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ADMISSIBILITY AND DESIGN ISSUES FOR DISCRETE FUZZY SINGULAR DELAY SYSTEMS WITH MULTIPLE DIFFERENCE MATRICES

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Abstract. *This paper studies the problem of admissibility analysis and stabilizing controller design for discrete fuzzy singular delay systems with multiple difference matrices. By utilizing matrix algebraic manipulating, an explicit admissibility analysis condition for the nominal discrete singular delay system is originally derived. The distinct condition is different from existing ones, which can cope with the fuzzy singular delay system with distinct difference matrices in the rules. Based on Lyapunov stability theory associated with the linear matrix inequalities (LMIs) approach, we thus can investigate the admissibility analysis and controller design for the regarded systems. Since the proposed criteria are expressed in terms of LMIs or parametric LMIs, we can handily evaluate them via current LMI solvers. Finally, illustrative examples are given to show the validity and practicability of the proposed method.*

Keywords: Fuzzy inference systems, Discrete singular systems, Delay state, Admissibility, PDC (parallel distributed compensation) design, Linear matrix inequalities (LMIs)

1. **Introduction.** Singular systems can represent a larger class of physical and engineering systems than traditional state space systems, due to the fact that they can include dynamic behaviors and nondynamic constraints simultaneously, e.g., electrical network analysis [1], large-scale systems [2], and economic systems [3,4]. Over the past years, many researches have deeply investigated the stability and stabilization issues for singular systems and have attained diversity developments. They are alternatively represented as descriptor systems, implicit systems, generalized state-space systems, differential-algebraic systems or semi-state systems [2,5,6]. It needs to be emphasized that study of singular systems is much more complicated than the regular ones, since we need to pay more exertion for ensuring the regularity and impulse-free (or causality for discrete-time cases) (see, for example [7-10] and the references therein). Furthermore, time delay naturally arises in realistic control systems [11]. Thus, admissibility and admissibilization for singular delay systems have further been addressed recently (see, for example [12-15] and the references therein).

Many obstinate nonlinear systems or uncertain systems can be simplified and treated via fuzzy control strategies with appropriate modeling in the fuzzy rule base [16-18]. Over the past two decades, the fuzzy control has been popularly and successfully carried on variety fields. Tanaka and Sugeno [19] originally proposed the T-S fuzzy model, where the consequent parts of the rules are represented by linear state equations. Thus, by concatenating the fruitful techniques of the traditional state-space systems, the stability

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and stabilization of the T-S fuzzy models (see, for example, [20-23]) and T-S fuzzy delay models (see, for example, [24-27], and the references therein) have been deeply studied in this community. Recently, the T-S fuzzy singular modes and T-S fuzzy singular delay modes are investigated by many works (see, for example, [28-35], and the references therein). For the discrete time cases, Huang [29] first presented a discrete singular fuzzy model and discussed its admissibility analysis issue. Then, Xu et al. [31] attained some less conservative results. However, both results involve intractable non-strict linear matrix inequalities (LMIs) with a possibility insufficient rank in the matrix terms, which cannot be directly evaluated by current LMI tools and need some extra treatment [36,37]. In the sequel, the discrete fuzzy singular systems with uncertainty and distinct difference matrices were proposed [33]. And, the D-stability and nonfragile control for discrete fuzzy singular systems with multiple delays were also addressed [34]. However, all the existing works of discussing the discrete fuzzy singular delay models assume the system with a common difference matrix E in the rules, where it causes a heavy restriction for the system modeling [32]. To the best of our knowledge, the discrete singular delay systems with multiple difference matrices E_i seem not to be addressed in the past.

In this paper, we focus on the admissibility analysis and the controller design for the discrete fuzzy singular delay systems with multiple difference matrices E_i. An admissibility analysis condition is first derived for the nominal discrete singular delay system. Then, based on the condition, we can address the admissibility analysis for the discrete fuzzy singular systems with delay state and multiple difference matrices. Furthermore, when utilizing fuzzy PDC control law, we further study the controller design for the resulting closed-loop system. For facilitating verification, all the proposed criteria are formulated in terms of the strict LMIs [36]. And, we thus can handily verify them via a current LMI solver [37]. At length, numerical examples are given to illustrate the effectiveness and practicability of the proposed methods. The rest of this paper is organized as follows. Section 2 presents problem formulation and preliminaries. Some definitions for admissibility and design issues for the regarded nominal models are also been given. In Section 3, based on the Lyapunov stability theory, we address the admissibility analysis for the unforced systems. And, by involving the PDC controller, the resulting closed-loop systems are further treated in Section 4. In Section 5, two numerical examples are given to demonstrate the validity of the proposed results. Some concluding remarks are collected in Section 6.

2. **Problem Statement and Preliminaries.**

Consider a discrete fuzzy singular delay model with multiple difference matrices in the rules. This model represents a set of fuzzy rules, which the consequent parts are the discrete singular delay models with the individual difference matrix E_i. The proposed singular model can retain more meaningful expression of inherent structure modeling from physical systems with embracing nonlinear or uncertain parameters [32,33]. An overall system can be approximately formed by fuzzy "blending" from these locally singular delay models. The discrete fuzzy singular delay model is presented by

$$\begin{aligned} &\text{Rule } i\text{: If } \theta_1(k) \text{ is } F_1^i \text{ and } \theta_2(k) \text{ is } F_2^i \text{ and } \dots \text{ and } \theta_n(k) \text{ is } F_n^i \\ &\qquad \text{Then } E_i x(k+1) = A_i x(k) + A_{di} x(k-d) + B_i u(k), \quad k > 0,\ d > 0 \\ &\qquad\qquad x(j) = \phi(j), \quad j = -d, -d+1, \dots, 0, \qquad i = 1, 2, \dots, r, \end{aligned} \tag{1}$$

where $x(\cdot) \in R^n$ is the state vector, $u(\cdot) \in R^p$ is the control input, and $\theta_j(\cdot)$ is the jth premise variable, $j = 1, 2, \dots, n$, F_j^i is a fuzzy set, $E_i \in R^{n\times n}$ is a difference matrix and may be singular, i.e., $rank(E_i) = m \le n$, $A_i \in R^{n\times n}$, $A_{di} \in R^{n\times n}$ and $B_i \in R^{n\times p}$ denote the system's matrices in the corresponding rules. Thus, an overall system can be lumped as

$$\begin{aligned}\sum_{i=1}^{r} h_i(\theta(k))E_i x(k+1) &= \frac{\sum_{i=1}^{r}\omega_i(\theta(k))\left[A_i x(k) + A_{di}x(k-d) + B_i u(k)\right]}{\sum_{i=1}^{r}\omega_i(\theta(k))} \\ &= \sum_{i=1}^{r} h_i(\theta(k))\left[A_i x(k) + A_{di}x(k-d) + B_i u(k)\right],\end{aligned} \tag{2}$$

where

$$\begin{cases}\omega_i(\theta(k)) = \prod_{j=1}^{n} F_j^i(\theta_j(k)) \geq 0 \\ \sum_{i=1}^{r}\omega_i(\theta(k)) > 0\end{cases} \quad i = 1, 2, \ldots, r,$$

$$\begin{cases}h_i(\theta(k)) = \dfrac{\omega_i(\theta(k))}{\sum_{i=1}^{r}\omega_i(\theta(k))} \geq 0 \\ \sum_{i=1}^{r} h_i(\theta(k)) = 1\end{cases} \quad i = 1, 2, \ldots, r$$

and $F_j^i(\theta_j(k))$ is the grade of membership of $\theta_j(k)$ in F_j^i. For simplification, we denote $\widetilde{E} \equiv \sum_{i=1}^{r} h_i(\theta(k))E_i x(k+1)$, $\widetilde{A} \equiv \sum_{i=1}^{r} h_i(\theta(k))A_i$, $\widetilde{A}_d \equiv \sum_{i=1}^{r} h_i(\theta(k))A_{di}$, and $\widetilde{B} \equiv \sum_{i=1}^{r} h_i(\theta(k))B_i$.

To investigate the stability and design issues for the system (2), definitions for discrete singular system are given as follows.

Definition 2.1. [12].

I. The pair (E, A) is said to be regular if $\det(zE - A)$ *is not identically zero.*
II. The pair (E, A) is said to be causal, if it is regular and $\deg(\det(zE-A)) = rank(E)$.
III. Let $\rho(E, A) \equiv \max_{\lambda\in\{z|\det(zE-A)=0\}} |\lambda|$*, the pair (E, A) is said to be stable if* $\rho(E, A) < 1$.
IV. The pair (E, A) is said to be admissible, if it is regular, causal, and stable.

Remark 2.1. *The stability issues of discrete singular systems are much more complicated than the state-space systems. In addition to verifying $\rho(E, A) < 1$ for stability assurance, we need to pay more exertion for ensuring the regularity and the causality simultaneously of the discrete singular system, where the admissibility issues for the nominal system are given in Definition 2.1.*

Definition 2.2. [12].

I. The triple (E, A, A_d) is said to be regular if $\det\left(z^{d+1}E - z^d A - A_d\right)$ *is not identically zero.*
II. The triple (E, A, A_d) is said to be causal, if it is regular and $\deg\left(z^{nd}\det\left(zE - A - z^{-d}A_d\right)\right) = nd + rank(E)$.
III. The discrete singular delay system $Ex(k+1) = Ax(k) + A_d x(k-d)$ is said to be stable, if $\rho(E, A, A_d) < 1$, where $\rho(E, A, A_d) \equiv \max_{\lambda\in\left\{z\,\middle|\,\det\left(z^{d+1}E-z^dA-A_d\right)=0\right\}} |\lambda|$.
IV. The discrete singular delay system $Ex(k+1) = Ax(k) + A_d x(k-d)$ (i.e., the triple (E, A, A_d)) is said to be admissible, if it is regular, causal, and stable.

Remark 2.2. *The stability issues in Definition 2.1 are extended to Definition 2.2 for further addressing the discrete singular delay system. In addition to verifying $\rho(E, A, A_d) < 1$ for stability assurance, we need to pay more exertion for ensuring the regularity and the* *causality simultaneously of the discrete singular system, where the admissibility issues for the nominal delay system are given in Definition 2.2.*

Lemma 2.1. [12]. *The nominal discrete singular delay system $Ex(k+1) = Ax(k) + A_d x(k-d)$ is regular, causal, and stable if and only if the pair (E, A) is regular, causal, and $\rho(E, A, A_d) < 1$.*

Lemma 2.2. [13]. *The triple (E, A, A_d) is admissible if and only if the corresponding transpose form $\left(E^T, A^T, A_d^T\right)$ is admissible.*

Based on Definition 2.1 and Definition 2.2 associated with Lemma 2.1, admissibility analysis issues can be addressed for the unforced discrete fuzzy singular delay systems, i.e., $u(k) \equiv 0$ in (2), in the following.

3. **Admissibility Analysis.** A sufficient condition for the unforced nominal singular delay system $Ex(k+1) = Ax(k) + A_d x(k-d)$ is derived in advance.

Theorem 3.1. *The nominal singular delay system $Ex(k+1) = Ax(k) + A_d x(k-d)$ is admissible if there exist matrices $P > 0$, $R > 0$ and Q with appropriate dimensions such that*

$$\begin{bmatrix} A^TPA - E^TPE + A^TSQ^T + QS^TA + R & \left(PA + SQ^T\right)^T A_d \\ A_d^T\left(PA + SQ^T\right) & A_d^TPA_d - R \end{bmatrix} < 0, \tag{3}$$

where $S \in R^{n\times(n-m)}$ is any matrix with full column rank and satisfies $E^TS = 0$.

Proof: Assume that matrices $P > 0$, $R > 0$ and Q satisfy (3). For the matrix E with $rank(E) = m \le n$, there exist nonsingular matrices M_1, $N_1 \in R^{n\times n}$ such that

$$M_1EN_1 = \begin{bmatrix} I_m & 0 \\ 0 & 0 \end{bmatrix}, \quad M_1AN_1 = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, \tag{4a}$$

$$M_1^{-T}PM_1^{-1} = \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix}, \quad M_1^{-T}SQ^TN_1 = \begin{bmatrix} 0 & 0 \\ S_1 & S_2 \end{bmatrix}, \quad N_1^TRN_1 = \begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix}. \tag{4b}$$

Using the expression in (4), deducing from the block $(1,1)$ of Equation (3) leads to

$$\begin{aligned}
&A^TPA - E^TPE + A^TSQ^T + QS^TA + R \\
&= N_1^{-T}\left(\begin{bmatrix} A_1^T & A_3^T \\ A_2^T & A_4^T \end{bmatrix}\begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix}\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} - \begin{bmatrix} I_m & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix}\begin{bmatrix} I_m & 0 \\ 0 & 0 \end{bmatrix}\right. \\
&\quad \left. + \begin{bmatrix} A_1^T & A_3^T \\ A_2^T & A_4^T \end{bmatrix}\begin{bmatrix} 0 & 0 \\ S_1 & S_2 \end{bmatrix} + \begin{bmatrix} 0 & S_1^T \\ 0 & S_2^T \end{bmatrix}\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} + \begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix}\right) N_1^{-1} \\
&= N_1^{-T}\left\{\begin{bmatrix} \Theta & \Theta \\ \Theta & A_2^TP_1A_2 + A_4^TP_2^TA_2 + A_2^TP_2A_4 + A_4^TP_3A_4 + A_4^TS_2 + S_2^TA_4 + R_3 \end{bmatrix}\right\} N_1^{-1} \\
&= N_1^{-T}\left\{\begin{bmatrix} \Theta & \Theta \\ \Theta & A_4^T\left(P_2^TA_2 + S_2\right) + \left(P_2^TA_2 + S_2\right)^T A_4 + \left(A_2^TP_1A_2 + A_4^TP_3A_4 + R_3\right) \end{bmatrix}\right\} N_1^{-1} \\
&< 0,
\end{aligned}$$

where 'Θ' represents a not utilized matrix. Due to $P_1 > 0$, $P_3 > 0$ and $R_3 > 0$, the last inequality implies $A_4^T\left(P_2^TA_2 + S_2\right) + \left(P_2^TA_2 + S_2\right)^T A_4 < 0$, i.e., the nonsigularity of $\left(P_2^TA_2 + S_2\right)^T A_4$, and so does A_4. From Definition 2.1 associated with Lemma 2.1, the nominal discrete singular delay system is assured to be regular and causal.

Let a Lyapunov-Krasovskii functional candidate as

$$V(k) = x^T(k)E^T PEx(k) + \sum_{j=1}^{d} x(k-j)^T Rx(k-j).$$

Based on the Lyapunov quadratic stability theory, the difference $V(k)$ can be manipulated as

$$\begin{aligned}
&\Delta V(k) \\
&= V(k+1) - V(k) \\
&= x^T(k+1)E^T PEx(k+1) + \sum_{j=1}^{d} x(k+1-j)^T Rx(k+1-j) - x^T(k)E^T PEx(k) \\
&\quad + \sum_{j=1}^{d} x(k-j)^T Rx(k-j) \\
&= x^T(k+1)E^T PEx(k+1) - x^T(k)E^T PEx(k) + x^T(k)Rx(k) - x^T(k-d)Rx(k-d).
\end{aligned}$$

By substituting the considered system into $\Delta V(k)$ and using the property $E^T S = 0$ lead to

$$\begin{aligned}
&\Delta V(k) \\
&= x^T(k+1)E^T PEx(k+1) - x^T(k)E^T PEx(k) + x^T(k)Rx(k) - x^T(k-d)Rx(k-d) \\
&\quad + x^T(k+1)E^T SQ^T x(k) + x(k)QS^T Ex(k+1) \\
&= (Ax(k) + A_d x(k-d))^T P (Ax(k) + A_d x(k-d)) - x^T(k)E^T PEx(k) + x^T(k)Rx(k) \\
&\quad - x^T(k-d)Rx(k-d) + (Ax(k) + A_d x(k-d))^T SQ^T x(k) \\
&\quad + x(k)QS^T (Ax(k) + A_d x(k-d)) \\
&= x^T(k) \left(A^T PA - E^T PE + A^T SQ^T + QS^T A + R\right) x(k) \\
&\quad + 2x^T(k) \left(A^T P + QS^T\right) A_d x(k-d) + x^T(k-d) \left(A_d^T PA_d - R\right) x(k-d) \\
&= \zeta^T(k) \begin{bmatrix} A^T PA - E^T PE + A^T SQ^T + QS^T A + R & \left(PA + SQ^T\right)^T A_d \\ A_d^T \left(PA + SQ^T\right) & A_d^T PA_d - R \end{bmatrix} \zeta(k),
\end{aligned}$$

where $\zeta(k) \equiv \begin{bmatrix} x^T(k) & x^T(k-d) \end{bmatrix}^T$. From (3), we obtain $\Delta V(k) < 0$, and the considered system is asserted to be stable. Together with regularity and causality, the system is thus concluded to be admissible according to Definition 2.2 associated with Lemma 2.1. □

Deducing from Lemma 2.2, the stability criteria in Theorem 3.1 for the triple (E, A, A_d) can also be represented for coping with the transpose form $\left(E^T, A^T, A_d^T\right)$ and presented as follows.

Corollary 3.1. *The nominal singular delay system $Ex(k+1) = Ax(k) + A_d x(k-d)$ is admissible if there exist matrices $P > 0$, $R > 0$ and Q with appropriate dimensions such that*

$$\begin{bmatrix} APA^T - EPE^T + ASQ^T + QS^T A^T + R & \left(PA^T + SQ^T\right)^T A_d^T \\ A_d \left(PA^T + SQ^T\right) & A_d PA_d^T - R \end{bmatrix} < 0,$$

where $S \in R^{n\times(n-m)}$ is any matrix with full column rank and satisfies $ES = 0$.

Thus, an admissibility criterion for the unforced discrete fuzzy singular system with delay state and multiple difference matrices, i.e., $u(k) \equiv 0$ in (2), is mainly derived as follows.

Theorem 3.2. *The unforced discrete fuzzy singular delay system (2) is admissible if there exist matrices $P > 0$, $R > 0$ and Q with appropriate dimensions such that*

$$\begin{bmatrix} 2A_i^TPA_i - E_g^TPE_t - E_t^TPE_g + 2A_i^TSQ^T + 2QS^TA_i + 2R & 2(PA_i + SQ^T)^TA_{dl} \\ 2A_{dl}^T(PA_i + SQ^T) & 2A_{dl}^TPA_{dl} - 2R \end{bmatrix} < 0$$

$$\forall i, l, g \le t, \tag{5}$$

where $S \in R^{n\times(n-m)}$ is any matrix with full column rank and satisfies $E_g^TS = 0$, $\forall g$.

Proof: Assume that there exist matrices $P > 0$, $R > 0$ and Q such that (5) holds. By (5) associated with $h_i \ge 0$, $\forall i$, $\sum_i h_i = 1$ defined in (2) with respect to A_i, we have

$$\begin{aligned}
&\begin{bmatrix} 2\widetilde{A}^TP\widetilde{A} - E_g^TPE_t - E_t^TPE_g + 2\widetilde{A}^TSQ^T + 2QS^T\widetilde{A} + 2R & 2\left(P\widetilde{A} + SQ^T\right)^TA_{dl} \\ 2A_{dl}^T\left(P\widetilde{A} + SQ^T\right) & 2A_{dl}^TPA_{dl} - 2R \end{bmatrix} \\
&= \begin{bmatrix} \begin{array}{l} 2\left(\sum_i h_iA_i\right)^TP\left(\sum_i h_iA_i\right) - E_g^TPE_t - E_g^TPE_t \\ +2\left(\sum_i h_iA_i\right)^TSQ^T + 2QS^T\left(\sum_i h_iA_i\right) + 2R \end{array} & 2\left(P\left(\sum_i h_iA_i\right)^T + SQ^T\right)^TA_{dl} \\ 2A_{dl}^T\left(P\left(\sum_i h_iA_i\right)^T + SQ^T\right) & 2A_{dl}^TPA_{dl} - 2R \end{bmatrix} \\
&= \sum_i h_i^2 \begin{bmatrix} 2A_i^TPA_i - E_g^TPE_t - E_h^TPE_t + 2A_i^TSQ^T + 2QS^TA_i + 2R & 2\left(PA_i + SQ^T\right)^TA_{dl} \\ 2A_{dl}^T\left(PA_i + SQ^T\right) & 2A_{dl}^TPA_{dl} - 2R \end{bmatrix} \\
&\quad + \sum_{i<j} h_ih_j \begin{bmatrix} \begin{array}{l} 2A_i^TPA_j + 2A_j^TPA_i - 2E_g^TPE_t - 2E_t^TPE_g \\ +2(A_i + A_j)^TSQ^T + 2QS^T(A_i + A_j) + 4R \end{array} & 2\left(P(A_i + A_j) + 2SQ^T\right)^TA_{dl} \\ 2A_{dl}^T\left(P(A_i + A_j) + 2SQ^T\right) & 4A_{dl}^TPA_{dl} - 4R \end{bmatrix} \\
&\le \sum_i h_i^2 \begin{bmatrix} 2A_i^TPA_i - E_g^TPE_t - E_t^TPE_g + 2A_i^TSQ^T + 2QS^TA_i + 2R & 2\left(PA_i + SQ^T\right)^TA_{dl} \\ 2A_{dl}^T\left(PA_i + SQ^T\right) & 2A_{dl}^TPA_{dl} - 2R \end{bmatrix} \\
&\quad + \sum_{i<j} h_ih_j \begin{bmatrix} \begin{array}{l} 2A_i^TPA_i + 2A_j^TPA_j - 2E_g^TPE_t - 2E_t^TPE_g \\ +2(A_i + A_j)^TSQ^T + 2QS^T(A_i + A_j) + 4R \end{array} & 2\left(P(A_i + A_j) + 2SQ^T\right)^TA_{dl} \\ 2A_{dl}^T\left(P(A_i + A_j) + 2SQ^T\right) & 4A_{dl}^TPA_{dl} - 4R \end{bmatrix} \\
&< 0, \quad \forall l, g \le h.
\end{aligned}$$

Similarly, by the above inequalities associated with $h_l \ge 0$, $\forall l$, $\sum_l h_l = 1$ defined in (2) with respect to A_{dl}, we have

$$\begin{aligned}
&\begin{bmatrix} 2\widetilde{A}^TP\widetilde{A} - E_g^TPE_t - E_t^TPE_g + 2\widetilde{A}^TSQ^T + 2QS^T\widetilde{A} + 2R & 2\left(P\widetilde{A} + SQ^T\right)^T\widetilde{A}_d \\ 2\widetilde{A}_d^T\left(P\widetilde{A} + SQ^T\right) & 2\widetilde{A}_d^TP\widetilde{A}_d - 2R \end{bmatrix} \\
&= \begin{bmatrix} 2\widetilde{A}^TP\widetilde{A} - E_g^TPE_t - E_t^TPE_g + 2\widetilde{A}^TSQ^T + 2QS^T\widetilde{A} + 2R & 2\left(P\widetilde{A} + SQ^T\right)^T\left(\sum_l h_lA_{dl}\right) \\ 2\left(\sum_l h_lA_{dl}\right)^T\left(P\widetilde{A} + SQ^T\right) & 2\left(\sum_l h_lA_{dl}\right)^TP\left(\sum_l h_lA_{dl}\right) - 2R \end{bmatrix} \\
&= \sum_l h_l^2 \begin{bmatrix} 2\widetilde{A}^TP\widetilde{A} - E_g^TPE_t - E_t^TPE_g + 2\widetilde{A}^TSQ^T + 2QS^T\widetilde{A} + 2R & 2\left(P\widetilde{A} + SQ^T\right)^TA_{dl} \\ 2A_{dl}^T\left(P\widetilde{A} + SQ^T\right) & 2A_{dl}^TPA_{dl} - 2R \end{bmatrix}
\end{aligned}$$

$$+\sum_{l<j} h_l h_j \begin{bmatrix} 4\widetilde{A}^T P\widetilde{A} - 2E_g^T PE_t - 2E_t^T PE_g + 4\widetilde{A}^T SQ^T \\ +4QS^T\widetilde{A} + 4R & 2\left(P\widetilde{A} + SQ^T\right)^T (A_{dl} + A_{dj}) \\ 2(A_{dl} + A_{dj})^T \left(P\widetilde{A} + SQ^T\right) & 2A_{dl}^T PA_{dj} + 2A_{dj}^T PA_{dl} - 4R \end{bmatrix}$$

$$\leq \sum_l h_l^2 \begin{bmatrix} 2\widetilde{A}^T P\widetilde{A} - E_g^T PE_t - E_t^T PE_g + 2\widetilde{A}^T SQ^T + 2QS^T\widetilde{A} + 2R & 2\left(P\widetilde{A} + SQ^T\right)^T A_{dl} \\ 2A_{dl}^T \left(P\widetilde{A} + SQ^T\right) & 2A_{dl}^T PA_{dl} - 2R \end{bmatrix}$$

$$+\sum_{l<j} h_l h_j \begin{bmatrix} 4\widetilde{A}^T P\widetilde{A} - 2E_g^T PE_t - 2E_t^T PE_g + 4\widetilde{A}^T SQ^T \\ +4QS^T\widetilde{A} + 4R & 2\left(P\widetilde{A} + SQ^T\right)^T (A_{dl} + A_{dj}) \\ 2(A_{dl} + A_{dj})^T \left(P\widetilde{A} + SQ^T\right) & 2A_{dl}^T PA_{dl} + 2A_{dj}^T PA_{dj} - 4R \end{bmatrix}$$

$$< 0, \quad \forall g \leq h.$$

Based on Theorem 3.1, by the above inequalities and involving the parameters $h_g \geq 0$, $\forall g$, $\sum_g h_g = 1$ with respect to E_g, deducing from (3) for the unforced system (2) leads to

$$\begin{bmatrix} \widetilde{A}^T P\widetilde{A} - \widetilde{E}^T P\widetilde{E} + \widetilde{A}^T SQ^T + QS^T\widetilde{A} + R & \left(P\widetilde{A} + SQ^T\right)^T \widetilde{A}_d \\ \widetilde{A}_d^T \left(P\widetilde{A} + SQ^T\right) & \widetilde{A}_d^T P\widetilde{A}_d - R \end{bmatrix}$$

$$= \begin{bmatrix} \widetilde{A}^T P\widetilde{A} - \left(\sum_g e_g E_g\right)^T P \left(\sum_g e_g E_g\right) + \widetilde{A}^T SQ^T + QS^T\widetilde{A} + R & \left(P\widetilde{A} + SQ^T\right)^T \widetilde{A}_d \\ \widetilde{A}_d^T \left(P\widetilde{A} + SQ^T\right) & \widetilde{A}_d^T P\widetilde{A}_d - R \end{bmatrix}$$

$$= \sum_g h_g^2 \begin{bmatrix} \widetilde{A}^T P\widetilde{A} - E_g^T PE_g + \widetilde{A}^T SQ^T + QS^T\widetilde{A} + R & \left(P\widetilde{A} + SQ^T\right)^T \widetilde{A}_d \\ \widetilde{A}_d^T \left(P\widetilde{A} + SQ^T\right) & \widetilde{A}_d^T P\widetilde{A}_d - R \end{bmatrix}$$

$$+\sum_{g<t} h_g h_t \begin{bmatrix} 2\widetilde{A}^T P\widetilde{A} - E_g^T PE_t - E_t^T PE_g + 2\widetilde{A}^T SQ^T \\ +2QS^T\widetilde{A} + 2R & 2\left(P\widetilde{A} + SQ^T\right)^T \widetilde{A}_d \\ 2\widetilde{A}_d^T \left(P\widetilde{A} + SQ^T\right) & 2\widetilde{A}_d^T P\widetilde{A}_d - 2R \end{bmatrix}$$

$$< 0.$$

Thus, the unforced discrete fuzzy singular delay system (2) is asserted to be admissible according to Theorem 3.1. □

According to Lemma 2.2, the stability criteria in Theorem 3.2 for the unforced system (2) can be alternatively represented of the symmetric manner and presented as follows.

Corollary 3.2. *The unforced discrete singular delay system (1) is admissible if there exist matrices $P > 0$, $R > 0$ and Q with appropriate dimensions such that*

$$\begin{bmatrix} 2A_i PA_i^T - E_g PE_t^T - E_t PE_g^T + 2A_i SQ^T + 2QS^T A_i^T + 2R & 2\left(PA_i^T + SQ^T\right)^T A_{dl}^T \\ 2A_{dl}\left(PA_i^T + SQ^T\right) & 2A_{dl} PA_{dl}^T - 2R \end{bmatrix} < 0$$

$$\forall i, l, g \leq h, \tag{6}$$

where $S \in R^{n\times(n-m)}$ is any matrix with full column rank and satisfies $E_g S = 0$, $\forall g$.

Deducing from Corollary 3.2, by involving the fuzzy parallel distributed compensation (PDC) control law [18,19], we further address the controller design for the resulting closed-loop system in (2) in the sequel.

4. **PDC Control Law.** By involving the fuzzy PDC control law, the fuzzy control rules share the same fuzzy sets with the fuzzy system in (1) and are given by

Rule i: If $\theta_1(k)$ is F_1^i and $\theta_2(k)$ is F_2^i and ... and $\theta_j(k)$ is F_j^i, Then $u(k) = K_i x(k)$, $i = 1, 2, \ldots, r$.

An overall fuzzy controller thus can be inferred as

$$u(k) = \frac{\sum_{i=1}^{r} \omega_i(\theta(k)) K_i x(k)}{\omega_i(\theta(k))} = \sum_{i=1}^{r} h_i(\theta(k)) K_i x(k). \tag{7}$$

Substituting (7) into (2) yields

$$\begin{aligned}
&\sum_{i=1}^{r} h_i(\theta(k)) E_i x(k+1) \\
&= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\theta(k)) h_j(\theta(k)) \left\{ (A_i + B_i K_j)\, x(k) + A_{di} x(k-d) \right\} \\
&= \left\{ \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\theta(k)) h_j(\theta(k)) \, (A_i + B_i K_j) \right\} x(k) + \sum_{i=1}^{r} h_i(\theta(k)) A_{di} x(k-d) \\
&= A_C x(k) + \sum_{i=1}^{r} h_i(\theta(k)) A_{di} x(k-d),
\end{aligned} \tag{8}$$

where $A_C \equiv \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\theta(k)) h_j(\theta(k)) \, (A_i + B_i K_j)$.

For systematic design of a stabilizing controller (7), a sufficient condition for the resulting closed-loop system (8) is derived in the following.

Theorem 4.1. *The resulting closed-loop discrete fuzzy singular delay system (8) is admissible, if there exist matrices $P > 0$, $R > 0$, Q with appropriate dimensions, and scalars σ_i such that*

$$\begin{bmatrix} \Psi_{11} & * & * \\ 2A_{dl}\left(PA_i^T + \sigma_i P E_i^T B_i B_i^T + SQ^T\right) & 2A_{dl} P A_{dl}^T - 2R & * \\ PA_i^T + \sigma_i P E_i^T B_i B_i^T & 0 & -\frac{1}{2}P \end{bmatrix} < 0 \quad \forall i, l, g \le t, \tag{9a}$$

$$\begin{bmatrix} \Psi_{12} & * & * \\ 2A_{dl}\left(P\left(A_i + A_j\right)^T + P\left(\sigma_j B_i B_j^T E_j + \sigma_i B_j B_i^T E_i\right)^T + 2SQ^T\right) & 4A_{dl} P A_{dl}^T - 4R & * \\ P\left(A_i + A_j\right)^T + P\left(\sigma_j B_i B_j^T E_j + \sigma_i B_j B_i^T E_i\right)^T & 0 & -P \end{bmatrix} < 0$$

$$\forall i < j, l, g \le t, \tag{9b}$$

where $\Psi_{11} \equiv 2A_i S Q^T + 2QS^T A_i^T - E_g P E_t^T - E_t P E_g^T + 2R$, $\Psi_{12} \equiv 2(A_i + A_j)SQ^T + 2QS^T(A_i + A_j)^T - 2E_g P E_t^T - 2E_t P E_g^T + 4R$, the matrix $S \in R^{n \times (n-m)}$ is of full column rank and satisfies $E_g S = 0$, $\forall g$. Then, a set of stabilizing PDC gains in (7) can be denoted as $K_i = \sigma_i B_i^T E_i$, $\forall i$.

Proof: Based on Corollary 3.2, the resulting closed-loop system (8) is admissible if there exist matrices $P > 0$, $R > 0$ and Q with appropriate dimensions such that

$$\begin{bmatrix} 2A_C P A_C^T - E_g P E_t^T - E_t P E_g^T + 2A_i S Q^T + 2QS^T A_i^T + 2R & 2\left(PA_C^T + SQ^T\right)^T A_{dl}^T \\ 2A_{dl}\left(PA_C^T + SQ^T\right) & 2A_{dl} P A_{dl}^T - 2R \end{bmatrix} < 0$$

$$\forall l, g \le t, \tag{10}$$

where $A_C \equiv \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(k))h_j(\theta(k))(A_i + B_iK_j)$ and $S \in R^{n\times(n-m)}$ is of full column rank and satisfies $E_gS = 0$, $\forall g$. By the Schur complement [36] it is equivalent to (10) that

$$\begin{bmatrix} 2A_CSQ^T + 2QS^TA_C^T - E_gPE_t^T - E_tPE_g^T + 2R & 2\left(PA_C^T + SQ^T\right)^T A_{dl}^T & A_CP \\ 2A_{dl}\left(PA_C^T + SQ^T\right) & 2A_{dl}PA_{dl}^T - 2R & 0 \\ PA_C^T & 0 & -\frac{1}{2}P \end{bmatrix} < 0$$

$$\forall l, g \le t. \tag{11}$$

Let $K_i = \sigma_i B_i^T E_i$, $\forall i$. Assume that there exist matrices $P > 0$, Q, and scalars σ_i such that Equation (9) holds. By (11), the parameters $h_i \ge 0$, $\forall i$, $\sum_i h_i = 1$ with respect to A_i, and $h_j \ge 0$, $\forall j$, $\sum_j h_j = 1$ with respect to K_j, deducing from (11) associated with $A_C = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(k))h_j(\theta(k))\,(A_i + B_iK_j)$ leads to

$$\begin{aligned}
&\begin{bmatrix} 2A_CSQ^T + 2QS^TA_C^T - E_gPE_t^T - E_tPE_g^T + 2R & 2\left(PA_C^T + SQ^T\right)^T A_{dl}^T & A_CP \\ 2A_{dl}\left(PA_C^T + SQ^T\right) & 2A_{dl}PA_{dl}^T - 2R & 0 \\ PA_C^T & 0 & -\frac{1}{2}P \end{bmatrix} \\
&= \begin{bmatrix} \begin{array}{l} 2\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(k))h_j(\theta(k)(A_i + B_iK_j)SQ^T \\ +2QS^T\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(k))h_j(\theta(k))\,(A_i + B_iK_j)^T - E_gPE_t^T - E_tPE_g^T + 2R \end{array} & * & * \\ 2A_{dl}\left(P\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(k))h_j(\theta(k))(A_i + B_iK_j)^T + SQ^T\right) & 2A_{dl}^PA_{dl}^T - 2R & * \\ P\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(k))h_j(\theta(k))(A_i + B_iK_j)^T & 0 & -\frac{1}{2}P \end{bmatrix} \\
&= \sum_i h_i^2\left(\begin{bmatrix} 2(A_i + B_iK_i)SQ^T + 2QS^T(A_i + B_iK_j) - E_gPE_t^T - E_tPE_g^T + 2R & * & * \\ 2A_{dl}\left((A_i + B_iK_i)^T + SQ^T\right) & 2A_{dl}PA_{dl}^T - 2R & * \\ P\,(A_i + B_iK_i)^T & 0 & -\frac{1}{2}P \end{bmatrix}\right) \\
&\quad + \sum_{i<j} h_ih_j\left(\begin{bmatrix} \begin{array}{l} 2(A_i + B_iK_j + A_j + B_jK_i)SQ^T \\ +2QS^T(A_i + B_iK_j + A_j + B_jK_i)^T - 2E_gPE_t^T - 2E_tPE_g^T + 4R \end{array} & * & * \\ 2A_{dl}\left((A_i + B_iK_j + A_j + B_jK_i)^T + 2SQ^T\right) & 4A_{dl}PA_{dl}^T - 4R & * \\ P(A_i + B_iK_j + A_j + B_jK_i)^T & 0 & -P \end{bmatrix}\right) \\
&\quad \sum_i h_i^2\left(\begin{bmatrix} \Psi_{11} & * & * \\ 2A_{dl}\left(PA_i^T + \sigma_iPE_i^TB_iB_i^T + SQ^T\right) & 2A_{dl}PA_{dl}^T - 2R & * \\ PA_i^T + \sigma_iPE_i^TB_iB_i^T & 0 & -\frac{1}{2}P \end{bmatrix}\right) \\
&\quad + \sum_{i<j} h_ih_j\left(\begin{bmatrix} \Psi_{12} & * & * \\ 2A_{dl}\left(P(A_i + A_j)^T + P\left(\sigma_jB_iB_j^TE_j + \sigma_iB_jB_i^TE_i\right)^T + 2SQ^T\right) & 4A_{dl}PA_{dl}^T - 4R & * \\ P(A_i + A_j)^T + P\left(\sigma_jB_iB_j^TE_j + \sigma_iB_jB_i^TE_i\right)^T & 0 & -P \end{bmatrix}\right) \\
&< 0 \quad \forall l, g \le t,
\end{aligned}$$

where $\Psi_{11} \equiv 2A_iSQ^T + 2QS^TA_i^T - E_gPE_t^T - E_tPE_g^T + 2R$, $\Psi_{12} \equiv 2(A_i + A_j)SQ^T + 2QS^T(A_i + A_j)^T - 2E_gPE_t^T - 2E_tPE_g^T + 4R$.

Thus, the resulting closed-loop system (8) is asserted to be admissible according to Corollary 3.2. □

Remark 4.1. *In Theorem 4.1, the PDC synthesis conditions are explicitly formulated in terms of parametric LMIs, so we can readily evaluate them via the LMI solver [37]. And, if a set of the feasible solutions is attained, we thus systematically evaluate a set of feedback gains as* $K_i = \sigma_i B_i^T E_i$, $\forall i$, *which achieve the resulting closed-loop system (8) with admissibility assurance.*

5. **Illustrative Examples.** We demonstrate the validity and practicability of the proposed approach by the following examples.

Example 5.1. *Consider a 3rd-order discrete fuzzy singular delay model in (2) with three rules. The systems' matrices of the singular delay models are respectively given as*

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0.6 & 0.4 & 0 \\ 0.3 & 0.4 & 0.4 \\ 0.2 & 0.5 & 0.6 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0.2 & 0.1 & 0 \\ 0.1 & 0.3 & 0.3 \\ 0.1 & 0.2 & 0.1 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.4 & -0.3 & 0 \\ 0.2 & -0.3 & 0.1 \\ 0.2 & 0.3 & 0.8 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -0.2 & -0.1 & 0 \\ 0.1 & -0.1 & 0 \\ 0.1 & 0.2 & 0.1 \end{bmatrix}$$

$$E_3 = \begin{bmatrix} 1 & 0.5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0.3 & 0.2 & 0.1 \\ -0.2 & 0.4 & 0 \\ -0.2 & 0 & 0.6 \end{bmatrix}, \quad A_{d3} = \begin{bmatrix} 0.1 & 0.1 & 0 \\ -0.1 & 0.2 & 0 \\ -0.1 & 0 & 0.1 \end{bmatrix}.$$

Since the considered discrete fuzzy delay singular model involves multiple matrices E_i in the rules, previous works [31,34] cannot be applied for admissibility analysis. Whereas, based on Theorem 3.2 with the given matrix $S = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$ satisfying $E_i^T S = 0$, $i = 1, 2, 3$, the analysis verification can be formed as a set of strict LMIs by (5). By utilizing the LMI solver [37] for evaluation, we then obtain a set of feasible solution and the satisfied matrices are shown as

$$P = \begin{bmatrix} 16.7314 & -2.2783 & -4.1417 \\ -2.2783 & 21.8618 & -4.9370 \\ -4.1417 & -4.9370 & 12.0321 \end{bmatrix} > 0, \quad R = \begin{bmatrix} 3.4791 & 2.0289 & 1.1545 \\ 2.0289 & 9.6080 & 5.4945 \\ 1.1545 & 5.4945 & 6.3226 \end{bmatrix} > 0,$$

$$Q = \begin{bmatrix} -2.2009 \\ -6.6986 \\ -13.9015 \end{bmatrix}.$$

Thus, the considered system is asserted to be admissible according to Theorem 3.2.

Example 5.2. *Consider a third-order discrete fuzzy singular system with delay state:*

Rule 1: If $x_1(k)$ is F_1
Then $E_1 x(k+1) = A_1 x(k) + A_{d1} x(k-1) + B_1 u(k)$,
Rule 2: If $x_1(k)$ is F_2
Then $E_2 x(k+1) = A_2 x(k) + A_{d2} x(k-1) + B_2 u(k)$,

where membership functions F_1 and F_2 are depicted in Figure 1. The systems' matrices are given as

$$E_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 1 & 1.2 & 0.2 \\ 0.3 & 1.4 & 0.2 \\ -0.2 & 0.3 & 0.5 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0.2 & 0.3 & 0 \\ 0.1 & 0.4 & 0.1 \\ -0.1 & 0.1 & 0.2 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1.5 & -1 & -0.4 \\ -1.2 & -0.7 & 0.2 \\ 0.2 & 0.5 & 0.8 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} -0.2 & -0.3 & 0 \\ 0.1 & -0.2 & -0.1 \\ 0.2 & 0.2 & 0.1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}.$$

For the unforced singular delay system, i.e., the regarded system with $u(t) \equiv 0$, the states' variables with a denoted initial condition $x(0) = [\,5 \;\; 3 \;\; 3.125\,]^T$ are firstly simulated and drawn in Figure 2. Observing on Figure 2, it apparently shows that the unforced

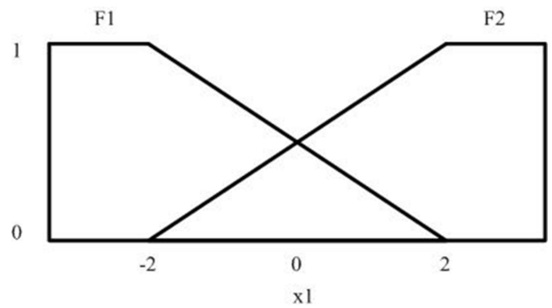

FIGURE 1. Membership functions for Example 5.2

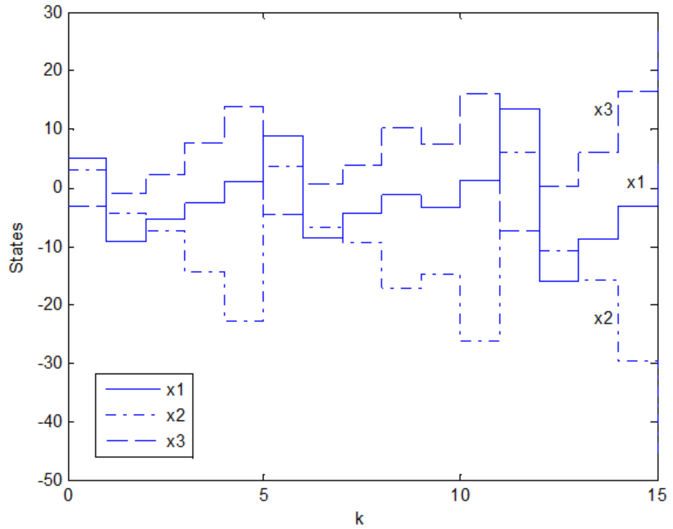

FIGURE 2. Uncompensated states' responses

system is unstable and a stabilizing control law needs to be equipped. Due to the fact that the regarded system includes multiple E_i*, the past works [31,34] cannot be utilized. Whereas, from the proposed result in Theorem 4.1 with giving a priori matrix* $S = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$ *satisfying* $E_iS = 0$, $\forall i$*, controller design criteria for the closed-loop model can be formulated by a set of parametric LMIs by (9). When evaluating them via a programming design associated with the LMI solver, and given parametric intervals as* $a_1 \in [-10, 10]$ *and* $a_2 \in [-10, 10]$*, we then achieve a set of feasible solutions as*

$$a_1 = -0.6, \quad a_2 = 0.4, \quad P = \begin{bmatrix} 8.2046 & -2.6428 & 2.8843 \\ -2.6428 & 3.7831 & -2.3892 \\ 2.8843 & -2.3892 & 6.9413 \end{bmatrix} \times 10^{-2} > 0,$$

$$R = \begin{bmatrix} 15.1296 & 3.9859 & -4.1270 \\ 3.9859 & 14.1881 & 7.8616 \\ -4.1270 & 7.8616 & 29.2497 \end{bmatrix} \times 10^{-3} > 0, \quad Q = \begin{bmatrix} 20.7367 \\ -8.6828 \\ -75.0535 \end{bmatrix} \times 10^{-3}.$$

And, a set of stabilizing PDC gains are thus evaluated as

$$K_1 = a_1 B_1^T E_1 = \begin{bmatrix} -0.6 & -1 & 2 \end{bmatrix}, \quad K_2 = a_2 B_2^T E_2 = \begin{bmatrix} 0.8 & 0.8 & 0 \end{bmatrix}.$$

Given the same initial condition as above, the regarded system equipped with the stabilizing PDC controller is simulated. And, the states' trajectories and the control input are depicted in Figure 3 and Figure 4, respectively. It is shown that all the compensated states $x_i(k)$, $\forall i$ *are toward to zero for a short period.*

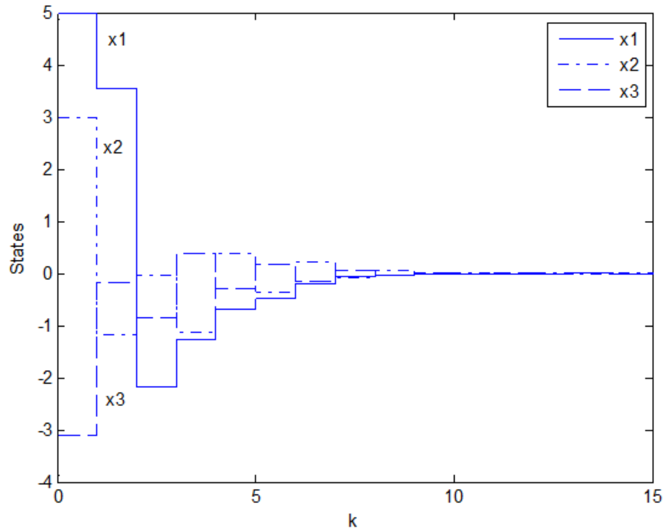

FIGURE 3. Compensated states' responses

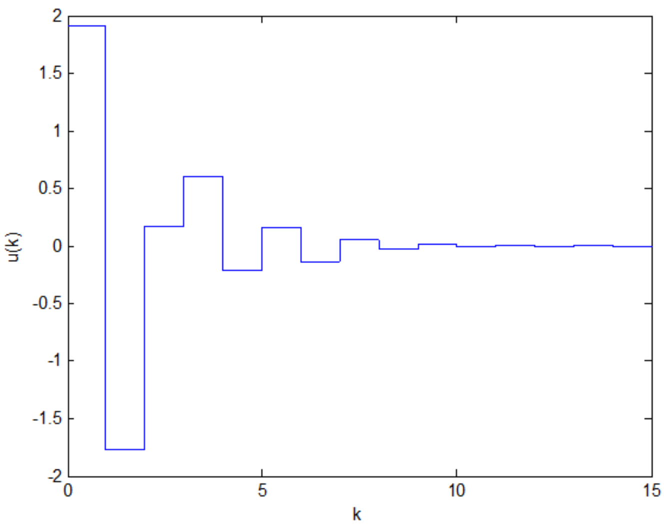

FIGURE 4. PDC control input

Remark 5.1. *From the illustrative examples, we conclude the main advantages of the proposed results as the following.*

i) In the past, it seems that there are no results on the stability and the PDC synthesis for the discrete fuzzy singular delay systems with multiple difference matrices E_i*'s. In this work, we devote to the admissibility analysis and PDC synthesis for the regarded system and present new criteria in Theorem 3.2 and Theorem 4.1, respectively.*

ii) Observing on Theorem 3.2 and Theorem 4.1, all the proposed criteria of admissibility analysis and PDC synthesis are explicitly formulated in terms of LMIs or parametric LMIs, so we can readily evaluate them via LMI solvers for the admissibility assurance and the PDC synthesis. The validity and efficiency of the proposed approach are demonstrated by the given examples.

6. **Conclusions.** In this study, we have discussed the admissibility analysis and the stabilizing controller design for the discrete singular fuzzy delay model with multiple difference

matrices. First, the admissibility condition for the nominal singular delay system was originally derived. Based on the distinct condition, we thus could deal with the admissibility analysis and the PDC controller design for the regarded delay systems with multiple difference matrices. Since all the proposed criteria are formulated by strict LMIs or parametric LMIs, we thus handily evaluated them via the current LMI programming tools. Finally, two illustrative examples demonstrated that the proposed results are effective and valid.

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