# SPHERICAL FERMATEAN INTERVAL VALUED FUZZY SOFT SET BASED ON MULTI CRITERIA GROUP DECISION MAKING 

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Received September 2021; revised January 2022


#### Abstract

Spherical fermatean interval valued fuzzy soft set is a more well organized, workable and generalized soft model to deal with uncertainty, as compared to fermatean interval valued fuzzy soft and spherical interval valued fuzzy soft models. This research article presents a novel multi criteria group decision making technique based on spherical fermatean interval valued fuzzy soft set. The multi criteria group decision making theory is well known for decision making and it is applied to evaluating best college education from alternatives. In this survey, data were collected related to attributes of the college education to demonstrate the significance of decision making in the case of ten colleges. Ten alternatives were considered with five attributes, i.e., campus environment, overall cost, academic quality, student/faculty relationship, career development of the college education. Finally, to illustrate the success of the present approach, a real life problem is presented where the evaluation information of the alternatives is given in terms of spherical fermatean interval valued fuzzy soft set.


Keywords: Spherical interval valued fuzzy set, Spherical fermatean interval valued fuzzy soft set, Multi criteria group decision making

1. Introduction. Decision making problem indicates the finding of best optional alternatives. Hwang and Yoon [2] discussed multiple criteria decision making (MCDM) methods. The matrix form of MCDM problem is as fallows:

$$
\mathscr{D}_{n \times m}=\begin{gathered}
\mathscr{B}_{1} \\
\mathscr{B}_{2} \\
\ldots
\end{gathered} \begin{gathered}
\mathscr{A}_{1} \\
\mathscr{A}_{2} \\
\vdots \\
\mathscr{A}_{n}
\end{gathered}\left(\begin{array}{cccc}
x_{11} & x_{12} & \ldots & x_{1 m} \\
x_{21} & x_{22} & \ldots & x_{2 m} \\
\vdots & \vdots & \ddots & \vdots \\
x_{n 1} & x_{n 2} & \ldots & x_{n m}
\end{array}\right)
$$

Here $\mathscr{A}_{1}, \mathscr{A}_{2}, \ldots, \mathscr{A}_{n}$ are called possible alternative mean with which decision makers have to choose, $\mathscr{B}_{1}, \mathscr{B}_{2}, \ldots, \mathscr{B}_{m}$ are called criteria mean for which alternative effecting is calculated and $x_{i j}$ means estimate of $\mathscr{A}_{i}$ with respect to $\mathscr{B}_{j}$. A fuzzy set was introduced by Zadeh [19] and it suggests that decision-makers are to be solving uncertain problems by considering membership degrees. The concept of an intuitionistic fuzzy set was introduced
by Atanassov and it is characterized by a degree of membership and non-membership satisfying the condition that sum of its membership degree and non-membership degree is not exceeding unity [1]. However, we may interact a problem in decision making (DM) events where the sum of the degree of membership and non-membership of a particular attribute is exceeding unity. So Yager [17] introduced the concept of Pythagorean fuzzy sets and it is characterized by the condition that the square sum of its degree of membership and non-membership is not to exceed unity. In 2019, spherical fuzzy sets were introduced by Gundogdu and Kahraman [21] as an extension of Pythagorean, neutrosophic and picture fuzzy sets. Ashraf et al. [22] discussed spherical fuzzy sets which is an advanced tool of the fuzzy sets and intuitionistic fuzzy sets. In decision making problems, times squares of sum of its degree of membership and non membership exceed unity. So Senapati and Yager introduced Fermatean fuzzy set [12]. Also, Fermatean fuzzy soft set is a generalization of the Pythagorean fuzzy soft set and it is characterized by the condition that the cubes of sum of its degree of membership and non membership is not to exceed unity.

In 2018, Garg et al. [23] proposed the algorithm for T-spherical fuzzy multi-attribute decision making (MADM) based on improved interactive aggregation operators. Ashraf and Abdullah [24] proposed spherical aggregation operators and applied them in multiattribute group decision making (MAGDM). Liu et al. [25] extended the generalized Maclaurin symmetric mean (GMSM) operator to the T-spherical fuzzy environment and proposed the T-spherical fuzzy GMSM operator (T-SFGMSM) and the T-spherical fuzzy weighted GMSM operator (T-SFWGMSM). In 2019, Quek et al. [26] developed a few new operational laws for T-spherical fuzzy sets, and based on these new operations. They proposed two types of Einstein aggregation operators, namely the Einstein interactive averaging aggregation operators and the Einstein interactive geometric aggregation operators under T-spherical fuzzy environment. In 2019, Gundogdu and Kahraman [21] introduced spherical fuzzy sets, their operational laws, and the spherical fuzzy TOPSIS method. An extension of WASPAS with spherical fuzzy sets, VIKOR method using spherical fuzzy sets and correlation coefficients were presented by Gundogdu and Kahraman [27]. Molodtsov [7] proposed the theory of soft sets. Soft sets more accurately reflect the objectivity and complexity of DM during actual situations. Maji et al. proposed the concept of fuzzy soft set [5] and intuitionistic fuzzy soft set [6]. In recent years, Peng et al. [10] have extended fuzzy soft set to Pythagorean fuzzy soft set. The soft set model solved a class of multi criteria group decision making consisting sum of the degree of membership and non-membership value which is exceeding unity but the sum of the squares is less than or equal to unity.

The paper is organized into five sections as follows. Section 1 is the introduction. Section 2 is followed by preliminaries. Section 3 follows the modified spherical fermatean interval valued fuzzy soft set theory based on multi criteria group decision making. Section 4 is followed by analysis for spherical fermatean interval valued fuzzy soft set models. Section 5 is followed by conclusion for the spherical fermatean interval valued fuzzy soft set models. Also, some numerical examples are given as in insert to evaluate the spherical fermatean interval valued fuzzy soft set.

## 2. Preliminaries.

Definition 2.1. Let $\mathbb{U}$ be a non-empty set of the universe, spherical interval valued fuzzy (SIVF) set in which $X$ in $\mathbb{U}$ is of the structure: $\vec{X}=\left\{u,\left(\overrightarrow{\alpha_{X}}(u), \overrightarrow{\beta_{X}}(u), \overrightarrow{\gamma_{X}}(u)\right) \mid u \in \mathbb{U}\right\}$, where $\overrightarrow{\alpha_{X}}(u)=\left[\alpha_{X}^{L}(u), \alpha_{X}^{U}(u)\right], \overrightarrow{\beta_{X}}(u)=\left[\beta_{X}^{L}(u), \beta_{X}^{U}(u)\right]$ and $\overrightarrow{\gamma_{X}}(u)=\left[\gamma_{X}^{L}(u), \gamma_{X}^{U}(u)\right]$ represent the degree of positive, neutral and negative-membership of $X$ respectively. Consider the mapping $\overrightarrow{\alpha_{X}}: \mathbb{U} \rightarrow D[0,1], \overrightarrow{\beta_{X}}: \mathbb{U} \rightarrow D[0,1], \overrightarrow{\gamma_{X}}: \mathbb{U} \rightarrow D[0,1]$ and $0 \leq$
$\left(\overrightarrow{\alpha_{X}}(u)\right)^{2}+\left(\overrightarrow{\beta_{X}}(u)\right)^{2}+\left(\overrightarrow{\gamma_{X}}(u)\right)^{2} \leq 1$ means $0 \leq \alpha_{X}^{2 U}(u)+\beta_{X}^{2 U}(u)+\gamma_{X}^{2 U}(u) \leq 1$. The degree of refusal is determined as $\overrightarrow{\pi_{X}}(u)=\left[\pi_{X}^{L}(u), \pi_{X}^{U}(u)\right]=\left[\sqrt{1-\alpha_{X}^{2 U}(u)-\beta_{X}^{2 U}(u)-\gamma_{X}^{2 U}(u)}\right.$, $\left.\sqrt{1-\alpha_{X}^{2 L}(u)-\beta_{X}^{2 L}(u)-\gamma_{X}^{2 L}(u)}\right]$. Here $\vec{X}=\left(\left[\alpha_{X}^{L}, \alpha_{X}^{U}\right],\left[\beta_{X}^{L}, \beta_{X}^{U}\right],\left[\gamma_{X}^{L}, \gamma_{X}^{U}\right]\right)$ is called a spherical interval valued fuzzy number (SIVFN).
Definition 2.2. Let $\mathbb{U}$ be a non-empty set of the universe, spherical fermatean interval valued fuzzy (SFIVF) set $X$ in $\mathbb{U}$ is of the structure: $\vec{X}=\left\{u,\left(\overrightarrow{\alpha_{X}}(u), \overrightarrow{\beta_{X}}(u), \overrightarrow{\gamma_{X}}(u)\right) \mid u \in \mathbb{U}\right\}$, where $\overrightarrow{\alpha_{X}}(u)=\left[\alpha_{X}^{L}(u), \alpha_{X}^{U}(u)\right], \overrightarrow{\beta_{X}}(u)=\left[\beta_{X}^{L}(u), \beta_{X}^{U}(u)\right]$ and $\overrightarrow{\gamma_{X}}(u)=\left[\gamma_{X}^{L}(u), \gamma_{X}^{U}(u)\right]$ represent the degree of positive, neutral and negative-membership of $X$ respectively. Consider the mapping $\overrightarrow{\alpha_{X}}: \mathbb{U} \rightarrow D[0,1], \overrightarrow{\beta_{X}}: \mathbb{U} \rightarrow D[0,1], \overrightarrow{\gamma_{X}}: \mathbb{U} \rightarrow D[0,1]$ and $0 \leq$ $\left(\overrightarrow{\alpha_{X}}(u)\right)^{3}+\left(\overrightarrow{\beta_{X}}(u)\right)^{3}+\left(\overrightarrow{\gamma_{X}}(u)\right)^{3} \leq 1$ means $0 \leq \alpha_{X}^{3 U}(u)+\beta_{X}^{3 U}(u)+\gamma_{X}^{3 U}(u) \leq 1$. The degree of refusal is determined as $\overrightarrow{\pi_{X}}(u)=\left[\pi_{X}^{L}(u), \pi_{X}^{U}(u)\right]=\left[\sqrt[3]{1-\alpha_{X}^{3 U}(u)-\beta_{X}^{3 U}(u)-\gamma_{X}^{3 U}(u)}\right.$, $\left.\sqrt[3]{1-\alpha_{X}^{3 L}(u)-\beta_{X}^{3 L}(u)-\gamma_{X}^{3 L}(u)}\right]$. Here $\vec{X}=\left(\left[\alpha_{X}^{L}, \alpha_{X}^{U}\right],\left[\beta_{X}^{L}, \beta_{X}^{U}\right],\left[\gamma_{X}^{L}, \gamma_{X}^{U}\right]\right)$ is called a spherical fermatean interval valued fuzzy number (SFIVFN).
Definition 2.3. Let $\mathbb{U}$ and $E$ be the universe and set of parameter, respectively. The pair $\overrightarrow{(\Theta, X)}$ or $\overrightarrow{\Theta_{X}}$ is called a spherical fermatean interval valued fuzzy soft (SFIVFS) set on $\mathbb{U}$ if $X \sqsubseteq E$ and $\Theta: X \rightarrow$ SFIVF ${ }^{\mathbb{U}}$, where SFIVF ${ }^{\mathbb{U}}$ denotes the set of all spherical fermatean interval valued fuzzy subsets of $\mathbb{U}$, i.e.,

$$
\begin{aligned}
& \overrightarrow{\Theta_{X}} \\
= & \left\{\left.\left(e,\left\{\frac{u}{\left(\left[\alpha_{\Theta_{X}}^{L}(u), \alpha_{\Theta_{X}}^{U}(u)\right],\left[\beta_{\Theta_{X}}^{L}(u), \beta_{\Theta_{X}}^{U}(u)\right],\left[\gamma_{\Theta_{X}}^{L}(u), \gamma_{\Theta_{X}}^{U}(u)\right]\right)}\right\}\right) \right\rvert\, e \in X, u \in \mathbb{U}\right\} .
\end{aligned}
$$

Remark 2.1. Let $\overrightarrow{r_{i j}}=\overrightarrow{\alpha_{\Theta X}}\left(e_{j}\right)\left(u_{i}\right), \overrightarrow{s_{i j}}=\overrightarrow{\beta_{\Theta_{X}}}\left(e_{j}\right)\left(u_{i}\right)$ and $\overrightarrow{t_{i j}}=\overrightarrow{\gamma_{\Theta_{X}}}\left(e_{j}\right)\left(u_{i}\right)$, where $i=1,2, \ldots, m$ and $j=1,2, \ldots, n$. Then the SFIVFS set $\overrightarrow{\Theta_{X}}$ is defined in the matrix form:

$$
\overrightarrow{\Theta_{X}}=\left[\left(\overrightarrow{r_{i j}}, \overrightarrow{s_{i j}}, \overrightarrow{t_{i j}}\right)\right]_{m \times n}=\left[\begin{array}{cccc}
\left(\overrightarrow{r_{11}}, \overrightarrow{s_{11}}, \overrightarrow{t_{11}}\right) & \left(\overrightarrow{r_{12}}, \overrightarrow{s_{12}}, \overrightarrow{t_{12}}\right) & \ldots & \left(\overrightarrow{r_{1 n}}, \overrightarrow{s_{1 n}}, \overrightarrow{t_{1 n}}\right) \\
\left(\overrightarrow{r_{21}}, \overrightarrow{s_{21}}, \overrightarrow{t_{21}}\right) & \left(\overrightarrow{r_{22}}, \overrightarrow{s_{22}}, \overrightarrow{t_{22}}\right) & \ldots & \left(\overrightarrow{r_{2 n}}, \overrightarrow{s_{2 n}}, \overrightarrow{t_{2 n}}\right) \\
\vdots & \vdots & \ddots & \vdots \\
\left(\overrightarrow{r_{m 1}}, \overrightarrow{s_{m 1}}, \overrightarrow{t_{m 1}}\right) & \left(\overrightarrow{r_{m 2}}, \overrightarrow{s_{m 2}}, \overrightarrow{t_{m 2}}\right) & \ldots & \left(\overrightarrow{r_{m n}}, \overrightarrow{s_{m n}}, \overrightarrow{t_{m n}}\right)
\end{array}\right]
$$

This matrix is called spherical fermatean interval valued fuzzy soft matrix (SFIVFSM).
Remark 2.2. Using fundamental operations of arithmetic leads to the following.
(i) $\left[a_{1}, a_{2}\right]+\left[a_{3}, a_{4}\right]=\left[a_{1}+a_{3}, a_{2}+a_{4}\right]$,
(ii) $\left[a_{1}, a_{2}\right]-\left[a_{3}, a_{4}\right]=\left[a_{1}-a_{4}, a_{2}-a_{3}\right]$,
(iii) $\left[a_{1}, a_{2}\right] \cdot\left[a_{3}, a_{4}\right]=\left[a_{1} a_{3}, a_{2} a_{4}\right]$, whenever $a_{1} \geq 0$ and $a_{2} \geq 0$,
(iv) $\frac{1}{\left[a_{1}, a_{2}\right]}=\left[\frac{1}{a_{2}}, \frac{1}{a_{1}}\right]$, whenever $0 \notin\left[a_{1}, a_{2}\right], a_{1}, a_{2}, a_{3}, a_{4} \in \mathbb{R}$.

## 3. Modified Spherical Fermatean Interval Valued Fuzzy Soft Set Models.

Definition 3.1. The cardinal set of the SFIVFS set $\overrightarrow{\Theta_{X}}$ over $\mathbb{U}$ is an SFIVFS set over $E$ and is defined as $\overrightarrow{c \Theta_{X}}=\left\{\left.\frac{e}{\left(\left[\alpha_{c \theta_{X}}^{L}(e), \alpha_{c \theta_{X}}^{U}(e)\right],\left[\beta_{c \tau_{X}}^{L}(e), \beta_{c \tau_{X}}^{U}(e)\right],\left[\gamma_{c{ }_{c}}^{L}(e), \gamma_{c \psi_{X}}^{U}(e)\right]\right)} \right\rvert\, e \in E\right\}=$
$\left\{\left.\frac{e}{\left(\vec{\alpha}_{c \theta_{X}}(e), \vec{\beta}_{c \tau_{X}}(e), \vec{\gamma}_{c \psi_{X}}(e)\right)} \right\rvert\, e \in E\right\}$, where $\vec{\alpha}_{c \theta_{X}}, \vec{\beta}_{c \tau_{X}}$ and $\vec{\gamma}_{c \psi_{X}}: E \rightarrow D[0,1]$ are mapping respectively, where $\vec{\alpha}_{c \theta_{X}}(e)=\frac{\left|\vec{\theta}_{X}(e)\right|}{|\mathbb{U}|}, \vec{\beta}_{c \tau_{X}}(e)=\frac{\left|\vec{\sigma}_{X}(e)\right|}{|\mathbb{U}|}$ and $\vec{\gamma}_{c \psi_{X}}(e)=\frac{\left|\vec{\psi}_{X}(e)\right|}{|\mathbb{U}|}$, where $\left|\vec{\theta}_{X}(e)\right|,\left|\vec{\tau}_{X}(e)\right|$ and $\left|\vec{\psi}_{X}(e)\right|$ denote the scalar cardinalities of the SFIVFS sets $\vec{\theta}_{X}(e), \vec{\tau}_{X}(e)$ and $\vec{\psi}_{X}(e)$ respectively, and $|\mathbb{U}|$ represents cardinality of $\mathbb{U}$. The collection of all cardinal sets of SFIVFS sets of $\mathbb{U}$ is represented as cSFIVF ${ }^{\mathbb{U}}$. If $X \subseteq E=\left\{e_{i} \mid\right.$ $i=1,2, \ldots, n\}$, then $\overrightarrow{c \Theta_{X}} \in c S F I V F^{\mathbb{U}}$ may be represented in the matrix form as

$$
\begin{aligned}
& {\left[\left(\left[r_{1 j}^{L}, r_{1 j}^{U}\right],\left[s_{1 j}^{L}, s_{1 j}^{U}\right],\left[t_{1 j}^{L}, t_{1 j}^{U}\right]\right)\right]_{1 \times n} } \\
= & {\left[\left(\left[r_{11}^{L}, r_{11}^{U}\right],\left[s_{11}^{L}, s_{11}^{U}\right],\left[t_{11}^{L}, t_{11}^{U}\right]\right),\left(\left[r_{12}^{L}, r_{12}^{U}\right],\left[s_{12}^{L}, s_{12}^{U}\right],\left[t_{12}^{L}, t_{12}^{U}\right]\right), \ldots,\right.} \\
& \left.\left(\left[r_{1 n}^{L}, r_{1 n}^{U}\right],\left[s_{1 n}^{L}, s_{1 n}^{U}\right],\left[t_{1 n}^{L}, t_{1 n}^{U}\right]\right)\right],
\end{aligned}
$$

where

$$
\left(\left[r_{1 j}^{L}, r_{1 j}^{U}\right],\left[s_{1 j}^{L}, s_{1 j}^{U}\right],\left[t_{1 j}^{L}, t_{1 j}^{U}\right]\right)=\left[\mu_{r \Theta_{X}}^{L}\left(e_{j}\right), \mu_{r \Theta_{X}}^{U}\left(e_{j}\right)\right], \quad \forall j=1,2, \ldots, n
$$

For our convenience matrix form as

$$
\left[\left(\vec{p}_{1 j}, \vec{q}_{1 j}, \vec{r}_{1 j}\right)\right]_{1 \times n}=\left[\left(\vec{p}_{11}, \vec{q}_{11}, \vec{r}_{11}\right),\left(\vec{p}_{12}, \vec{q}_{12}, \vec{r}_{12}\right), \ldots,\left(\vec{p}_{1 n}, \vec{q}_{1 n}, \vec{r}_{1 n}\right)\right]
$$

where

$$
\left(\vec{p}_{1 j}, \vec{q}_{1 j}, \vec{r}_{1 j}\right)=\vec{\mu}_{c \Theta_{X}}\left(e_{j}\right), \quad \forall j=1,2, \ldots, n
$$

This matrix is called as cardinal matrix of $\overrightarrow{c \Theta_{X}}$ of $E$.
Definition 3.2. Let $\overrightarrow{\Theta_{X}} \in$ SFIVF $^{\mathbb{U}}$ and $\overrightarrow{c \Theta_{X}} \in c S F I V F^{\mathbb{U}}$. The SFIVFS set aggregation operator SFIVFS ${ }_{\text {agg }}: c S F I V F^{\mathbb{U}} \times \operatorname{SFIVF}^{\mathbb{U}} \rightarrow \operatorname{SFIVFS}(\mathbb{U}, E)$ is defined as
$\operatorname{SFIVFS}_{\text {agg }}\left(\overrightarrow{c \Theta_{X}}, \overrightarrow{\Theta_{X}}\right)=\left\{\left.\frac{u}{\vec{\mu}_{\Theta_{X}^{*}}(u)} \right\rvert\, u \in \mathbb{U}\right\}=\left\{\left.\frac{u}{\left({\overrightarrow{\alpha_{\theta}^{X}}}^{\theta_{X}^{*}}(u), \vec{\beta}_{\tau_{X}^{*}}(u), \vec{\gamma}_{\psi_{X}^{*}}(u)\right)} \right\rvert\, u \in \mathbb{U}\right\}$.
This collection is called aggregate SFIVFS set $\overrightarrow{\Theta_{X}}$. The positive membership function is defined as

$$
\vec{\alpha}_{\theta_{X}^{*}}(u): \mathbb{U} \rightarrow D[0,1] \text { by } \vec{\alpha}_{\theta_{X}^{*}}(u)=\frac{1}{|E|} \sum_{e \in E}\left(\vec{\alpha}_{c \theta_{X}}(e), \vec{\alpha}_{\theta_{X}}(e)\right)(u),
$$

neutral membership function is defined as

$$
\vec{\beta}_{\tau_{\dot{X}}^{*}}(u): \mathbb{U} \rightarrow D[0,1] \text { by } \vec{\beta}_{\tau_{X}^{*}}(u)=\frac{1}{|E|} \sum_{e \in E}\left(\vec{\beta}_{c \tau_{X}}(e), \vec{\beta}_{\tau_{X}}(e)\right)(u)
$$

and negative membership function is defined as

$$
\vec{\gamma}_{\psi_{X}^{*}}(u): \mathbb{U} \rightarrow D[0,1] \text { by } \vec{\gamma}_{\psi_{x}^{*}}(u)=\frac{1}{|E|} \sum_{e \in E}\left(\vec{\gamma}_{c \psi_{X}}(e), \vec{\gamma}_{\psi_{X}}(e)\right)(u)
$$

The set SFIVFS agg $\left(\overrightarrow{c \Theta_{X}}, \overrightarrow{\Theta_{X}}\right)$ is expressed in matrix form as follows:

$$
\left[\left(\left[r_{i 1}^{L}, r_{i 1}^{U}\right],\left[s_{i 1}^{L}, s_{i 1}^{U}\right],\left[t_{i 1}^{L}, t_{i 1}^{U}\right]\right)\right]_{m \times 1}=\left[\begin{array}{c}
\left(\left[r_{11}^{L}, r_{11}^{U}\right],\left[s_{11}^{L}, s_{11}^{U}\right],\left[t_{11}^{L}, t_{11}^{U}\right]\right) \\
\left(\left[r_{21}^{L}, r_{21}^{U}\right],\left[s_{21}^{L}, s_{21}^{U}\right],\left[t_{21}^{L}, t_{21}^{U}\right]\right) \\
\vdots \\
\left(\left[r_{m 1}^{L}, r_{m 1}^{U}\right],\left[s_{m 1}^{L}, s_{m 1}^{U}\right],\left[t_{m 1}^{L}, t_{m 1}^{U}\right]\right)
\end{array}\right]
$$

where $\left[\left(\left[r_{i 1}^{L}, r_{i 1}^{U}\right],\left[s_{i 1}^{L}, s_{i 1}^{U}\right],\left[t_{i 1}^{L}, t_{i 1}^{U}\right]\right)\right]=\left[\mu_{\Theta_{X}^{*}}^{L}\left(u_{i}\right), \mu_{\Theta_{X}^{*}}^{U}\left(u_{i}\right)\right], \forall i=1,2, \ldots$, m. This ma-


Theorem 3.1. Let $\overrightarrow{\Theta_{X}}$ be the SFIVFS set. Suppose that $\overrightarrow{M_{\Theta_{X}}}, \overrightarrow{M_{c \Theta_{X}}}, \overrightarrow{M_{\Theta_{X}}^{*}}$ are matrices of $\overrightarrow{\Theta_{X}}, \overrightarrow{c \Theta_{X}}, \overrightarrow{\Theta_{X}^{*}}$ respectively, then $\overrightarrow{M_{\Theta_{X}}} \times \overrightarrow{M_{c \Theta_{X}}^{T}}=\overrightarrow{M_{\Theta_{X}}^{*}} \times|E|$, where $\overrightarrow{M_{\Theta_{X}}} \times \overrightarrow{M_{c \Theta_{X}}^{T}}$ is called a SFIVFSM-product and $\overrightarrow{M_{c \Theta_{X}}^{T}}$ is the transpose of $\overrightarrow{M_{c \Theta_{X}}}$.

Proof: The proof follows from Definition 3.1 and Definition 3.2.
We can make a multi criteria group decision making based on modified spherical fermatean interval valued fuzzy soft set models by the following algorithms.
3.1. Modified spherical fermatean interval valued fuzzy soft set model-I (Proposed). An algorithm for DM problems using SFIVFS set model is explained. The algorithm for the selection of the best choice is given as follows.
Step 1. Input the values for SFIVFS set $\overrightarrow{\Theta_{X}}$ over $\mathbb{U}$.
Step 2. Calculate the cardinalities and cardinal set $\overrightarrow{c \Theta_{X}}$ of $\overrightarrow{\Theta_{X}}$.
Step 3. Compute the aggregate SFIVFS set $\overrightarrow{\Theta_{X}^{*}}$ of $\overrightarrow{\Theta_{X}}$.
Step 4. Find the score function $S_{c}(u)=\frac{\left(\alpha_{u}^{3 L}-\beta_{u}^{3 U}-\gamma_{u}^{3 U}\right)+\left(\alpha_{u}^{3 U}-\beta_{u}^{3 L}-\gamma_{u}^{3 L}\right)}{2}$ and $u \in \mathbb{U}$.
Step 5. Output for the best alternative is $\max _{i} S_{c}\left(u_{i}\right)$.
Step 6. End.
3.2. Survey study. Decision making can be applied in various fields such as selection of best washing machine, laptop, engineerings and two wheeler motor bike of which choosing a college for education is to be discussed. In the selection of college for under going teaching education, the evaluation of teacher education is carried out according to various standards of experts. There are various studies, primarily conducted that have investigated the reasons why parents select a particular college, which they think best suits for their college student's needs and parental aspirations. We identify a factor regarded as parental decision making: Academic factor which is divided into five identified elements namely campus environment, overall cost, academic quality, student/faculty relationship and career development. Our goal is to select the optimal one out of a great number of alternatives based on the assessment of experts against the criteria. A parent intends to choose the best college education. Here we intend to choose ten colleges that are nominated. The score of the college education evaluated by the experts is represented by $E=\left\{e_{1}\right.$ : campus environment, $e_{2}$ : overall cost, $e_{3}$ : academic quality, $e_{4}$ : student/faculty relationship, $e_{5}$ : career development $\}$. After discussion each college is evaluated to a subset of parameters $X=\left\{e_{1}, e_{2}, e_{3}, e_{4}\right\} \subseteq E$. We apply the above-mentioned algorithm as follows.
Step-1: Form SFIVFS set $\overrightarrow{\Theta_{X}}$ of $\mathbb{U}$ is defined below:

$$
\begin{aligned}
\overrightarrow{\Theta_{X}}= & \left\{\left(e_{1},\left\{\frac{S_{1}}{[0.35,0.4],[0.2,0.25],[0.35,0.4]}, \frac{S_{2}}{[0.3,0.5],[0.3,0.45],[0.3,0.35]},\right.\right.\right. \\
& \frac{S_{4}}{[0.2,0.3],[0.25,0.35],[0.4,0.45]}, \frac{S_{5}}{[0.15,0.35],[0.25,0.3],[0.1,0.2]}, \\
& \frac{S_{7}}{[0.25,0.3],[0.3,0.4],[0.4,0.5]}, \frac{S_{9}}{[0.25,0.4],[0.4,0.5],[0.2,0.25]},
\end{aligned}
$$

$$
\begin{aligned}
& \left.\left.\frac{S_{10}}{[0.35,0.45],[0.2,0.25],[0.3,0.35]}\right\}\right) \text {, } \\
& \left(e_{2},\left\{\frac{S_{2}}{[0.3,0.45],[0.25,0.3],[0.3,0.5]}, \frac{S_{3}}{[0.4,0.5],[0.15,0.2],[0.3,0.4]},\right.\right. \\
& \frac{S_{5}}{[0.2,0.35],[0.3,0.45],[0.15,0.45]}, \frac{S_{6}}{[0.3,0.35],[0.4,0.6],[0.3,0.35]} \text {, } \\
& \left.\left.\frac{S_{8}}{[0.15,0.2],[0.45,0.5],[0.3,0.4]}, \frac{S_{10}}{[0.2,0.25],[0.3,0.4],[0.2,0.25]}\right\}\right), \\
& \left(e_{3},\left\{\frac{S_{3}}{[0.45,0.55],[0.25,0.3],[0.35,0.4]}, \frac{S_{4}}{[0.4,0.45],[0.1,0.25],[0.2,0.3]},\right.\right. \\
& \frac{S_{6}}{[0.4,0.45],[0.25,0.35],[0.5,0.55]}, \frac{S_{8}}{[0.5,0.6],[0.2,0.25],[0.15,0.2]}, \\
& \left.\left.\frac{S_{9}}{[0.3,0.35],[0.35,0.45],[0.5,0.6]}\right\}\right), \\
& \left(e_{4},\left\{\frac{S_{1}}{[0.5,0.55],[0.2,0.25],[0.3,0.5]}, \frac{S_{2}}{[0.2,0.25],[0.45,0.5],[0.25,0.45]},\right.\right. \\
& \frac{S_{4}}{[0.35,0.45],[0.15,0.2],[0.3,0.45]}, \frac{S_{5}}{[0.2,0.25],[0.4,0.45],[0.25,0.3]}, \\
& \frac{S_{7}}{[0.3,0.4],[0.2,0.35],[0.25,0.3]}, \frac{S_{8}}{[0.25,0.35],[0.3,0.6],[0.3,0.35]}, \\
& \left.\left.\left.\frac{S_{10}}{[0.15,0.3],[0.3,0.7],[0.15,0.2]}\right\}\right)\right\} .
\end{aligned}
$$

Step-2: The cardinal set of $\overrightarrow{\Theta_{X}}$ as

$$
\begin{aligned}
& \overrightarrow{c \Theta_{X}} \\
= & \left\{\frac{e_{1}}{([0.185,0.27],[0.19,0.25],[0.205,0.25])}, \frac{e_{2}}{([0.155,0.21],[0.185,0.245],[0.155,0.235])},\right. \\
& \left.\frac{e_{3}}{([0.205,0.24],[0.115,0.16],[0.17,0.205])}, \frac{e_{4}}{([0.195,0.255],[0.2,0.305],[0.18,0.255])}\right\} .
\end{aligned}
$$

Step-3: By Theorem 3.1, the aggregate SFIVFS set $\overrightarrow{\Theta_{X}^{*}}$ of $\overrightarrow{\Theta_{X}}$ is

$$
\begin{aligned}
\overrightarrow{M_{\Theta_{X}^{*}}} & =\frac{\overrightarrow{M_{\Theta_{X}}} \times \overrightarrow{M_{c \Theta_{X}}^{t}}}{|E|} \\
& =\frac{1}{5}\left\{\begin{array}{ccccc}
{\left[\begin{array}{ccccc}
{[0.35,0.4]} & {[0,0]} & {[0,0]} & {[0.5,0.55]} & {[0,0]} \\
{[0.3,0.5]} & {[0.3,0.45]} & {[0,0]} & {[0.2,0.25]} & {[0,0]} \\
{[0,0]} & {[0.4,0.5]} & {[0.45,0.55]} & {[0,0]} & {[0,0]} \\
{[0.2,0.3]} & {[0,0]} & {[0.4,0.45]} & {[0.35,0.45]} & {[0,0]} \\
{[0.15,0.35]} & {[0.2,0.35]} & {[0,0]} & {[0.2,0.25]} & {[0,0]} \\
{[0,0]} & {[0.3,0.35]} & {[0.4,0.45]} & {[0,0]} & {[0,0]} \\
{[0.25,0.3]} & {[0,0]} & {[0,0]} & {[0.3,0.4]} & {[0,0]} \\
{[0,0]} & {[0.15,0.2]} & {[0.5,0.6]} & {[0.25,0.35]} & {[0,0]} \\
{[0.155,0.21]} \\
{[0.205,0.24]} \\
{[0.25,0.4]} & {[0,0]} & {[0.3,0.35]} & {[0,0]} & {[0,0]} \\
{[0.35,0.45]} & {[0.2,0.25]} & {[0,0]} & {[0.15,0.3]} & {[0,0]}
\end{array}\right],}
\end{array}\right]=\left[\begin{array}{l}
{[0,0]}
\end{array}\right]
\end{aligned}
$$

$$
\left.\left.\begin{array}{l}
{\left[\begin{array}{ccccc}
{[0.2,0.25]} & {[0,0]} & {[0,0]} & {[0.2,0.25]} & {[0,0]} \\
{[0.3,0.45]} & {[0.25,0.3]} & {[0,0]} & {[0.45,0.5]} & {[0,0]} \\
{[0,0]} & {[0.15,0.2]} & {[0.25,0.3]} & {[0,0]} & {[0,0]} \\
{[0.25,0.35]} & {[0,0]} & {[0.1,0.25]} & {[0.15,0.2]} & {[0,0]} \\
{[0.25,0.3]} & {[0.3,0.45]} & {[0,0]} & {[0.4,0.45]} & {[0,0]} \\
{[0,0]} & {[0.4,0.6]} & {[0.25,0.35]} & {[0,0]} & {[0,0]} \\
{[0.3,0.4]} & {[0,0]} & {[0,0]} & {[0.2,0.35]} & {[0,0]} \\
{[0,0]} & {[0.45,0.5]} & {[0.2,0.25]} & {[0.3,0.6]} & {[0,0]} \\
{[0.4,0.5]} & {[0,0]} & {[0.35,0.45]} & {[0,0]} & {[0,0]} \\
{[0.2,0.25]} & {[0.3,0.4]} & {[0,0]} & {[0.3,0.7]} & {[0,0]}
\end{array}\right]}
\end{array}\right] \begin{array}{c}
{[0.19,0.25]} \\
{[0.185,0.245]} \\
{[0.115,0.16]} \\
{[0.2,0.305]} \\
{[0,0]}
\end{array}\right]
$$

Hence,

$$
\begin{aligned}
& \overrightarrow{\Theta_{X}^{*}} \\
= & \left\{\frac{S_{1}}{([0.039,0.054],[0.04,0.05],[0.041,0.051])}, \frac{S_{2}}{([0.039,0.054],[0.04,0.061],[0.041,0.051])},\right. \\
& \frac{S_{4}}{([0.041,0.048],[0.03,0.04],[0.034,0.047])}, \frac{S_{3}}{([0.041,0.054],[0.038,0.05],[0.041,0.051])}, \\
& \frac{S_{5}}{([0.039,0.054],[0.04,0.061],[0.036,0.051])}, \frac{S_{7}}{([0.041,0.048],[0.037,0.049],[0.034,0.047])}, \\
& \frac{S_{8}}{([0.039,0.054],[0.04,0.061],[0.041,0.051])}, \frac{S_{8}}{([0.041,0.051],[0.04,0.061],[0.036,0.051])}, \\
& \left.\frac{S_{9}}{([0.041,0.054],[0.038,0.05],[0.04,0.05])}, \frac{S_{10}}{([0.037,0.054],[0.04,0.061],[0.041,0.05])}\right\} .
\end{aligned}
$$

Step-4: The score function $S_{c}\left(C_{i}\right)$ as follows:

Score value $=$| Colleges | $S_{c}\left(C_{i}\right)$ |
| :---: | :---: |
| $C_{1}$ | -0.000087 |
| $C_{2}$ | -0.000138 |
| $C_{3}$ | -0.000027 |
| $C_{4}$ | -0.000078 |
| $C_{5}$ | -0.000127 |
| $C_{6}$ | -0.000066 |
| $C_{7}$ | -0.000138 |
| $C_{8}$ | -0.000134 |
| $C_{9}$ | -0.000071 |
| $C_{10}$ | -0.000138 |

3.3. Modified spherical fermatean interval valued fuzzy soft set model-II (Proposed).
Step 1. Input the values for SFIVFSM on the basis of the parameters.
Step 2. Case-I: Obtain the choice matrix for the positive, neutral and negative-membership of SFIVFSM (weights are equal).
Case-II: Find the choice matrix for the positive, neutral and negative-membership of SFIVFSM (weights are unequal).
Step 3. Compute the score value $S_{c}(u)=\frac{\left(\alpha_{u}^{3 L}-\beta_{u}^{3 U}-\gamma_{u}^{3 U}\right)+\left(\alpha_{u}^{3 U}-\beta_{u}^{3 L}-\gamma_{u}^{3 L}\right)}{2}$ and $u \in \mathbb{U}$.
Step 4. Output for the best alternative is $\max _{i} S_{c}\left(u_{i}\right)$.
Step 5. End.
Case-I: Let $\vec{X}=\left(\vec{\alpha}_{i j}, \vec{\beta}_{i j}, \vec{\gamma}_{i j}\right) \in S F I V F S M_{m \times n}$. If weights are equal, then choice matrix of SFIVFSM $\vec{X}$ is given as

$$
\overrightarrow{\mathcal{C}(X)}=\left[\left(\frac{\sum_{j=1}^{n}\left(\vec{\alpha}_{i j}\right)^{3}}{n}, \frac{\sum_{j=1}^{n}\left(\vec{\beta}_{i j}\right)^{3}}{n}, \frac{\sum_{j=1}^{n}\left(\vec{\gamma}_{i j}\right)^{3}}{n}\right)\right]_{m \times 1}, \quad \forall i
$$

We may appeal to above survey, and the choice matrix can be obtained as

Score value $=$|  | Colleges |  | $S_{c}\left(C_{i}\right)$ |
| :---: | :---: | :---: | :---: |
|  | $C_{1}$ |  |  |
| $C_{2}$ | 0.000788 |  |  |
| $C_{3}$ | 0.001828 |  |  |
| $C_{4}$ | -0.000029 |  |  |
| $C_{5}$ | -0.001216 |  |  |
| $C_{6}$ | -0.00228 |  |  |
| $C_{7}$ | -0.00064 |  |  |
| $C_{8}$ | -0.001352 |  |  |
| $C_{9}$ | -0.002325 |  |  |
| $C_{10}$ | -0.00335 |  |  |

Case-II: Let $\vec{X}=\left(\vec{\alpha}_{i j}, \vec{\beta}_{i j}, \vec{\gamma}_{i j}\right) \in$ SFIVFSM $_{m \times n}$. If weights are unequal, then weighted choice matrix of SFIVFSM $\vec{X}$ is

$$
\overrightarrow{\mathcal{C}_{w}(X)}=\left[\left(\frac{\sum_{j=1}^{n} \vec{w}_{j}\left(\vec{\alpha}_{i j}\right)^{3}}{\sum \overrightarrow{w_{j}}}, \frac{\sum_{j=1}^{n} \vec{w}_{j}\left(\vec{\beta}_{i j}\right)^{3}}{\sum \overrightarrow{w_{j}}}, \frac{\sum_{j=1}^{n} \vec{w}_{j}\left(\vec{\gamma}_{i j}\right)^{3}}{\sum \overrightarrow{w_{j}}}\right)\right]_{m \times 1}, \quad \forall i .
$$

Put weights $\left(\vec{w}_{j}\right)=\{[0.16,0.165],[0.14,0.145],[0.18,0.19],[0.17,0.175],[0.15,0.155]\}$. We may appeal to above survey, and the weighted choice matrix can be obtained as

$$
\text { Score value }=\begin{array}{cc}
\hline \text { Colleges } & S_{c}\left(C_{i}\right) \\
\cline { 2 - 3 } & C_{1} \\
C_{2} & 0.000046 \\
C_{3} & 0.000096 \\
C_{4} & 0.0000008 \\
C_{5} & -0.000044 \\
C_{6} & -0.000114 \\
C_{7} & -0.00002 \\
C_{8} & -0.000077 \\
C_{9} & -0.000144 \\
C_{10} & -0.000354 \\
\hline
\end{array}
$$

### 3.4. Modified spherical fermatean interval valued fuzzy soft set model-III (Proposed).

Step 1. Input the values for SFIVFSM on the basis of parameters.

Step 2. Find the spherical fermatean interval valued fuzzy weighted averaging numbers (SFIVFWANs) under aggregated operation,

$$
\overrightarrow{\mathcal{C}(X)}=\left(\sum_{j=1}^{n} \overrightarrow{w_{j}} \overrightarrow{\alpha_{i j}}, \sum_{j=1}^{n} \overrightarrow{w_{j}} \overrightarrow{\beta_{i j}}, \sum_{j=1}^{n} \overrightarrow{w_{j}} \overrightarrow{\gamma_{i j}}\right) .
$$

Step 3. Compute the score function $S_{c}(u)=\frac{\left(\alpha_{u}^{3 L}-\beta_{u}^{3 U}-\gamma_{u}^{3 U}\right)+\left(\alpha_{u}^{3 U}-\beta_{u}^{3 L}-\gamma_{u}^{3 L}\right)}{2}$, $u \in \mathbb{U}$.
Step 4. Output for the best alternative is $\max _{i} S_{c}\left(u_{i}\right)$.
Step 5. End.
Put weights $\left(\vec{w}_{j}\right)=\{[0.16,0.165],[0.14,0.145],[0.18,0.19],[0.17,0.175],[0.15,0.155]\}$. We may appeal to above survey,

$$
\overrightarrow{\mathcal{C}(X)}=\left\{\begin{array}{c}
{\left[\begin{array}{c}
{[0.141,0.16225]} \\
{[0.124,0.1915]} \\
{[0.137,0.177]} \\
{[0.1635,0.21375]} \\
{[0.086,0.15225]} \\
{[0.114,0.13625]} \\
{[0.091,0.1195]} \\
{[0.1535,0.20425]} \\
{[0.094,0.1325]} \\
{[0.1095,0.163]}
\end{array}\right]}
\end{array}\right]\left[\begin{array}{c}
{[0.066,0.085]} \\
{[0.1595,0.20525]} \\
{[0.066,0.086]} \\
{[0.0835,0.14025]} \\
{[0.15,0.1935]} \\
{[0.101,0.1535]} \\
{[0.082,0.12725]} \\
{[0.15,0.225]} \\
{[0.127,0.168]} \\
{[0.125,0.22175]}
\end{array}\right],\left[\begin{array}{c}
{[0.107,0.1535]} \\
{[0.1325,0.209]} \\
{[0.105,0.134]} \\
{[0.105,0.21]} \\
{[0.0795,0.15075]} \\
{[0.132,0.15525]} \\
{[0.1065,0.135]} \\
{[0.12,0.15725]} \\
{[0.122,0.15525]} \\
{[0.1015,0.129]}
\end{array}\right]
$$

Table 1. Analysis of above-proposed models

| Models | Ranking of alternatives | Alternatives |
| :---: | :--- | :---: |
| Model-I | $C_{2}=C_{7}=C_{10} \leq C_{8} \leq C_{5} \leq C_{1} \leq C_{4} \leq C_{9} \leq C_{6} \leq C_{3}$ | $C_{3}$ |
| Model-II Case-(i) | $C_{10} \leq C_{9} \leq C_{6} \leq C_{2} \leq C_{8} \leq C_{5} \leq C_{7} \leq C_{4} \leq C_{1} \leq C_{3}$ | $C_{3}$ |
| Model--II Case-(ii) | $C_{10} \leq C_{9} \leq C_{6} \leq C_{2} \leq C_{8} \leq C_{5} \leq C_{7} \leq C_{4} \leq C_{1} \leq C_{3}$ | $C_{3}$ |
| Model-III | $C_{2} \leq C_{10} \leq C_{5} \leq C_{9} \leq C_{8} \leq C_{6} \leq C_{7} \leq C_{4} \leq C_{1} \leq C_{3}$ | $C_{3}$ |

## 4. Analysis for Modified Spherical Fermatean Interval Valued Fuzzy Soft Set

Models. It is observed that the third college education is the best for students in effective manner. Finally, parents select the third college because of the following reasons.

1) Campus environment is found to be the best among the other colleges.
2) Overall cost is better than the any other colleges in the list.
3) Academic quality is evaluated to be falling in line with the expectation of parents.
4) Students/faculty relationship is found to be the best among the remaining colleges.
5) Career development is stronger than the any other colleges.


Figure 1. SFIVFS set models based on MCGDM
5. Conclusion. In this present communication, the above three spherical fermatean interval valued fuzzy soft set models followed by multi criteria group decision making are explained. The main focus of this study is the awareness of spherical fermatean interval valued fuzzy soft set models among college education, so that parents can learn and apply some decision making methods in practical applications. The results indicate that the first choice comes out to be ten colleges with the three spherical fermatean interval valued fuzzy soft set models which have five attributes. We discussed campus environment, overall cost, academic quality, students/faculty relationship and career development are found to be the best among the other colleges. Also, we have inserted a few sorts of statistical charts to image the rankings of alternatives under consideration.

Acknowledgment. The authors would like to thank the anonymous referee who provided useful and detailed comments on a previous/earlier version of the manuscript.

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