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H_∞ FAULT DETECTION AND CONTROL FOR CONTINUOUS-TIME TAKAGI-SUGENO FUZZY SYSTEMS

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Abstract. *This paper studies the fault detection and control problem for nonlinear continuous-time fuzzy systems. The nonlinear control systems are modeled through a Takagi-Sugeno model. The main idea focuses on designing a fault detection observer and controller such that the impact of the unknown inputs and the faults on the system is minimized in the sense of the H_∞ norm. To develop sufficient condition of exponential stability, we design an observer to construct the residual signal and the controller by using the Lyapunov function and the H_∞ performance. The results are established in terms of linear matrix inequalities (LMIs). Finally, simulations results are provided to verify the effectiveness of the presented scheme.*

Keywords: Fault detection, Control, Lyapunov function, Linear matrix inequality, Takagi-Sugeno fuzzy model

1. **Introduction.** The fault detection (FD) in dynamic systems has been an active field of research during the past decades because of an increasing demand for higher safety and reliability standards [1-8]. A fault represents any kind of malfunction in plant, as actuator faults or sensor faults. The presence of faults in the system leads to degradation of performances of the overall system behavior. Without taken into account, by detecting and tolerating this fault, the degradation may turn into instability of the system. The objective of fault detection (FD) is to detect the fault signal accurately whenever it appears. The basic idea to do this is to use a state observer or filter to construct a residual signal. The residual evaluation function is compared with a predefined threshold. When the residual function has a value larger than the threshold, an alarm of faults is generated. On the other hand, it is well known that control inputs, unavoidable unknown inputs, and faults are coupled in many industrial systems. This fact can lead to potential sources of false alarm. This means that FD systems have to be robust to control inputs and unknown inputs and, at the same time, enhance the sensitivity to the faults [8]. Various model-based fault-detection techniques have been proposed; see [3-6,8]. In [6], robust fault detection problem of discrete-time networked systems with unknown input and multiple state delays is studied. However, most of the techniques mentioned above rely on the system parameters to be known. Many approaches have been applied in FD and control for systems with faults [7,9-13].

One of the main difficulties in designing a fault-detection system for nonlinear dynamical systems is that a rigorous mathematical model may be very difficult to obtain, if

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not impossible. However, many physical systems can be expressed either in some form of mathematical model locally or as an aggregation of a set of mathematical models. Fuzzy system theory enables us to utilize qualitative, linguistic information about a highly complex nonlinear system to construct a mathematical model for it. The Takagi-Sugeno fuzzy model was introduced in [14], and it showed its efficiency to control complex control system problems and was used in many applications [13,15-22]. It is described by the fuzzy rules of type IF-THEN, and such a modeling strategy provides a possible manner to develop systematic approaches to analysis and synthesis of general nonlinear systems. Theories of linear control system can be employed to resolve the nonlinear problems directly.

During the past decade, observer-based fault detection in nonlinear systems represented by T-S fuzzy models has been increasingly attracting the attention of many researchers. The fault detection or estimation problems for continuous-time T-S fuzzy nonlinear systems have also attracted considerable attention [23-30]. In [23], the authors address the problem of learning-observers based reconstruction of actuator faults in T-S fuzzy systems. To reduce the conservatism of the system with the above observer design several approaches are considered, such as H_∞ performance in [24], H_- performance in [29] and mixed H_-/H_∞ in [25,26,30], but, no one of the approaches in the literature has utilized the H_∞ performance with new idea developed in [4] where all the variables are put together in an augmented unknown matrix.

Many results in the literature have studied the exponential stability of dynamical systems (see, [31-34]). To the best of our knowledge, the exponential stability with the design of an FD observer system for continuous-time T-S fuzzy systems has not been studied in the literature. However, it is more important to study the exponential stability since the transient process of a system may be better described if the decay rate is determined which motivates this study.

The present work investigates the design of an FD observer system for continuous-time nonlinear T-S fuzzy system affected by sensor faults and unknown disturbances by using the Lyapunov function. The main focus of this study is to design a robust fault detection system under relaxed LMIs conditions. The robust fault detection observer has a good robustness to disturbances and sensitivity to faults. The observer gains and the residual weighting matrix are obtained through the H_∞ approach and the exponential stability property. Sufficient conditions for the H_∞ ault detection observer are expressed in terms of linear matrix inequalities (LMIs). In addition, all the variables are put together in an augmented unknown one matrix to avoid obtaining product terms between Lyapunov matrix P and observer and controller matrices. Hence, the conservatism could be reduced by our proposed approach. Simulation examples are given to show the effectiveness of the proposed approach.

Summarizing the above discussion, we aim at addressing the problem of fault detection by a T-S fuzzy approach. The main contributions of the current work are listed as follows.

i) We have utilized construction of a residual signal approach to solve the fault detection design problem in the case that it could be made sensitive to faults. Also, the exponential asymptotic stability and H_∞ performance attenuation of the fuzzy augmented system are obtained.

ii) Based on the parameter dependent Lyapunov functional approach and the T-S fuzzy technique, a sufficient condition has been obtained and formulated in terms of linear matrix inqualities (LMIs).

The rest of the paper is organized as follows. In Section 2, the problem is formulated and useful preliminaries are introduced. The main results are derived in Section 3. Illustrative examples are presented in Section 4 to show the effectiveness and potential of the proposed design techniques. Finally, conclusion and perspectives notes are given in Section 5.

Notations. Standard notations are used in this paper. For a matrix A, A^T denotes its transpose. $A > 0$ and $A < 0$ denote positive-definite and negative-definite matrices, respectively. $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ represent the set of real n-vectors and $m \times n$ matrices, respectively. The symbol * within a matrix represents the symmetric entries. 0, I denote the null matrix and identity matrix with appropriate dimensions, respectively.

2. **Problem Statement.** The T-S fuzzy dynamic model of a continuous-time nonlinear system is described by some fuzzy IF-THEN rules, each of which represents a local linear input-output relation of the system. The i-th rule of the T-S fuzzy model is of the following form,

$$\text{Plant rule } i\text{: IF } \mu_1(t) \text{ is } \eta_{i1} \text{ and } \cdots \text{ and } \mu_q(t) \text{ is } \eta_{iq}, \text{ THEN}$$
$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + E_i d(t) + F_i f(t) \\ y(t) = C_i x(t) + N_i d(t) + M_i f(t) \end{cases} \tag{1}$$

where $\mu(t) = [\mu_1(t), \ldots, \mu_q(t)]^T$ are the premise variables, η_{ij} $(i = 1, \ldots, r;\ j = 1, \ldots, q)$ are fuzzy sets that are characterized by the membership functions; r and q are the number of IF-THEN rules and of the premise variables, respectively; $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^p$, are the system states, control inputs, output measurement, respectively, $d(t) \in \mathbb{R}^v$ is the external disturbances, and faults $f(t) \in \mathbb{R}^m$. A_i, B_i, E_i, F_i, C_i, N_i and M_i are known constant matrices with appropriate dimensions.

Since the state $x(t)$ is unavailable and the disturbance $d(t)$ and the faults are unknown, the objective of this study is to synthesize a fuzzy observer to estimate the state $x(t)$ in order to design a controller that can tolerate the faults and reduces the effects of the disturbance. Hence, consider the following fuzzy observer:

$$\text{IF } \mu_1(t) \text{ is } \eta_{i1} \text{ and } \cdots \text{ and } \mu_q(t) \text{ is } \eta_{iq}, \text{ THEN}$$
$$\begin{cases} \dot{\hat{x}}(t) = G_i \hat{x}(t) + L_i y(t) \\ \hat{y}(t) = C_i \hat{x}(t) \end{cases} \tag{2}$$

By using the center-average defuzzifier, the dynamics of T-S fuzzy systems (1) and (2) are inferred as follows, respectively:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \eta_i(\mu(t)) \left\{ A_i x(t) + B_i u(t) + E_i d(t) + F_i f(t) \right\} \\ y(t) = \sum_{i=1}^{r} \eta_i(\mu(t)) \left\{ C_i x(t) + N_i d(t) + M_i f(t) \right\} \end{cases} \tag{3}$$

and,

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} \eta_i(\mu(t)) (G_i \hat{x}(t) + L_i y(t)) \\ \hat{y}(t) = \sum_{i=1}^{r} \eta_i(\mu(t)) C_i \hat{x}(t) \end{cases} \tag{4}$$

where $\eta_i(\mu(t))$ is defined as:

$$\eta_i(\mu(t)) = \frac{\beta_i(\mu(t))}{\sum_{i=1}^{r} \beta_i(\mu(t))}, \quad \beta_i(\mu(t)) = \prod_{j=1}^{q} \rho_{ij}(\mu_j(t))$$

in which $\rho_{ij}(\mu_j(t))$ is the grade of membership of $\mu_j(t)$ in ρ_{ij}. It can be seen that for all t,

$$\eta_i(\mu(t)) \geq 0; \quad i = 1, \ldots, r, \quad \sum_{i=1}^{r} \eta_i(\mu(t)) = 1, \quad \forall t$$

The observer-based controller of the system is described by the following IF-THEN rules,

Control Rule i:

$$\text{IF } \mu_1(t) \text{ is } \eta_{i1} \text{ and } \cdots \text{ and } \mu_q(t) \text{ is } \eta_{iq}, \text{ THEN } u(t) = k_i\hat{x}(t), \ (i = 1, 2, \ldots, r) \quad (5)$$

where k_i $(i = 1, 2, \ldots, r)$ are the local controller gains to be determined. The design of the fuzzy controller consists in determining the feedback gains k_i $(i = 1, 2, \ldots, r)$ such that the closed-loop system is asymptotically stable with the presence of perturbation and faults.

The defuzzified output of the observer-based controller is given by

$$u(t) = \sum_{i=1}^{r} \eta_i(\mu(t))k_i\hat{x}(t) \quad (6)$$

Note that the state is assumed to be available to be used in feedback control.

Obviously, this fact introduces indirectly the fault in the control. While the fault occurs, the stabilizing controller will compensate the gap introduced by the fault.

The observer-based residual generator is defined by

$$\text{IF } \mu_1(t) \text{ is } \eta_{i1} \text{ and } \cdots \text{ and } \mu_q(t) \text{ is } \eta_{iq}, \text{ THEN } r(t) = V_i\left(y(t) - \hat{y}(t)\right) \quad (7)$$

The defuzzified output of the observer-based residual is given by

$$r(t) = \sum_{i=1}^{r} \eta_i(\mu(t))V_i\left(y(t) - \hat{y}(t)\right) \quad (8)$$

Matrices V_i are also to be computed. By combining the above structures of the observer, the controller and the residual, and defining $e(t) = x(t) - \hat{x}(t)$, the following augmented continuous-time system is obtained:

$$\begin{cases} \dot{\tilde{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \eta_i(\mu(t))\eta_j(\mu(t))\left[\tilde{A}_{ij}\tilde{x}(t) + \tilde{B}_{ij}w(t)\right] \\ r(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \eta_i(\mu(t))\eta_j(\mu(t))\left[\tilde{C}_{ij}\tilde{x}(t) + \tilde{D}_{ij}w(t)\right] \end{cases} \quad (9)$$

where,

$$\tilde{A}_{ij} = \begin{pmatrix} A_i + B_iK_j & -B_iK_j \\ A_i - L_iC_j + B_iK_j - G_i & G_i - B_iK_j \end{pmatrix}, \ \tilde{B}_{ij} = \begin{pmatrix} E_i & F_i \\ E_i - L_iN_j & F_i - L_iM_j \end{pmatrix}$$

$$\tilde{C}_{ij} = \begin{bmatrix} 0 & V_iC_j \end{bmatrix}, \ \tilde{D}_{ij} = \begin{bmatrix} V_iN_j & V_iM_j \end{bmatrix}, \ \tilde{x}(t) = \begin{pmatrix} x(t) \\ e(t) \end{pmatrix}, \ w(t) = \begin{pmatrix} d(t) \\ f(t) \end{pmatrix} \quad (10)$$

Now, the problem of fault detection observer design and a controller for continuous-time T-S fuzzy system can be formulated as follows: develop an observer of the form (4) for system (3) such that the augmented system (9) is stable when $w(t) = 0$ and satisfy

$$l = \int_0^\infty \left(r^T(t)r(t) - \gamma^2 w^T(t)w(t)\right) dt < 0 \quad (11)$$

For all $w(t) \neq 0$. The scalar γ is a prescribed positive scalar and indicates an H_∞ disturbance attenuation index.

With this inequality, we ensure that the effect of the perturbation on the residual signal is minimized. So by minimizing γ, the observer will generate the residual signals that have the best robustness to disturbances and the best sensitivity to faults.

Remark 2.1. *In practice, the effect of disturbance should be taken into account in the modeled system. In addition, a fault signal is considered for solving the fault detection and control problem. In this paper, we have taken into consideration of all these practical situations and dressed the problem of FD and controller with a new approach.*

Remark 2.2. *For the fault detection problem several approaches are considered to solve it such as H_∞, H_- and mixed H_-/H_∞ criterion. For continuous-time nonlinear dynamic systems, applying H_∞ optimization techniques is proposed in [24]. In [29], the authors have presented stability conditions with local nonlinear models and H_- performance. In [9,10], the authors proposed mixed H_-/H_∞ performance for discrete-time T-S fuzzy model. Fault detection filter in multi-objective problem is designed in [25]. However, very few papers have treated the fault detection observer and controller problem [26,30], and in these papers the authors solved this problem by using the mixed H_-/H_∞ performance. These approaches have established less conservative results, but this increases the computational complexity, which is clear on the decoupling during the calculation of the H_∞ performance and H_- norm and then the mixing. To show that our approach is less conservative than these references, the following table gives a comparison of the number of the decision variables.*

Table 1. Comparison of number of variables

Method	Number of variables
[24]	$3 + 5n^2 + n$
[25]	$4 + 5n^2 + n$
[26]	$2 + 6n^2 + n$
[29]	$2 + 5n^2 + n$
[30]	$3 + 18n^2 + 6n$
Our approach	$2 + 4n^2 + n\frac{(n+1)}{2}$

Consequently, with our results the computational demand on searching for the solution of the stabilizing conditions can be alleviated. A second advantage of our approach is that we expect a reduced conservatism. This is illustrated in the examples.

As used in the literature of fault detection [1,6], the identification of the fault $f(t)$ is not necessary. One can use the following residual evaluation function:

$$J_r(\tau) = \|r(t)\|_2 = \left(\int_{t_0}^{t_0+\tau} r^T(\theta) r(\theta) d\theta \right)^{1/2} \tag{12}$$

where t_0 denotes the initial evaluation time instant, and τ is the evaluation time steps. The occurrence of faults can be detected by comparing $J_r(\tau)$ to a threshold value J_{th} according to the following logic relationship:

$$J_r(\tau) > J_{th} \Rightarrow \textit{Faults} \Rightarrow \textit{Alarm}$$
$$J_r(\tau) < J_{th} \Rightarrow \textit{No Faults}.$$

The threshold value can be calculated during the fault-free system operation as indicated in [6].

$$J_{th} = \sup_{d(t)\in\ell_2[0,+\infty),\, f(t)=0} \|r(t)\|_2 \tag{13}$$

Definition 2.1. *Given $\alpha > 0$. System (9) is called α-exponentially stable if there exists a scalar $\vartheta > 0$ such that for all continuous initial function $x(0)$, the solution of (9) satisfies the following condition:*

$$\|x(t)\| \leq \vartheta \|x(0)\| e^{-\alpha t} \quad \forall t \geq 0$$

3. **Main Results.** The objective of this section is to obtain an LMI enabling to synthesize the T-S fuzzy model observer together with its corresponding residual and controller.

Theorem 3.1. *Given positive scalars α and γ, the system (9) is exponentially stable and guarantees the performance index (11) for all $w(t) \in \ell_2[0,\infty)$, if there exist matrices $P = P^T > 0$, G_i, L_i, K_i and V_i, $i = 1,\ldots,r$ such that*

$$\Omega_{ij} = \begin{pmatrix} P\tilde{A}_{ij} + \tilde{A}_{ij}^T P + \alpha P + \tilde{C}_{ij}^T \tilde{C}_{ij} & P\tilde{B}_{ij} + \tilde{C}_{ij}^T \tilde{D}_{ij} \\ * & \tilde{D}_{ij}^T \tilde{D}_{ij} - \gamma^2 I \end{pmatrix} < 0 \tag{14}$$

Proof: Consider the quadratic Lyapunov function candidate as

$$V(\tilde{x}(t)) = \tilde{x}^T(t) P \tilde{x}(t) \tag{15}$$

where $P = P^T > 0$. The derivative of the Lyapunov function (15) over the dynamics (9) is as follows:

$$\dot{V}(\tilde{x}(t)) = 2\tilde{x}^T(t) P \dot{\tilde{x}}(t) \tag{16}$$

$$\dot{V}(\tilde{x}(t)) = \sum_{i=1}^{r}\sum_{j=1}^{r} \eta_i(\mu(t))\eta_j(\mu(t)) \left\{ \tilde{x}^T(t) \left[P\tilde{A}_{ij} + \tilde{A}_{ij}^T P \right] \tilde{x}(t) + 2\tilde{x}^T(t) P\tilde{B}_{ij} w(t) \right\} \tag{17}$$

Let

$$J(t) = \dot{V}(\tilde{x}(t)) + \alpha V(\tilde{x}(t)) + r^T(t) r(t) - \gamma^2 w^T(t) w(t) \tag{18}$$

Using (9) and (17), we have

$$\begin{aligned} J(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} \eta_i(\mu(t))\eta_j(\mu(t)) \left\{ \tilde{x}^T(t) \left[P\tilde{A}_{ij} + \tilde{A}_{ij}^T P \right] \tilde{x}(t) + 2\tilde{x}^T(t) P\tilde{B}_{ij} w(t) \right\} \\ &\quad + \sum_{i=1}^{r}\sum_{j=1}^{r} \eta_i(\mu(t))\eta_j(\mu(t)) \sum_{k=1}^{r}\sum_{l=1}^{r} \eta_k(\mu(t))\eta_l(\mu(t)) \Big\{ \Big[\tilde{C}_{ij}\tilde{x}(t) \\ &\quad + \tilde{D}_{ij} w(t) \Big]^T \left[\tilde{C}_{kl}\tilde{x}(t) + \tilde{D}_{kl} w(t) \right] \Big\} + \alpha \tilde{x}^T(t) P \tilde{x}(t) - \gamma^2 w(t) w(t) \qquad (19) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} \eta_i(\mu(t))\eta_j(\mu(t)) \Big\{ \tilde{x}^T(t) \left[P\tilde{A}_{ij} + \tilde{A}_{ij}^T P + \alpha P + \tilde{C}_{ij}^T \tilde{C}_{ij} \right] \tilde{x}(t) \\ &\quad + 2\tilde{x}^T(t) \left[P\tilde{B}_{ij} + \tilde{C}_{ij}^T \tilde{D}_{ij} \right] w(t) + w^T(t) \left[\tilde{D}_{ij}^T \tilde{D}_{ij} - \gamma^2 I \right] w(t) \Big\} \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} \eta_i(\mu(t))\eta_j(\mu(t)) \xi^T(t) \Omega_{ij} \xi(t) \qquad (20) \end{aligned}$$

where,

$$\xi(t) = \begin{pmatrix} \tilde{x}(t) \\ w(t) \end{pmatrix} \tag{21}$$

If (14) holds, we obtain $J(t) < 0$. Since $r^T(t)r(t) \geq 0$, for $w(t) = 0$ we have $\dot{V}(\tilde{x}(t)) + \alpha V(\tilde{x}(t)) < 0$ which implies the exponential asymptotic stability of system (9).

Since (14) holds, we have $\int_0^{+\infty} J(t)dt < 0$. From the exponential asymptotic stability, $V(\tilde{x}(\infty)) = 0$, and from the zero initial condition, it can be easily seen that the condition (11) is satisfied. This completes the proof. □

Remark 3.1. *Condition (14) of Theorem 3.1 guarantees the exponential stability of system (9). Consequently, the exponential convergence of the observation error to zero is guaranteed. However, condition (14) cannot be tested by LMI toolbox of Matlab since it contains nonlinearities. To overcome this problem, we state the following theorem.*

Theorem 3.2. *For given scalars γ and $\alpha > 0$, the closed-loop system (9) is exponentially stable with H_∞ performance γ if there exist P, $Q > 0$, G_i, L_i, K_j and V_i, $i, j = 1, \ldots, r$, such that*

$$\begin{pmatrix} \Upsilon_{ij} + \alpha P & P\bar{E}_i + P\bar{L}_i\bar{N}_j & \tilde{C}_{ij}^T \\ * & -\gamma^2 I & \tilde{D}_{ij}^T \\ * & * & -I \end{pmatrix} < 0 \tag{22}$$

$$P\bar{B}_i = \bar{B}_i Q \tag{23}$$

where,

$$\Upsilon_{ij} = \bar{A}_i^T P + P\bar{A}_i + \bar{B}_i U_j + U_j^T \bar{B}_i^T + P\bar{L}_i\bar{C}_j + \bar{C}_j^T\bar{L}_i^T P$$

$$U_j = Q\bar{K}_j$$

$$\left[\begin{array}{l} \bar{A}_i = \begin{pmatrix} A_i & 0 \\ A_i & 0 \end{pmatrix}, \bar{B}_i = \begin{pmatrix} 0 & B_i \\ -I & B_i \end{pmatrix}, \bar{K}_{ij} = \begin{pmatrix} G_i & -G_i \\ K_j & -K_j \end{pmatrix}, \bar{C}_j = \begin{pmatrix} 0 & 0 \\ C_j & 0 \end{pmatrix} \\ \bar{E}_i = \begin{pmatrix} E_i & F_i \\ E_i & F_i \end{pmatrix}, \bar{L}_i = \begin{pmatrix} 0 & 0 \\ 0 & -L_i \end{pmatrix}, \bar{N}_j = \begin{pmatrix} 0 & 0 \\ N_j & M_j \end{pmatrix} \end{array}\right. \tag{24}$$

Remark 3.2. *For the fault detection problem, it is not necessary to estimate the fault. By considering the deviation of $J_r(\tau)$ between fault-free case and faulty case, we can decide whether or not the fault has occurred. Compared with the fault estimation methods [1,10,12,15,23], the proposed technique reduces the requirement of the observer and leads to a less overall complexity in practice.*

Proof: It is noted that $\tilde{A}_{ij} = \bar{A}_i + \bar{B}_i\bar{K}_j + \bar{L}_i\bar{C}_j$ and $\tilde{B}_{ij} = \bar{E}_i + \bar{L}_i\bar{N}_j$. Taking account of $\tilde{A}_{ij}$ *and* $\tilde{B}_{ij}$ from (14), the following inequality is obtained:

$$\bar{\Omega}_{ij} = \begin{pmatrix} \bar{\Upsilon}_{ij} + \alpha P + \tilde{C}_{ij}^T\tilde{C}_{ij} & P\bar{E}_i + P\bar{L}_i\bar{N}_j + \tilde{C}_{ij}^T\tilde{D}_{ij} \\ * & \tilde{D}_{ij}^T\tilde{D}_{ij} - \gamma^2 I \end{pmatrix} < 0 \tag{25}$$

where

$$\bar{\Upsilon}_{ij} = \bar{A}_i^T P + P\bar{A}_i + P\bar{B}_i\bar{K}_j + \bar{K}_j^T\bar{B}_i^T P + P\bar{L}_i\bar{C}_j + \bar{C}_j^T\bar{L}_i^T P \tag{26}$$

It is clear that in (26) there exists a nonlinearity at the terms $P\bar{B}_i\bar{K}_j$, $\bar{K}_j^T\bar{B}_i^T P$, to eliminate this nonlinearity we use Equality (23). It is obvious that $\bar{\Omega}_{ij} < 0$ implies the following inequality:

$$\bar{\Upsilon}_{ij} + \alpha P + \tilde{C}_{ij}^T\tilde{C}_{ij} < 0 \tag{27}$$

which implies that

$$\tilde{A}_{ij}^T P + P\tilde{A}_{ij} + \alpha P < 0 \tag{28}$$

Consider the quadratic Lyapunov function presented in (15). Then, for all nonzero trajectories $\tilde{x}(t)$ of the system (9) and $w(t)$, we have

$$\dot{V}(\tilde{x}(t)) + \alpha V(\tilde{x}(t)) = \dot{\tilde{x}}^T(t)P\tilde{x}(t) + \tilde{x}^T(t)P\dot{\tilde{x}}(t) + \alpha\tilde{x}^T(t)P\tilde{x}(t)$$

$$= \sum_{i=1}^{r}\sum_{j=1}^{r}\eta_i(\mu(t))\eta_j(\mu(t))\left[\tilde{x}^T(t)\left(\tilde{A}_{ij}^T P + P\tilde{A}_{ij} + \alpha P\right)\tilde{x}(t)\right] \quad (29)$$

Then, according to (28), we obtain

$$\dot{V}(\tilde{x}(t)) + \alpha V(\tilde{x}(t)) < 0 \quad (30)$$

It follows that

$$\dot{V}(\tilde{x}(t)) < -\alpha V(\tilde{x}(t)) \quad (31)$$

The Lyapunov function (15) is positive definite and bounded. Taking account of (31), we can easily deduce that

$$\|\tilde{x}(t)\| < \sqrt{\frac{\kappa}{\upsilon}}e^{-\alpha t}\|\tilde{x}(0)\| \quad \forall t \in \mathbb{R}^+ \quad (32)$$

where $\upsilon = \lambda_{\min}(P)$, $\kappa = \lambda_{\max}(P)$ and $\alpha > 0$ is the decay rate. This implies the closed-loop system is exponentially asymptotically stable.

Since (25) holds, by schur complement and by using (23), we obtain (22).

Now, for any nonzero $w(t) \in \ell_2[0,\infty)$ and zero initial condition $\tilde{x}(0)$, considering $J(t)$ given by (18), we obtain from the above analysis that

$$J(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\eta_i(\mu(t))\eta_j(\mu(t))\xi^T(t)\Omega_{ij}\xi(t) < 0$$

The following inequality is true:

$$\bar{J} = \int_0^{+\infty} J(t)dt = \int_0^{+\infty}\left(\dot{V}(\tilde{x}(t)) + \alpha V(\tilde{x}(t)) + r^T(t)r(t) - \gamma^2 w^T(t)w(t)\right)dt < 0 \quad (33)$$

which implies that

$$\bar{J} = \int_0^{+\infty}\xi^T(t)\Omega\xi(t)dt \text{ with } \Omega = \sum_{i=1}^{r}\sum_{j=1}^{r}\eta_i(\mu(t))\eta_j(\mu(t))\Omega_{ij}$$

According to the zero initial condition, we obtain that the H_∞ performance index (11) is satisfied, which implies the closed-loop system is exponentially stable and satisfies the H_∞ performance.

The proof is completed. □

Remark 3.3. *In Matlab LMI Toolbox, we do not have the possibility to program directly Equality (23) in Theorem 3.2. To this end, we propose to convert (23) into an LMI presented in (34) by setting η as a sufficiently small positive scalar:*

$$\begin{pmatrix} -\eta I & \bar{B}_i^T P - Q^T\bar{B}_i^T \\ * & -I \end{pmatrix} \leq 0 \quad (34)$$

Remark 3.4. *The feasibility of LMIs presented in (22) allows us to determine the parameters G_i, L_i, K_i and V_i and to synthesize the observer. The determination of V_i allows to determine the corresponding residual generator and to detect the defect. Also the knowledge of K_i makes if possible to stabilize the system and to check the H_∞ performance.*

4. Examples.

In this section, two examples are presented to illustrate the effectiveness of the proposed method.

Example 4.1. *In order to illustrate the effectiveness of the proposed method in this work, we take the system (1) with the following data:*

$$A_1 = \begin{bmatrix} -4 & 1 & -3 \\ -1 & -7.5 & -1 \\ 2 & 1 & -6 \end{bmatrix},\ B_1 = \begin{bmatrix} 0.8 \\ 0 \\ 0 \end{bmatrix},\ C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},\ M_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ E_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

$$A_2 = \begin{bmatrix} -4 & 1 & -3 \\ -1 & -6.5 & -1 \\ 2 & 1 & -6 \end{bmatrix},\ B_2 = \begin{bmatrix} 1.2 \\ 0 \\ 0 \end{bmatrix},\ C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},\ M_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ E_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

$$A_3 = \begin{bmatrix} -1 & 1 & -3 \\ -1 & -7.5 & -1 \\ 2 & 1 & -6 \end{bmatrix},\ B_3 = \begin{bmatrix} 0.8 \\ 0 \\ 0 \end{bmatrix},\ C_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},\ M_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ E_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

$$A_4 = \begin{bmatrix} -1 & 1 & -3 \\ -1 & -6.5 & -1 \\ 2 & 1 & -6 \end{bmatrix},\ B_4 = \begin{bmatrix} 1.2 \\ 0 \\ 0 \end{bmatrix},\ C_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},\ M_4 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ E_4 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

And the membership functions are $\eta_1 = \frac{1-\sin(x_2(t))}{2}$, $\eta_2 = \frac{1+\sin(x_2(t))}{2}$, $\eta_3 = \frac{1-\sin(x_3(t))}{2}$, $\eta_4 = \frac{1+\sin(x_3(t))}{2}$.

Given $\alpha = 1$, *based on Theorem 3.2, the matrices of the observer, the controller and the residual generator are obtained as follows:*

$$L_1 = \begin{bmatrix} 0 & 0 \\ -0.0096 & 7.0081 \\ -0.0096 & 7.0089 \end{bmatrix},\ K_1 = \begin{bmatrix} -5.6218 & -0.0033 & 0.0056 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0.0020 & -7.0088 & 0 \\ 0.0020 & -7.0088 & 0 \end{bmatrix},\ V_1 = 1.7468$$

$$L_2 = \begin{bmatrix} 0 & 0 \\ -0.0127 & 8.2742 \\ -0.0127 & 8.2743 \end{bmatrix},\ K_2 = \begin{bmatrix} -4.2516 & -0.0006 & 0.0022 \end{bmatrix},$$

$$G_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0.0014 & -8.2742 & 0 \\ 0.0014 & -8.2742 & 0 \end{bmatrix},\ V_2 = 1.8382$$

$$L_3 = \begin{bmatrix} 0 & 0 \\ -0.0121 & 6.6053 \\ -0.0121 & 6.6053 \end{bmatrix},\ K_3 = \begin{bmatrix} -5.8179 & -0.0013 & 0.0037 \end{bmatrix},$$

$$G_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0.0014 & -6.6052 & 0 \\ 0.0014 & -6.6053 & 0 \end{bmatrix},\ V_3 = 1.7582$$

$$L_4 = \begin{bmatrix} 0 & 0 \\ -0.0120 & 5.4972 \\ -0.0120 & 5.4973 \end{bmatrix},\ K_4 = \begin{bmatrix} -3.4333 & -0.0008 & 0.0024 \end{bmatrix},$$

$$G_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0.0020 & -5.4972 & 0 \\ 0.0020 & -5.4972 & 0 \end{bmatrix},\ V_4 = 1.7242$$

TABLE 2. H_∞ norm bound

Methods	γ
Theorem 8 in [26]	0.5
Theorem 1 [30]	0.8
Theorem 3.2 in this paper	5.7582×10^{-6}

The values of γ generated by Theorem 3.2, [26] and [30] are shown in Table 2.

We can see that the obtained value of γ from Table 2 by our approach is less than that of [26] and [30], which illustrates the merit of Theorem 3.2.

Example 4.2. *In this example, we apply the above analysis technique to a mass-spring-damper system, which is borrowed from [24]. It is assumed that the stiffness coefficient of the spring, the damping coefficient of the damper and the input term have nonlinearity. The differential equation describing the system is given by*

$$M\ddot{x} + g(x, \dot{x}) + f(x) = \phi(\dot{x})\, u,$$

where M is the mass, $g(x, \dot{x}) = D(c_1 x + c_2 \dot{x})$ is the nonlinear term with respect to the damper, $f(x) = c_3 x + c_4 x^3$ is the nonlinear term with respect to the spring, and $\phi(\dot{x}) = 1 + c_5 \dot{x}$ is the nonlinear term with respect to the input term.

Assume $x \in [-1, 1]$ and $\dot{x} \in [-1, 1]$, and the parameters are set as follows:

$$M = 1,\ D = 1,\ c_1 = 0.5,\ c_2 = 1.726,\ c_3 = 0.5,\ c_4 = 0.67,\ C_5 = 0.$$

Let $x = [x, \dot{x}]^T = [x_1, x_2]^T$, f is the fault input, and $\phi(t) \in \mathbb{R}^{n_\phi}$ is the sector-bounded nonlinear function and unmeasurbale proposed in [29].

Then the system is written as a T-S fuzzy system as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} \eta_i(\mu(t)) \left[A_i x(t) + Bu(t) + Ed(t) + Ff(t)\right] \\ y(t) = Cx(t) + Nd(t) + Mf(t) \end{cases} \tag{35}$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & -1.726 \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & 1 \\ -1.67 & -1.726 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C = \begin{bmatrix} 0 & 1 \end{bmatrix},$$
$$E = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix},\ F = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix},\ N = 0.5,\ M = 0.3,\ \eta_1 = 1 - x^2,\ \eta_2 = x^2.$$

Applying Theorem 3.2, we obtain the values for the gain matrices L_1, L_2, K_1, K_2, G_1, G_2, V_1 and V_2 as follows:

$$L_1 = \begin{bmatrix} -3.4763 \\ 37.2117 \end{bmatrix},\ L_2 = \begin{bmatrix} -3.7656 \\ 36.4730 \end{bmatrix},$$
$$K_1 = \begin{bmatrix} -0.6349 & -0.6094 \end{bmatrix},\ K_2 = \begin{bmatrix} -1.1086 & -0.6387 \end{bmatrix},$$
$$G_1 = \begin{bmatrix} 2.5420 & 19.8174 \\ -24.9417 & -92.1179 \end{bmatrix},\ G_2 = \begin{bmatrix} -16.7431 & 1.9839 \\ -24.2542 & -81.2234 \end{bmatrix},$$
$$V_1 = 6.03 \times 10^{-6} \text{ and } V_2 = 1.1 \times 10^{-5}.$$

And the obtained H_∞ norm bounds are reported in Table 3.

From Table 3, we can see that the value of γ obtained by our approach presented in Theorem 3.2 has the least value compared with the values in the literature researches

TABLE 3. H_∞ norm bound

Methods	γ
Theorem 1 in [24]	1.6901
Theorem 1 [29]	0.5692
Theorem 3.2 in this paper	8.24×10^{-6}

presented in [24] and [29]. This shows that our approach is conservative less than that of [24] and [29], which illustrates the merit of Theorem 3.2.

Then for the simulation purpose, we let the occurrence of the fault $f(t) = 1$ from 40 to 60 s and the threshold value $J_{th} = 0.1079.10^{-4}$.

Figure 1 presents the residual signal while the evolution of the system states is shown in Figure 3. From this figure, it is seen that the trajectories of the system states converge exponentially to equilibrium point when the fault is discarded. Figure 2 presents the evaluation function J_r in which the solid line is fault-free case, while the dashed line is the faulty case, it is seen that $J_r(t) > J_{th}$ when $t \geq 40$ s, which means that the fault $f(t)$ can be immediately and effectively detected when it occurs under the disturbance input.

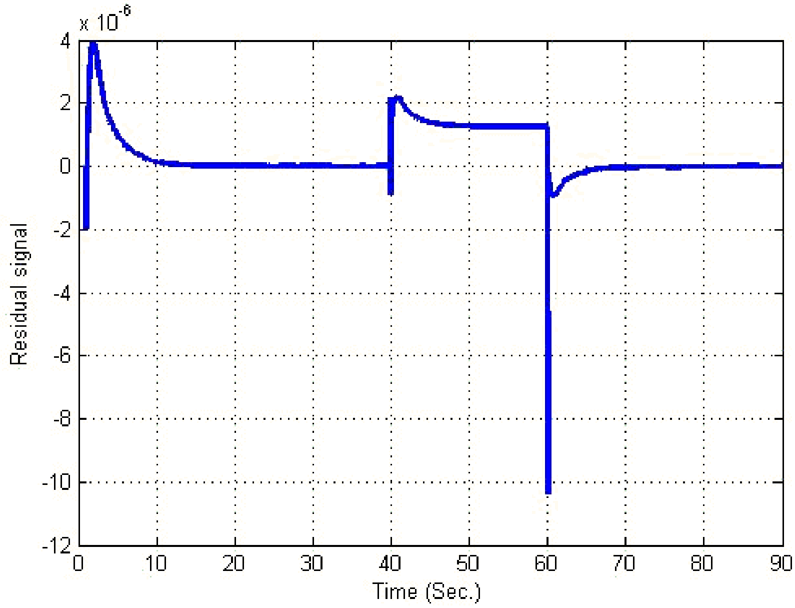

FIGURE 1. Residual signal $r(t)$ when the faults from 40 to 60 s

5. **Conclusion.** In this paper, the fault detection and control problem for continuous-time nonlinear T-S fuzzy systems has been investigated. The controller-based observer has been designed to maintain the exponenetial stability of the faulty nonlinear T-S fuzzy system and can be considered as a fault-tolerant control. This technique uses a new method assembling in a compact form all the variables in one augmented matrix. The obtained conditions have been worked out in a simple way to obtain new LMIs, and the desired observer and controller matrices can be constructed easily through the solution of LMIs. Finally, two illustrative examples are provided to show the effectiveness and applicability of the proposed results. Future work will extend the developed method for T-S fuzzy systems with actuator saturation.

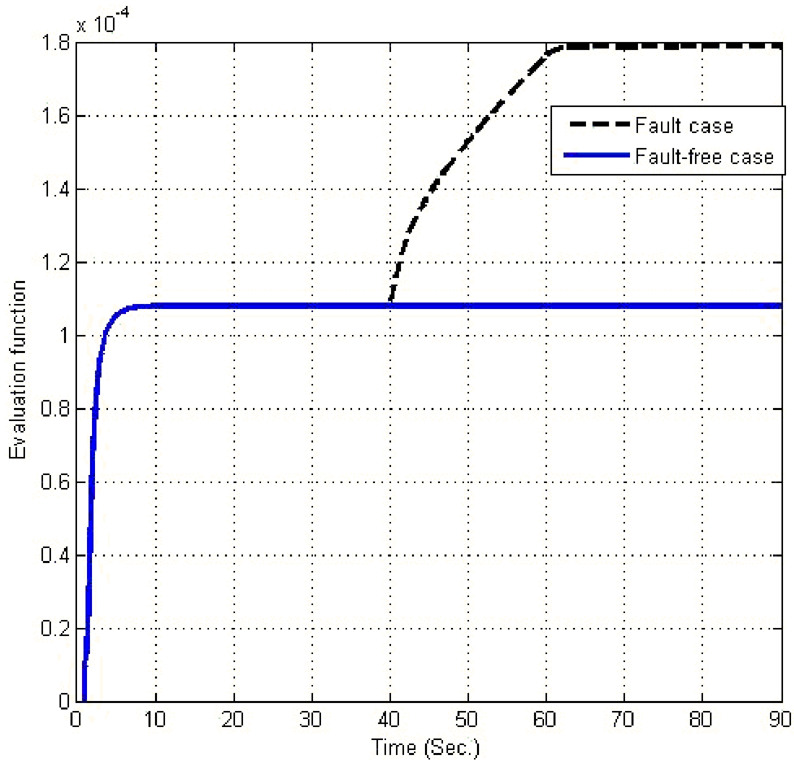

FIGURE 2. The residual evaluation function $J_r(t)$

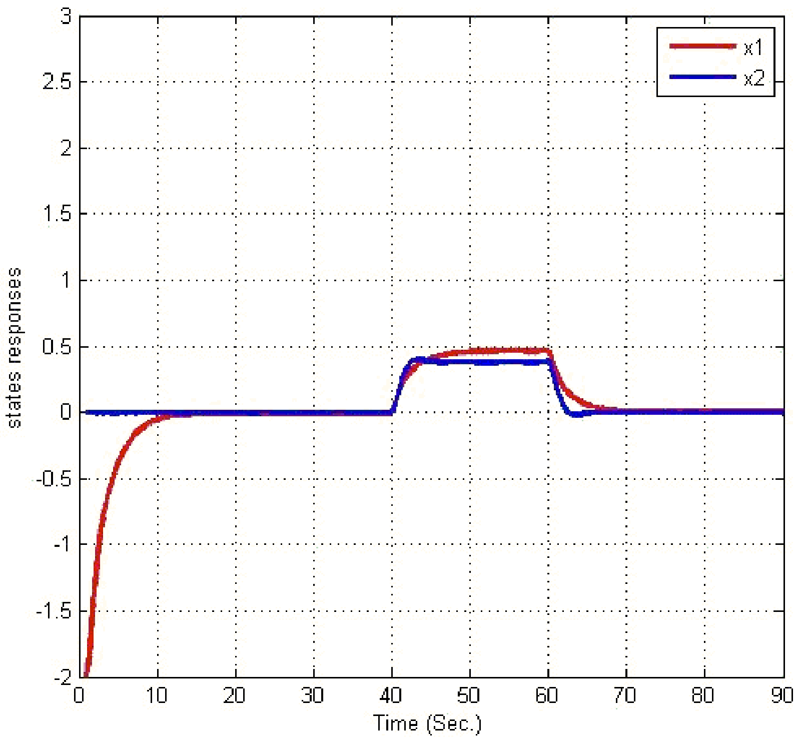

FIGURE 3. State responses with input control

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