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DESIGN AIDS FOR RECTANGULAR CROSS-SECTION BEAMS WITH STRAIGHT HAUNCHES: PART 1

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Abstract. *This paper shows design aids for rectangular cross-section beams with straight haunches subjected to a uniformly distributed load to obtain the fixed-end moments factors, carry-over factors and stiffness factors, which is novelty of this research. The design aids are developed using the equations previously described by the writers in a companion paper. The simplifications made are as follows: the height of straight haunches on the left side and the right side are equal, because this is the most used by architectural forms (aesthetic) and the beam height in the central part is the 10% of beam length that is proposed by the code of the ACI to control deflections. The results of the considered problem are compared with the equations previously described by the writers in a companion paper, and these are the same with an approximation of three digits. Therefore, design aids provide a great tool for structural engineers by saving time.*
Keywords: Design aids, Rectangular cross-section beams, Straight haunches, Uniformly distributed load, Fixed-end moments factors, Carry-over factors, Stiffness factors

1. **Introduction.** The beams of reinforced concrete with haunches in its ends are differentiated from prismatic because the beam height varies in three different parts, where the height being greater at the ends of the beam and in central part is constant. The main application is in high-rise buildings, bridges and viaducts that have beams of big long, because the maximum moments in absolute value are presented at the ends of the beams.

Design aids are represented by means of tables, graphs and descriptions of procedures with which it is intended to shorten the routine work of structural designers. Design aids are the result of a selection of the available materials (cross-sections), to offer only the one considered to be the latest and most frequently used.

The most relevant works on the topic of design aids for structural analysis are: Design aids of concrete cantilever retaining walls are shown in tables based on the strength design method to simplify the iterative process of design [1]. Design aids for a reinforced concrete beam based on the minimum cost concept by graphics are presented for different strengths of concrete and different beam widths [2]. Design aids for fixed support reinforced concrete cylindrical shells under uniformly distributed loads consider the following: 1) The stress normal to the surface is neglected; 2) Reinforced concrete shell subjected to uniformly distributed load varying sinusoidally along its length; 3) The different symmetrical edge loads are considered along their longitudinal boundaries [3]. Design aids for trapezoidal

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and rectangular rigid footings of symmetrical shape resting on the soil to determine their maximum axial load and biaxial moment capacities without exceeding the bearing capacity of the soil for three different types of pressure distribution of the soil on footing are considered: uniform, linear, and parabolic [4]. Design aids for determining deflection of beams reinforced along part of their length by graphics to obtain the maximum deflection of a beam are presented [5]. Design aids of concrete columns are presented by interaction diagrams for columns of circular and rectangular cross-section with different proportions and different strengths of concrete and steel [6,7]. Design aids for seismic strengthening of reinforced concrete beams help to identify the amount of longitudinal and transverse steel required for posed seismic demand on the reinforced concrete beam [8]. Design aids for limit state design of reinforced concrete members (slabs, beams, columns, footings, retaining walls, water tanks) are represented by tables, charts and figures [9]. Design aids for prestressed concrete double tee beams with web opening associated with the variation of its compressive strength are shown [10]. Design aids for beam-column design of structural steel by the code IS800:2007 are presented [11]. Design aids for the simplified stress design of Helicoidal HSS beams using design equations expressed in terms of beam cross section (section module in "X" and "Y") providing the curves fitted for different cross sections [12]. An approach to develop curves is presented to illustrate primarily the relation between stresses in steel reinforcement not yielded, coefficient of resistance moment and the percentage of tensile and compressive reinforcement for reinforced concrete sections [13].

Other works for variable sections (non-prismatic members) are: Modeling for "I" cross section beams with straight haunches subjected to uniformly distributed load and/or concentrated load localized at anywhere on beam [14,15]; A new method is presented to solve the bending vibration equation of viscoelastic cantilever beam for variable cross section [16]; Modeling for rectangular cross section beams with parabolic haunches subjected to uniformly distributed load and/or concentrated load localized at anywhere on beam [17,18].

Also, main works on the topic of rectangular cross section beams with straight haunches are: A mathematical model for rectangular beams subjected to a uniformly distributed load with symmetric straight haunches to obtain the fixed-end moments, the carry-over factors and the stiffness factors (This work does not consider shear deformations) [19]; A mathematical model for rectangular beams with symmetric straight haunches subjected to a concentrated load localized at anywhere on beam to obtain the fixed-end moments (This work does not consider shear deformations) [20]; A mathematical model for rectangular beams with straight haunches (General case: symmetrical and/or nonsymmetrical) subjected to a uniformly distributed load to obtain the fixed-end moments, the carry-over factors and the stiffness factors (This work considers the bending and shear deformations) [21]; A mathematical model for rectangular beams with straight haunches (General case: symmetrical and/or nonsymmetrical) subjected to a concentrated load localized at anywhere on beam to obtain the fixed-end moments (This work considers the bending and shear deformations) [22].

Therefore, the review of the literature clearly shows that there is no close relationship with the topic of design aids for rectangular members with straight haunches under a uniformly distributed load to obtain the fixed-end moments factors, carry-over factors and stiffness factors that are addressed in this paper.

This paper shows design aids for rectangular members with straight haunches under a uniformly distributed load to obtain the fixed-end moments factors, carry-over factors and stiffness factors, which is novelty of this research. The design aids are developed using the equations previously described [21]. The simplifications made are as follows: u

(height of straight haunches on the left side) = z (height of straight haunches on the right side) because this is the most used by architectural forms (aesthetic), h (beam height in the central part) = $0.1L$ (beam length) that is proposed by the code of the ACI to control deflections. An example is presented to show the simplicity and effectiveness of the proposed design aids in the analysis of rectangular cross-section beams with straight haunches subjected to a uniformly distributed load.

The paper is organized as follows. Section 2 describes the beam of rectangular cross-section with straight haunches. Subsection 2.1 shows the design aids for the fixed-end moments factors of a uniformly distributed load. Subsection 2.2 presents the design aids for the carry-over factors. Subsection 2.3 shows the design aids for the stiffness factors. Section 3 shows the example of application of design aids. Results are presented in Section 4. Conclusion (Section 5) completes the paper.

2. **Design Aids.** Figure 1 shows an isometric member of rectangular section-cross with straight haunches in each end, where a is the horizontal length of the left haunch, c is the horizontal length of the right haunch and b is the width of the beam.

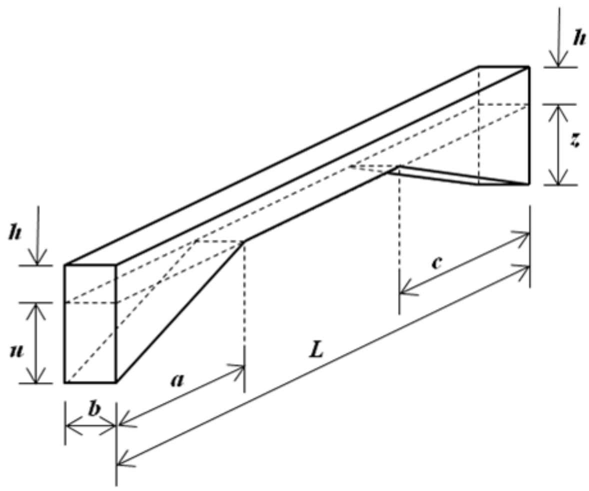


FIGURE 1. Rectangular isometric member with straight haunches

The design aids for members of rectangular section-cross with straight haunches consist in obtaining the fixed-end moments factors of a uniformly distributed load, carry-over factors and stiffness factors through simplified diagrams [21].

The simplifications made are as follows: $u = z$ because this is the most used by architectural forms (aesthetic); $h = 0.1L$ that is proposed by the code of the ACI to control deflections.

2.1. **Design aids for fixed-end moments.** The equations for fixed-end moments for uniformly distributed load are obtained as follows [21]:

$$M_{AB} = m_{AB} w L^2 \tag{1}$$

$$M_{BA} = m_{BA} w L^2 \tag{2}$$

where M_{AB} and M_{BA} are fixed-end moments, m_{AB} and m_{BA} are fixed-end moment factors, w is uniformly distributed load and L is the length of the member.

Design aids for fixed-end moment factors are obtained from Equations (13) and (14) taking into account $u = z = \beta h$ (β takes values of 0.5, 1.0, 1.5, 2.0); $a = \alpha L$ (α takes any value of 0 to 1); $c = \lambda L$ (λ takes values of 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9); $h = 0.1L$; $v = 0.20$ of concrete [21]. For $c = 0L$ it is considered $u = \beta h$; $z = 0$.

Design aids for fixed-end moment factors are obtained as follows.

1) The material and geometric properties of the beam are defined ($c = \lambda L$, $h = 0.1L$, $v = 0.20$, $u = z = \beta h$, $a = \alpha L$), and for $c = 0L$ it is considered $u = \beta h$ and $z = 0$.

2) Substitute each value of λ into Equation (13) to obtain m_{AB} and into Equation (14) to obtain m_{BA} [21]. These graphs are shown in function of α.

3) Simplified equations are obtained using the Derive software.

4) Now, the graphs are obtained using the Maple software.

Simplified diagrams are shown in function of α as follows (see Figure 2).

2.2. **Design aids for carry-over factors.** The equations for carry-over factors are obtained as follows [21]:

$$C_{AB} = \frac{M_{BA}}{M_{AB}} \tag{3}$$

$$C_{BA} = \frac{M_{AB}}{M_{BA}} \tag{4}$$

where M_{AB} is considered the member simply supported to the support A (free rotation) and M_{BA} is considered the member fixed in the support B (zero rotation) for Equation (3), M_{BA} is considered the member simply supported to the support B (free rotation) and M_{AB} is considered the member fixed in the support A (zero rotation) for Equation (4).

Design aids for carry-over factors are obtained from Equations (19) and (20) taking into account $u = z = \beta h$; $a = \alpha L$; $c = 0.0L$, $0.1L$, $0.2L$, $0.3L$, $0.4L$, $0.5L$, $0.6L$, $0.7L$, $0.8L$, $0.9L$; $h = 0.1L$; $v = 0.20$ of concrete [21]. For $c = 0L$ it is considered $u = \beta h$; $z = 0$.

The procedure to obtain the design aids for carry-over factors is the same with that used by fixed-end moment factors. Simplified diagrams are shown in function of α as follows (see Figure 3).

2.3. **Design aids for stiffness factors.** The equations for stiffness factors are obtained as follows [21]:

$$K_{AB} = \frac{k_{AB}EI}{L} \tag{5}$$

$$K_{BA} = \frac{k_{BA}EI}{L} \tag{6}$$

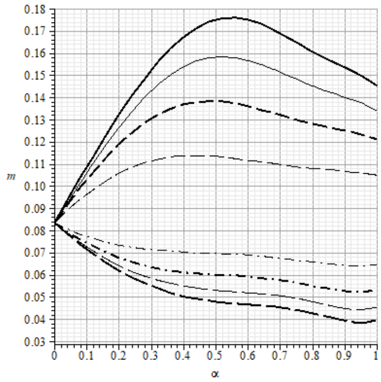
where k_{AB} and k_{BA} are the stiffness factors, E is the modulus of elasticity, I moment of inertia of the central part.

Design aids for stiffness factors are obtained from Equations (23) and (24) taking into account $u = z = \beta h$; $a = \alpha L$; $c = 0.0L$, $0.1L$, $0.2L$, $0.3L$, $0.4L$, $0.5L$, $0.6L$, $0.7L$, $0.8L$, $0.9L$; $h = 0.1L$; $v = 0.20$ of concrete [21]. For $c = 0L$ it is considered $u = \beta h$; $z = 0$.

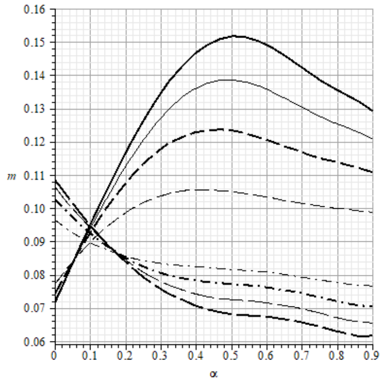
The procedure to obtain the design aids for stiffness factors is the same with that used by fixed-end moment factors. Simplified diagrams are shown in function of α as follows (see Figure 4).

3. Example of Application by Design Aids.

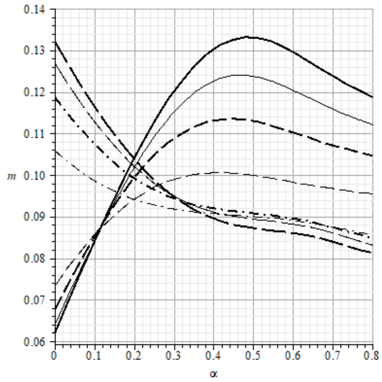
In Figure 5 is illustrated a continuous beam of three span of rectangular cross section with straight haunches in its ends on the internal supports. The first and third span (A-B and C-D) are of 12.00 m, and these have straight haunches in the support B and C, and in the support A and D have not straight haunches. The second span (B-C) is 12.00 m, and the straight haunches are perfectly symmetrical. Constant data on all the cross sections are: $v = 0.20$ for concrete, $w = 10$ kN/m, $b = 0.50$ m. Figure 6 shows the three beams separately and their fixed-end moments for uniformly distributed load in each support. The final moments are obtained



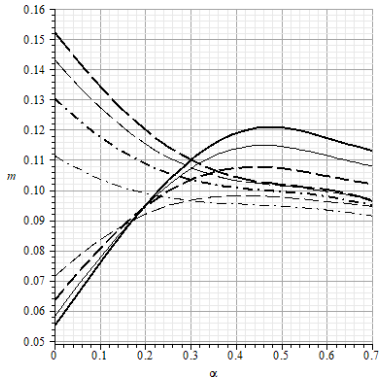
(a) $a = \alpha L$, $c = 0.0L$



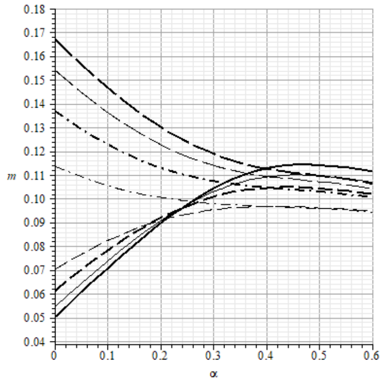
(b) $a = \alpha L$, $c = 0.1L$



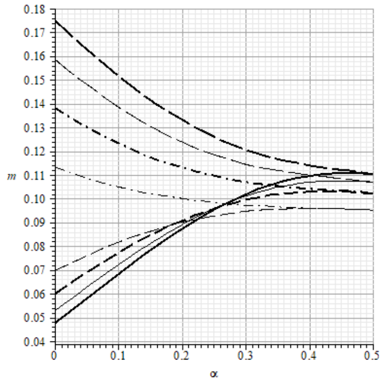
(c) $a = \alpha L$, $c = 0.2L$



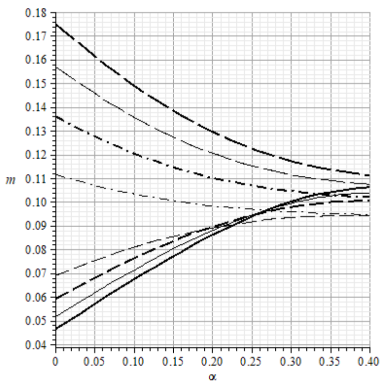
(d) $a = \alpha L$, $c = 0.3L$



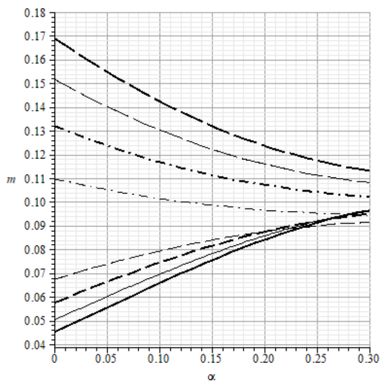
(e) $a = \alpha L$, $c = 0.4L$



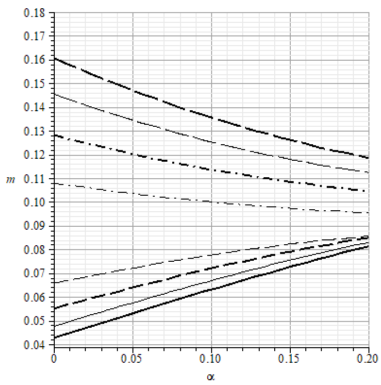
(f) $a = \alpha L$, $c = 0.5L$



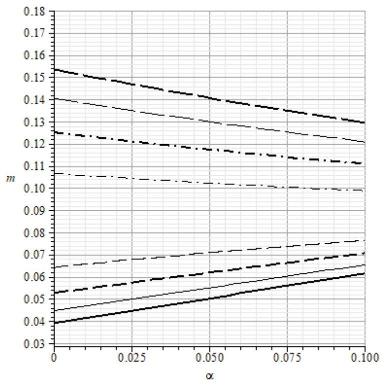
(g) $a = \alpha L$, $c = 0.6L$



(h) $a = \alpha L$, $c = 0.7L$



(i) $a = \alpha L$, $c = 0.8L$



(j) $a = \alpha L$, $c = 0.9L$

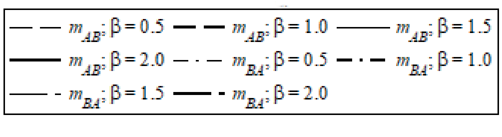
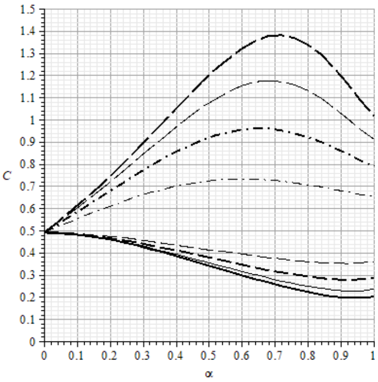
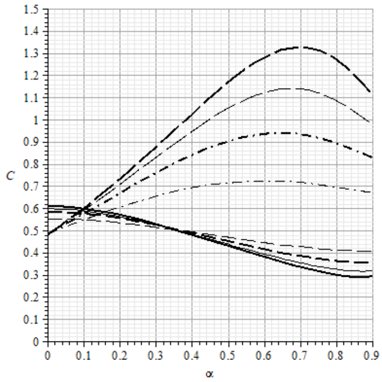


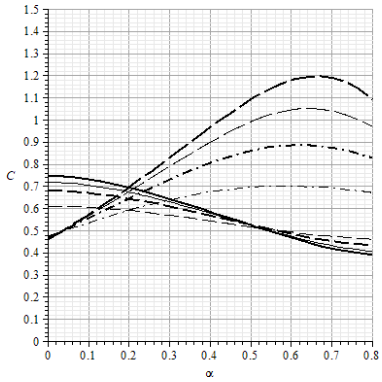
FIGURE 2. Fixed-ends moment factors for uniformly distributed load



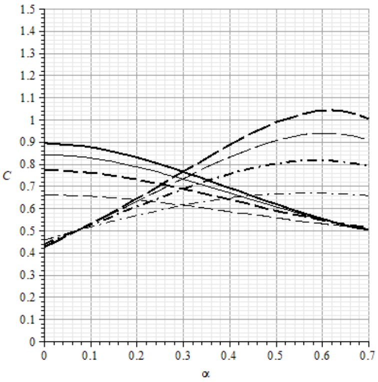
(a) $a = \alpha L$, $c = 0.0L$



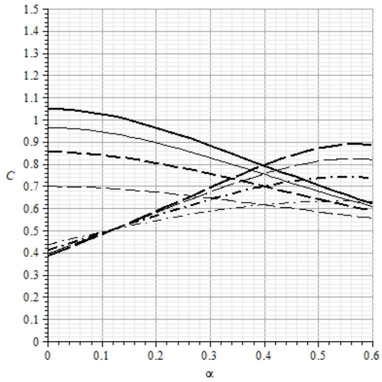
(b) $a = \alpha L$, $c = 0.1L$



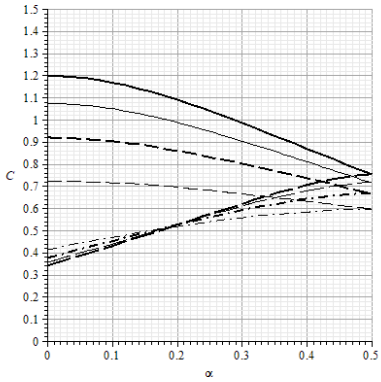
(c) $a = \alpha L$, $c = 0.2L$



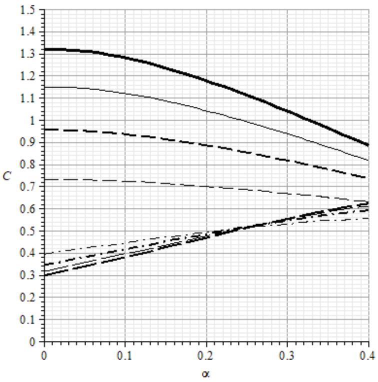
(d) $a = \alpha L$, $c = 0.3L$



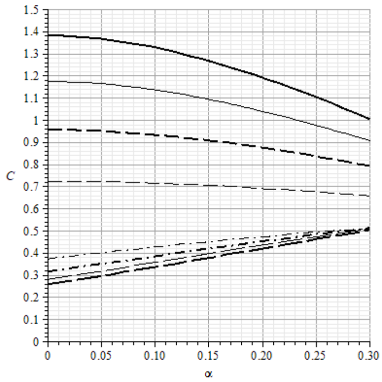
(e) $a = \alpha L$, $c = 0.4L$



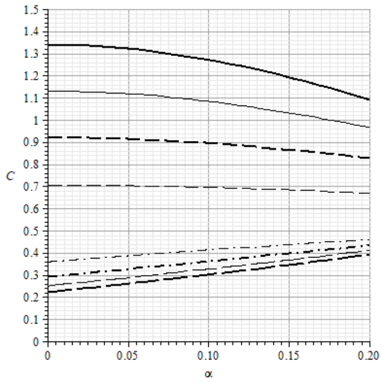
(f) $a = \alpha L$, $c = 0.5L$



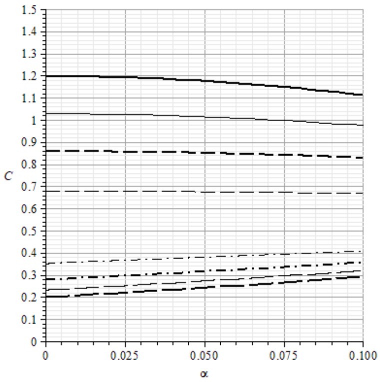
(g) $a = \alpha L$, $c = 0.6L$



(h) $a = \alpha L$, $c = 0.7L$



(i) $a = \alpha L$, $c = 0.8L$



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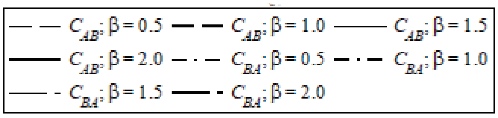
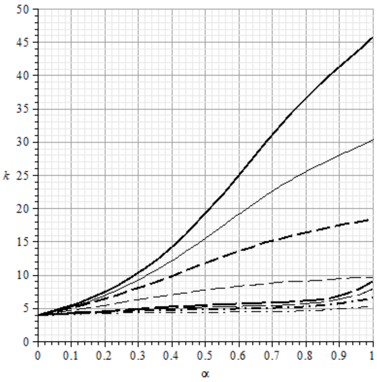
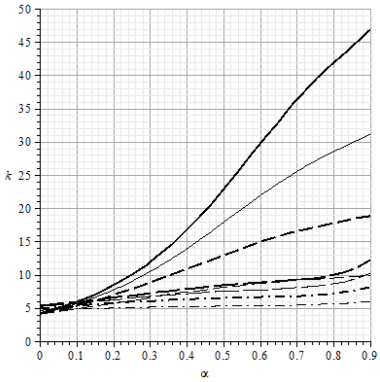


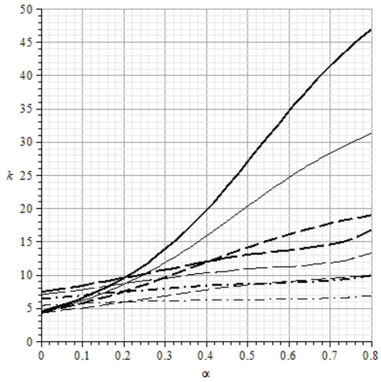
FIGURE 3. Carry-over factors



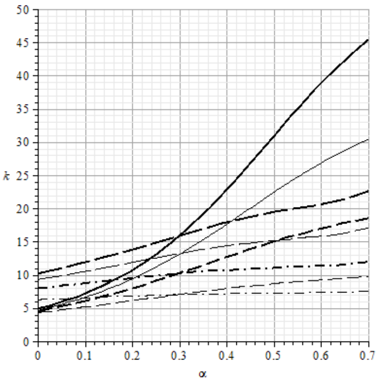
(a) $a = \alpha L$, $c = 0.0L$



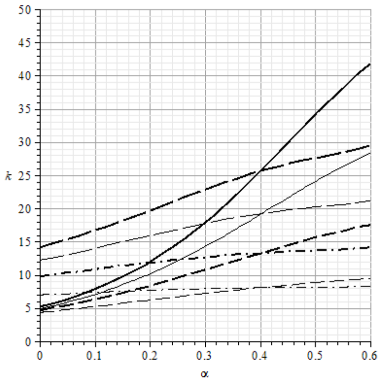
(b) $a = \alpha L$, $c = 0.1L$



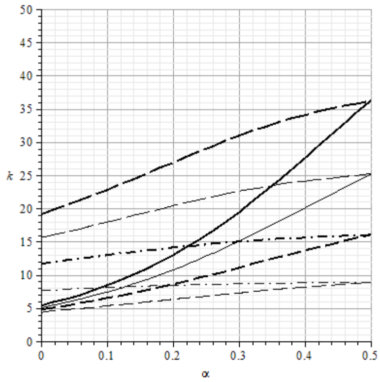
(c) $a = \alpha L$, $c = 0.2L$



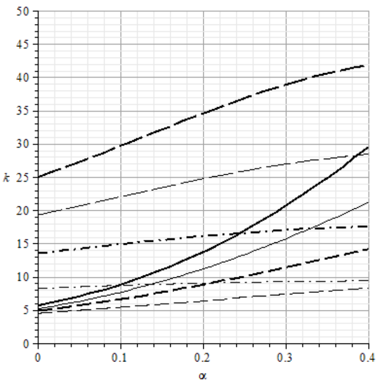
(d) $a = \alpha L$, $c = 0.3L$



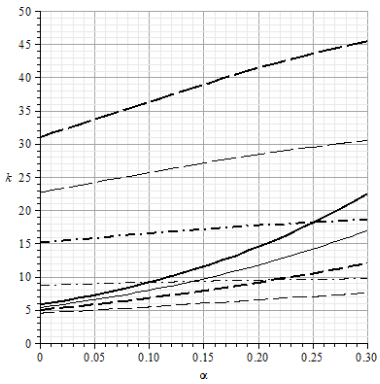
(e) $a = \alpha L$, $c = 0.4L$



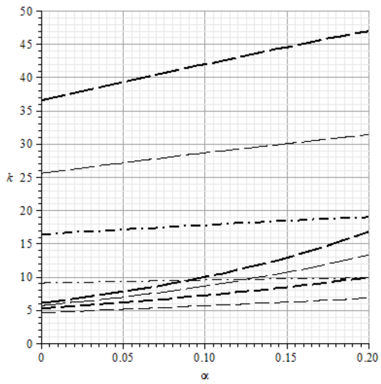
(f) $a = \alpha L$, $c = 0.5L$



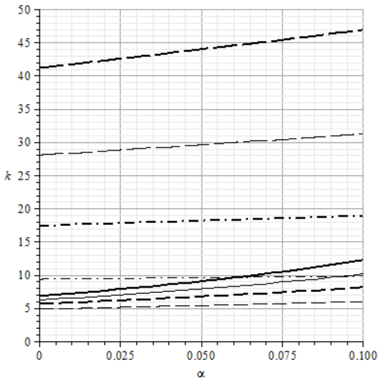
(g) $a = \alpha L$, $c = 0.6L$



(h) $a = \alpha L$, $c = 0.7L$



(i) $a = \alpha L$, $c = 0.8L$



(j) $a = \alpha L$, $c = 0.9L$

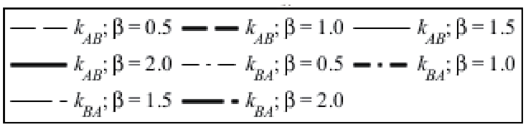


FIGURE 4. Stiffness factors

using the design aids proposed in this paper and the solution is obtained by the matrix methods.

Note: The horizontal distance of the haunches is considered to 3 meters because generally the inflection points are located to 1/4 of the length of the beam from its supports.

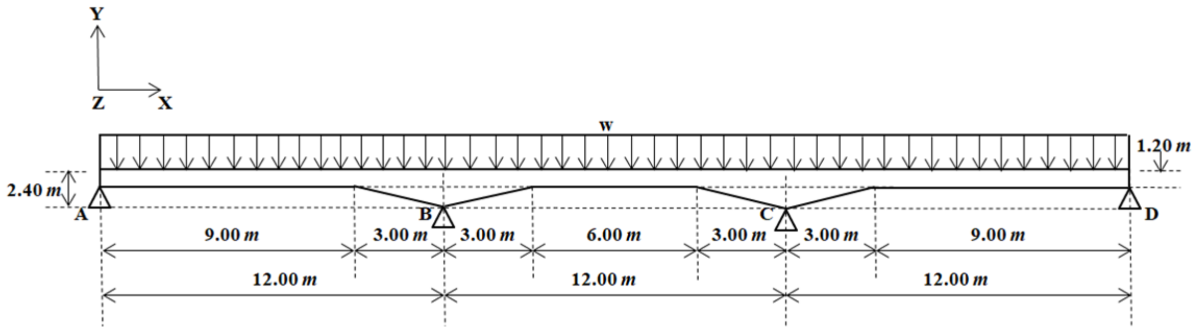


FIGURE 5. Continuous rectangular beam with straight haunches

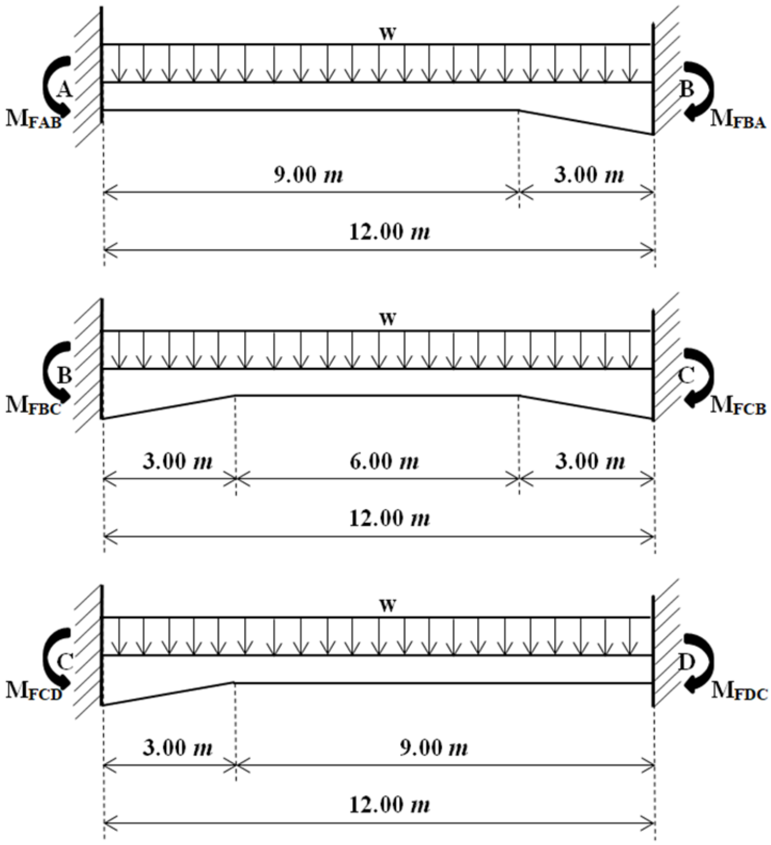


FIGURE 6. Three separate beams

The data for beam A-B are: $a = 0.00$ m; $c = 3.00$ m; $h = 1.20$ m; $u = 0.00$ m; $z = 1.20$ m; $L = 12.00$ m; $w = 10$ kN/m. To use the design aids, these are inverted in the supports because the haunches are located on the right side and in the graphics are located on the left side. The graphs shown in Figure 2(a) (Moments factors), Figure 3(a) (Carry-over factors), and Figure 4(a) (Stiffness factors) can be used to find the values. Now, to use the design aids, the following inverted values are: $\alpha = a/L = 0.25$; $\lambda = c/L = 0$; $\beta = u/h = 1.00$; $z = 0$. The graph shown in Figure 2(a) is used to find the moments factors at both ends ($\beta = 1.00$), and the fixed-end moments factors inverted in the supports are obtained: $m_{AB} = 0.065$ and $m_{BA} = 0.125$. The fixed-end moments are: $M_{FAB} = 93.60$ kN-m and $M_{FBA} = 180.00$ kN-m. The graph shown in Figure 3(a) is used to find the carry-over factors at both ends ($\beta = 1.00$), and the carry-over factors inverted in the supports are obtained: $C_{AB} = 0.72$ and $C_{BA} = 0.45$. The graph shown in Figure 4(a) is used to find the stiffness factors at both ends ($\beta = 1.00$), and the stiffness factors inverted in the supports are obtained: $k_{AB} = 4.50$ and $k_{BA} = 7.20$; now the absolute stiffnesses are: $K_{AB} = 4.50EI/L$ and $K_{BA} = 7.20EI/L$.

The data for beam B-C are: $a = 3.00$ m; $c = 3.00$ m; $h = 1.20$ m; $u = 1.20$ m; $z = 1.20$ m; $L = 12.00$ m; $w = 10$ kN/m. Now, to use the design aids, the following values are: $\alpha = a/L = 0.25$; $\lambda = c/L = 0.25$; $\beta = u/h = z/h = 1.00$. The graph shown in Figure 2(c) for $a = 0.2L$ and $c = 0.2L$ ($m_{BC} = 0.099$ and $m_{CB} = 0.099$) and Figure 2(d) for $a = 0.3L$ and $c = 0.3L$ ($m_{BC} = 0.104$ and $m_{CB} = 0.104$) by interpolation for $c = 0.25L$ are obtained the fixed-end moments factors at both ends ($\beta = 1.00$), and their values are: $m_{BC} = 0.1015$ and $m_{CB} = 0.1015$, and the fixed-end moments are: $M_{FBC} = 146.16$ kN-m and $M_{FCB} = 146.16$ kN-m. The graph shown in Figure 3(c) for $a = 0.2L$ and $c = 0.2L$ ($C_{BC} = 0.64$ and $C_{CB} = 0.64$) and Figure 3(d) for $a = 0.3L$ and $c = 0.3L$ ($C_{BC} = 0.69$ and $C_{CB} = 0.69$) by interpolation for $c = 0.25L$ are obtained the carry-over factors at both ends ($\beta = 1.00$), and their values are: $C_{BC} = 0.665$ and $C_{CB} = 0.665$. The graph shown in Figure 4(c) for $a = 0.2L$ and $c = 0.2L$ ($k_{BC} = 7.5$ and $k_{CB} = 7.5$) and Figure 4(d) for $a = 0.3L$ and $c = 0.3L$ ($k_{BC} = 10.3$ and $k_{CB} = 10.3$) by interpolation for $c = 0.25L$ are obtained the stiffness factors at both ends ($\beta = 1.00$), and their values are: $k_{BC} = 8.90$ and $k_{CB} = 8.90$; now the absolute stiffnesses are: $K_{BC} = 8.90EI/L$ and $K_{CB} = 8.90EI/L$.

The data for beam C-D are: $a = 3.00$ m; $c = 0.00$ m; $h = 1.20$ m; $u = 1.20$ m; $z = 0.00$ m; $L = 12.00$ m; $w = 10$ kN/m. Now to use the design aids, the following values are: $\alpha = a/L = 0.25$; $\lambda = c/L = 0$; $\beta = u/h = 1.00$; $z = 0$. The graph shown in Figure 2(a) is used to find the moments factors at both ends ($\beta = 1.00$), and the fixed-end moments factors are obtained: $m_{CD} = 0.125$ and $m_{DC} = 0.065$. The fixed-end moments are: $M_{FCD} = 180.00$ kN-m and $M_{FDC} = 93.60$ kN-m. The graph shown in Figure 3(a) is used to find the carry-over factors at both ends ($\beta = 1.00$), and the carry-over factors are obtained: $C_{CD} = 0.45$ and $C_{DC} = 0.72$. The graph shown in Figure 4(a) is used to find the stiffness factors at both ends ($\beta = 1.00$), and the stiffness factors are obtained: $k_{CD} = 7.20$ and $k_{DC} = 4.50$; now the absolute stiffnesses are: $K_{CD} = 7.20EI/L$ and $K_{DC} = 4.50EI/L$.

The stiffness matrix of the beam "A-B" is

$$K_{AB} = \begin{bmatrix} k_{11}^{AB} & k_{12}^{AB} \\ k_{21}^{AB} & k_{22}^{AB} \end{bmatrix} = \begin{bmatrix} 4.50 & 3.24 \\ 3.24 & 7.20 \end{bmatrix} \frac{EI_x}{L}$$

where $k_{11}^{AB} = K_{AB}$; $k_{22}^{AB} = K_{BA}$; $k_{12}^{AB} = C_{AB}K_{AB}$; $k_{21}^{AB} = C_{BA}K_{BA}$; $k_{12}^{AB} = k_{21}^{AB}$.

The stiffness matrix of the beam "B-C" is

$$K_{BC} = \begin{bmatrix} k_{11}^{BC} & k_{12}^{BC} \\ k_{21}^{BC} & k_{22}^{BC} \end{bmatrix} = \begin{bmatrix} 8.90 & 5.92 \\ 5.92 & 8.90 \end{bmatrix} \frac{EI_x}{L}$$

where $k_{11}^{BC} = K_{BC}$; $k_{22}^{BC} = K_{CB}$; $k_{12}^{BC} = C_{BC}K_{BC}$; $k_{21}^{BC} = C_{CB}K_{CB}$; $k_{12}^{BC} = k_{21}^{BC}$.

The stiffness matrix of the beam "C-D" is

$$K_{CD} = \begin{bmatrix} k_{11}^{CD} & k_{12}^{CD} \\ k_{21}^{CD} & k_{22}^{CD} \end{bmatrix} = \begin{bmatrix} 7.20 & 3.24 \\ 3.24 & 4.50 \end{bmatrix} \frac{EI_x}{L}$$

where $k_{11}^{CD} = K_{CD}$; $k_{22}^{CD} = K_{DC}$; $k_{12}^{CD} = C_{CD}K_{CD}$; $k_{21}^{CD} = C_{DC}K_{DC}$; $k_{12}^{CD} = k_{21}^{CD}$.

General stiffness matrix "K_G" of the continuous beam is

$$K_G = \begin{bmatrix} k_{11}^{AB} & k_{12}^{AB} & 0 & 0 \\ k_{21}^{AB} & k_{22}^{AB} + k_{11}^{BC} & k_{12}^{BC} & 0 \\ 0 & k_{21}^{BC} & k_{22}^{BC} + k_{11}^{CD} & k_{12}^{CD} \\ 0 & 0 & k_{21}^{CD} & k_{22}^{CD} \end{bmatrix} = \begin{bmatrix} 4.50 & 3.24 & 0 & 0 \\ 3.24 & 16.10 & 5.92 & 0 \\ 0 & 5.92 & 16.10 & 3.24 \\ 0 & 0 & 3.24 & 4.50 \end{bmatrix} \frac{EI_x}{L}$$

Fixed-end moments of the beams (phase 1) are

$$\begin{bmatrix} M_{AB} \\ M_{BA} \end{bmatrix} = \begin{bmatrix} +93.60 \\ -180.00 \end{bmatrix}; \quad \begin{bmatrix} M_{BC} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} +146.16 \\ -146.16 \end{bmatrix}; \quad \begin{bmatrix} M_{CD} \\ M_{DC} \end{bmatrix} = \begin{bmatrix} +180.00 \\ -93.60 \end{bmatrix}$$

The vector of effective moments that act on the continuous beam is

$$\begin{bmatrix} M_A \\ M_B \\ M_C \\ M_D \end{bmatrix} = \begin{bmatrix} -93.60 \\ +180.00 - 146.16 \\ +146.16 - 180.00 \\ +93.60 \end{bmatrix} = \begin{bmatrix} -93.60 \\ +33.84 \\ -33.84 \\ +93.60 \end{bmatrix}$$

Force-displacement relationship is

$$[P] = [K][d]$$

where $[P]$ is the vector of effective moments that acts on the continuous beam, $[K]$ is the general stiffness matrix, and $[d]$ is the vector of displacements.

$$\begin{bmatrix} -93.60 \\ +33.84 \\ -33.84 \\ +93.60 \end{bmatrix} = \begin{bmatrix} 4.50 & 3.24 & 0 & 0 \\ 3.24 & 16.10 & 5.92 & 0 \\ 0 & 5.92 & 16.10 & 3.24 \\ 0 & 0 & 3.24 & 4.50 \end{bmatrix} \frac{EI_x}{L} \begin{bmatrix} \theta_A \\ \theta_B \\ \theta_C \\ \theta_D \end{bmatrix}$$

The solution of the system is

$$\begin{bmatrix} \theta_A \\ \theta_B \\ \theta_C \\ \theta_D \end{bmatrix} = \begin{bmatrix} -30.0883 \\ +12.9004 \\ -12.9004 \\ +30.0883 \end{bmatrix} \frac{L}{EI_x}$$

The mechanical elements associated to the analysis moments (phase 2) are

$$\begin{bmatrix} M_{AB} \\ M_{BA} \end{bmatrix} = \begin{bmatrix} k_{11}^{AB} & k_{12}^{AB} \\ k_{21}^{AB} & k_{22}^{AB} \end{bmatrix} \begin{bmatrix} \theta_A \\ \theta_B \end{bmatrix} = \begin{bmatrix} 4.50 & 3.24 \\ 3.24 & 7.20 \end{bmatrix} \frac{EI_x}{L} \begin{bmatrix} -30.0883 \\ +12.9004 \end{bmatrix} \frac{L}{EI_x} = \begin{bmatrix} -93.60 \\ -4.60 \end{bmatrix}$$

$$\begin{bmatrix} M_{BC} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} k_{11}^{BC} & k_{12}^{BC} \\ k_{21}^{BC} & k_{22}^{BC} \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} 8.90 & 5.92 \\ 5.92 & 8.90 \end{bmatrix} \frac{EI_x}{L} \begin{bmatrix} +12.9004 \\ -12.9004 \end{bmatrix} \frac{L}{EI_x} = \begin{bmatrix} +38.44 \\ -38.44 \end{bmatrix}$$

$$\begin{bmatrix} M_{CD} \\ M_{DC} \end{bmatrix} = \begin{bmatrix} k_{11}^{CD} & k_{12}^{CD} \\ k_{21}^{CD} & k_{22}^{CD} \end{bmatrix} \begin{bmatrix} \theta_C \\ \theta_D \end{bmatrix} = \begin{bmatrix} 7.20 & 3.24 \\ 3.24 & 4.50 \end{bmatrix} \frac{EI_x}{L} \begin{bmatrix} -12.9004 \\ +30.0883 \end{bmatrix} \frac{L}{EI_x} = \begin{bmatrix} +4.60 \\ +93.60 \end{bmatrix}$$

The moments that result of the sum of phases 1 and 2 (final moments that act on each of the beams) are

$$\begin{bmatrix} M_{AB} \\ M_{BA} \end{bmatrix} = \begin{bmatrix} +93.60 \\ -180.00 \end{bmatrix} + \begin{bmatrix} -93.60 \\ -4.60 \end{bmatrix} = \begin{bmatrix} 0 \\ -184.60 \end{bmatrix}$$

$$\begin{bmatrix} M_{BC} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} +146.16 \\ -146.16 \end{bmatrix} + \begin{bmatrix} +38.44 \\ -38.44 \end{bmatrix} = \begin{bmatrix} +184.60 \\ -184.60 \end{bmatrix}$$

$$\begin{bmatrix} M_{CD} \\ M_{DC} \end{bmatrix} = \begin{bmatrix} +180.00 \\ -93.60 \end{bmatrix} + \begin{bmatrix} +4.60 \\ +93.60 \end{bmatrix} = \begin{bmatrix} +184.60 \\ 0 \end{bmatrix}$$

Therefore, these final moments are the moments for the design of the beams.

4. **Results.** Figures 2, 3 and 4 show the fixed-end moments factors (m_{AB} and m_{BA}), the carry-over factors (C_{AB} and C_{BA}) and the stiffness factors (k_{AB} and k_{BA}) in function of α.

Figure 2 shows the graphs for the fixed-end moments factors (m_{AB} and m_{BA}) subjected to a uniformly distributed load. The graphs show the following.

1) When the section is constant Figure 2(a) should be used, employing $\alpha = 0$ to obtain $m_{AB} = 0.083$ and $m_{BA} = 0.083$.

2) When the section is symmetrical, the values match in m_{AB} and m_{BA} for any height of the straight haunches (see Figure 2(b) to 2(f)).

3) When the values of λ increase, the fixed-end moments factors (m_{AB}) in $\alpha = 0$ decrease and the fixed-end moments factors (m_{BA}) in $\alpha = 0$ increase until $c = 0.6L$ (Figure 2(g)) and then decrease.

4) The maximum value of fixed-end moment factor "m_{AB}" is presented in Figure 2(a) ($\alpha = 0.55$, $\lambda = 0$ and $\beta = 2.00$) of 0.176.

5) The maximum value of fixed-end moment factor "m_{BA}" is presented in Figure 2(f) ($\alpha = 0$, $\lambda = 0.5$ and $\beta = 2.00$) of 0.175.

Figure 3 shows the graphs for the carry-over factors (C_{AB} and C_{BA}). The graphs show the following.

1) When the section is constant Figure 3(a) should be used, employing $\alpha = 0$ to obtain $C_{AB} = 0.49$ and $C_{BA} = 0.49$.

2) When the section is symmetrical, the values match in C_{AB} and C_{BA} for any height of the straight haunches (see Figure 3(b) to 3(f)).

3) When the values of λ increase, the carry-over factors (C_{AB}) in $\alpha = 0$ increase until $c = 0.7L$ (Figure 3(h)) and then decrease and the carry-over factors (C_{BA}) in $\alpha = 0$ decrease.

4) The maximum value of carry-over factor "C_{AB}" is presented in Figure 3(h) ($\alpha = 0$, $\lambda = 0.7$ and $\beta = 2.00$) of 1.38.

5) The maximum value of carry-over factor "C_{BA}" is presented in Figure 3(a) ($\alpha = 0.7$, $\lambda = 0$ and $\beta = 2.00$) of 1.38.

Figure 4 shows the graphs for the stiffness factors (k_{AB} and k_{BA}). The graphs show the following.

1) When the section is constant Figure 4(a) should be used, employing $\alpha = 0$ to obtain $k_{AB} = 3.90$ and $k_{BA} = 3.90$.

2) When the section is symmetrical, the values match in k_{AB} and k_{BA} for any height of the straight haunches (see Figure 4(b) to 4(f)).

3) When the values of λ increase, the stiffness factors (k_{AB}) in $\alpha = 0$ increase, and the stiffness factors (k_{BA}) in $\alpha = 0$ increase.

4) The maximum value of stiffness factor "k_{AB}" is presented in Figure 4(c) ($\alpha = 0.8$, $\lambda = 0.2$ and $\beta = 2.00$) of 47.

5) The maximum value of stiffness factor "k_{BA}" is presented in Figure 4(i) ($\alpha = 0.2$, $\lambda = 0.8$ and $\beta = 2.00$) of 47.

Figure 7 shows in detail the steps to obtain the diagrams of shear forces and moments of the numerical example. Steps are shown as follows.

Step 1: End reactions due to applied loading are obtained by balance of the isolated beams due to the loads, i.e., each of the beams is solved separately.

Step 2: End reactions due to end moments are obtained by balance of the isolated beams due to end moments, i.e., each of the beams is solved separately.

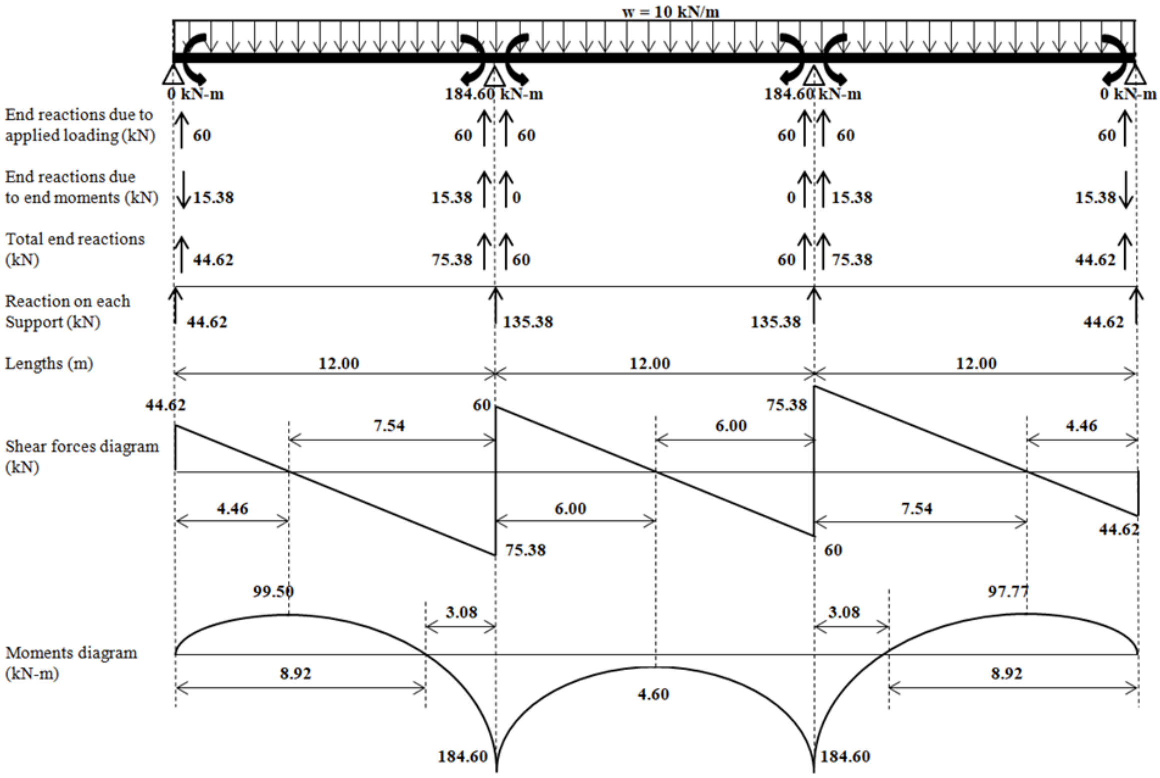


Figure 7. Diagrams of shear forces and moments

Step 3: Total end reactions are obtained by sum of the end reactions due to the applied loading (Step 1) and the end reactions due to end moments (Step 2).

Step 4: Reaction on each support is obtained by sum of the reactions generate of the beam on the left side and the beam on the right side.

Step 5: Shear force diagram is generated from the reactions on each support and the uniformly distributed load.

Step 6: Moments diagram is obtained from the shear force diagram, i.e., the shear force diagram area produces the moments diagram. Also, when the shear force has zero value, the moment is maximum positive.

The results of the considered problem in this paper are compared with Equations (13) and (14) to obtain the fixed-end moment factors, with Equations (19) and (20) to obtain the carry-over factors and with Equations (23) and (24) to obtain the stiffness factors [21]. These are the same with an approximation of three digits. Therefore, the degree of speed to obtain these factors by means of design aids is not compared with the equations, because the six equations are very complex and some software must be used to obtain the values.

5. **Conclusions.** Design aids for rectangular cross section beams with straight haunches in its ends subjected to a uniformly distributed load are developed to obtain the fixed-end moments factors, carry-over factors and stiffness factors.

The graphics presented in this paper simplify the problem of designing a rectangular cross section beam with straight haunches in its ends; these are accurate and efficient.

The purpose of this paper is to give practicing engineers some way of reducing the design time required for projects, while still complying with the ACI Standard 318, Building Code Requirements for Structural Concrete. Here the uniformly distributed load could be the live load and the dead load for building construction.

An example is presented that demonstrates the validity, applicability, simplicity and effectiveness of the proposed design aids in the analysis of rectangular cross-section beams with straight haunches subjected to a uniformly distributed load.

Suggestions for future research may be to obtain the design aids for the fixed-end moments factors for uniformly distributed load, carry-over factors and stiffness factors for different cross sections such as T or I, and for straight or parabolic haunches.

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