frac{k}{J_m}(\theta_1 - \theta_m) - \frac{b}{J_m}\omega_m + \frac{k_\tau}{J_m}u \\ \dot{\theta}_1 &= \omega_1 \\ \dot{\omega}_1 &= \frac{k}{J_1}(\theta_1 - \theta_m) - \frac{mgh}{J_1}\sin(\theta_1) \end{aligned} \tag{35}$$

where J_m and J_1 are the inertia of the motor and the connecting rod. The dynamics of the system is nonlinear. The parameters of the system (1) are given as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 10 \\ 1.95 & 0 & -1.95 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix} \quad E = \begin{bmatrix} 0 \\ 12.5 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad g(t, x(t)) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.333\sin x_3 \end{bmatrix}$$

After Laplace transformation, the reconstructed system parameters are given as follows:

$$\bar{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 & 21.6 \\ 0 & 0 & 0 & 10 & 0 \\ 1.95 & 0 & -1.95 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \quad \bar{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \bar{E} = \begin{bmatrix} 0 \\ 12.5 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\bar{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad \bar{g}(t, x(t)) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.333 \sin x_3 \\ 0 \end{bmatrix}$$

Matrices L_i $(i = 1, 2, 3)$ and K_i $(i = 1, 2)$ are selected as follows

$$L_1 = \begin{bmatrix} 10 & 105.4 \\ -5943 & 58.5 \\ -391.8 & 6.2 \\ 105.2 & 1.1 \\ 23.8 & 1 \end{bmatrix} \quad K_1 = -0.48 \quad K_2 = \begin{bmatrix} 0.032 & 0.035 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 0.0758 & 0.2848 & -0.0262 & -0.050 & 0.1420 \end{bmatrix} \quad L_3 = 1.8$$

By solving LMI (34), it can be formulated as follows

$$P = \begin{bmatrix} 1.1518 & 0.0089 & -5.998 & -0.0213 & -0.1162 \\ * & 0.0254 & -0.0176 & -0.0260 & -0.0004 \\ * & * & 0.6672 & -0.1215 & 0.0540 \\ * & * & * & 2.5170 & -0.0976 \\ * & * & * & * & 1.1769 \end{bmatrix}$$

In this fault diagnosis experiment, two different fault types are considered. In the first case, the constant fault is considered. The fault time is 20s to 100s, and the fault will change in 50s. In the second case, time-varying fault is considered, which occurs in 50s. The specific failure modes are given as follows:

Case 1: Constant fault

$$f(t) = \begin{cases} 0 & 0 \le t < 20 \\ 0.5 & 20 \le t < 50 \\ 1.5 & t \ge 50 \end{cases}$$

Case 2: Time-varying fault

$$f(t) = \begin{cases} 0 & 0 \le t < 20 \\ 0.5 & 20 \le t < 50 \\ 1.5 - e^{-0.15(t-50)} & t \ge 50 \end{cases}$$

The desired trajectory of the manipulator is set as $Y_d(t) = \begin{bmatrix} \sin t \\ \cos t \end{bmatrix}$.

The simulation results of Case 1 are shown in Figure 1. Figure 2 gives the fault diagnosis results of Case 2. The fault occurrence time of this simulation experiment is 20s to 100s, and the fault changes in 50s. From Figure 1 and Figure 2, it can be seen that the fault estimation can track the change of fault. In order to prove the robustness of the algorithm, the external disturbance is added to the time-varying fault diagnosis. It can be seen from Figure 2 and Figure 3 that the algorithm has good robustness. Figure 4 and Figure 5 show the actual output and expected output without fault. It can be seen that when no fault occurs, the actual output can track the expected output exactly. Figure 6 and Figure 7 show the actual output and the expected output with fault-tolerant controller. From Figure 6 and Figure 7, it can be seen that, in the normal operation of the system, the expected output tracks the actual output. After the fault occurs in 20s and changes in 50s suddenly, the actual output fluctuates a little and then the actual output keeps up with the expected output with the proposed fault-tolerant tracking method. The actual output and expected output without fault-tolerant control are shown in Figure 8 and

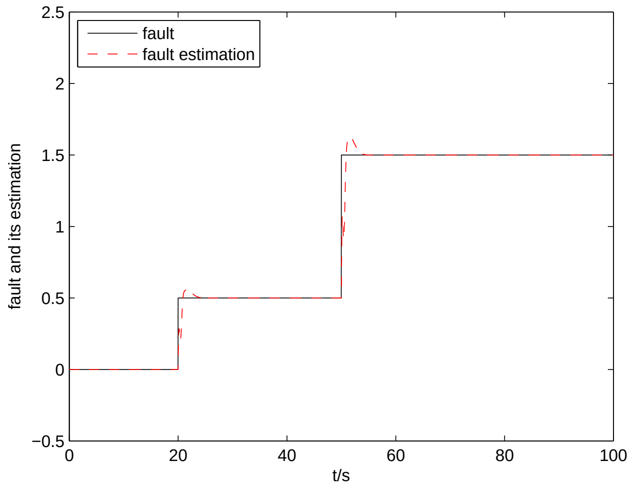

FIGURE 1. Constant fault and fault estimation

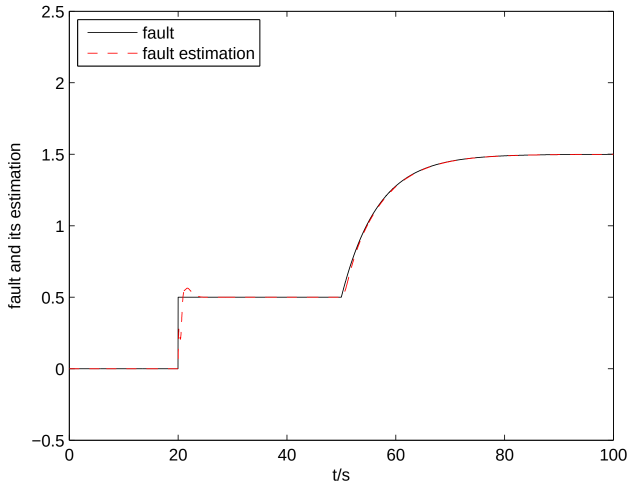

FIGURE 2. Time-varying fault and fault estimation

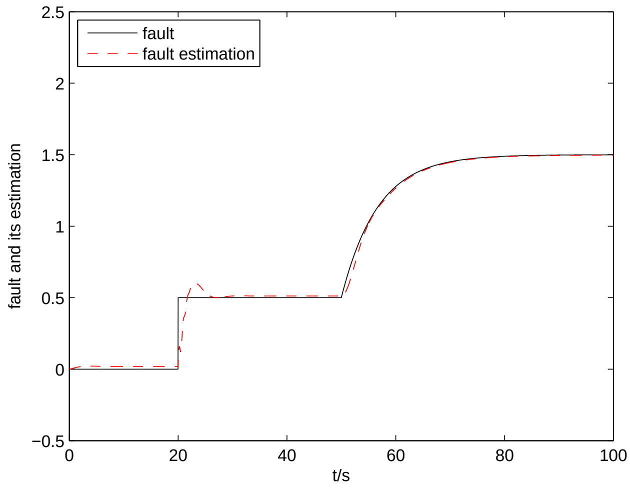

FIGURE 3. Time-varying fault and fault estimation with disturbance

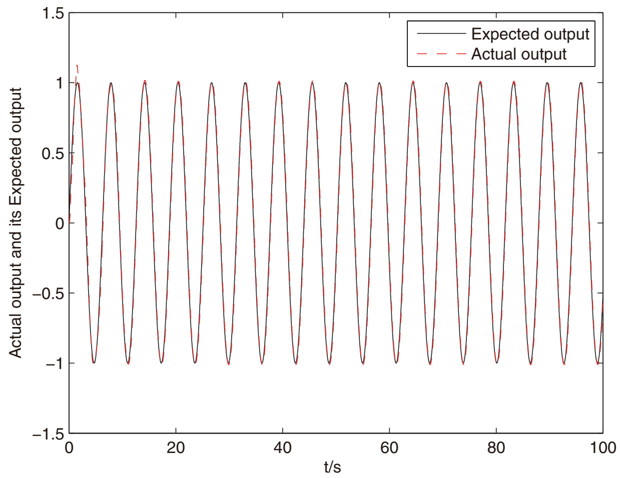

FIGURE 4. The actual output Y_1 and expected output Yd_1 without fault

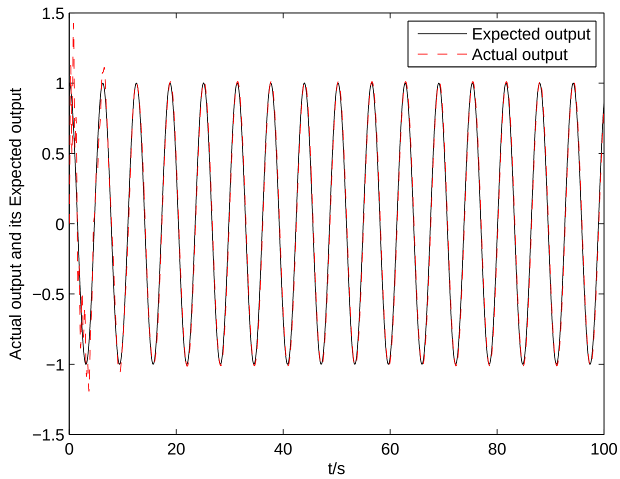

FIGURE 5. The actual output Y_2 and expected output Yd_2 without fault

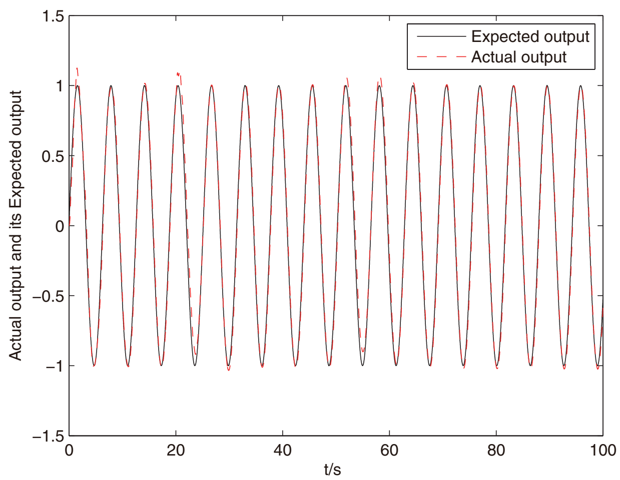

FIGURE 6. The actual output Y_1 and expected output Yd_1 with fault-tolerant controller

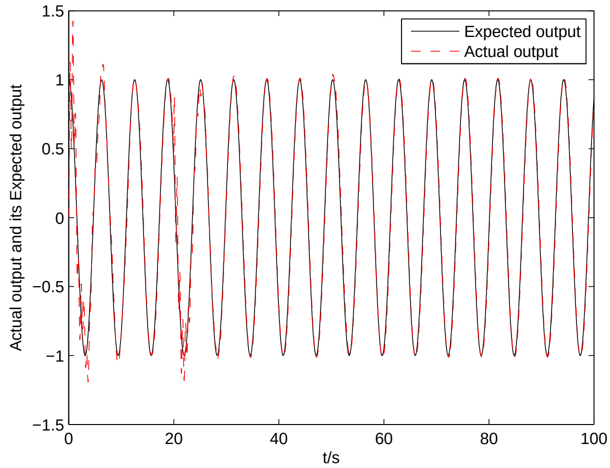

FIGURE 7. The actual output Y_2 and expected output Yd_2 with fault-tolerant controller

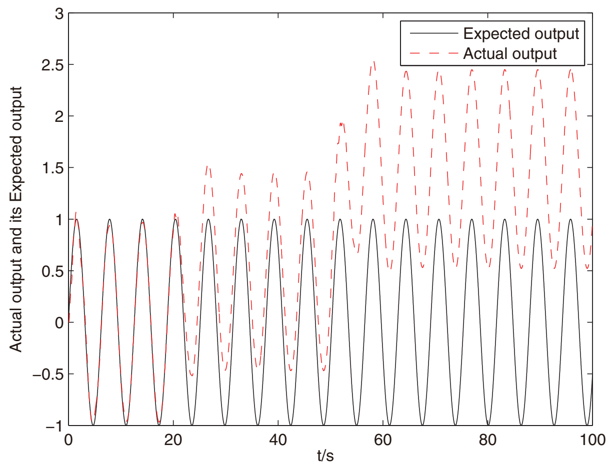

FIGURE 8. The actual output Y_1 and expected output Yd_1 without fault-tolerant controller

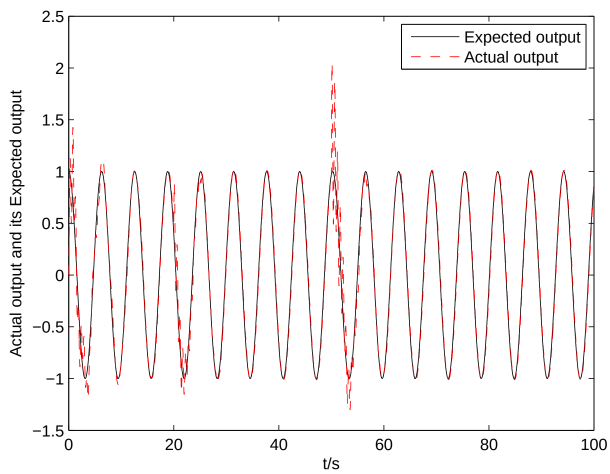

FIGURE 9. The actual output Y_2 and expected output Yd_2 without fault-tolerant controller

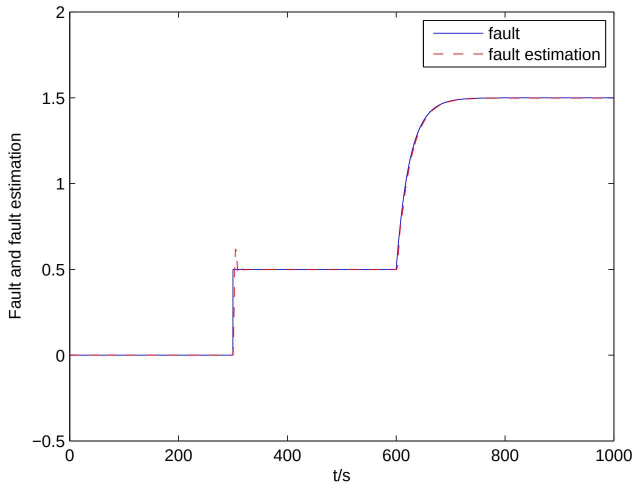

FIGURE 10. Fault and fault estimation in [24]

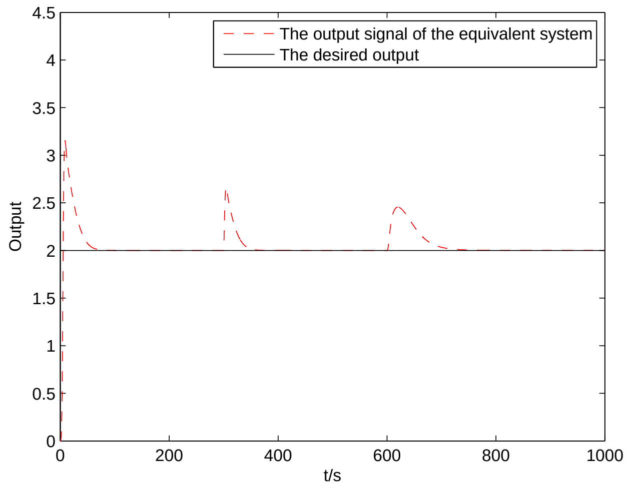

FIGURE 11. The actual output and expected output with fault-tolerant controller in [24]

Figure 9. Through the comparison of Figure 6 and Figure 8, Figure 7 and Figure 9, the effect of fault-tolerant control can be clearly shown.

In order to prove the simulation effect, this paper is compared with another paper on fault diagnosis and fault-tolerant control effect of the networked control system. In [24], an adaptive fault diagnosis observer and a PI-based fault-tolerant controller is used. The simulation results are shown in Figure 10 and Figure 11. From Figures 10 and 11, in the case of similar diagnosis results, the fault-tolerant effect of this paper is better than this method.

6. **Conclusions.** In this paper, a new fault diagnosis and fault-tolerant control algorithm is designed for nonlinear networked control systems. Different from the previous fault diagnosis algorithm, a Laplace transform strategy is used to analyze the time delay problem of the nonlinear networked control system. Besides, a novel fault diagnosis algorithm based on learning observer is proposed. The linear matrix inequality (LMI) approach is used to calculate the required observer gain and adaptive tuning rule for fault

diagnosis. Based on the fault estimation information and other measurement information, a fault-tolerant controller based on state feedback is designed for the system output trajectory to track the desired trajectory. Finally, encouraging results have been obtained by computer simulations.

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REFERENCES

[1] R. Isermann, Supervision, fault-detection and fault-diagnosis methods – An introduction, *Control Engineering Practice*, vol.5, no.5, pp.639-652, 1997.

[2] R. Isermann, Estimation of physical parameters for dynamic processes with application to an industrial robot, *International Journal of Control*, vol.55, no.6, pp.1287-1298, 1992.

[3] J. Zhang, Z. Wu, G. Cui, P. Fan, J. Cao and M. Ning, Ensor fault diagnosis based on dorrentropy filter and probabilistic neural network, *ICIC Express Letters*, vol.13, no.8, pp.743-751, 2019.

[4] H. Yan, S. Yan, H. Zhang and X. Zhao, An overview of networked control of complex dynamic systems, *Mathematical Problems in Engineering*, vol.2014, no.2, pp.741-764, 2014.

[5] Y. Zhang, F. Wang, S. J. Qin, X. Yang and Y. Chen, Fault detection for MIMO networked control system, *Fault Detection, Supervision and Safety of Technical Processes 2006*, vol.39, no.13, pp.1080-1084, 2006.

[6] D. W. Zhang, Z. Y. Zhou and X. C. Jia, Networked fuzzy output feedback control for discrete-time Takagi-Sugeno fuzzy systems with sensor saturation and measurement noise, *Information Sciences*, vol.457, no.8, pp.182-194, 2018.

[7] Y. L. Wang, L. Q. Han and T. B. Wang, Fault detection filter design for data reconstruction-based continuous-time networked control systems, *Information Sciences: An International Journal*, vol.328, pp.577-594, 2016.

[8] Q. Y. Ai, C. S. Liu and B. Jiang, Robust fault detection for a class of nonlinear network control system with communication delay, *Journal of Systems Engineering and Electronics*, vol.20, no.5, pp.1024-1030, 2009.

[9] W. Zhang and K. Duan, Event-triggered fault-tolerant control for networked systems with dynamic quantiser, *IET Control Theory and Applications*, vol.10, no.9, pp.1088-1096, 2016.

[10] X. A. Chu and M. G. Li, Event-triggered fault estimation and sliding mode fault-tolerant control for a class of nonlinear networked control systems, *Journal of the Franklin Institute*, vol.355, no.13, pp.5475-5502, 2018.

[11] R. Shahnazi and Q. Zhao, Adaptive fuzzy descriptor sliding mode observer-based sensor fault estimation for uncertain nonlinear systems, *Asian Journal of Control*, vol.18, no.5, pp.1478-1488, 2016.

[12] A. B. Brahim, S. Dhahri, F. B. Hmida and A. Sellami, Simultaneous actuator and sensor faults reconstruction based on robust sliding mode observer for a class of nonlinear systems, *Asian Journal of Control*, vol.19, no.1, pp.1-10, 2017.

[13] J. Yang, F. Zhu, X. Wang and X. Bu, Robust sliding-mode observer-based sensor fault estimation, actuator fault detection and isolation for uncertain nonlinear systems, *International Journal of Control, Automation and Systems*, vol.13, no.5, pp.1037-1046, 2015.

[14] Q. X. Jia, C. Wen, Y. C. Zhang and H. Y. Li, Fault reconstruction for Takagi-Sugeno fuzzy systems via learning observers, *International Journal of Control*, vol.89, no.3, pp.564-578, 2016.

[15] Q. X. Jia, C. Wen, Y. C. Zhang and H. Y. Li, Fault reconstruction for continuous-time systems via learning observers, *Asian Journal of Control*, vol.18, no.2, pp.549-561, 2016.

[16] N. Li, H. Y. Sun and Q. L. Zhang, Robust passive adaptive fault tolerant control for stochastic wing flutter via delay control, *European Journal of Control*, vol.48, no.2, pp.74-82, 2019.

[17] H. R. Patel and V. A. Shah, Passive fault-tolerant control tracking for nonlinear system with intermittent fault and time delay, *IFAC-PapersOnLine*, vol.87, no.8, pp.1707-1715, 2019.

[18] S. M. Mahmoud, A. M. Memon and P. Shi, Observer-based fault-tolerant control for a class of nonlinear networked control systems, *International Journal of Control*, vol.87, no.8, pp.1707-1715, 2014.

[19] K. Schenk, B. Glbitti and L. Z. Jan, Cooperative fault tolerant control of network control system, *IFAC-PapersOnLine*, vol.51, no.24, pp.570-577, 2018.

[20] W. Wu, Fault-tolerant control of uncertain non-linear networked control systems with time-varying delay, packet dropout and packet disordering, *IET Control Theory and Applications*, vol.11, no.7, pp.973-984, 2017.
[21] X. D. Wang, Z. Y. Fei, Z. H. Wang and X. Y. Liu, Event-triggered fault estimation and sliding mode fault-tolerant control for a class of nonlinear networked control systems, *Journal of the Franklin Institute*, vol.356, no.8, pp.4420-4441, 2019.
[22] E. Last, Linear matrix inequalities in system and control theory, SAM, *Proc. of the IEEE*, vol.86, no.12, pp.2473-2474, 1994.
[23] M. W. Spong, Modeling and control of elastic joint robots, *Journal of Dynamic Systems Measurement and Control*, vol.109, no.4, pp.310-319, 1987.
[24] L. L. Fan, L. N. Yao, H. K. Jin and L. J. Zhou, Fault diagnosis and fault-tolerant control based on Laplace transform for nonlinear networked control systems with random delay, *International Journal of Robust and Nonlinear Control*, vol.30, no.3, pp.1223-1239, 2020.