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DESIGN AIDS FOR RECTANGULAR CROSS-SECTION BEAMS WITH STRAIGHT HAUNCHES: PART 2

Laura Leticia Gaona Tamez, Arnulfo Luévanos Rojas*
Sandra López Chavarría, Manuel Medina Elizondo
and Martin Jaramillo Rosales

Instituto de Investigaciones Multidisciplinaria
Facultad de Contaduría y Administración
Universidad Autónoma de Coahuila, Unidad Torreón
Blvd. Revolución 151 Ote. CP 27000, Torreón, Coahuila, México
*Corresponding author: arnulfol_2007@hotmail.com

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Abstract. *This paper shows the design aids for rectangular cross-section beams with straight haunches subjected to a concentrated load localized anywhere of the beam to obtain the fixed-end moments factors, which is novelty of this research. The part 1 of this paper shows the design aids for rectangular cross-section beams with straight haunches subjected to a uniformly distributed load to obtain the fixed-end moments factors, carry-over factors and stiffness factors. The design aids are developed using the equations previously described by the writers in a companion paper. The simplifications made are the same as in part 1. The results of the considered problem are compared with the equations previously described by the writers in a companion paper, and these are the same with an approximation of three digits. Therefore, design aids provide a great tool for structural engineers by saving time.*
Keywords: Design aids, Rectangular cross-section beams, Straight haunches, Concentrated load, Fixed-end moments factors

1. **Introduction.** The beams with haunches are defined as structural elements that vary in section at the ends of the connection with the columns in order to reduce deflections and positive moments when negative ones increase.

A beam with haunches is used for basically two reasons:

1) Architecture: When the architecture of the structure demands such as churches and chapels;
2) Structural: When the construction has large spans and high loads, in order to reduce deflections and positive moments, and consequently increase negative moments.

In the calculation of the beams it is obtained, sometimes, that the stresses to support in its joining to the column are considerable. To absorb these stresses, it would be enough to increase the reinforced steel section in these dangerous areas. However, this is not always economical and another solution is used: the beam with haunches in its ends to obtain an increase in the concrete section (greater effective depth or effective cant), increasing, the lever arm of the reinforcing steel rods.

Design aids are represented by means of tables, graphs and descriptions of procedures with which it is intended to shorten the routine work of structural designers.

The most relevant works on the topic of design aids for structural analysis are shown in Part 1 [1-13]. Other works for variable sections (non-prismatic members) are presented

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in Part 1 [14-18]. Also, main works on the topic of rectangular cross section beams with straight haunches are shown in Part 1 [19-22].

This paper shows the design aids for rectangular cross-section beams with straight haunches subjected to a concentrated load localized anywhere of the beam to obtain the fixed-end moments factors, which is novelty of this research. The design aids are developed using the equations previously described [22]. The simplifications made are the same as in part 1. The design aids of the fixed-end moments factors are presented in function of ξ ($\xi = e/L$ = the position of the concentrated load measured from support "A" divided between the beam length). An example is presented to show the simplicity and effectiveness of the proposed design aids in the analysis of rectangular cross-section beams with straight haunches subjected to a uniformly distributed load and various concentrated loads.

The paper is organized as follows. Section 2 describes the beam of rectangular cross-section with straight haunches, and shows the design aids for the fixed-end moment factors of a concentrated load localized anywhere of the beam. Section 3 shows the example of application of design aids. Results are presented in Section 4. Conclusion (Section 5) completes the paper.

2. **Design Aids.** Figure 1 of part 1 shows an isometric member of rectangular section-cross with straight haunches in each end.

The design aids for members of rectangular section-cross with straight haunches consist in obtaining the fixed-end moments factors of a concentrated load localized anywhere of the beam through simplified diagrams [22].

The simplifications made are as follows: $u = z$ because this is the most used by architectural forms (aesthetic); $h = 0.1L$ that is proposed by the code of the ACI to control deflections.

The equations for fixed-end moments for concentrated load localized anywhere of the beam are obtained as follows [22]:

$$M_{AB} = m_{AB}PL \tag{1}$$

$$M_{BA} = m_{BA}PL \tag{2}$$

where M_{AB} and M_{BA} are fixed-end moments, m_{AB} and m_{BA} are fixed-end moment factors, P is concentrated load and L is the length of the member.

The procedure is developed taking into account the three sections, the first is where the left straight haunches are located, the second is in the central part, and the third is on the right straight haunches. The three curves are grouped together in a single diagram, because the concentrated load can be located anywhere on the beam.

Design aids for fixed-end moment factors are obtained from Equations (22) and (23) for the first section, from Equations (24) and (25) for the second section, from Equations (26) and (27) for the third section, taking into account $u = z = \beta h$ (β takes values of 0.5, 1.0, 1.5, 2.0); $a = \alpha L$ (α takes values of 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1); $c = \lambda L$ (λ takes values of 0.1, 0.2, 0.3, 0.4, 0.5); $h = 0.1L$; $\xi = e/L$ (ξ takes any value of 0 to 1, and e is the location of the load); $v = 0.20$ of concrete [22]. For $c = 0L$ it is considered $u = \beta h$; $z = 0$.

Design aids for fixed-end moment factors for concentrated load are obtained as follows:

1. The material and geometric properties of the beam are defined ($c = \lambda L$, $h = 0.1L$, $v = 0.20$, $u = z = \beta h$, $a = \alpha L$), and for $c = 0L$ it is considered $u = \beta h$ and $z = 0$.

2. Substitute each value of α and λ into Equations (22), (24) and (26) to obtain m_{AB}, and into Equations (23), (25) and (27) to obtain m_{BA} [22]. These graphs are shown in function of ξ.

The way to generate these graphs is as follows:

2.1. Equations (22) and (23) are fixed from α and λ, Equations (24) and (25) are fixed from α and λ, and Equations (26) and (27) are fixed from α and λ;

2.2. The graphs of Equations (22) and (23) are taken of 0 to α (left straight haunches), Equations (24) and (25) are taken of α to $1-\lambda$ (central parts), and Equations (26) and (27) are taken of $1-\lambda$ to 1 (right straight haunches).

3. Simplified equations are obtained using the Derive software.

4. Now, the graphs are obtained using the Maple software.

All simplified diagrams are shown as a function of ξ to obtain the fixed-end moment factors (m_{AB} and m_{BA}).

Figure 1 shows the graphs for $a = (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 \text{ and } 1.0)L$ and $c = 0.0L$.

Figure 2 presents the graphs for $a = (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 \text{ and } 0.9)L$ and $c = 0.1L$.

Figure 3 shows the graphs for $a = (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7 \text{ and } 0.8)L$ and $c = 0.2L$.

Figure 4 presents the graphs for $a = (0.1, 0.2, 0.3, 0.4, 0.5, 0.6 \text{ and } 0.7)L$ and $c = 0.3L$.

Figure 5 shows the graphs for $a = (0.1, 0.2, 0.3, 0.4, 0.5 \text{ and } 0.6)L$ and $c = 0.4L$.

Figure 6 presents the graphs for $a = (0.1, 0.2, 0.3, 0.4 \text{ and } 0.5)L$ and $c = 0.5L$.

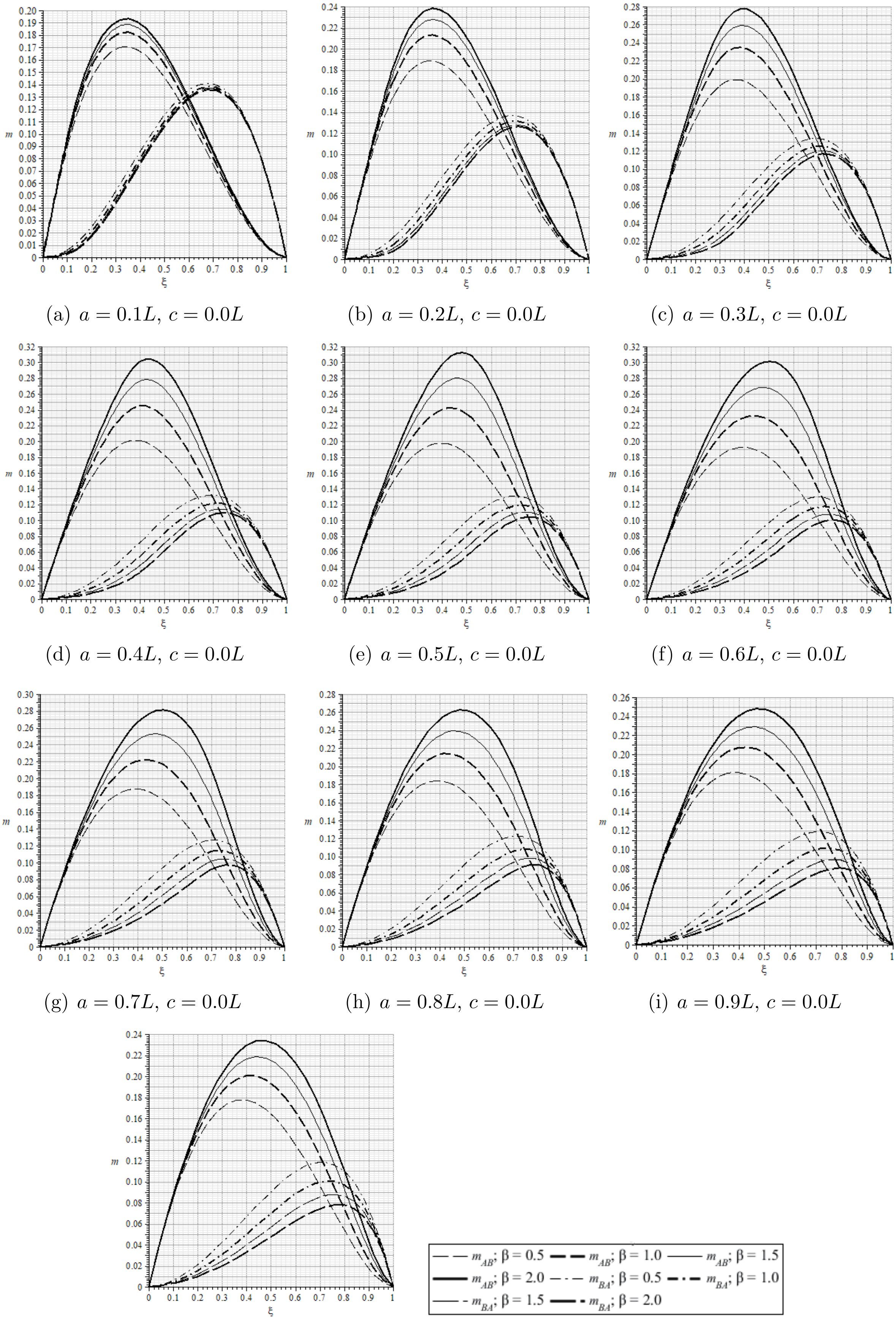
3. **Example of Application by Design Aids.** A road bridge continuous beam of three span of rectangular cross section with straight haunches in its ends on the internal supports is shown in Figure 5 of part 1. Constant data on the cross sections are: $v = 0.20$ for concrete, $w = 5$ kN/m (Dead load), $b = 0.50$ m. Figure 6 of part 1 shows the three beams separately and their fixed-end moments for uniformly distributed load (Dead load) in each support. Figure 7 shows the critical position of the live loads for the three beams separately of the highway bridge and their fixed-end moments for concentrated load (Live load) in each support; these live loads are obtained from the specifications for the design of bridges (AASHTO 2014) [23]. The final moments are obtained using the design aids of part 1 for the uniformly distributed load and the design aids for the concentrated loads proposed in this paper and the solution is obtained by the matrix methods.

Note: The horizontal distance of the haunches is considered to 3 meters because generally the inflection points are located to 1/4 of the length of the beam from its supports.

The data for beam A-B are: For dead load (uniformly distributed load), all factors are the same as shown in part 1; therefore, the moment factors are: $m_{AB} = 0.065$ and $m_{BA} = 0.125$, and the fixed-end moments are: $M_{FAB} = 46.80$ kN-m and $M_{FBA} = 90.00$ kN-m. The carry-over factors are: $C_{AB} = 0.72$ and $C_{BA} = 0.45$. The stiffness factors are: $k_{AB} = 4.50$ and $k_{BA} = 7.20$, and the absolute stiffnesses are: $K_{AB} = 4.50EI/L$ and $K_{BA} = 7.20EI/L$. Now, the data for the live load (concentrated load) are: $a = 0.00$ m; $c = 3.00$ m; $h = 1.20$ m; $u = 0.00$ m; $z = 1.20$ m; $L = 12.00$ m; $P = 35$ kN located in $e = 0.97$ m ($\xi = 0.08$); $P = 145$ kN located in $e = 5.27$ m ($\xi = 0.44$) and $P = 145$ kN located in $e = 9.57$ m ($\xi = 0.80$). To use the design aids, these are inverted in the supports because the haunches are located on the right side and in the graphics are located on the left side. Now, to use the design aids, the following inverted values are: $a = 0.25L$; $c = 0L$; $\beta = u/h = 1.00$; $z = 0$; $P = 35$ kN and $\xi = 0.92$; $P = 145$ kN and $\xi = 0.56$; $P = 145$ kN and $\xi = 0.20$ ($\beta = 1.00$).

The graph shown in Figure 1(b) for $a = 0.2L$ and $c = 0.0L$, and in Figure 1(c) for $a = 0.3L$ and $c = 0L$ are used to obtain the factors by interpolation.

For $P = 35$ kN and $\xi = 0.92$ with $a = 0.2L$ and $c = 0.0L$, the factors are: $m_{AB} = 0.065$ and $m_{BA} = 0.010$, and with $a = 0.3L$ and $c = 0.0L$, the factors are: $m_{AB} = 0.065$ and $m_{BA} = 0.012$. The fixed-end moments factors at both ends by interpolation for $a = 0.25L$



(a) $a = 0.1L$, $c = 0.0L$
(b) $a = 0.2L$, $c = 0.0L$
(c) $a = 0.3L$, $c = 0.0L$
(d) $a = 0.4L$, $c = 0.0L$
(e) $a = 0.5L$, $c = 0.0L$
(f) $a = 0.6L$, $c = 0.0L$
(g) $a = 0.7L$, $c = 0.0L$
(h) $a = 0.8L$, $c = 0.0L$
(i) $a = 0.9L$, $c = 0.0L$
(j) $a = 1.0L$, $c = 0.0L$

FIGURE 1. Fixed-ends moment factors for concentrated load for $c = 0.0L$

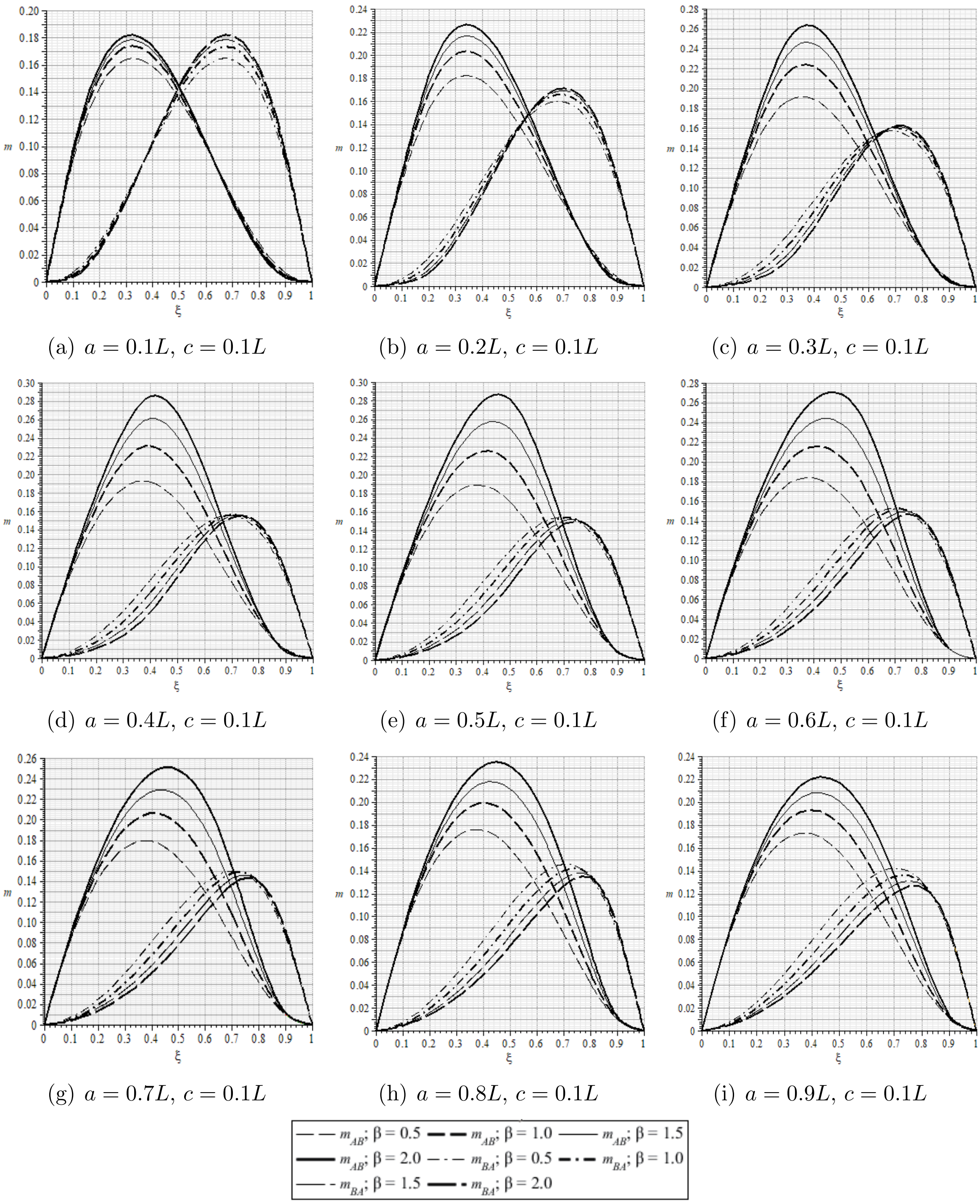
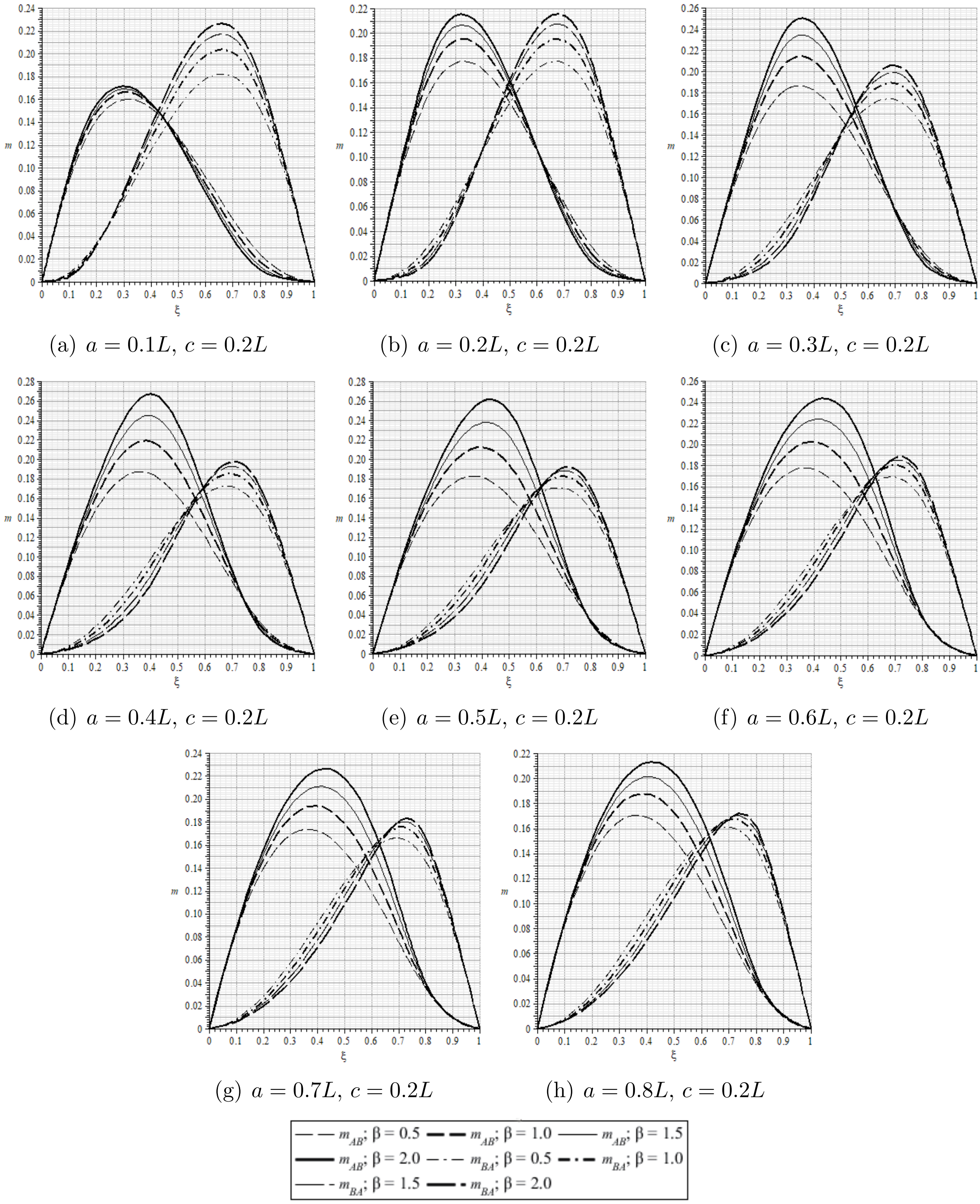


FIGURE 2. Fixed-ends moment factors for concentrated load for $c = 0.1L$

and $c = 0.0L$ are: $m_{AB} = 0.065$ and $m_{BA} = 0.011$, and the fixed-end moments are $M_{FAB} = 27.30$ kN-m and $M_{FBA} = 4.62$ kN-m.

For $P = 145$ kN and $\xi = 0.56$ with $a = 0.2L$ and $c = 0.0L$, the factors are: $m_{AB} = 0.114$ and $m_{BA} = 0.164$, and with $a = 0.3L$ and $c = 0.0L$, the factors are: $m_{AB} = 0.105$ and $m_{BA} = 0.189$. The fixed-end moments factors at both ends by interpolation for $a = 0.25L$ and $c = 0.0L$ are: $m_{AB} = 0.110$ and $m_{BA} = 0.176$, and the fixed-end moments are $M_{FAB} = 191.40$ kN-m and $M_{FBA} = 306.24$ kN-m.



(a) $a = 0.1L$, $c = 0.2L$ (b) $a = 0.2L$, $c = 0.2L$ (c) $a = 0.3L$, $c = 0.2L$

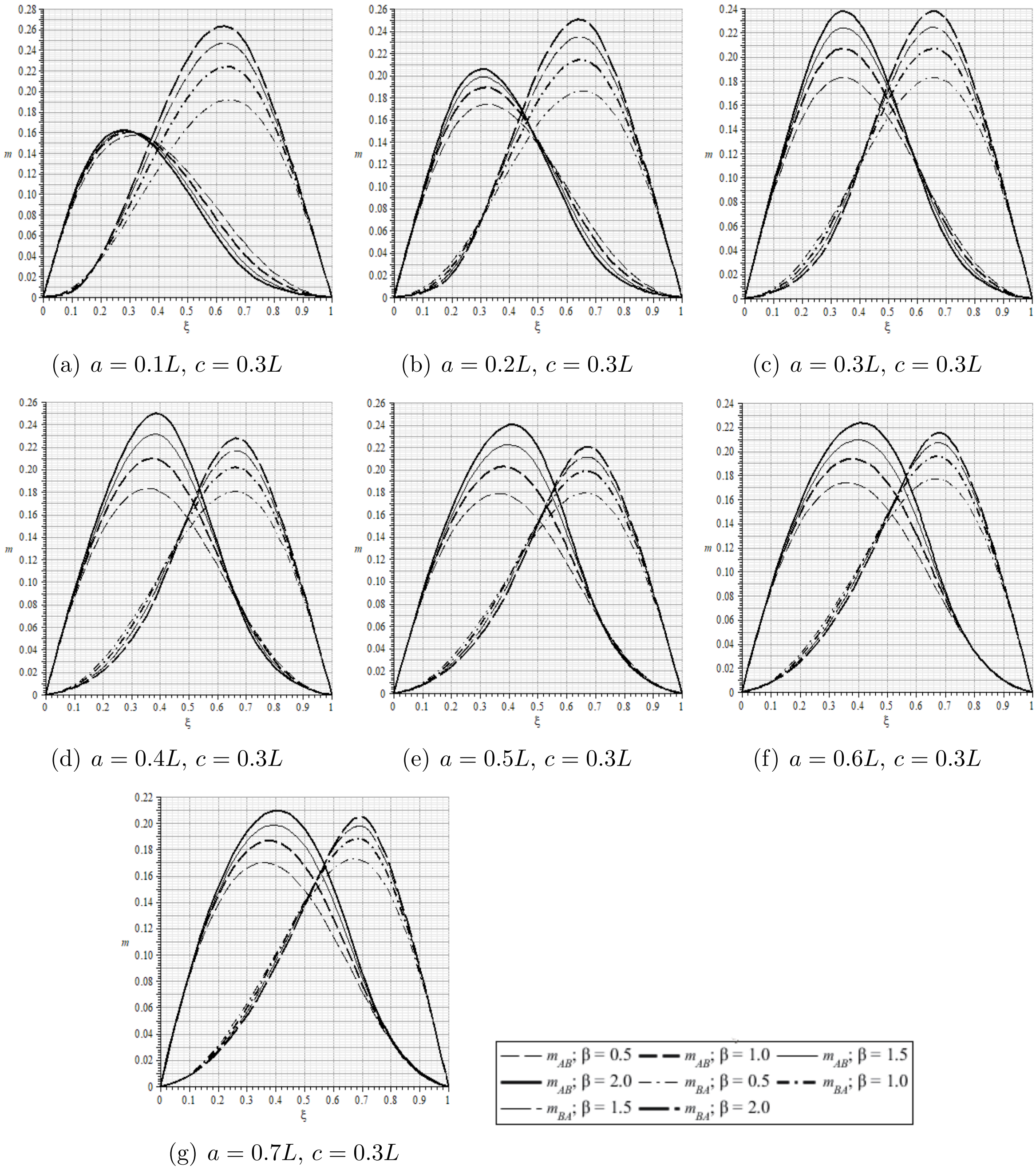
(d) $a = 0.4L$, $c = 0.2L$ (e) $a = 0.5L$, $c = 0.2L$ (f) $a = 0.6L$, $c = 0.2L$

(g) $a = 0.7L$, $c = 0.2L$ (h) $a = 0.8L$, $c = 0.2L$

FIGURE 3. Fixed-ends moment factors for concentrated load for $c = 0.2L$

For $P = 145$ kN and $\xi = 0.20$ with $a = 0.2L$ and $c = 0.0L$, the factors are: $m_{AB} = 0.014$ and $m_{BA} = 0.169$, and with $a = 0.3L$ and $c = 0.0L$, the factors are: $m_{AB} = 0.015$ and $m_{BA} = 0.170$. The fixed-end moments factors at both ends by interpolation for $a = 0.25L$ and $c = 0.0L$ are: $m_{AB} = 0.0145$ and $m_{BA} = 0.1695$, and the fixed-end moments are $M_{FAB} = 25.23$ kN-m and $M_{FBA} = 294.93$ kN-m.

Now, the total fixed-end moments are $M_{FAB} = 243.93$ kN-m and $M_{FBA} = 605.79$ kN-m.



(a) $a = 0.1L$, $c = 0.3L$ (b) $a = 0.2L$, $c = 0.3L$ (c) $a = 0.3L$, $c = 0.3L$

(d) $a = 0.4L$, $c = 0.3L$ (e) $a = 0.5L$, $c = 0.3L$ (f) $a = 0.6L$, $c = 0.3L$

(g) $a = 0.7L$, $c = 0.3L$

FIGURE 4. Fixed-ends moment factors for concentrated load for $c = 0.3L$

The data for beam B-C are: For dead load (uniformly distributed load), all factors are the same as shown in part 1; therefore, the moment factors are: $m_{BC} = 0.1015$ and $m_{CB} = 0.1015$, and the fixed-end moments are: $M_{FBC} = 73.08$ kN-m and $M_{FCB} = 73.08$ kN-m. The carry-over factors are: $C_{BC} = 0.665$ and $C_{CB} = 0.665$. The stiffness factors are: $k_{BC} = 8.90$ and $k_{CB} = 8.90$, and the absolute stiffnesses are: $K_{BC} = 8.90EI/L$ and $K_{CB} = 8.90EI/L$. Now, the data for the live load (concentrated load) are: $a = 3.00$ m; $c = 3.00$ m; $h = 1.20$ m; $u = 1.20$ m; $z = 1.20$ m; $L = 12.00$ m; $P = 35$ kN located in $e = 0.97$ m ($\xi = 0.08$); $P = 145$ kN located in $e = 5.27$ m ($\xi = 0.44$) and $P = 145$ kN located in $e = 9.57$ m ($\xi = 0.80$). Now, to use the design aids, the following values are: $a = 0.25L$; $c = 0.25L$; $\beta = u/h = 1.00$; $P = 35$ kN and $\xi = 0.08$; $P = 145$ kN and $\xi = 0.44$; $P = 145$ kN and $\xi = 0.80$.

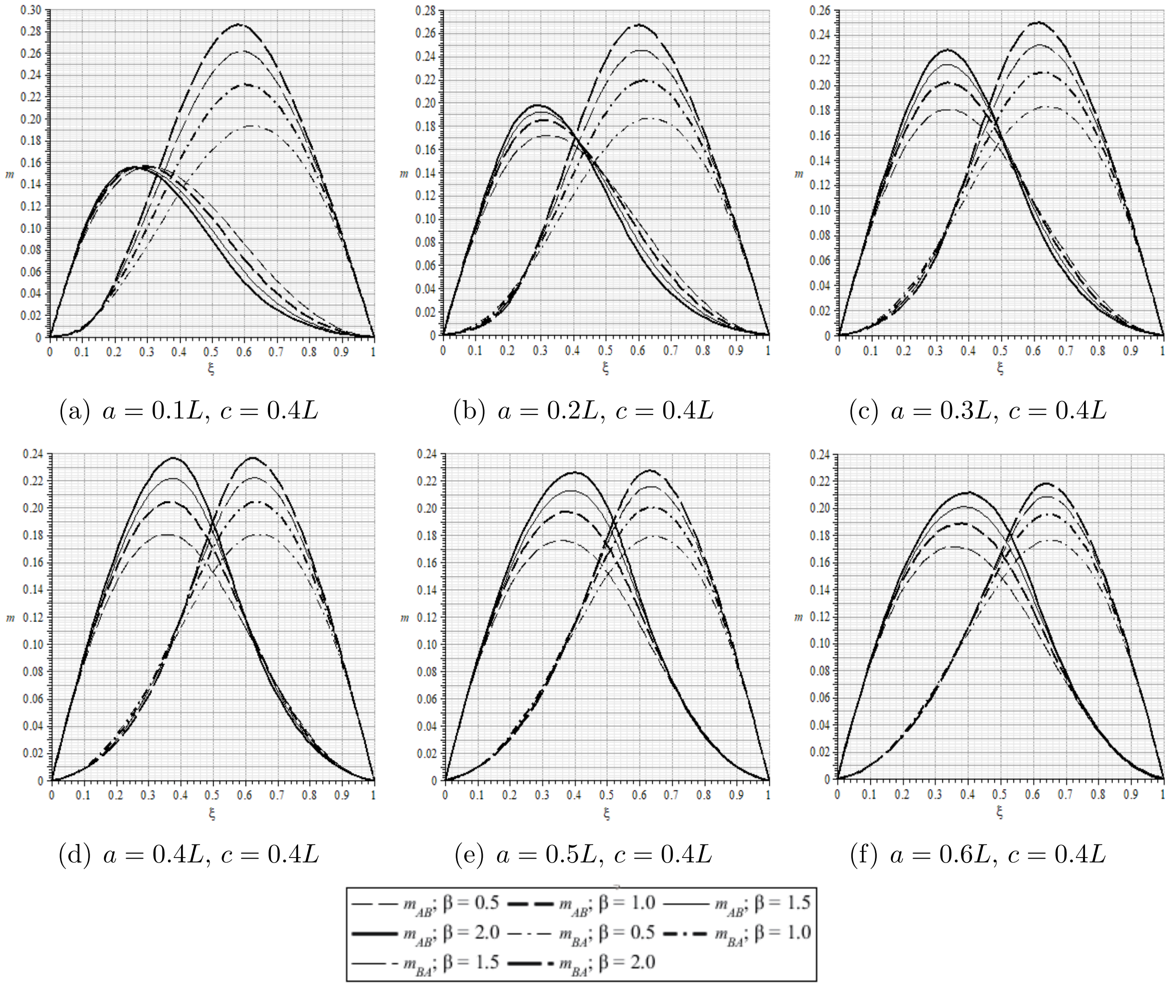


FIGURE 5. Fixed-ends moment factors for concentrated load for $c = 0.4L$

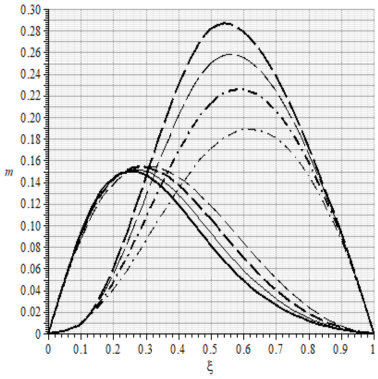
The graph shown in Figure 3(b) for $a = 0.2L$ and $c = 0.2L$, and in Figure 4(c) for $a = 0.3L$ and $c = 0.3L$ are used to obtain the factors by interpolation.

For $P = 35$ kN and $\xi = 0.08$ with $a = 0.2L$ and $c = 0.2L$, the factors are: $m_{BC} = 0.075$ and $m_{CB} = 0.004$, and with $a = 0.3L$ and $c = 0.3L$, the factors are: $m_{BC} = 0.074$ and $m_{CB} = 0.005$. The fixed-end moments factors at both ends by interpolation for $a = 0.25L$ and $c = 0.25L$ are: $m_{BC} = 0.0745$ and $m_{CB} = 0.0045$, and the fixed-end moments are $M_{FBC} = 31.29$ kN-m and $M_{FCB} = 1.89$ kN-m.

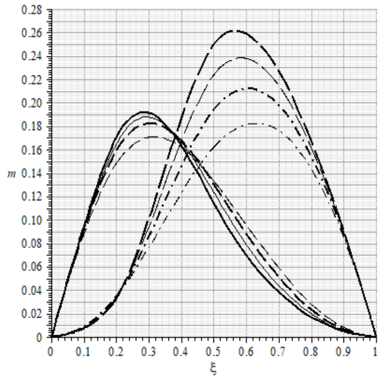
For $P = 145$ kN and $\xi = 0.44$ with $a = 0.2L$ and $c = 0.2L$, the factors are: $m_{BC} = 0.176$ and $m_{CB} = 0.126$, and with $a = 0.3L$ and $c = 0.3L$, the factors are: $m_{BC} = 0.189$ and $m_{CB} = 0.134$. The fixed-end moments factors at both ends by interpolation for $a = 0.25L$ and $c = 0.25L$ are: $m_{BC} = 0.1825$ and $m_{CB} = 0.130$, and the fixed-end moments are $M_{FBC} = 317.55$ kN-m and $M_{FCB} = 226.20$ kN-m.

For $P = 145$ kN and $\xi = 0.80$ with $a = 0.2L$ and $c = 0.2L$, the factors are: $m_{BC} = 0.023$ and $m_{CB} = 0.164$, and with $a = 0.3L$ and $c = 0.3L$, the factors are: $m_{BC} = 0.027$ and $m_{CB} = 0.162$. The fixed-end moments factors at both ends by interpolation for $a = 0.25L$ and $c = 0.25L$ are: $m_{BC} = 0.025$ and $m_{CB} = 0.163$, and the fixed-end moments are $M_{FBC} = 43.50$ kN-m and $M_{FCB} = 283.62$ kN-m.

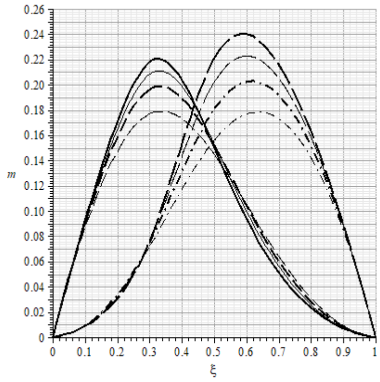
Now, the total fixed-end moments are $M_{FBC} = 392.34$ kN-m and $M_{FCB} = 511.71$ kN-m.



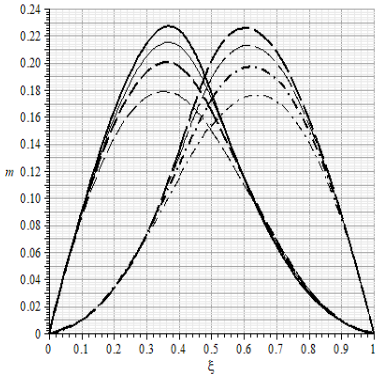
(a) $a = 0.1L$, $c = 0.5L$



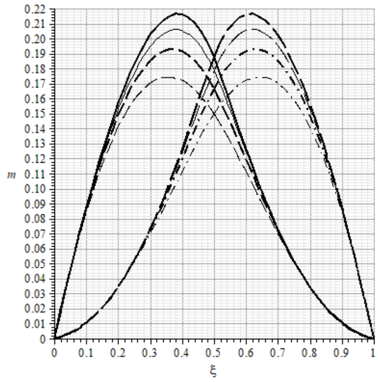
(b) $a = 0.2L$, $c = 0.5L$



(c) $a = 0.3L$, $c = 0.5L$



(d) $a = 0.4L$, $c = 0.5L$



(e) $a = 0.5L$, $c = 0.5L$

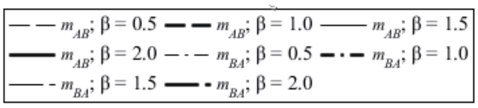


FIGURE 6. Fixed-ends moment factors for concentrated load for $c = 0.5L$

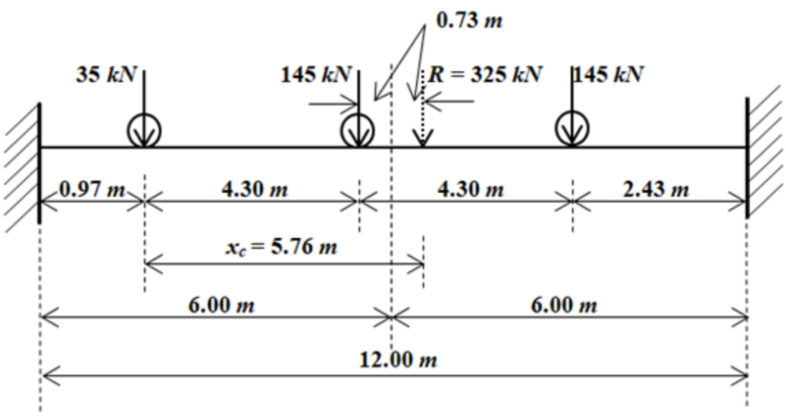


FIGURE 7. Critical position for the three beams

The data for beam C-D are: For dead load (uniformly distributed load), all factors are the same as shown in part 1; therefore, the moment factors are: $m_{CD} = 0.125$ and $m_{DC} = 0.065$, and the fixed-end moments are: $M_{FCD} = 90.00$ kN-m and $M_{FDC} = 46.80$ kN-m. The carry-over factors are: $C_{CD} = 0.45$ and $C_{DC} = 0.72$. The stiffness factors are: $k_{CD} = 7.20$ and $k_{DC} = 4.50$, and the absolute stiffnesses are: $K_{CD} = 7.20EI/L$ and

$K_{DC} = 4.50EI/L$. Now, the data for the live load (concentrated load) are: $a = 3.00$ m; $c = 0.00$ m; $h = 1.20$ m; $u = 1.20$ m; $z = 0$ m; $L = 12.00$ m; $P = 35$ kN located in $e = 0.97$ m ($\xi = 0.08$); $P = 145$ kN located in $e = 5.27$ m ($\xi = 0.44$) and $P = 145$ kN located in $e = 9.57$ m ($\xi = 0.80$). Now, to use the design aids, the following values are: $a = 0.25L$; $c = 0L$; $\beta = u/h = 1.00$; $z = 0$; $P = 35$ kN and $\xi = 0.08$; $P = 145$ kN and $\xi = 0.44$; $P = 145$ kN and $\xi = 0.80$.

The graph shown in Figure 1(b) for $a = 0.2L$ and $c = 0.0L$, and in Figure 1(c) for $a = 0.3L$ and $c = 0.0L$ are used to obtain the factors by interpolation.

For $P = 35$ kN and $\xi = 0.08$ with $a = 0.2L$ and $c = 0.0L$, the factors are: $m_{CD} = 0.076$ and $m_{DC} = 0.002$, and with $a = 0.3L$ and $c = 0.0L$, the factors are: $m_{CD} = 0.075$ and $m_{DC} = 0.002$. The fixed-end moments factors at both ends by interpolation for $a = 0.25L$ and $c = 0.0L$ are: $m_{CD} = 0.0755$ and $m_{DC} = 0.002$, and the fixed-end moments are $M_{FCD} = 31.71$ kN-m and $M_{FDC} = 0.84$ kN-m.

For $P = 145$ kN and $\xi = 0.44$ with $a = 0.2L$ and $c = 0.0L$, the factors are: $m_{CD} = 0.204$ and $m_{DC} = 0.080$, and with $a = 0.3L$ and $c = 0.0L$, the factors are: $m_{CD} = 0.229$ and $m_{DC} = 0.071$. The fixed-end moments factors at both ends by interpolation for $a = 0.25L$ and $c = 0.0L$ are: $m_{CD} = 0.2165$ and $m_{DC} = 0.0755$, and the fixed-end moments are $M_{FCD} = 376.71$ kN-m and $M_{FDC} = 131.37$ kN-m.

For $P = 145$ kN and $\xi = 0.80$ with $a = 0.2L$ and $c = 0.0L$, the factors are: $m_{CD} = 0.051$ and $m_{DC} = 0.119$, and with $a = 0.3L$ and $c = 0.0L$, the factors are: $m_{CD} = 0.060$ and $m_{DC} = 0.116$. The fixed-end moments factors at both ends by interpolation for $a = 0.25L$ and $c = 0.0L$ are: $m_{CD} = 0.0555$ and $m_{DC} = 0.1175$, and the fixed-end moments are $M_{FCD} = 96.57$ kN-m and $M_{FDC} = 204.45$ kN-m.

Now, the total fixed-end moments are $M_{FCD} = 504.99$ kN-m and $M_{FDC} = 336.66$ kN-m.

The stiffness matrix of each beam is the same, and the general stiffness matrix "K_G" of the continuous beam is the same of part 1.

Fixed-end moments of the beams (phase 1) are obtained by adding of the moment due to the uniformly distributed load and the moment due to the concentrated loads:

$$\begin{bmatrix} M_{AB} \\ M_{BA} \end{bmatrix} = \begin{bmatrix} +290.73 \\ -695.79 \end{bmatrix}; \quad \begin{bmatrix} M_{BC} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} +465.42 \\ -584.79 \end{bmatrix}; \quad \begin{bmatrix} M_{CD} \\ M_{DC} \end{bmatrix} = \begin{bmatrix} +594.99 \\ -383.46 \end{bmatrix}$$

The vector of effective moments that act on the continuous beam is

$$\begin{bmatrix} M_A \\ M_B \\ M_C \\ M_D \end{bmatrix} = \begin{bmatrix} -290.73 \\ +695.79 - 465.42 \\ +584.79 - 594.99 \\ +383.46 \end{bmatrix} = \begin{bmatrix} -290.73 \\ +230.37 \\ -10.20 \\ +383.46 \end{bmatrix}$$

Force-displacement relationship is

$$[P] = [K][d]$$

where $[P]$ is the vector of effective moments that acts on the continuous beam, $[K]$ is the general stiffness matrix, and $[d]$ is the vector of displacements.

$$\begin{bmatrix} -290.73 \\ +230.37 \\ -10.20 \\ +383.46 \end{bmatrix} = \begin{bmatrix} 4.50 & 3.24 & 0 & 0 \\ 3.24 & 16.10 & 5.92 & 0 \\ 0 & 5.92 & 16.10 & 3.24 \\ 0 & 0 & 3.24 & 4.50 \end{bmatrix} \frac{EI_x}{L} \begin{bmatrix} \theta_A \\ \theta_B \\ \theta_C \\ \theta_D \end{bmatrix}$$

The solution of the system is

$$\begin{bmatrix} \theta_A \\ \theta_B \\ \theta_C \\ \theta_D \end{bmatrix} = \begin{bmatrix} -100.7174 \\ +50.1537 \\ -42.3617 \\ +115.7137 \end{bmatrix} \frac{L}{EI_x}$$

The mechanical elements associated to the analysis moments (phase 2) are

$$\begin{bmatrix} M_{AB} \\ M_{BA} \end{bmatrix} = \begin{bmatrix} k_{11}^{AB} & k_{12}^{AB} \\ k_{21}^{AB} & k_{22}^{AB} \end{bmatrix} \begin{bmatrix} \theta_A \\ \theta_B \end{bmatrix} = \begin{bmatrix} 4.50 & 3.24 \\ 3.24 & 7.20 \end{bmatrix} \frac{EI_x}{L} \begin{bmatrix} -100.7174 \\ +50.1537 \end{bmatrix} \frac{L}{EI_x} = \begin{bmatrix} -290.73 \\ +34.78 \end{bmatrix}$$

$$\begin{bmatrix} M_{BC} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} k_{11}^{BC} & k_{12}^{BC} \\ k_{21}^{BC} & k_{22}^{BC} \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} 8.90 & 5.92 \\ 5.92 & 8.90 \end{bmatrix} \frac{EI_x}{L} \begin{bmatrix} +50.1537 \\ -42.3617 \end{bmatrix} \frac{L}{EI_x} = \begin{bmatrix} +195.59 \\ -80.11 \end{bmatrix}$$

$$\begin{bmatrix} M_{CD} \\ M_{DC} \end{bmatrix} = \begin{bmatrix} k_{11}^{CD} & k_{12}^{CD} \\ k_{21}^{CD} & k_{22}^{CD} \end{bmatrix} \begin{bmatrix} \theta_C \\ \theta_D \end{bmatrix} = \begin{bmatrix} 7.20 & 3.24 \\ 3.24 & 4.50 \end{bmatrix} \frac{EI_x}{L} \begin{bmatrix} -42.3617 \\ +115.7137 \end{bmatrix} \frac{L}{EI_x} = \begin{bmatrix} +69.91 \\ +383.46 \end{bmatrix}$$

The moments that result of the sum of phases 1 and 2 (final moments that act on each of the beams) are

$$\begin{bmatrix} M_{AB} \\ M_{BA} \end{bmatrix} = \begin{bmatrix} +290.73 \\ -695.79 \end{bmatrix} + \begin{bmatrix} -290.73 \\ +34.78 \end{bmatrix} = \begin{bmatrix} 0 \\ -661.01 \end{bmatrix}$$

$$\begin{bmatrix} M_{BC} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} +465.42 \\ -584.79 \end{bmatrix} + \begin{bmatrix} +195.59 \\ -80.11 \end{bmatrix} = \begin{bmatrix} +661.01 \\ -664.90 \end{bmatrix}$$

$$\begin{bmatrix} M_{CD} \\ M_{DC} \end{bmatrix} = \begin{bmatrix} +594.99 \\ -383.46 \end{bmatrix} + \begin{bmatrix} +69.91 \\ +383.46 \end{bmatrix} = \begin{bmatrix} +664.90 \\ 0 \end{bmatrix}$$

Therefore, these final moments are the moments for the design of the beams.

4. **Results.** Figures 1 to 6 show the fixed-end moments factors (m_{AB} and m_{BA}) in function of ξ, which is the location of the concentrated load measured from the left support of the beam.

Figure 1 shows the graphs for thefixed-end moments factors (m_{AB} and m_{BA}) subjected to a concentrated load for $c = 0.0L$. The graphs show the following.

1) The maximum value of fixed-end moment factor "m_{AB}" is presented in Figure 1(e) ($\alpha = 0.5$, $\lambda = 0$, $\beta = 2.00$ and $\xi = 0.48$) of 0.302.

2) The maximum value of fixed-end moment factor "m_{BA}" is presented in Figure 1(a) ($\alpha = 0.1$, $\lambda = 0$, $\beta = 0.50$ and $\xi = 0.66$) of 0.141.

Figure 2 shows the graphs for the fixed-end moments factors (m_{AB} and m_{BA}) subjected to a concentrated load for $c = 0.1L$. The graphs show the following.

1) The maximum fixed-end moments factor "m_{AB}" is presented in Figure 2(e) ($\alpha = 0.5$, $\lambda = 0.1$, $\beta = 2.00$ and $\xi = 0.46$) of 0.288.

2) The maximum fixed-end moments factor "m_{BA}" is presented in Figure 2(a) ($\alpha = 0.1$, $\lambda = 0.1$, $\beta = 2.00$ and $\xi = 0.68$) of 0.182.

3) The fixed-end moments factors (m_{AB} and m_{BA}) are equal in Figure 2(a) ($\alpha = 0.1$, $\lambda = 0.1$, $\beta = 0.50$, 1.00, 1.50, 2.00 and $\xi = 0.50$). This means that the beam is symmetrical in load and in geometry, i.e., the concentrated load is located in the center of the beam.

Figure 3 shows the graphs for the fixed-end moments factors (m_{AB} and m_{BA}) subjected to a concentrated load for $c = 0.2L$. The graphs show the following.

1) The maximum fixed-end moments factor "m_{AB}" is presented in Figure 3(d) ($\alpha = 0.4$, $\lambda = 0.2$, $\beta = 2.00$ and $\xi = 0.40$) of 0.268.

2) The maximum fixed-end moments factor "m_{BA}" is presented in Figure 3(a) ($\alpha = 0.1$, $\lambda = 0.2$, $\beta = 2.00$ and $\xi = 0.66$) of 0.228.

3) The fixed-end moments factors (m_{AB} and m_{BA}) are equal in Figure 3(b) ($\alpha = 0.2$, $\lambda = 0.2$, $\beta = 0.50$, 1.00, 1.50, 2.00 and $\xi = 0.50$). This means that the beam is symmetrical in load and in geometry, i.e., the concentrated load is located in the center of the beam.

Figure 4 shows the graphs for the fixed-end moments factors (m_{AB} and m_{BA}) subjected to a concentrated load for $c = 0.3L$. The graphs show the following.

1) The maximum fixed-end moments factor "m_{AB}" is presented in Figure 4(d) ($\alpha = 0.4$, $\lambda = 0.3$, $\beta = 2.00$ and $\xi = 0.38$) of 0.250.

2) The maximum fixed-end moments factor "m_{BA}" is presented in Figure 4(a) ($\alpha = 0.1$, $\lambda = 0.3$, $\beta = 2.00$ and $\xi = 0.62$) of 0.265.

3) The fixed-end moments factors (m_{AB} and m_{BA}) are equal in Figure 4(c) ($\alpha = 0.3$, $\lambda = 0.3$, $\beta = 0.50$, 1.00, 1.50, 2.00 and $\xi = 0.50$). This means that the beam is symmetrical in load and in geometry, i.e., the concentrated load is located in the center of the beam.

Figure 5 shows the graphs for the fixed-end moments factors (m_{AB} and m_{BA}) subjected to a concentrated load for $c = 0.4L$. The graphs show the following.

1) The maximum fixed-end moments factor "m_{AB}" is presented in Figure 5(d) ($\alpha = 0.4$, $\lambda = 0.4$, $\beta = 2.00$ and $\xi = 0.37$) of 0.238.

2) The maximum fixed-end moments factor "m_{BA}" is presented in Figure 5(a) ($\alpha = 0.1$, $\lambda = 0.4$, $\beta = 2.00$ and $\xi = 0.58$) of 0.288.

3) The fixed-end moments factors (m_{AB} and m_{BA}) are equal in Figure 5(d) ($\alpha = 0.4$, $\lambda = 0.4$, $\beta = 0.50$, 1.00, 1.50, 2.00 and $\xi = 0.50$). This means that the beam is symmetrical in load and in geometry, i.e., the concentrated load is located in the center of the beam.

Figure 6 shows the graphs for the fixed-end moments factors (m_{AB} and m_{BA}) subjected to a concentrated load for $c = 0.5L$. The graphs show the following.

1) The maximum fixed-end moments factor "m_{AB}" is presented in Figure 6(d) ($\alpha = 0.5$, $\lambda = 0.5$, $\beta = 2.00$ and $\xi = 0.37$) of 0.228.

2) The maximum fixed-end moments factor "m_{BA}" is presented in Figure 6(a) ($\alpha = 0.1$, $\lambda = 0.5$, $\beta = 2.00$ and $\xi = 0.55$) of 0.288.

3) The fixed-end moments factors (m_{AB} and m_{BA}) are equal in Figure 6(e) ($\alpha = 0.5$, $\lambda = 0.5$, $\beta = 0.50$, 1.00, 1.50, 2.00 and $\xi = 0.50$). This means that the beam is symmetrical in load and in geometry, i.e., the concentrated load is located in the center of the beam.

Figure 8 shows in detail the steps to obtain the diagrams of shear forces and moments. Steps are same as that in part 1.

The results of the considered problem in this paper were compared by means of Equations (22) and (23) for the first section, Equations (24) and (25) for the central part, Equations (26) and (27) for the third section [22], and these are the same with an approximation of three digits. Therefore, the degree of speed to obtain these factors by means of design aids is not compared with the equations, because the six equations are very complex and some software must be used to obtain the values.

5. **Conclusions.**

Design aids for rectangular cross section beams with straight haunches in its ends subjected to a concentrated load located anywhere of the beam are developed to obtain the fixed-end moments factors.

The graphics presented in this paper simplify the problem of designing a rectangular cross section beam with straight haunches in its ends; these are accurate and efficient.

The purpose of this paper is to give practicing engineers some way of reducing the design time required for projects, while still complying with the ACI Standard 318, Building Code Requirements for Structural Concrete. Here the dead load is the uniformly distributed load and the live load is the concentrated load due to rolling loads for highway bridges.

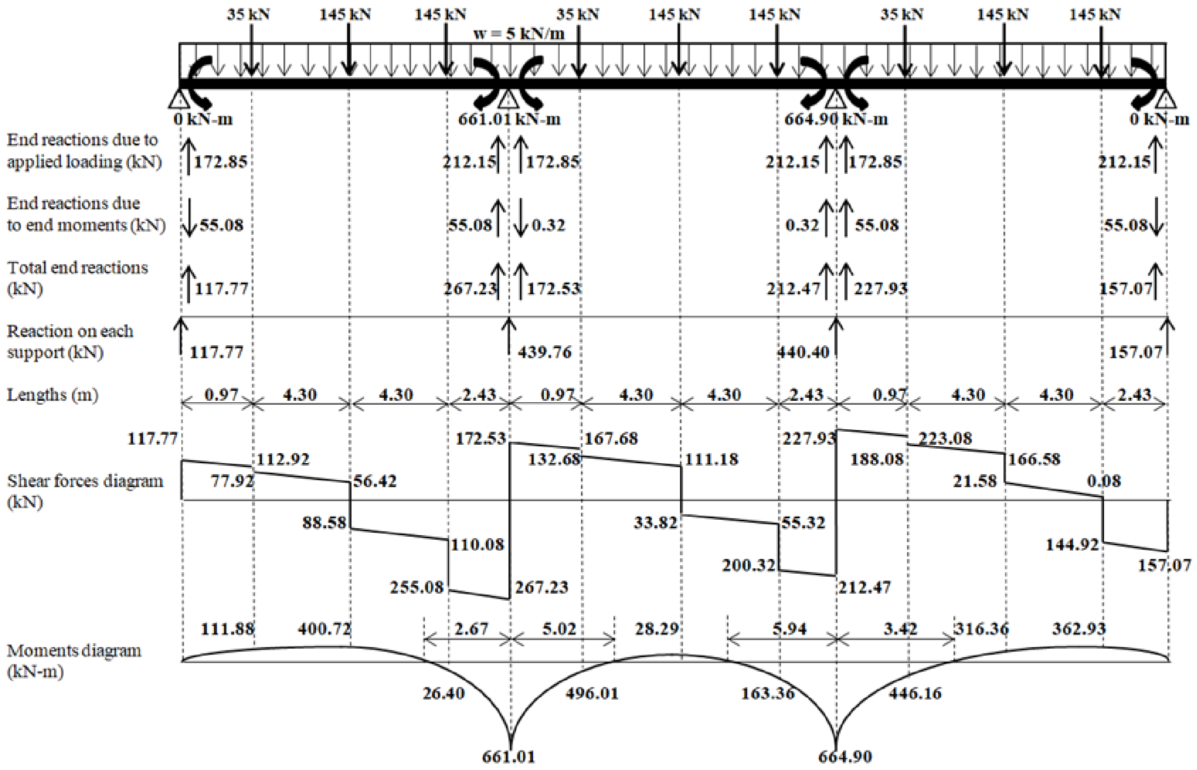


Figure 8. Diagrams of shear forces and moments

An example is presented to demonstrate the validity, applicability, simplicity and effectiveness of the proposed design aids in the analysis of rectangular cross-section beams with straight haunches subjected to various concentrated loads that are the rolling loads.

Suggestions for future research may be to obtain the design aids for the fixed-end moments factors subjected to concentrated load for different cross sections such as T or I, and for straight or parabolic haunches.

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