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PLACEMENT OF SURFACE SENSORS FOR COOPERATIVE LOCALIZATION OF SUBMERGED AUTONOMOUS UNDERWATER VEHICLES

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Abstract. *Cooperative localization using surface sensors is an effective method for localization of submerged Autonomous Underwater Vehicles (AUVs), especially for long duration and wide area operations. However, placement of surface sensors such that the corresponding sensor formation maximizes the observability of AUVs is a challenging task. In this paper, a new method for obtaining optimal formation of surface sensors has been presented for multiple AUVs cooperative localization system. Most of the real world constraints/situations such as characteristics of acoustic signals in water, multiple-AUVs cooperative localization and computation complexity with the increase in number of AUVs have been considered. Firstly, an evaluation function based on acoustic signal transmission characteristics and Fisher information matrix theory has been formulated for AUVs whose position is known. In order to obtain the optimal formation for a given system, the value of evaluation function must reach its maximum. A recursive optimization algorithm has been used for the solution of the evaluation function. Simulation examples are then presented to calculate optimal formation for single AUV and multi-AUVs systems. Finally, to verify the effectiveness of the proposed method, various simulations have been performed using Extended Kalman Filter (EKF) to compare the localization error between optimal formations and random formations. The simulation results indicate that the optimal formation is able to decrease the noise error and enhance the observability of the system.*

Keywords: AUVs, Cooperative localization, Optimal formation, Sensor placement

1. **Introduction.** Localization of submerged AUVs is one of the most challenging tasks in AUVs operation due to not availability of Global Positioning System (GPS) signals and severe attenuation of radio frequency waves in water [1,2]. Therefore, Inertial Navigation System (INS) forms an integral part of an AUV's localization system. However, inertial sensors accumulate error with the passage of time, causing unbounded error in localization of the AUVs. In order to maintain the accuracy of INS, different techniques have been proposed in recent past [3-5]. Cooperative localization is one of the most recent techniques that are being researched for localization of AUVs [6-9]. In cooperative localization of AUVs, sensor AUVs are deployed at or near water surface and their positions are accurately known with the help of GPS. The localization of submerged AUVs (target AUVs) is performed by measuring their relative range or bearing with respect to the sensor AUVs. Besides other challenges of cooperative localization, accurate measurement of the target's relative position using sound waves is a challenging task due to higher transmission losses, multiple path effect, varying speed of sound in water and dependence of

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sound propagation on environmental conditions. While most of these factors are uncontrollable, the accuracy of cooperative localization can still be optimized by maximizing the observability of sensor AUVs by keeping them at maximum observable location (called optimal formation of sensor AUVs).

Extensive research has been carried out in the field of cooperative localization in recent past. However, optimal formation for cooperative navigation is relatively a new field and only little literature is available. Optimal sensor formation was studied in [10] for static localization of target using Time of Arrival (TOA), range only and bearing only measurements. It was concluded that the target-sensor geometries are not unique in general. The analysis of optimal formation of linear arrays was presented in [11] for passive localization using range and Angle of Arrival (AoA) measurement. Fisher Information Matrix (FIM) and Cramer-Rao Lower Bound (CRLB) were taken as optimality criteria to characterize localization performance of the sensors. In [12], sensor placement and motion coordination were studied for target tracking application using range measurements in 2D and 3D scenarios. In [13], optimal formation for underwater AUV localization was presented. The evaluation function was derived on the basis of CRLB and range dependent measurement noise. The effect of separation angle and range was studied on placement of sensors and it was concluded that range deviation has more influence on localization performance of the sensors than separation angle. Similarly in [14], optimal formation for multiple AUVs was proposed for both 2D and 3D based on virtual structure. By looking at the literature review, we can observe that although few researchers have presented some solutions to the problem of optimal sensors placement, research on many aspects of the field is still missing. For example, in most of the previous research literature, only optimal formations have been proposed for surface sensors. However, the effect of the proposed formations on actual accuracy of underwater localization has never been analyzed.

In this paper, we have presented a novel approach for estimating optimal formation of the sensor AUVs for cooperative localization of multiple target AUVs, simultaneously. The sensor AUVs are supposed to be placed on surface so that their accurate position may be known using GPS. To solve this problem, we have built an evaluation function based on special characteristics of acoustics signals and the theory of CRLB/FIM. Then, a stepping recursive strategy has been followed to solve the evaluation function for its maximum value for multiple vessels. This strategy ensured that the computation complexity should remain limited even when number of AUVs is increased.

The remaining paper is organized as follows. In the next section, the proposed evaluation function based on FIM and underwater acoustics characteristics is derived. To keep the computational complexity limited, a recursive stepping strategy has been proposed in Section 3. In Section 4, simulation results for two different examples are presented and analyzed. To determine whether the proposed evaluation function is able to actually decrease the measurement noise in real system, simulations based on Extended Kalman Filter (EKF) are presented in Section 5. Finally, the paper is concluded in Section 6.

2. Derivation of Evaluation Function.

Let us consider that point $p_i = [x_i, y_i, z_i]^T$ be the position of the i-th sensor where $i = 1, 2, 3, \ldots, n$ and n is the total number of sensor AUVs and $q_j = [x_j, y_j, z_j]^T$ be the position of the j-th target where $j = 1, 2, 3, \ldots, m$ and m is the total number of target AUVs. Then the measured distance between the i-th sensor and the j-th target is given as follows:

$$l_{ij} = r_{ij} + \omega_{ij} \tag{1}$$

where ω_{ij} is the measurement noise and r_{ij} is the actual range of the target and is given by distance formula:

$$r_{ij} = \|p_i - q_j\| = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + d^2} \tag{2}$$

where d is the depth of the target.

Considering the special characteristic of underwater acoustic signal, the measurement noise is directly related to range of the target. The measurement noise may be defined as [15]:

$$\omega_{ij} = \sigma\left(1 + \eta r_{ij}^{\gamma}\right) \tag{3}$$

where σ is measurement noise constant and η and γ are constants related to distance. Let $w_j = [\omega_{1j}, \omega_{2j}, \omega_{3j}, \ldots, \omega_{ij}]^T$, where $i = 1, 2, 3, \ldots, n$, $j = 1, 2, 3, \ldots, m$. So we can obtain a conclusion that $E\left(w_j \cdot w_j^T\right) = \sum_j = \sigma^2\left(1 + \eta r_{ij}^{\gamma}\right) \cdot I$, where I is an n-by-n identity matrix.

For cooperative localization of AUVs, the state vector likelihood function is given as follows [12]:

$$P(q_j) = \frac{1}{(2\pi)^{\frac{n}{2}}|\Sigma_j|^{\frac{1}{2}}} \exp\left\{-\frac{1}{2}(L_j - R_j)^T \Sigma_j^{-1}(L_j - R_j)\right\} \tag{4}$$

where $L_j = [l_{1j}, l_{2j}, l_{3j}, \ldots, l_{ij}]^T \sim (R_j, \Sigma_j)$ and $R_j = [r_{1j}, r_{2j}, r_{3j}, \ldots, r_{ij}]^T$.

By definition of Fisher information matrix, in the multiple AUV navigation system, the j-th target's FIM is given by

$$FIM(q_j) = E\left\{\left(\nabla_{q_j} \ln(P(q_j))\right) \cdot \left(\nabla_{q_j} \ln(P(q_j))\right)^T\right\} \tag{5}$$

Taking natural logarithm of Equation (4), we get

$$\ln(P(q_j)) = -\frac{1}{2}\left(n \cdot \ln(2\pi) + \ln\left(|\Sigma_j|\right)\right) - \frac{1}{2}(L_j - R_j)^T \Sigma_j^{-1}(L_j - R_j) \tag{6}$$

Take partial derivative of Equation (6) w.r.t. q_j:

$$\nabla_{q_j} \ln(P(q_j)) = (\nabla_{q_j} R_j)^T \Sigma_j^{-1}(L_j - R_j) \tag{7}$$

From (7) and (5), we can derive FIM as

$$FIM(q_j) = \frac{1}{\sigma^2}\sum_{i=1}^{n}\begin{pmatrix} (u_{ijx})^2 & (u_{ijx})(u_{ijy}) \\ (u_{ijx})(u_{ijy}) & (u_{ijy})^2 \end{pmatrix} \tag{8}$$

where,

$$u_{ij} = \left[\frac{\partial r_{ij}}{\partial x_j} \cdot \frac{1}{\left(1 + \eta r_{ij}^{\gamma}\right)}, \frac{\partial r_{ij}}{\partial y_j} \cdot \frac{1}{\left(1 + \eta r_{ij}^{\gamma}\right)}\right]^T \tag{9}$$

It is important to highlight that the variance of any unbiased estimator can only approach a lower limit called CRLB which is given as

$$CRB(q_j) = FIM(q_j)^{-1}.$$

When the determinant of FIM will reach its maximum, the measurement noise covariance will be minimum; hence, the corresponding formation will be the optimal formation of the sensor AUVs with respect to the given target AUV. In order to obtain the optimal formation for all targets, we select the sum of logarithm of FIM determinant as the evaluation function F. The function is given by

$$F = \sum_{j=1}^{m} \ln\left(|FIM(q_j)|\right) \tag{10}$$

In order to estimate the optimal position of a sensor for the entire target AUVs, the value of evaluation function must reach its maximum. To solve for the optimal solution, we will

compute the derivatives of evaluation function with respect to the position coordinates of the sensor and equate it to zero.

3. **Recursive Optimization Algorithm.** It can be clearly seen from Equation (10) that despite the evaluation function "F" combines every target AUVs FIM, it only provides optimal position of single sensor AUV. In order to calculate optimal formation of the whole system, evaluation function for all the sensors AUVs is to be solved simultaneously. For multiple AUVs system, when the number of sensors increases, the corresponding unknown quantity is also increased; this drastically increases the computation complexity to solve the evaluation function. In order to cater this problem, a recursive optimization algorithm is devised to ensure the calculation is achievable. The algorithm is implemented as follows.

Step 1) FIM based on all targets' positions is calculated for each sensor. Then, sum of logarithm of the FIM determinant is calculated as per Equation (10) to achieve the evaluation function in the current step, denoted as $F[t]$.

Step 2) Derivative of the evaluation function ($\partial F/\partial x_i$ and $\partial F/\partial y_i$) with respect to horizontal and vertical coordinates of each sensor position is calculated.

Step 3) These results are used to update the position coordinates of sensor AUVs with following equations:

$$\begin{aligned} x_i[t+1] &= x_i[t] + \mu^{\lambda[t]} \cdot \frac{\partial F}{\partial x_i[t]} \\ y_i[t+1] &= y_i[t] + \mu^{\lambda[t]} \cdot \frac{\partial F}{\partial y_i[t]} \end{aligned} \tag{11}$$

where $\mu \in (0,1)$ is a constant; $\lambda[0] = 1$ and $\lambda[t+1] = \lambda[t] + 1$.

Step 4) Calculate the evaluation function value based on the updated sensor positions and there are two possible situations.

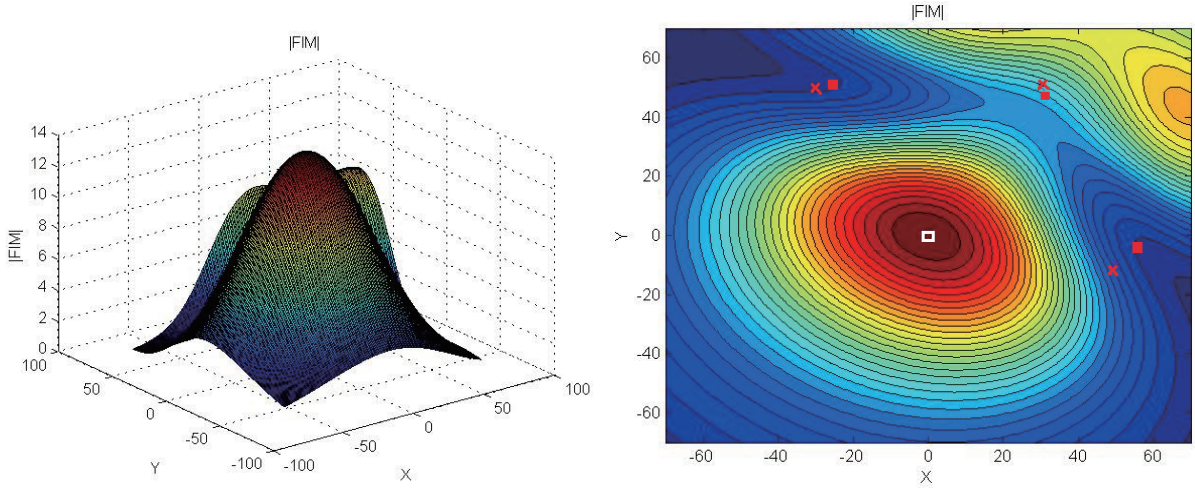
Step 5) $F[t+1] > F[t]$ which means the updated positions of sensors can enhance the observability of system. In this case, we go back to Step 1) to start the next calculation based on the updated sensors' positions.

Step 6) $F[t+1] < F[t]$ which means the updated positions of leaders cannot enhance the observability of system. Current position of sensor AUVs is the optimal formation of cooperation localization. The algorithm is to be terminated.

It is noteworthy that there is a constant exponential calculation in Step 3). In this calculation, by incrementing the constant in every step, the result value is reduced. In this way, the recursive moving distance of leader is reduced every step. Therefore, accuracy of the final result is increased.

4. **Simulation Results and Analysis.** In this section, the proposed algorithm is explained with two simulation examples: one for single target AUV localization and the other for multiple target AUVs localization.

4.1. **Optimal formation – Single target cooperative localization.** First of all, the simplest situation of one target and three sensors is taken into consideration. The value of σ depends upon the measurement sensors being used while the values of η and γ are dependent upon the underwater environmental conditions. In our simulation examples, we have used the values as 0.1, 0.05 and 1, respectively [1]. The target AUV is assumed to be placed just below origin at depth of 50 meters. Figure 1(a) shows the plot of $|FIM|$ in xy-plane. The plot shows maximum value at $(0,0)$ which is exactly where the target AUV is located. The optimal positions estimated for the three sensor AUVs are summarized in Table 1.



(a) (b)

FIGURE 1. (color online) (a) 3D grid map of evaluation function; (b) 2D contour map of evaluation function for single target localization

TABLE 1. Estimated optimal position of sensors (single-target system)

Coordinate	Sensor 1	Sensor 2	Sensor 3
x (m)	-25.40	31.01	55.92
y (m)	50.31	47.04	-3.64

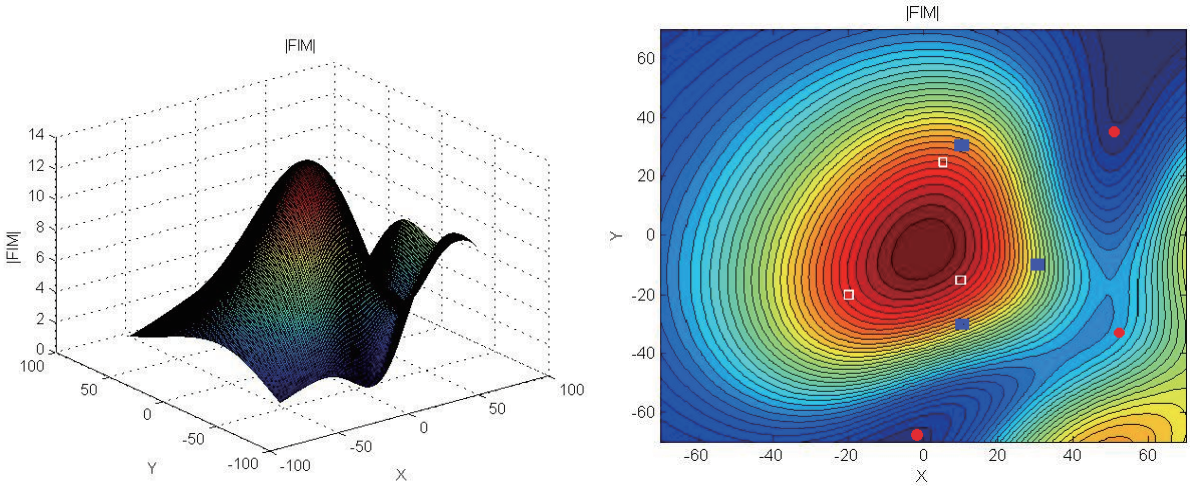
In Figure 1(b), evaluation function has been shown on a contour plot such that the value of evaluation function increases from blue to red color. The white hollow square represents the two-dimensional projection position of the target AUV; the red cross represents the two-dimensional projection of the starting position for the sensors and the red square represents their final optimal position.

4.2. **Optimal formation – Multiple target cooperative localization.** To check the efficacy of the proposed algorithm, simulation of three targets and three sensor AUVs system is performed for optimal formation. The measurement noise characteristics are taken same as previous example. The targets are assumed to be placed at $(-20, -20)$, $(10, -15)$ and $(5, 25)$ at depth of 50 meters. The optimal positions estimated for the three sensor AUVs are summarized in Table 2.

TABLE 2. Estimated optimal position of sensors (multi-target system)

Coordinate	Sensor 1	Sensor 2	Sensor 3
x (m)	-2.18	50.46	52.13
y (m)	-67.65	34.82	-33.19

Evaluation function plotted in a 3D map against the x-y coordinates is shown in Figure 2(a). In Figure 2(b), evaluation function has been shown on a contour plot such that the value of evaluation function increases from blue to red color. The white hollow square represents the two-dimensional projection position of target AUVs; the blue squares represent the starting positions for sensor in the recursive strategy and the red dots represent their final optimal positions.



(a) (b)

FIGURE 2. (color online) (a) 3D grid map of evaluation function for multiple target localization; (b) 2D contour map of evaluation function for multiple target localization

5. **Extended Kalman Filter Simulation Results.** To determine whether the evaluation function is able to actually decrease the measurement noise in real system, several simulations are performed based on Extended Kalman Filter (EKF). For this purpose, we have compared the measurement noises of the optimal formation and any random formation of the sensors. Considering the second example of three targets and three sensors, we choose two formations to simulate. Setting the simulation time for 1000 seconds for each target in different formations, we calculate the average value of noises in this period of time and combine three values from different sensors. By repeating the simulation 200 times, we have plotted the measurement noise for both the formations in Figure 3. The graph with blue plus sign represents the EKF noise of a random sensor formation and the graph with red diamond represents the EKF error of the optimal sensor formation obtained from the second example of Section 4. Figures 3(a), 3(b) and 3(c) show the average EKF error to the individual target AUVs with different sensor formations while Figure 3(d) shows the sum of the average EKF error of all the three targets.

From the above figure we can conclude that the optimal formation has maintained better accuracy for targets localization, not only for each target but also for the whole system. In 200 simulation cycles, the average noise error of random formation is 1.6745 meters and in the optimal formation the number has dropped to 1.3077 meters. In conclusion, the optimal formation is able to decrease the noise error and enhance the observability of the system.

6. **Conclusions.** In this paper, a method was proposed to achieve the optimal formation for multiple AUVs system under the influence of distance dependent acoustic noise. By using Cramer-Rao lower bound and Fisher information matrix theory, the evaluation function was derived and a recursive optimization algorithm was used to calculate the optimal solution for formation. Based on the theoretical analysis, we also obtained the numerical solution through evaluation function and extended Kalman filter. After several simulations with EKF, the conclusion is drawn that the optimal observability is achieved. Future work will focus on studying the sensors' trajectory planning for cooperative localization of moving underwater vehicles.

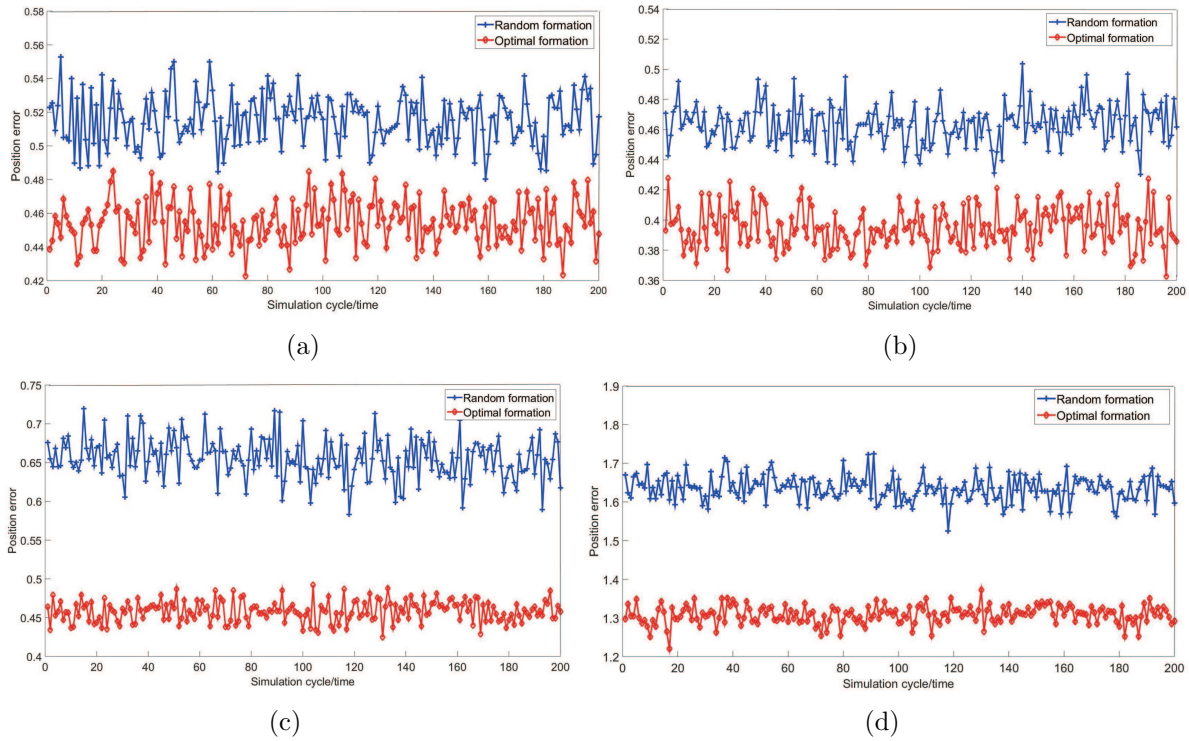


FIGURE 3. Mean EKF localization error for different formations

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