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A DESIGN METHOD FOR STRONGLY STABILIZING CONTROLLERS

Tan Akuzawa[1], Dayu Zhang[1], Tatsuya Hoshikawa[1], Jinghui Li[1]
Yuuko Tatsumi[1], Kotaro Hashikura[2], Md Abdus Samad Kamal[2]
Takaaki Suzuki[2] and Kou Yamada[2]

[1]Graduate School of Science and Technology
[2]Division of Mechanical Science and Technology
Gunma University
1-5-1 Tenjincho, Kiryu 376-8515, Japan
{ t14302081; t192b601; t10801250; t161b070; t08302053 }@gunma-u.ac.jp
{ k-hashikura; maskamal; suzuki.taka; yamada }@gunma-u.ac.jp

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Abstract. *When the control system is stabilized by a stable controller, the controller is said to be a strongly stabilizing controller. While there are many design methods of stabilizing controllers, most existing methods do not consider the stability of stabilizing controllers. Because the instability of the stabilizing controllers makes closed-loop systems very sensitive to disturbances and reduces the tracking performance to reference inputs, it is required in practice to use the stable stabilizing controller whenever it is possible. Youla et al. showed that not every plant can be stabilized by a stable controller and examined a design procedure for stable stabilizing controllers. However, it is better in practice to design controllers that make use of the individuality of given plants. From this perspective, Hoshikawa et al. clarified the class of strongly stabilizable plants that can be stabilized by a stable controller. However, they have not given a design method of strongly stabilizing controllers for the class of strongly stabilizable plants. This is the purpose of this paper, and to propose a design method, we clarify the parameterization of all strongly stabilizing controllers for strongly stabilizable plants.*
Keywords: Strong stabilization, Strongly stabilizable plants, Controller parameterization, Closed-loop characteristics

1. **Introduction.** In this paper, we examine a design method for stable stabilizing controllers using the parameterization of all plants that can be stabilized by a stable controller. The parameterization problem is ensuring all stabilizing controllers for a plant [1-8] and those plants can be stabilized [9]. Because this parameterization can successfully search for all proper stabilizing controllers, it is used as a tool for many control problems.

For an unstable plant, the parameterization of all stabilizing controllers has been solved by Youla et al. [1, 2]. The structure of the parameterization of all stabilizing controllers for unstable plants has full-order state feedback, including a full-order observer [5]. Glaria and Goodwin [6] gave a simple parameterization for single-input/single-output minimum-phase systems. However, two difficulties remain. One is that the parameterization of all stabilizing controllers given by Glaria and Goodwin generally includes improper controllers. In practical application, the controller must be proper. The other is that they do not give the parameterization of all internally stabilizing controllers. Yamada overcame these problems and proposed the parameterization of all proper internally stabilizing controllers for single-input/single-output minimum-phase systems [7].

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For a stable plant, the parameterization of all stabilizing controllers with an internal model control structure that has advantages such as closed-loop stability is ensured simply by choosing a stable internal model controller parameter, and closed-loop performance characteristics are related directly to the controller parameters, making online tuning of the internal model controller convenient [8]. However, there is a question as to whether stabilizing controllers for unstable plants can be represented by an internal model control structure. For this question, Morari and Zafiriou [8] examined the parameterization of all stabilizing internal model controllers for unstable plants. However, their parameterization involves two difficulties. First, their internal model is not necessarily proper. In addition, their parameterization includes improper internal model controllers. To overcome these problems, Chen et al. proposed the simple parameterization of all proper stabilizing internal model controllers for minimum-phase unstable plants [10]. Zhang et al. [11] proposed a new parameterization that does not need the coprime factorization and has a form similar to Youla parameterization. In this way, the theory of the parameterization of all stabilizing controllers has been advancing.

However, little attention has been paid to the stability of stabilizing controllers. If an unstable stabilizing controller is employed, the unstable poles of the stabilizing controller make the closed-loop transfer function have zeros in the right half plane. This results in the closed-loop system being very sensitive to disturbances and reduces the tracking performance to reference inputs [4, 12]. In addition, if the feedback-loop of the control system is cut, that is, the control system breaks down to a feedforward control system, the unstable poles of the stabilizing controller produce the unstable poles of the control system. Thus, the control system becomes unstable even if the plant is stable. Based on the presented reasons, it is desirable in practice that the control system is stabilized by a stable stabilizing controller [12]. Therefore, several design methods of a stable stabilizing controller, which is considered a strongly stabilizing controller, have been considered [4, 12-20, 22, 23].

There exist plants that cannot be stabilized by any stable controller. Youla et al. showed that a plant is strongly stabilizable if and only if it satisfies the parity interlacing property condition and examined a design procedure of stable stabilizing controllers [13]. However, while they gave a method to identify strongly stabilizable plants, they did not specify a specific form of strongly stabilizable plants.

Wakaiki et al. examined the sensitivity reduction problem with stable controllers for linear time-invariant multiple-input/multiple-output distributed parameter systems [17]. Wakaiki et al. considered the strong and robust stabilization problem that a class of plants has finitely many simple unstable zeros, but possibly infinitely many unstable poles stabilized by a stable controller in linear time-invariant single-input/single-output infinitely dimensional systems [18]. However, they do not clarify the class of strongly stabilizable plants. If the class is clarified, we can obtain the parameterization of all stable stabilizing controllers. From this perspective, Hoshikawa et al. clarified the parameterization of all strongly stabilizable plants [21]. Although Hoshikawa et al. indicated the possibility of obtaining the parameterization of all stable stabilizing controllers based on their results [21], no paper has derived a parameterization of all stable stabilizing controllers. To design a stable stabilizing controller, the parameterization of all stable stabilizing controllers should be a powerful tool. This motivates us to obtain the parameterization of all stable stabilizing controllers.

The strong stabilization can be applied to control systems working in an environment where the feedback interconnection is easy to break. Examples are power plants and communication networks.

The purpose of this paper is to propose a design method for stable stabilizing controllers for plants that can be stabilized by a stable controller in [21] using the parameterization of all stable stabilizing controllers. To propose a design method of stable stabilizing controllers, we clarify the parameterization of all strongly stabilizing controllers for strongly stabilizable plants in [21]. In addition, we investigate the characteristics of the closed-loop system resulting from the parameterization of all stable stabilizing controllers.

This paper is organized as follows. In Section 2, we formulate the problem considered in this study. In Section 3, we propose the parameterization of all stable stabilizing controllers for strongly stabilizable plants. In Section 4, we show the control system characteristics with the parameterization of all stable stabilizing controllers. In Section 5, we propose a design method of stable stabilizing controllers based on the parameterization of all stable stabilizing controllers given in Section 3. In Section 6, we provide a numerical example to illustrate the effectiveness of the proposed design method. Section 7 gives concluding remarks.

Notation

R	The set of real numbers.
$R(s)$	The set of real rational functions with the variable s.
RH_∞	The set of stable proper real rational functions.
$\mathcal{U}$	The set of unimodular functions on RH_∞. That is, $U(s) \in \mathcal{U}$ implies both $U(s) \in RH_\infty$ and $U^{-1}(s) \in RH_\infty$.

2. **Problem Formulation.**

Consider the control system in

$$\begin{cases} y(s) = G(s)u(s) + d(s) \\ u(s) = C(s)\left(r(s) - y(s)\right) \end{cases}, \tag{1}$$

where $G(s) \in R(s)$ is the plant, $C(s) \in R(s)$ is the controller, $y(s) \in R(s)$ is the output, $u(s) \in R(s)$ is the control input, $d(s) \in R(s)$ is the disturbance, and $r(s) \in R(s)$ is the reference input.

The strong stabilization is the control method that makes the control system in (1) stable by using stable controllers. Therefore, if the plant $G(s)$ in (1) can be stabilized by a stable controller $C(s)$, we call the plant $G(s)$ a strongly stabilizable plant. According to [21], the parameterization of all strongly stabilizable plants is given by

$$G(s) = \frac{Q_1(s)}{1 - Q_1(s)Q_2(s)}, \tag{2}$$

where $Q_1(s) \in RH_\infty$ and $Q_2(s) \in RH_\infty$ are some stable proper real rational functions.

According to [8], the stabilizing controllers can be represented by the internal model control structure, that is, the stabilizing controller is parameterized in the form of

$$C(s) = \frac{Q(s)}{1 - Q(s)G(s)}, \tag{3}$$

where $Q(s) \in RH_\infty$ is the Youla parameter. However, this parameterization does not ensure the stability of the resulting controller $C(s)$. The purpose of this paper is to obtain the parameterization of all stable stabilizing controllers for the plant $G(s)$ given in the form of (2) and to propose a design method of stable stabilizing controllers based on the obtained parameterization.

3. **The Parameterization of All Stable Stabilizing Controllers for Strongly Stabilizable Plants.**

In this section, we derive the parameterization of all stable stabilizing controllers for strongly stabilizable plants. This parameterization is summarized in the following theorem.

Theorem 3.1. *Consider the strongly stabilizable plant $G(s)$ represented in the form of (2). Then, all stable stabilizing controllers $C(s)$ are parameterized by*

$$C(s) = \frac{Q_2(s) + (1 - Q_1(s)Q_2(s))\, Q(s)}{1 - Q_1(s)Q(s)}, \tag{4}$$

where $Q(s) \in RH_\infty$ is given by

$$Q(s) = \frac{1 - \hat{Q}(s)}{Q_1(s)}, \tag{5}$$

with $\hat{Q}(s) \in \mathcal{U}$ making $Q(s)$ proper and satisfying

$$\hat{Q}(s_i) = 1 \quad (\forall i = 1, \dots, n) \tag{6}$$

for the unstable zeros s_i $(i = 1, \dots, n)$ of $Q_1(s)$.

Proof: From [4], all stabilizing controllers are parameterized by

$$C(s) = \frac{X(s) + D(s)Q(s)}{Y(s) - N(s)Q(s)}, \tag{7}$$

where $N(s)$ and $D(s)$ are coprime factors of $G(s)$ over RH_∞ satisfying

$$G(s) = \frac{N(s)}{D(s)}, \tag{8}$$

$X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ are some transfer functions satisfying

$$N(s)X(s) + D(s)Y(s) = 1 \tag{9}$$

and $Q(s) \in RH_\infty$ is arbitrary. Noting that the strongly stabilizable plant $G(s)$ is represented in the form of (2), we choose the coprime factors $N(s)$ and $D(s)$ as

$$N(s) = Q_1(s) \tag{10}$$

and

$$D(s) = 1 - Q_1(s)Q_2(s), \tag{11}$$

respectively. From (10) and (11), it is straightforward to verify that the following pair of $X(s)$ and $Y(s)$ satisfy (9):

$$X(s) = Q_2(s) \tag{12}$$

and

$$Y(s) = 1. \tag{13}$$

Substituting (10), (11), (12) and (13) into (7), we have

$$C(s) = \frac{Q_2(s) + (1 - Q_1(s)Q_2(s))\, Q(s)}{1 - Q_1(s)Q(s)}. \tag{14}$$

In this way, when the strongly stabilizable plant $G(s)$ is represented in the form of (2), all strongly stabilizing controller $C(s)$ is parameterized by (4).

Next, we confirm that $C(s)$ given by (4) is stable. Because $Q_1(s) \in RH_\infty$, $Q_2(s) \in RH_\infty$, and $Q(s) \in RH_\infty$, the resulting controller $C(s)$ in (4) is stable if and only if $Q(s)$ in (5) makes $1 - Q_1(s)Q(s)$ belong to $\mathcal{U}$. Therefore, we prove that $Q(s)$ in (5) makes $1 - Q_1(s)Q(s)$ belong to $\mathcal{U}$.

First, the necessity is shown. That is, we show that if $Q(s)$ in (4) makes $1 - Q_1(s)Q(s)$ belong to $\mathcal{U}$, then $Q(s) \in RH_\infty$ is written by (5). Based on the assumption that $Q(s)$ in (4) makes $1 - Q_1(s)Q(s)$ belong to $\mathcal{U}$, using $\hat{Q}(s) \in \mathcal{U}$, we have

$$1 - Q_1(s)Q(s) = \hat{Q}(s). \tag{15}$$

When s_i $(i = 1, \ldots, n)$ denote the unstable zeros of $Q_1(s)$, the condition

$$1 - Q_1(s_i)Q(s_i) = 1 \tag{16}$$

holds true. Therefore, $\hat{Q}(s)$ satisfies (6). From easy manipulation, (15) is rewritten as

$$Q(s) = \frac{1 - \hat{Q}(s)}{Q_1(s)}. \tag{17}$$

The fact that $Q(s)$ in (17) belongs to RH_∞ is confirmed as follows: From $\hat{Q}(s) \in \mathcal{U}$, if $Q(s)$ in (17) is unstable, the unstable poles of $Q(s)$ are equal to the unstable zeros s_i and $(i = 1, \ldots, n)$ of $Q_1(s)$. Because $\hat{Q}(s)$ satisfies (6),

$$1 - \hat{Q}(s_i) = 0 \quad (\forall i = 1, \ldots, n) \tag{18}$$

hold true. This means that unstable zeros s_i $(i = 1, \ldots, n)$ of $Q_1(s)$ are not equal to the unstable poles of $Q(s)$. Therefore, $Q(s)$ is stable. That is, when we select $\hat{Q}(s)$ to make $Q(s)$ proper, $Q(s)$ in (17) belongs to RH_∞. Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, if $Q(s)$ in (4) is given by (5), then $1 - Q_1(s)Q(s)$ belongs to $\mathcal{U}$. Using $Q(s)$ in (5), $1 - Q_1(s)Q(s)$ is rewritten as

$$1 - Q_1(s)Q(s) = \hat{Q}(s). \tag{19}$$

Because $\hat{Q}(s) \in \mathcal{U}$, $1 - Q_1(s)Q(s)$ belongs to $\mathcal{U}$. Thus, the sufficiency has been shown. We have thus proved the theorem. □

In this section, we obtained the parameterization (4) of the strongly stabilizing controllers. For further development, the characteristics of the closed-loop system resulting from the parameterization should be investigated to derive the design method of stable stabilizing controllers.

4. Characteristics of Closed-Loop System.

In this section, we investigate the characteristics of the closed-loop system with the stabilizing controller given by (4).

First, the reference tracking characteristic is considered. If the parameterization of all stable stabilizing controllers in (4) is employed, the transfer function from the reference input $r(s)$ to the output $y(s)$ of the control system in (1) is given by

$$\frac{y(s)}{r(s)} = 1 - (1 - Q_1(s)Q_2(s))\,\hat{Q}(s). \tag{20}$$

Therefore, for the output $y(s)$ to follow the step reference input $r(s) = 1/s$ without steady-state error,

$$(1 - Q_1(0)Q_2(0))\,\hat{Q}(0) = 0 \tag{21}$$

must be satisfied. Because $\hat{Q}(s) \in \mathcal{U}$, $\hat{Q}(s)$ has no zero at the origin, there is no $\hat{Q}(s)$ satisfying (21). However, if $\hat{Q}(s)$ is chosen to satisfy

$$\hat{Q}(0) \simeq 0, \tag{22}$$

the output $y(s)$ follows the step reference input $r(s) = 1/s$ with a small steady-state error.

Next, we consider the disturbance attenuation characteristic. The transfer function from the reference input $d(s)$ to the output $y(s)$ is given by

$$\frac{y(s)}{d(s)} = (1 - Q_1(s)Q_2(s))\,\hat{Q}(s). \tag{23}$$

Therefore, to attenuate the step disturbance $d(s) = 1/s$ completely,

$$(1 - Q_1(0)Q_2(0))\,\hat{Q}(0) = 0 \tag{24}$$

must be satisfied. Because $\hat{Q}(s) \in \mathcal{U}$, $\hat{Q}(s)$ has no zero at the origin, there is no $\hat{Q}(s)$ satisfying (24). However, if $\hat{Q}(s)$ is chosen to satisfy

$$\hat{Q}(0) \simeq 0, \tag{25}$$

the step disturbance $d(s) = 1/s$ is attenuated effectively.

5. **Design Method of Stable Stabilizing Controllers.** In this section, we propose a design method of stable stabilizing controllers. From Theorem 3.1, to design a stable stabilizing controller $C(s)$, we need to settle $\hat{Q}(s) \in \mathcal{U}$ that satisfies (6) and makes $Q(s)$ in (5) proper. In addition, for the output $y(s)$ to follow the reference input $r(s) = 1/s$ with a small steady-state error, $\hat{Q}(s)$ needs to satisfy (22). A design method of $\hat{Q}(s) \in \mathcal{U}$ satisfying (6) and (22), making $Q(s)$ in (5) proper, and stabilizing controller $C(s)$ is summarized as follows.

1) Let $Q_1(s)$ be factorized as

$$Q_1(s) = Q_{1i}(s)Q_{1o}(s), \tag{26}$$

where $Q_{1i}(s) \in RH_\infty$ is the inner function satisfying $Q_{1i}(0) = 1$ and $Q_{1o}(s) \in RH_\infty$ is the outer function.

2) Using $Q_{1o}(s)$, design $\bar{Q}(s) \in RH_\infty$ as

$$\bar{Q}(s) = \frac{k}{Q_{1o}(s)(\tau s + 1)^\alpha}, \tag{27}$$

where $\tau \in R$, α is an arbitrary positive integer to make $\bar{Q}(s)$ proper, that is α is greater than or equal to the relative degree of the plant $G(s)$, and k is a constant satisfying $k < 1$ and $k \simeq 1$.

3) Using $\bar{Q}(s)$ in (27), define $\hat{Q}(s) \in \mathcal{U}$ by

$$\hat{Q}(s) = 1 - Q_1(s)\bar{Q}(s) = 1 - Q_{1i}(s)\frac{k}{(\tau s + 1)^\alpha}, \tag{28}$$

satisfying (6) and (22) and making $Q(s)$ proper.

4) Using $\hat{Q}(s)$ in (28), settle $Q(s)$ in (5) by

$$Q(s) = \bar{Q}(s) = \frac{k}{Q_{1o}(s)(\tau s + 1)^\alpha}. \tag{29}$$

5) Using $Q(s)$ in (29), fix a stable stabilizing controller $C(s)$ in (4).

The small gain theorem ensures that $\hat{Q}(s)$ in (28) belongs to $\mathcal{U}$. Next, we confirm that $\hat{Q}(s)$ in (28) satisfies (6) and (22) and makes $Q(s)$ proper. First, we show that $\hat{Q}(s)$ in (28) satisfies (6). $\hat{Q}(s)$ in (28) is rewritten as (28). Because the unstable zeros s_i $(i = 1, \dots, n)$ of $Q_1(s)$ are equal to that of $Q_{1i}(s)$, it follows that $Q_{1i}(s_i) = 0$ $(\forall i = 1, \dots, n)$. Therefore, $\hat{Q}(s)$ in (28) satisfies (6). Next, we show that $\hat{Q}(s)$ in (28) satisfies (22). Because $Q_{1i}(0) = 1$ and $k \simeq 1$, we have

$$\hat{Q}(0) = 1 - k \simeq 0. \tag{30}$$

Therefore, $\hat{Q}(s)$ in (28) satisfies (22). Finally, we show that $\hat{Q}(s)$ in (28) makes $Q(s)$ proper. Because $Q(s)$ is determined by (29) with $\bar{Q}(s)$ in (27), $Q(s)$ is proper. In this way, we find that $\hat{Q}(s) \in \mathcal{U}$ defined by (28) is guaranteed to satisfy (6) and (22) and to make $Q(s)$ proper.

6. **Numerical Example.** This section presents a numerical example to illustrate the features of the proposed design method. Consider the strong stabilization of the controlled plant

$$G(s) = \frac{(s-7)(s+1)}{(s-1)(s+3)(s+5)}. \tag{31}$$

We note that $G(s)$ is rewritten in the form of (2):

$$G(s) = \frac{\frac{s-7}{(s+2)(s+3)}}{1 + \frac{s-7}{(s+2)(s+3)}\frac{s+3}{s+1}}, \tag{32}$$

where $Q_1(s)$ and $Q_2(s)$ in (2) are chosen as

$$Q_1(s) = \frac{s-7}{(s+2)(s+3)} \tag{33}$$

and

$$Q_2(s) = -\frac{s+3}{s+1}. \tag{34}$$

Therefore, $G(s)$ in (31) is strongly stabilizable.

Using the design method described in Section 5, we obtain a stable stabilizing controller $C(s)$ to make the output $y(s)$ follow the step reference input $r(s) = 1/s$ with a small steady-state error and to attenuate effectively the step disturbance $d(s) = 1/s$. First $Q_1(s)$ in (33) is factorized by (26), where

$$Q_{1i}(s) = -\frac{s-7}{s+7} \tag{35}$$

and

$$Q_{1o}(s) = -\frac{s+7}{(s+2)(s+3)}. \tag{36}$$

Using $Q_{1o}(s)$ in (36), $\bar{Q}(s) \in RH_\infty$ is given by (27), where k, α, and τ are settled by

$$k = 0.999, \tag{37}$$

$$\alpha = 1 \tag{38}$$

and

$$\tau = 0.1. \tag{39}$$

Then, $\bar{Q}(s)$ is obtained as

$$\bar{Q}(s) = -\frac{(s+2)(s+3)}{s+7}\frac{0.999}{0.1s+1}. \tag{40}$$

$\hat{Q}(s) \in \mathcal{U}$ is given by (28) and written by

$$\hat{Q}(s) = 1 + \frac{s-7}{s+7}\frac{0.999}{0.1s+1}. \tag{41}$$

Substituting (41) into (5), we have

$$Q(s) = -\frac{(s+2)(s+3)}{s+7}\frac{0.999}{0.1s+1}. \tag{42}$$

From (42) and (4), we obtain a stable stabilizing controller $C(s)$ for the strongly stabilizable plant $G(s)$ in (31) as

$$C(s) = \frac{-10.99(s+0.3798)(s+3)(s+4.803)}{(s+2.594\times 10^{-3})(s+1)(s+26.99)}. \tag{43}$$

Using the stable stabilizing controller $C(s)$ in (43), the response of the output $y(t)$ of the closed-loop system (1) for the step reference input $r(t) = 1$ is depicted in Figure 1. Figure 1 verifies that the closed-loop system in (1) is stable and that the output $y(t)$ follows the step reference input $r(t) = 1$ with a small steady-state error.

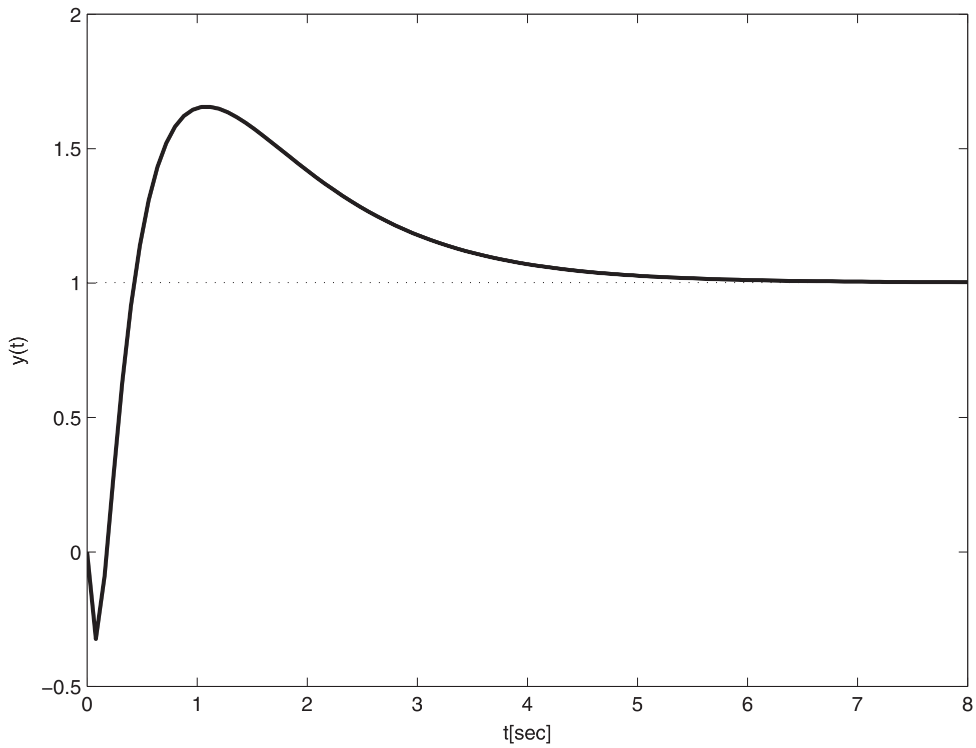

FIGURE 1. Response of the output $y(t)$ for the step reference input $r(t) = 1$ with a nominal controlled plant

In contrast, when the step disturbance $d(t) = 1$ is exerted, the response of the output $y(t)$ of the closed-loop system (1) is depicted in Figure 2. Figure 2 verifies that the disturbance $d(t) = 1$ is effectively attenuated.

The presented example shows that the proposed method can design a stable stabilizing controller $C(s)$ based on reference tracking and the disturbance attenuation characteristics.

To check the robustness of the proposed design method, we consider the situation where a minimum-phase controller $C(s)$ in (43) is required to stabilize the plant

$$G_1(s) = \frac{(s - 7.3)(s + 1)}{(s - 1)(s + 3)(s + 5)}, \tag{44}$$

which is obtained by perturbing the plant $G(s)$. In this situation, the response of the output $y(t)$ for the step reference input $r(t) = 1$ is depicted in Figure 3. The response of the output $y(t)$ for the step disturbance $d(t) = 1$ is depicted in Figure 4. Figures 3 and 4 indicate that the closed-loop system with the proposed controller (43) possesses robustness against the plant perturbation.

Furthermore, we compare the proposed design method with that of [8], by which $C(s)$ is parametrized by

$$C(s) = \frac{Q(s)}{1 - Q(s)G(s)}. \tag{45}$$

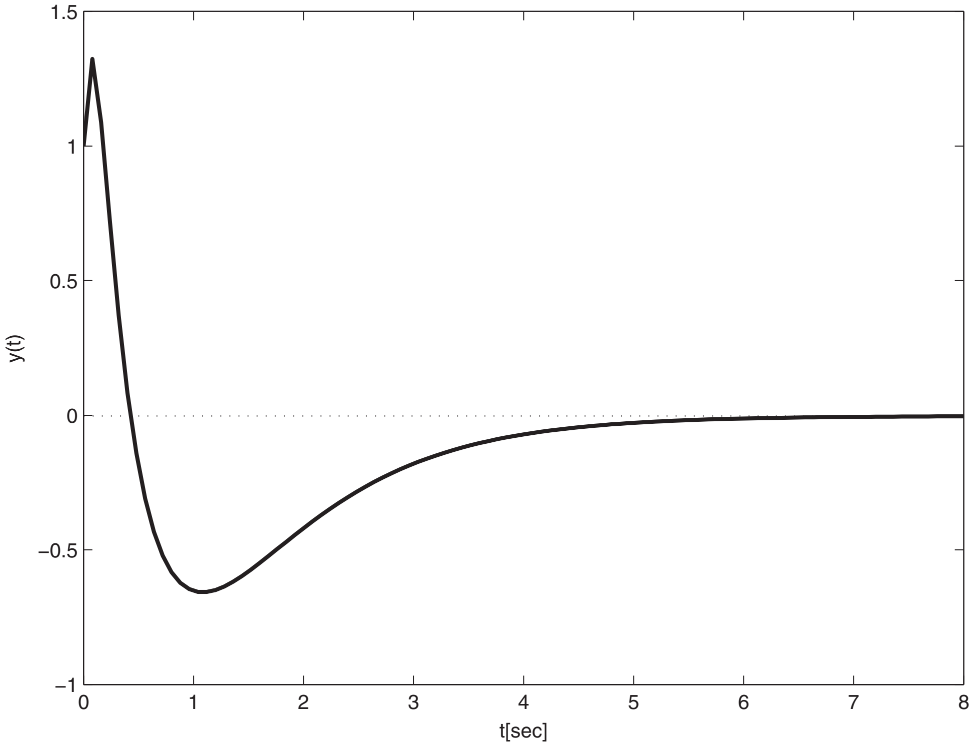

FIGURE 2. Response of the output $y(t)$ for the step disturbance $d(t) = 1$ with a nominal controlled plant

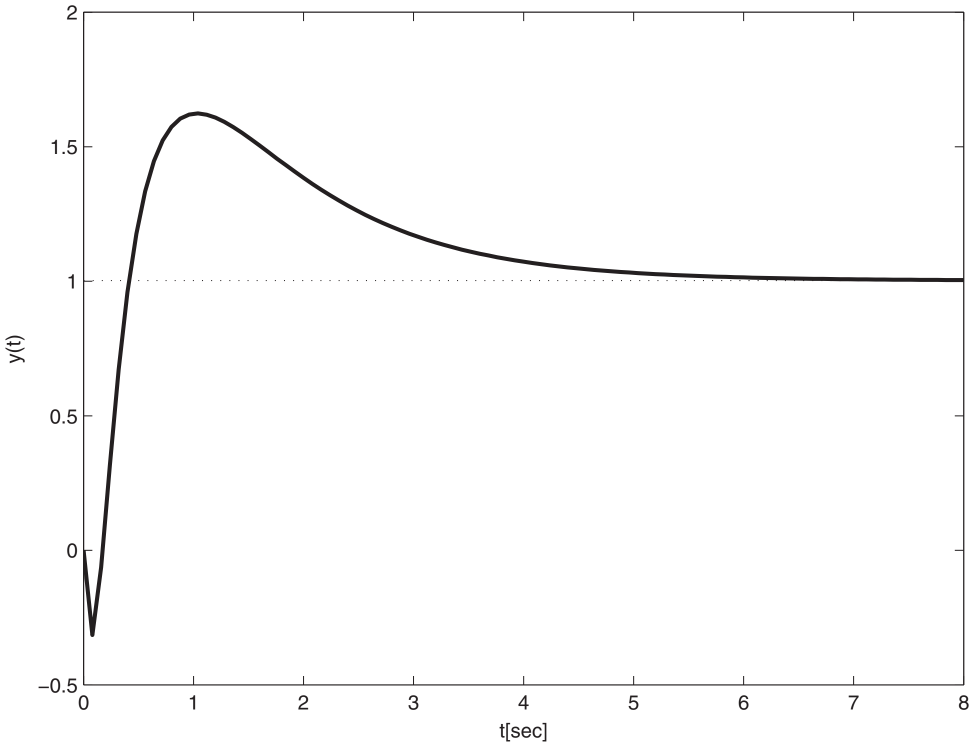

FIGURE 3. Response of the output $y(t)$ of for step reference input $r(t) = 1$ with a perturbed controlled plant

By using the same Youla parameter $Q(s)$ with (5), we obtain another controller

$$C(s) = \frac{-9.99(s+2)(s+3)(s+5)(s-1)}{(s+27.66)(s-1.782)\left(s^2+5.116s+9.942\right)}. \tag{46}$$

Here, the poles of $C(s)$ contain 1.782, and $C(s)$ in (46) is obviously unstable. Therefore, the controllers that we obtain by the design method [8] are not necessarily stable.

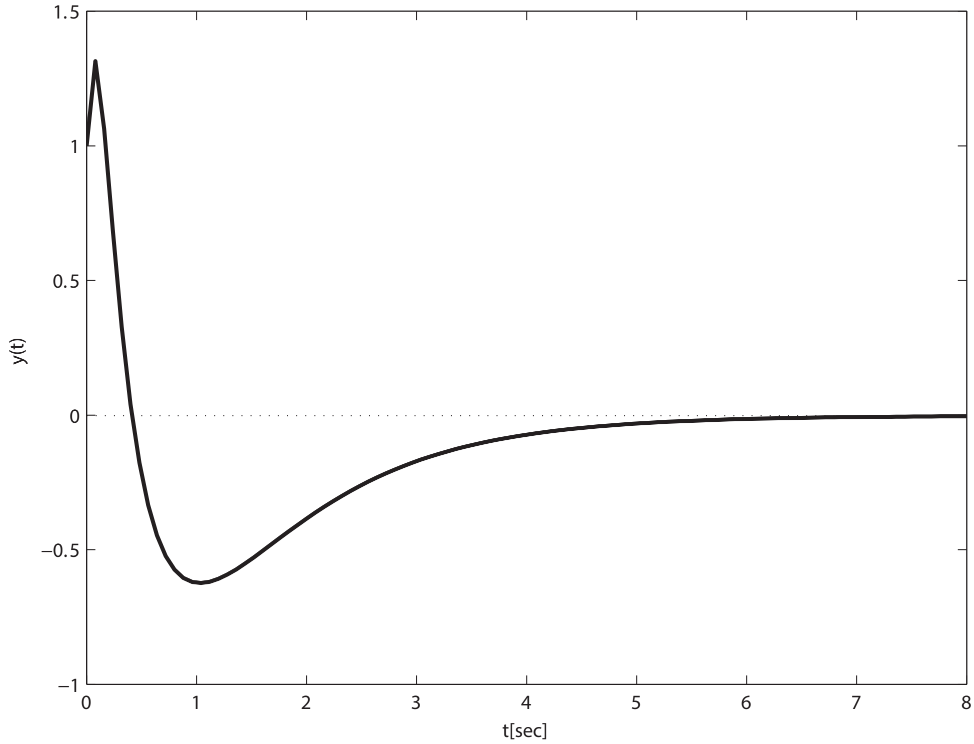

FIGURE 4. Response of the output $y(t)$ for step disturbance $d(t) = 1$ with a perturbed controlled plant

7. **Conclusions.** We newly revealed the parameterization (4) of all stable stabilizing controllers and investigated the characteristics of the resulting closed-loop system. Furthermore, based on the results, we proposed a design method of stable stabilizing controllers such that the steady-state error caused by the step reference and disturbance inputs is kept as small as possible. The features of the proposed design method were illustrated through the numerical example. In this paper, we only addressed the stability requirement on the controller's poles, but not on the controller's zeros. In a forthcoming paper, we will clarify the parameterization of all minimum-phase stabilizing controllers and study the characteristics of the resulting closed-loop system.

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