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FUZZY PARAMETERIZED RELATIVE FUZZY SOFT SETS IN DECISION-MAKING PROBLEMS

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Abstract. *In this paper, we define fuzzy parameterized relative fuzzy soft sets (fprf-soft sets) which are generalization of the fuzzy parameterized fuzzy soft set and the relative soft set. We give some their operations such as a complement, a union and an intesection and give some their properties together with presented and supported examples. Moreover, we construct the method based on a score value of the fprf-soft sets in decision-making problems. Finally, the example presented in this paper demonstrates that the model is practical for solving decision-making problems.*
Keywords: Fuzzy parameterized relative fuzzy soft sets, Relative soft set, Decision-making problems

1. **Introduction.** The solution to real-life problems in many fields involves data that contain the uncertainties. When dealing with uncertainties in data, there are theories that can be applied, such as the probability theory, fuzzy sets [1], rough sets [2], and other mathematical tools. Molodtsov [3] created the soft set theory in 1999, which is a completely new way for modeling ambiguity and uncertainty that is devoid of issues. He showed soft set theory applications in a variety of domains. Many authors have investigated the soft set theory since then [4-8]. Maji et al. [9] extended soft set theory to fuzzy soft set theory. They presented the fuzzy soft sets, fuzzy soft operations such as the union, intersection and investigated their properties. In 2007, Roy and Maji [10] discussed the application of the algorithm of fuzzy soft sets in decision-making problems. The score value of fuzzy soft sets in decision-making problems was computed using the comparison table in the algorithm. Later, Çağman et al. [11] defined fuzzy parameterized fuzzy soft sets and gave some its properties. They discussed the algorithm in decision-making problem fuzzy parameterized fuzzy soft set. In 2013, Balami and Musa [12] defined relative soft sets and its basic properties and the generalization of soft sets. Moreover, they discussed its operation such as union and intersection. Syafaruddin et al. [14] presented the application of adaptive neuro-fuzzy inference system (ANFIS) method to determine the optimal output power of crystalline Silicon photo voltaic (PV) modules technology. Next, Rotjanasom et al. [13] defined the fuzzy parameterized relative soft sets and investigated their properties. Moreover, the decision-making problems were proposed by Yang and He [15]. They presented a novel method based on fixed point iteration and improved TOPSIS method for multi-attribute groups decision making.

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The theory of soft set and fuzzy soft set is a good mathematical tool when it comes to dealing with uncertainty. However, it is also a new notion when it comes to applying it to abstract algebraic structures. In 2011, Yang [17] established fuzzy soft sets to fuzzy soft semigroups. He defined a fuzzy soft [left, right] ideal and a fuzzy soft semigroup over a semigroup. He provided sufficient and necessary conditions for α-level set, intersection and union of fuzzy soft [left, right] ideals. In 2015, Siripitukdet and Suebsan [18] defined semiprime, prime and strongly prime fuzzy soft bi-ideals over semigroups and presented their properties. Fuzzy soft bi-ideals over semigroups were also studied by Suebsan and Siripitukdet [19] in 2018, followed by the presentation of their properties which proved that the image of fuzzy soft bi-ideals over semigroups is the fuzzy soft bi-ideals over semigroups. In 2020, Yiarayong [16] applied the concept of picture fuzzy sets to semigroup theory. He characterized different classes regular (resp. intra-regular and semisimple) semigroups in terms of picture fuzzy left and right ideals (resp. picture fuzzy ideals).

When it comes to existing researches of fuzzy soft sets in decision-making problems, they are limited to some extent. The algorithm of Roy and Maji [10] is not easy to judge in case which have different factors of the choice value and score value. Moreover, we observe that Roy and Maji's method is designed for one universe. The fuzzy parameterized relative fuzzy soft set plays a key role in dealing with problems specified above.

Motivated and inspired by the works above, we are interested in an application of the relative soft sets in decision-making problems. In this paper, we propose the concept of fuzzy parameterized relative fuzzy soft sets (*fprf*-soft sets) as a hybrid model of the fuzzy parameterized fuzzy soft set and the relative soft set which is generalization of the fuzzy parameterized fuzzy set and the relative soft set. We discuss the score value in the model of the *fprf*-soft set in decision-making problem. The results show that the model of the *fprf*-soft set is practical for solving in decision-making problems.

The organization of the paper is as follows. In Section 2, basic notions about a soft set, a fuzzy soft set, a fuzzy parameterized fuzzy soft set and a relative soft set are reviewed. Moreover, we review the Roy and Maji's method based on the comparison table. Section 3 focuses on fuzzy parameterized relative fuzzy soft sets with relevant examples. We also presented the basic properties of the operations. Section 4 constructs the algorithm of fuzzy parameterized relative fuzzy soft sets (*fprf*-soft sets) in decision-making problem. Section 5 presents a numerical example and comparison analysis to illustrate the validity of the proposed method. The final section discusses the conclusion of this paper.

2. **Preliminaries.**

In this section, we summarize the preliminary definitions, and results that are required later in this paper.

Molodtsov [3] firstly initiated a mathematical tool which is soft set theory for modeling uncertainty. Next, Çağman and Enginoğlu [5] rewrote the definition of the soft set of Molodtsov [3] as follows.

Definition 2.1. [5] *Let U be an initial universe, $P(U)$ be the power set of U, E be the set of all parameters and $A \subseteq E$. $\widehat{F}_A$ is called a soft set over U, where*

$$\widehat{F}_A = \left\{ \left(x, \widehat{f}_A(x)\right) : x \in E,\ \widehat{f}_A(x) \in P(U) \right\},$$

where $\widehat{f}_A$ is called an approximate function given by

$$\widehat{f}_A : E \to P(U) \text{ such that } \widehat{f}_A(x) = \emptyset \text{ if } x \notin A.$$

The value $\widehat{f}_A(x)$ is a set called x-element of the soft set $\widehat{F}_A$ for all $x \in E$. The set of all soft sets over U will be denoted by $S(U)$.

Definition 2.2. [11] *Let U be an initial universe. X is called a fuzzy set over U, where*

$$X = \{(\mu_X(u)/u) : u \in U,\ \mu_X(u) \in [0, 1]\},$$

where μ_X is called a membership function of X given by $\mu_X : U \to [0, 1]$. The value $\mu_X(u)$ is called the grade of membership of $u \in U$. The value represents the degree of u belonging to the fuzzy set X. The set of all fuzzy sets over U will be denoted by $Fuz(U)$.

Maji et al. [9] presented the definition of a fuzzy soft set.

Definition 2.3. [9] *Let U be an initial universe set, E be a set of parameters and let $\emptyset \neq A \subseteq E$. A pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by $F : A \to Fuz(U)$ and $Fuz(U)$ is the set of all fuzzy sets on U.*

Definition 2.4. [9] *Let (F, A) and (G, B) be two fuzzy soft sets over a common universe U. The* AND *operation of two fuzzy soft sets (F, A) and (G, B), "(F, A) AND (G, B)", denoted by $(F, A) \widetilde{\wedge} (G, B)$, is defined by $(F, A) \widetilde{\wedge} (G, B) := (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \wedge G(\beta)$, for all $(\alpha, \beta) \in A \times B$.*

Note that $(F(\alpha) \wedge G(\beta))(x) = F(\alpha)(x) \wedge G(\beta)(x) = \min\{F(\alpha)(x), G(\beta)(x)\}$ for all $x \in U$.

Later, Çağman et al. [11] rewrote the definition of the fuzzy soft set of Maji et al. [9] as follows.

Definition 2.5. [11] *Let U be an initial universe, E be the set of all parameters, $A \subseteq E$. $\widehat{\Gamma}_A$ is called a fuzzy soft set over U, where*

$$\widehat{\Gamma}_A = \{(x, \widehat{\gamma}_A(x)) : x \in E,\ \widehat{\gamma}_A(x) \in Fuz(U)\},$$

where $\widehat{\gamma}_A$ is called a fuzzy approximate function given by

$$\widehat{\gamma}_A : E \to Fuz(U) \text{ such that } \widehat{\gamma}_A(x) = \emptyset \text{ if } x \notin A.$$

The value $\widehat{\gamma}_A(x)$ is a fuzzy set called x-element of the fuzzy soft set $\widehat{\Gamma}_A$ for all $x \in E$. The set of all fuzzy soft sets over U will be denoted by $FS(U)$.

Definition 2.6. [20] *Let U be an initial universe, $P(U)$ be the power set of U, E be the set of all parameters and X be a fuzzy set over E with the membership function $\mu_X : E \to [0, 1]$. $\widehat{F}_X$ is called a fuzzy parameterized soft set (fps-set) over U, where*

$$\widehat{F}_X = \left\{ \left(\mu_X(x)/x, \widehat{f}_X(x) \right) : x \in E,\ \widehat{f}_X(x) \in P(U),\ \mu_X(x) \in [0, 1] \right\},$$

where $\widehat{f}_X$ is called an approximate function of the fps-set $\widehat{F}_X$ given by

$$\widehat{f}_X : E \to P(U) \text{ such that } \widehat{f}_X(x) = \emptyset \text{ if } \mu_X(x) = 0.$$

The value $\widehat{f}_X(x)$ is a set called x-element of the fps-set $\widehat{F}_X$ for all $x \in E$. The sets of all fps-set over U will be denoted by $FPS(U)$.

Definition 2.7. [11] *Let U be an initial universe, E be the set of all parameters and X be a fuzzy set over E with the membership function $\mu_X : E \to [0, 1]$ and $\widetilde{\Gamma}_X(x)$ be a fuzzy set over U for all $x \in E$. $\widetilde{\Gamma}_X$ is called a fuzzy parameterized fuzzy soft set (fpfs-set) over U, where*

$$\widetilde{\Gamma}_X = \{(\mu_X(x)/x, \widetilde{\gamma}_X(x)) : x \in E,\ \gamma_X(x) \in Fuz(U),\ \mu_X(x) \in [0, 1]\},$$

where $\widetilde{\gamma}_X(x)$ is called a fuzzy approximate function of the fpfs-set $\widetilde{\Gamma}_X$ given by

$$\widetilde{\gamma}_X : E \to Fuz(U) \text{ such that } \widetilde{\gamma}_X(x) = \emptyset \text{ if } \mu_X(x) = 0.$$

The value $\widetilde{\gamma}_X(x)$ *is a fuzzy set called* x*-element of the fpfs-set* $\widetilde{\Gamma}_X$ *for all* $x \in E$*. The set of all fpfs-sets over* U *will be denoted by* $FPFS(U)$*.*

Example 2.1. *Assume that* $U = \{u_1, u_2, u_3, u_4, u_5\}$ *is a universal set and* $E = \{x_1, x_2, x_3, x_4\}$ *is a set of all parameters. If* $X = \{0.3/x_1, 0.6/x_2, 0.9/x_4\}$ *and*

$$\begin{aligned}\widetilde{\gamma}_X(x_1) &= \{0.2/u_1, 0.6/u_2, 0.5/u_3, 0.8/u_4, 0.4/u_5\},\\ \widetilde{\gamma}_X(x_2) &= \{0.3/u_1, 0.7/u_2, 0.8/u_3, 0.4/u_4, 0.5/u_5\},\\ \widetilde{\gamma}_X(x_4) &= \{0.9/u_1, 0.4/u_2, 0.6/u_3, 0.9/u_4, 0.2/u_5\},\end{aligned}$$

then the fpfs-set $\widetilde{\Gamma}_X$ *is written by*

$$\begin{aligned}\widetilde{\Gamma}_X = \{&(0.3/x_1, \{0.2/u_1, 0.6/u_2, 0.5/u_3, 0.8/u_4, 0.4/u_5\}),\\ &(0.6/x_2, \{0.3/u_1, 0.7/u_2, 0.8/u_3, 0.4/u_4, 0.5/u_5\}),\\ &(0.9/x_4, \{0.9/u_1, 0.4/u_2, 0.6/u_3, 0.9/u_4, 0.2/u_5\})\}.\end{aligned}$$

The concept of relative soft sets was introduced by Balami and Musa [12].

Let $\{U_i :\in I\}$ be a collection of universe such that $\bigcap_{i\in I} U_i = \emptyset$ and let $\{E_{U_i} : i \in I\}$ be a collection of set of parameters. $U = P(U_i)$ denotes the power set of U_i, $E = E_{U_i}$ and $A \subseteq E$.

Definition 2.8. [12] *A pair* (F, A) *is celled a relative soft set over* U *where* F *is a mapping given by* $F : A \to U$*.*

In other words, a relative soft set over U *is a parameterized family of subset of the universe* U*. For* $e \in A$*,* $F(e)$ *may be considered an* e*-approximate element of the soft set* (F, A)*. Based on the above definition, any change in the ordering of the universe will produce a different relative soft set.*

Example 2.2. *As an illustration, suppose that there are two universes* U_1 *and* U_2*, and let us consider a relative soft set* (F, A) *which describes the "attractiveness of cloths", and "shoes" that Mr. X is going to buy. Let* $U_1 = \{C_1, C_2, C_3, C_4, C_5\}$ *be the set of cloths and* $U_2 = \{S_1, S_2, S_3, S_4\}$ *be the set of shoes.*

Let $E_U = \{E_{U_1}, E_{U_2}\}$ *be the collection of sets decision parameters, where* $E_{U_1} = \{e_{U_1}, 1 = \textit{expensive}, e_{U_1}, 2 = \textit{cheap}, e_{U_1}, 3 = \textit{beautiful}\}$*, and* $E_{U_2} = \{e_{U_2}, 1 = \textit{expensive}, e_{U_2}, 2 = \textit{made in Italy}, e_{U_2}, 3 = \textit{black}\}$*.*

Let $U = P(U_i)$*,* $E = E_{U_i}$ *and* $A \subseteq E$ *such that* $i = 1, 2$*.* $A = \{a_1 = (e_{U_1}, 1, e_{U_2}, 1), a_2 = (e_{U_1}, 1, e_{U_2}, 2), a_3 = (e_{U_1}, 2, e_{U_2}, 3), a_4 = (e_{U_1}, 3, e_{U_2}, 2)\}$*.*

Suppose that

$$\begin{aligned}&F(a_1) = (\{C_2, C_3\}, \{S_1, S_4\}), \quad F(a_2) = (\{C_1, C_3\}, \{S_2, S_3\}),\\ &F(a_3) = (\{C_1, C_4, C_5\}, \emptyset), \quad F(a_4) = (\{C_2, C_5\}, \{S_2, S_3\}).\end{aligned}$$

Then we can see the relative soft set (F, A) *as consisting of the following approximations,*

$$\begin{aligned}(F, A) = \{&(a_1, (\{C_2, C_3\}, \{S_1, S_4\})), (a_2, (\{C_1, C_3\}, \{S_2, S_3\})),\\ &(a_3, (\{C_1, C_4, C_5\}, \emptyset)), (a_4, (\{C_2, C_5\}, \{S_2, S_3\}))\}.\end{aligned}$$

We can see that each approximate has two parts, viz. a predicate and an approximate value set. The illustration can logically be explained as follows: for $F(a_1) = (\{C_2, C_3\}, \{S_1, S_4\})$*, if* $\{C_2, C_3\}$ *is the set of expensive cloths to Mr. X, then the set of relatively expensive shoes to him is* $\{S_1, S_4\}$*. It has been shown that relative soft set is a conditional relation.*

Now, we briefly review the algorithms in decision-making problems.

Roy and Maji [10] used the definition of the fuzzy soft set of Maji et al. [9] and defined the comparison table approach in decision-making problems.

Let $U = \{o_1, o_2, \ldots, o_n\}$ be an object set and let $E = \{e_1, e_2, \ldots, e_k\}$ be a set of parameters.

The comparison table is a square table in which the numbers of rows and columns are equal, rows and columns both are labelled by the object names $o_1, o_2, \ldots, o_n$ of U, and the entries are c_{ij}, $i, j \in \{1, 2, \ldots, n\}$ given by c_{ij} = the number of parameters for which the membership value of o_i exceeds or equal to the membership value of o_j. Obviously, $0 \leq c_{ij} \leq k$, and $c_{ii} = k$, for all i, j, where k is the number of all parameters in a fuzzy soft set. Thus, c_{ij} indicates a numerical measure, which is an integer number and o_i dominates o_j in c_{ij} number of parameters out of k parameters.

Roy and Maji [10] used the comparison table in the algorithm as follows:

Algorithm in [10]

Step 1. Input the fuzzy soft sets (F, A), (G, B) and (H, C).

Step 2. Input the parameter set P.

Step 3. Compute the resultant fuzzy soft set (S, P) from (F, A), (G, B) and (H, C).

Step 4. Compute the comparison table of (S, P) and compute t_i and r_i for o_i for all i.

Step 5. Compute the score value ($S_i = r_i - c_i$) of o_i for all i.

Step 6. The decision is S_k if $S_k = \max_i S_i$.

Step 7. We choose only o_k if k has more than one value.

They computed (S, P) by "AND" operations.

Example 2.3. [21] *Let* $U = \{o_1, o_2, o_3, o_4, o_5, o_6\}$ *be a set of objects,* $E = \{a_1 :=$ *white,* $a_2 :=$ *green,* $a_3 :=$ *pink,* $a_4 :=$ *blue,* $b_1 :=$ *long,* $b_2 :=$ *very long,* $b_3 :=$ *short,* $b_4 :=$ *very short,* $b_5 :=$ *medium,* $c_1 :=$ *rough,* $c_2 :=$ *moderately fine,* $c_3 :=$ *fine,* $c_4 :=$ *extra fine*$\}$ *be a set of parameters and* $A = \{a_1, a_2, a_3, a_4\}$, $B = \{b_1, b_2, b_3, b_4, b_5\}$ *and* $C = \{c_1, c_2, c_3, c_4\}$. *Suppose that the company is looking to select the object.*

Step 1. *We consider the fuzzy soft set* (F, A), (G, B) *and* (H, C) *in tabular forms as in Table 1, Table 2, and Table 3.*

Table 1. The tabular form of the fuzzy soft set (F, A)

U/A	a_1	a_2	a_3	a_4
o_1	0.4	0.5	0.7	1.0
o_2	0.4	1.0	0.4	0.6
o_3	0.5	0.6	0.9	0.8
o_4	0.9	0.3	0.5	0.9
o_5	0.8	0.4	0.7	0.6
o_6	1.0	0.3	0.5	0.4

Table 2. The tabular form of the fuzzy soft set (G, B)

U/B	b_1	b_2	b_3	b_4	b_5
o_1	0.5	0.3	0.9	0.7	0.6
o_2	0.9	0.7	0.4	0.2	0.8
o_3	0.7	0.5	0.5	0.2	0.8
o_4	1.0	0.9	0.3	0.2	0.5
o_5	0.3	0.2	1.0	0.9	0.6
o_6	0.4	0.3	0.9	0.7	0.4

We compute $(F, A) \,\widetilde{\wedge}\, (G, B)$. *We will have* $4 \times 5 = 20$ *parameters of the form* e_{ij}, *where* $e_{ij} = a_i \wedge b_j$ *for all* $i \in \{1, 2, 3, 4\}$ *and* $j \in \{1, 2, 3, 4, 5\}$. *We require the fuzzy soft set for the parameters* $R = \{e_{11}, e_{15}, e_{21}, e_{24}, e_{33}, e_{44}, e_{45}\}$.

TABLE 3. The tabular form of the fuzzy soft set (H, C)

U/C	c_1	c_2	c_3	c_4
o_1	0.4	0.5	0.2	1.0
o_2	0.7	0.6	0.5	0.6
o_3	0.6	0.7	0.4	0.7
o_4	0.8	0.7	0.7	0.4
o_5	0.7	0.7	0.6	0.5
o_6	0.9	0.8	0.8	1.0

Let (K, R) be the resultant fuzzy soft set for the fuzzy soft sets (F, A) and (G, B). Therefore, after performing the $(F, A) \widetilde{\wedge} (G, B)$ for some parameters R, we can represent this fuzzy soft set (K, R) in a tabular form as in Table 4.

TABLE 4. The tabular form of the fuzzy soft set (K, R)

U/e_{ij}	e_{11}	e_{15}	e_{21}	e_{24}	e_{33}	e_{44}	e_{45}
o_1	0.4	0.4	0.5	0.5	0.7	0.7	0.6
o_2	0.4	0.4	0.9	0.2	0.4	0.2	0.7
o_3	0.5	0.5	0.6	0.2	0.5	0.2	0.8
o_4	0.9	0.5	0.3	0.2	0.3	0.2	0.5
o_5	0.3	0.8	0.3	0.4	0.7	0.6	0.6
o_6	0.4	0.6	0.3	0.3	0.5	0.4	0.4

Step 2. *Suppose that $P = \{e_{11} \wedge c_1, e_{15} \wedge c_3, e_{21} \wedge c_2, e_{24} \wedge c_4, e_{33} \wedge c_3, e_{44} \wedge c_3, e_{45} \wedge c_4\}$ is the set of choice parameters of the observer.*
Step 3. *We can represent the resultant fuzzy soft set (S, P) in a tabular form as in Table 5.*

TABLE 5. The tabular form of the fuzzy soft set (S, P)

$U\backslash e_{ij}$	$e_{11} \wedge c_1$	$e_{15} \wedge c_3$	$e_{21} \wedge c_2$	$e_{24} \wedge c_4$	$e_{33} \wedge c_3$	$e_{44} \wedge c_3$	$e_{45} \wedge c_4$	Choice value
o_1	0.4	0.2	0.5	0.5	0.2	0.2	0.6	2.6
o_2	0.4	0.4	0.6	0.2	0.4	0.2	0.6	2.8
o_3	0.5	0.4	0.6	0.2	0.4	0.2	0.7	3.0
o_4	0.8	0.5	0.3	0.2	0.3	0.2	0.4	2.7
o_5	0.3	0.6	0.3	0.4	0.6	0.6	0.5	3.3
o_6	0.4	0.6	0.3	0.3	0.5	0.4	0.4	2.9

Step 4. *The comparison table of the resultant fuzzy soft set (S, P) is as below.*

TABLE 6. Table of the comparison

	o_1	o_2	o_3	o_4	o_5	o_6
o_1	7	4	2	4	4	4
o_2	6	7	5	5	3	3
o_3	6	7	7	5	3	3
o_4	4	4	4	7	2	3
o_5	3	4	4	6	7	6
o_6	4	5	4	6	3	7

Step 5. *We compute the row sum, column sum and the score for each o_i as shown below.*

TABLE 7. Table of the score value

	Row sum	Column sum	Score value
o_1	25	30	-5
o_2	29	31	-2
o_3	31	26	5
o_4	24	33	-9
o_5	30	22	8
o_6	29	26	3

Step 6. *From the above score value table, it is clear that the maximum score is 8, scored by o_5. Hence, the decision is favour of selecting o_5.*

Assume that the company is looking to select the object of ten universes $U_1, U_2, \ldots, U_{10}$. Roy and Maji's method must compute ten times because Roy and Maji's method is designed for one universe U. We can extend one universe U to multiple universes $\{U_i : i \in I\}$. The fuzzy parameterized relative fuzzy soft set plays a key role in dealing with problems specified above.

In order to deal with the proposed problems, we define fuzzy parameterized relative fuzzy soft sets (*fprf*-soft set) and present an algorithm in decision-making problems based on the *fprf*-soft set in the next section.

3. **Fuzzy Parameterized Relative Fuzzy Soft Sets.** In this section, we define the fuzzy parameterized relative fuzzy soft set (*fprf*-soft set) as a hybrid model of the fuzzy parameterized fuzzy soft sets and the relative soft sets.

Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i\in I} U_i = \emptyset$ and let $\{E_i : i \in I\}$ be a collection of set of parameters. Let $\{X_i : i \in I\}$ be a collection of a set of fuzzy sets over E_{U_i}. $Fuz(U_i)$ denotes a set of all fuzzy sets over U_i, $E = E_{U_i}$, $X = X_{U_i}$ and $A \subseteq E$.

Definition 3.1. *A fuzzy parameterized relative fuzzy soft set (fprf-soft set) Γ_X over U is defined by the set of ordered pair*

$$\Gamma_X = \{(\mu_X(x)/x, \gamma_X(x)) : x \in E,\ \gamma_X(x) \in Fuz(U_i),\ \mu_X(x) \in [0,1]\},$$

where the function $\gamma_X : E \to Fuz(U_i)$ is called a fuzzy approximate function of the fprf-soft set Γ_X, and the value $\gamma_X(x)$ is a fuzzy set called x-element of the fprf-soft set Γ_X for all $x \in E$. If $A \subseteq E$, then we denote by Γ_X^A a fprf-soft set respect to A. The sets of all fprf-soft sets over U will be denoted by $FPRF(U)$.

Example 3.1. *Suppose that there are two universes U_1 and U_2 and let us consider a fprf-soft set Γ_X^A which describes the attractiveness of "Diamond necklace" and "Diamond ring" that Miss K. is going to buy. Let $U_1 = \{N_1, N_2, N_3, N_4\}$ be the set of Diamond necklace and $U_2 = \{R_1, R_2, R_3, R_4, R_5, R_6\}$ be the set of Diamond ring. Let $E_{U_i} = \{E_{U_1}, E_{U_2}\}$ be the collection of decision parameters, where*

$$E_{U_1} = \{e_{11}\{\textit{carat light weight}\}, e_{12}\{\textit{beautiful}\}, e_{13}\{\textit{expentive}\}\},$$
$$E_{U_2} = \{e_{21}\{\textit{made in Belgium}\}, e_{22}\{\textit{carat light weight}\}, e_{23}\{\textit{modern}\}\}.$$

Let $E = E_{U_i}$, $i = 1, 2$ and $A \subseteq E$. Let $X = \{X_{U_1}, X_{U_2}\}$ be a collection of the set of fuzzy set E, where $X_{U_1} = \{0.5/e_{11}, 0.8/e_{12}, 0.7/e_{13}\}$ and $X_{U_2} = \{0.3/e_{21}, 0.5/e_{22}, 0.7/e_{23}\}$.

Let $A = \{a_1 = (0.5/e_{11}, 0.7/e_{23}), a_2 = (0.8/e_{12}, 0.3/e_{21}), a_3 = (0.7/e_{13}, 0.5/e_{22}), a_4 = (0.8/e_{12}, 0.7/e_{23})\}$. *Suppose that*

$$\begin{aligned} F(a_1) &= (\{0.6/N_1, 0.3/N_2, 0.5/N_3, 0.8/N_4\},\\ &\quad \{0.3/R_1, 0.2/R_2, 0.3/R_3, 0.2/R_4, 0.7/R_5, 0.1/R_6\}),\\ F(a_2) &= (\{0.4/N_1, 0.3/N_2, 0.5/N_3, 0.6/N_4\},\\ &\quad \{0.8/R_1, 0.3/R_2, 0.4/R_3, 0.6/R_4, 0.5/R_5, 0.8/R_6\}),\\ F(a_3) &= (\{0.8/N_1, 0.1/N_2, 0.5/N_3, 0.4/N_4\},\\ &\quad \{0.5/R_1, 0.7/R_2, 0.6/R_3, 0.5/R_4, 0.3/R_5, 0.7/R_6\}),\\ F(a_4) &= (\{0.9/N_1, 0.3/N_2, 0.5/N_3, 0.6/N_4\},\\ &\quad \{0.3/R_1, 0.3/R_2, 0.6/R_3, 0.2/R_4, 0.7/R_5, 0.9/R_6\}). \end{aligned}$$

Then the Γ_X^A *is written by*

$$\begin{aligned} \Gamma_X^A = \{&((0.5, 0.7)/a_1, (\{0.6/N_1, 0.3/N_2, 0.5/N_3, 0.8/N_4\},\\ &\{0.3/R_1, 0.2/R_2, 0.3/R_3, 0.2/R_4, 0.7/R_5, 0.1/R_6\})),\\ &((0.8, 0.3)/a_2, (\{0.4/N_1, 0.3/N_2, 0.5/N_3, 0.6/N_4\},\\ &\{0.8/R_1, 0.3/R_2, 0.4/R_3, 0.6/R_4, 0.5/R_5, 0.8/R_6\})),\\ &((0.7, 0.5)/a_3, (\{0.8/N_1, 0.1/N_2, 0.5/N_3, 0.4/N_4\},\\ &\{0.5/R_1, 0.7/R_2, 0.6/R_3, 0.5/R_4, 0.3/R_5, 0.7/R_6\})),\\ &((0.8, 0.7)/a_4, (\{0.9/N_1, 0.3/N_2, 0.5/N_3, 0.6/N_4\},\\ &\{0.3/R_1, 0.3/R_2, 0.6/R_3, 0.2/R_4, 0.7/R_5, 0.9/R_6\}))\}. \end{aligned}$$

Definition 3.2. *Let* $\Gamma_X^A, \Gamma_X^B \in FPRF(U)$. *Then,* Γ_X^A *is a fprf-subset of* Γ_X^B*, denoted by* $\Gamma_X^A \tilde{\subseteq} \Gamma_X^B$*, if* $\mu_X^A(x) \le \mu_X^B(x)$ *and* $\gamma_X^A(x) \subseteq \gamma_X^B(x)$ *for all* $x \in A$.

Definition 3.3. *Let* $\Gamma_X^A, \Gamma_X^B \in FPRF(U)$. *Then* Γ_X^A, Γ_X^B *are fuzzy parameterized relative fuzzy soft sets equally written as* $\Gamma_X^A = \Gamma_X^B$ *if* $A = B$ *and* $\mu_X^A(x) = \mu_X^B(x)$ *and* $\gamma_X^A(x) = \gamma_X^B(x)$ *for all* $x \in A$.

Definition 3.4. *Let* $\Gamma_X^A \in FPRF(U)$. *Then, the complement of* Γ_X^A*, denoted by* $\Gamma_X^{A^{\tilde{c}}}$*, is defined by*

$$\mu_{X^{A^c}}(x) = 1 - \mu_X^A(x) \text{ and } \gamma_{X^{A^{\tilde{c}}}} = \gamma_X^{A^c}(x)$$

for all $x \in A$*, where* $\gamma_X^{A^c}(x)$ *is the complement of the set* $\gamma_X^A(x)$*, that is,* $\gamma_X^{A^c}(x) = U \setminus \gamma_X^A(x)$ *for every* $x \in A$.

Example 3.2. *From Example 3.1, we then have*

$$\begin{aligned} \Gamma_X^{A^{\tilde{c}}} = \{&((0.5, 0.3)/a_1, (\{0.4/N_1, 0.7/N_2, 0.5/N_3, 0.2/N_4\},\\ &\{0.7/R_1, 0.8/R_2, 0.7/R_3, 0.8/R_4, 0.3/R_5, 0.9/R_6\})),\\ &((0.2, 0.7)/a_2, (\{0.6/N_1, 0.7/N_2, 0.5/N_3, 0.4/N_4\},\\ &\{0.2/R_1, 0.7/R_2, 0.6/R_3, 0.4/R_4, 0.5/R_5, 0.2/R_6\})),\\ &((0.3, 0.5)/a_3, (\{0.2/N_1, 0.9/N_2, 0.5/N_3, 0.6/N_4\},\\ &\{0.5/R_1, 0.3/R_2, 0.4/R_3, 0.5/R_4, 0.7/R_5, 0.3/R_6\})),\\ &((0.2, 0.3)/a_4, (\{0.1/N_1, 0.7/N_2, 0.5/N_3, 0.4/N_4\},\\ &\{0.7/R_1, 0.7/R_2, 0.4/R_3, 0.8/R_4, 0.3/R_5, 0.1/R_6\}))\}. \end{aligned}$$

Proposition 3.1. *Let* Γ_X^A, Γ_X^B *and* Γ_X^C *be three fprf-soft sets over* U. *Then*

i) $\Gamma_X^A \subseteq \Gamma_X^A$,

ii) If $\Gamma_X^A \subseteq \Gamma_X^B$ *and* $\Gamma_X^B \subseteq \Gamma_X^C$*, then* $\Gamma_X^A \subseteq \Gamma_X^C$*,*
iii) If $\Gamma_X^A = \Gamma_X^B$ *and* $\Gamma_X^B = \Gamma_X^C$*, then* $\Gamma_X^A = \Gamma_X^C$*,*
iv) If $\Gamma_X^A \subseteq \Gamma_X^B$ *and* $\Gamma_X^B \subseteq \Gamma_X^A$*, then* $\Gamma_X^A = \Gamma_X^B$*,*
v) $\left(\Gamma_X^{A^{\tilde{c}}}\right)^{\tilde{c}} = \Gamma_X^A$.

Definition 3.5. *Let* $\Gamma_X^A, \Gamma_X^B \in FPRF(U)$*. Then an intersection of* Γ_X^A *and* Γ_X^B*, denoted by* $\Gamma_X^A \,\widetilde{\cap}\, \Gamma_X^B$*, is defined by* $\mu_X^{A\widetilde{\cap}B}(x) = \min\left\{\mu_X^A(x), \mu_X^B(x)\right\}$ *and* $\gamma_X^{A\widetilde{\cap}B}(x) = \gamma_X^A(x) \cap \gamma_X^B(x)$ *for all* $x \in E$*.*

Example 3.3. *From Example 3.1, let* $A = \{a_1 = (0.5/e_{11}, 0.7/e_{23}), a_2 = (0.8/e_{12}, 0.3/e_{21}), a_3 = (0.7/e_{13}, 0.5/e_{22}), a_4 = (0.8/e_{12}, 0.7/e_{23})\}$*. Suppose that*

$$\begin{aligned}\Gamma_X^A = \{&((0.5, 0.7)/a_1, (\{0.6/N_1, 0.3/N_2, 0.5/N_3, 0.8/N_4\},\\ &\{0.3/R_1, 0.2/R_2, 0.3/R_3, 0.2/R_4, 0.7/R_5, 0.1/R_6\})),\\ &((0.8, 0.3)/a_2, (\{0.4/N_1, 0.3/N_2, 0.5/N_3, 0.6/N_4\},\\ &\{0.8/R_1, 0.3/R_2, 0.4/R_3, 0.6/R_4, 0.5/R_5, 0.8/R_6\})),\\ &((0.7, 0.5)/a_3, (\{0.8/N_1, 0.1/N_2, 0.5/N_3, 0.4/N_4\},\\ &\{0.5/R_1, 0.7/R_2, 0.6/R_3, 0.5/R_4, 0.3/R_5, 0.7/R_6\})),\\ &((0.8, 0.7)/a_4, (\{0.9/N_1, 0.3/N_2, 0.5/N_3, 0.6/N_4\},\\ &\{0.3/R_1, 0.3/R_2, 0.6/R_3, 0.2/R_4, 0.7/R_5, 0.9/R_6\}))\}.\end{aligned}$$

Let $B = \{b_1 = (0.6/e_{11}, 0.8/e_{23}), b_2 = (0.9/e_{12}, 0.4/e_{21}), b_3 = (0.6/e_{13}, 0.4/e_{22}), b_4 = (0.7/e_{12}, 0.8/e_{23})\}$*. Suppose that*

$$\begin{aligned}\Gamma_X^B = \{&((0.6, 0.8)/b_1, (\{0.5/N_1, 0.6/N_2, 0.4/N_3, 0.8/N_4\},\\ &\{0.1/R_1, 0.4/R_2, 0.6/R_3, 0.3/R_4, 0.8/R_5, 0.9/R_6\})),\\ &((0.9, 0.4)/b_2, (\{0.3/N_1, 0.2/N_2, 0.4/N_3, 0.8/N_4\},\\ &\{0.8/R_1, 0.4/R_2, 0.5/R_3, 0.7/R_4, 0.4/R_5, 0.8/R_6\})),\\ &((0.6, 0.4)/b_3, (\{0.5/N_1, 0.3/N_2, 0.9/N_3, 0.4/N_4\},\\ &\{0.6/R_1, 0.3/R_2, 0.7/R_3, 0.1/R_4, 0.7/R_5, 0.5/R_6\})),\\ &((0.7, 0.8)/b_4, (\{0.8/N_1, 0.3/N_2, 0.4/N_3, 0.2/N_4\},\\ &\{0.6/R_1, 0.3/R_2, 0.5/R_3, 0.7/R_4, 0.8/R_5, 0.6/R_6\}))\}.\end{aligned}$$

Thus,

$$\begin{aligned}\Gamma_X^A \,\widetilde{\cap}\, \Gamma_X^B = \Gamma_X^C = \{&((0.5, 0.7)/c_1, (\{0.5/N_1, 0.3/N_2, 0.4/N_3, 0.8/N_4\},\\ &\{0.1/R_1, 0.2/R_2, 0.3/R_3, 0.2/R_4, 0.7/R_5, 0.1/R_6\})),\\ &((0.8, 0.3)/c_2, (\{0.3/N_1, 0.2/N_2, 0.4/N_3, 0.6/N_4\},\\ &\{0.8/R_1, 0.3/R_2, 0.4/R_3, 0.6/R_4, 0.4/R_5, 0.8/R_6\})),\\ &((0.6, 0.4)/c_3, (\{0.5/N_1, 0.1/N_2, 0.5/N_3, 0.4/N_4\},\\ &\{0.5/R_1, 0.3/R_2, 0.6/R_3, 0.1/R_4, 0.3/R_5, 0.5/R_6\})),\\ &((0.7, 0.7)/c_4, (\{0.8/N_1, 0.3/N_2, 0.4/N_3, 0.2/N_4\},\\ &\{0.3/R_1, 0.3/R_2, 0.5/R_3, 0.2/R_4, 0.7/R_5, 0.6/R_6\}))\}.\end{aligned}$$

Definition 3.6. *Let* $\Gamma_X^A, \Gamma_X^B \in FPRF(U)$*. Then a union of* Γ_X^A *and* Γ_X^B*, denoted by* $\Gamma_X^A \,\widetilde{\cup}\, \Gamma_X^B$*, is defined by* $\mu_X^{A\widetilde{\cup}B}(x) = \max\left\{\mu_X^A(x), \mu_X^B(x)\right\}$ *and* $\gamma_X^{A\widetilde{\cup}B}(x) = \gamma_X^A(x) \cup \gamma_X^B(x)$ *for all* $x \in E$*.*

Example 3.4. *From Example 3.3, then*

$$\begin{aligned}\Gamma_X^A \,\widetilde{\cup}\, \Gamma_X^B = \Gamma_X^C = \{&((0.6, 0.8)/c_1, (\{0.6/N_1, 0.6/N_2, 0.5/N_3, 0.8/N_4\},\\ &\{0.3/R_1, 0.4/R_2, 0.6/R_3, 0.3/R_4, 0.8/R_5, 0.9/R_6\})),\\ &((0.9, 0.4)/c_2, (\{0.4/N_1, 0.2/N_2, 0.5/N_3, 0.8/N_4\},\\ &\{0.8/R_1, 0.4/R_2, 0.5/R_3, 0.7/R_4, 0.5/R_5, 0.8/R_6\})),\\ &((0.7, 0.5)/c_3, (\{0.8/N_1, 0.3/N_2, 0.9/N_3, 0.4/N_4\},\\ &\{0.6/R_1, 0.7/R_2, 0.7/R_3, 0.5/R_4, 0.7/R_5, 0.7/R_6\})),\\ &((0.8, 0.8)/c_4, (\{0.9/N_1, 0.3/N_2, 0.5/N_3, 0.6/N_4\},\\ &\{0.6/R_1, 0.3/R_2, 0.6/R_3, 0.7/R_4, 0.8/R_5, 0.9/R_6\}))\}.\end{aligned}$$

Proposition 3.2. *Let Γ_X^A, Γ_X^B and Γ_X^C be three fprf-soft sets over U. If Γ_X^A, Γ_X^B and Γ_X^C are conformable for the union and intersection, then*

i) $\Gamma_X^A \,\widetilde{\cup}\, \Gamma_X^A = \Gamma_X^A$,
ii) $\Gamma_X^A \,\widetilde{\cup}\, \Gamma_X^B = \Gamma_X^B \,\widetilde{\cup}\, \Gamma_X^A$,
iii) $\Gamma_X^A \,\widetilde{\cup}\, \left(\Gamma_X^B \,\widetilde{\cup}\, \Gamma_X^C\right) = \left(\Gamma_X^A \,\widetilde{\cup}\, \Gamma_X^B\right) \widetilde{\cup}\, \Gamma_X^C$,
iv) $\Gamma_X^A \,\widetilde{\cap}\, \Gamma_X^A = \Gamma_X^A$,
v) $\Gamma_X^A \,\widetilde{\cap}\, \Gamma_X^B = \Gamma_X^B \,\widetilde{\cap}\, \Gamma_X^A$,
vi) $\Gamma_X^A \,\widetilde{\cap}\, \left(\Gamma_X^B \,\widetilde{\cap}\, \Gamma_X^C\right) = (\Gamma_X^A \,\widetilde{\cap}\, \Gamma_X^B) \,\widetilde{\cap}\, \Gamma_X^C$,
vii) $\Gamma_X^A \,\widetilde{\cup}\, \left(\Gamma_X^B \,\widetilde{\cap}\, \Gamma_X^C\right) = \left(\Gamma_X^A \,\widetilde{\cup}\, \Gamma_X^B\right) \widetilde{\cap} \left(\Gamma_X^A \,\widetilde{\cup}\, \Gamma_X^C\right)$,
viii) $\Gamma_X^A \,\widetilde{\cap}\, \left(\Gamma_X^B \,\widetilde{\cup}\, \Gamma_X^C\right) = \left(\Gamma_X^A \,\widetilde{\cap}\, \Gamma_X^B\right) \widetilde{\cup} \left(\Gamma_X^A \,\widetilde{\cap}\, \Gamma_X^C\right)$.

4. **Algorithm.** In this section, we define a multiply weight value of Γ_X^A and construct a model for solving a decision-making problem based on *fprf*-soft sets.

Definition 4.1. *Let $\Gamma_X^A \in FPRF(U)$. Then the multiply weight value, denoted by $\Gamma_X^{A^*}$, is defined by $\Gamma_X^{A^*} = \left\{\mu_{\Gamma_X^{A^*}}(u)/u : u \in U\right\}$, which is a fuzzy set over U. Here, the membership degree $\mu_{\Gamma_X^{A^*}}(u)$ of u is defined as follows:*

$$\mu_{\Gamma_X^{A^*}}(u) = \mu_X^A(a)\mu_{\gamma_X^A(a)}(u) \text{ for all } a \in A.$$

We shall adapt the algorithm of Roy and Maji's method [10] to a new algorithm based on *fprf*-soft set as follows:

Algorithm

Step 1. Construction of the *fprf*-soft sets Γ_X^A and Γ_X^B over U_i.

Step 2. Computation of the union or intersection Γ_X^C of Γ_X^A and Γ_X^B over U_i.

Step 3. Find the multiply weight value $\Gamma_X^{C^*}$ and computation of the choice value.

Step 4. Construction of the comparison table.

Step 5. Computation of the row sum (r_i) and the column sum (c_i) and the score value $(S_i = r_i - c_i)$ of o_i for all i of U_i.

Step 6. The decision is S_k if $S_k = \max_i S_i$ of U_i.

Step 7. Computation of the consistency test (CI) and the consistency ratio (CR). The consistency test $CI = \frac{S_i - m}{m-1}$, where m is the number of the universes and the consistency ratio $CR = \frac{CI}{RI}$, where RI is the random indices corresponding to the number of the universes.

The consistency ratio (CR) is acceptable if it does not exceed 0.10 [22].

Step 8. We choose only o_k if k has more than one value corresponding to the choice value.

Table 8. The tabular form of the random indices RI

m	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

5. Numerical Example.

In this section, we present an example using the new algorithm in a decision-making problem.

The following example uses the new algorithm in a decision-making problem.

Example 5.1. *Suppose that there are there universes U_1, U_2 and U_3. Let us consider the fprs-soft set Γ_X^A which describes the condition of some states in a country. Mr. X and Mr. Y with enough capital are considering for the location of his shopping mall.*

Let $U_1 = \{S_1, S_2, S_3\}$ be a set of states with availability of land, $U_2 = \{S_4, S_5, S_6\}$ be a set of states with availability of labour, and $U_3 = \{S_7, S_8, S_9\}$ be a set of states with availability of safety. Let $E_{U_i} = \{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of the set of parameters related to the above universes, where

$$E_{U_1} = \{e_{11}\{\textit{densely populated state}\}, e_{12}\{\textit{near the sky train station}\}, e_{13}\{\textit{on the main road}\}, e_{14}\{\textit{with an area of more than 4 acres}\}\}.$$

$$E_{U_2} = \{e_{21}\{\textit{speak English}\}, e_{22}\{\textit{love service}\}, e_{23}\{\textit{highly education}\}, e_{24}\{\textit{criminal record}\}\}.$$

$$E_{U_3} = \{e_{31}\{\textit{security alarm}\}, e_{32}\{\textit{CCTV}\}, e_{33}\{\textit{security guard}\}, e_{34}\{\textit{with a walk through metal detector}\}\}.$$

Let $E = E_{U_i}$, $i = 1, 2, 3$ and $A \subseteq E$. Let $X = \{X_{U_1}, X_{U_2}, X_{U_3}\}$ be a collection of the set of fuzzy set over E, where $X_{U_1} = \{0.6/e_{11}, 0.5/e_{12}, 0.3/e_{13}, 0.8/e_{14}\}$, $X_{U_2} = \{0.3/e_{21}, 0.7/e_{22}, 0.2/e_{23}, 0.1/e_{24}\}$ and $X_{U_3} = \{0.5/e_{31}, 0.3/e_{32}, 0.7/e_{33}, 0.8/e_{44}\}$.

Step 1. *Let $A = \{a_1 = (0.6/e_{11}, 0.3/e_{21}, 0.5/e_{31}), a_2 = (0.5/e_{12}, 0.7/e_{22}, 0.3/e_{32}), a_3 = (0.3/e_{13}, 0.1/e_{24}, 0.7/e_{33}), a_4 = (0.8/e_{14}, 0.2/e_{23}, 0.8/e_{34}), a_5 = (0.8/e_{14}, 0.7/e_{22}, 0.8/e_{34})\}$.*

Suppose that Γ_X^A is a fprf-soft set which Mr. X is considering for the location of his shopping mall as follows:

$$\begin{aligned}\Gamma_X^A = \{&((0.6, 0.3, 0.5)/a_1, (\{0.6/S_1, 0.3/S_2, 0.1/S_3\},\\ &\{0.2/S_4, 0.5/S_5, 0.4/S_6\}, \{0.8/S_7, 0.3/S_8, 0.5/S_9\})),\\ &((0.5, 0.7, 0.3)/a_2, (\{0.3/S_1, 0.5/S_2, 0.7/S_3\},\\ &\{0.3/S_4, 0.2/S_5, 0.6/S_5\}, \{0.3/S_7, 0.7/S_8, 0.8/S_9\})),\\ &((0.3, 0.1, 0.7)/a_3, (\{0.1/S_1, 0.6/S_2, 0.7/S_3\},\\ &\{0.2/S_4, 0.3/S_5, 0.5/S_6\}, \{0.4/S_7, 0.6/S_8, 0.6/S_9\})),\\ &((0.8, 0.2, 0.8)/a_4, (\{0.4/S_1, 0.6/S_2, 0.8/S_3\},\\ &\{0.7/S_4, 0.5/S_5, 0.6/S_6\}, \{0.3/S_7, 0.5/S_8, 0.4/S_9\})),\\ &((0.8, 0.7, 0.8)/a_5, (\{0.6/S_1, 0.3/S_2, 0.8/S_3\},\\ &\{0.3/S_4, 0.2/S_5, 0.6/S_6\}, \{0.3/S_7, 0.5/S_8, 0.4/S_9\}))\}.\end{aligned}$$

Let $B = \{b_1 = (0.7/e_{11}, 0.4/e_{21}, 0.6/e_{31}), b_2 = (0.5/e_{12}, 0.8/e_{22}, 0.4/e_{32}), b_3 = (0.4/e_{13}, 0.1/e_{24}, 0.7/e_{33}), b_4 = (0.8/e_{14}, 0.1/e_{23}, 0.7/e_{34}), b_5 = (0.9/e_{14}, 0.8/e_{22}, 0.8/e_{34})\}$.

Suppose that Γ_X^B is a fprf-soft set which Mr. Y is considering for the location of his shopping mall as follows:

$$\begin{aligned}\Gamma_X^B = \{&((0.7, 0.4, 0.6)/b_1, (\{0.7/S_1, 0.4/S_2, 0.2/S_3\},\\ &\{0.3/S_4, 0.5/S_5, 0.4/S_6\}, \{0.9/S_7, 0.4/S_8, 0.5/S_9\})),\\ &((0.5, 0.8, 0.4)/b_2, (\{0.3/S_1, 0.6/S_2, 0.7/S_3\},\end{aligned}$$

$\{0.4/S_4, 0.2/S_5, 0.7/S_5\}, \{0.4/S_7, 0.7/S_8, 0.8/S_9\})),$
$((0.4, 0.1, 0.7)/b_3, (\{0.2/S_1, 0.6/S_2, 0.7/S_3\},$
$\{0.3/S_4, 0.3/S_5, 0.6/S_6\}, \{0.5/S_7, 0.6/S_8, 0.7/S_9\})),$
$((0.8, 0.1, 0.7)/b_4, (\{0.5/S_1, 0.6/S_2, 0.8/S_3\},$
$\{0.7/S_4, 0.6/S_5, 0.6/S_6\}, \{0.4/S_7, 0.5/S_8, 0.4/S_9\})),$
$((0.9, 0.8, 0.8)/b_5, (\{0.6/S_1, 0.4/S_2, 0.8/S_3\},$
$\{0.4/S_4, 0.2/S_5, 0.6/S_6\}, \{0.4/S_7, 0.5/S_8, 0.4/S_9\}))\}.$

Step 2. *We compute the intersection Γ_X^C of Γ_X^A and Γ_X^B. Thus*

$$\begin{aligned}\Gamma_X^A \widetilde{\cap} \Gamma_X^B = \Gamma_X^C = \{&((0.6, 0.3, 0.5)/a_1, (\{0.6/S_1, 0.3/S_2, 0.1/S_3\},\\ &\{0.2/S_4, 0.5/S_5, 0.4/S_6\}, \{0.8/S_7, 0.3/S_8, 0.5/S_9\})),\\ &((0.5, 0.7, 0.3)/a_2, (\{0.3/S_1, 0.5/S_2, 0.7/S_3\},\\ &\{0.3/S_4, 0.2/S_5, 0.6/S_5\}, \{0.3/S_7, 0.7/S_8, 0.8/S_9\})),\\ &((0.3, 0.1, 0.7)/a_3, (\{0.1/S_1, 0.6/S_2, 0.7/S_3\},\\ &\{0.2/S_4, 0.3/S_5, 0.5/S_6\}, \{0.4/S_7, 0.6/S_8, 0.6/S_9\})),\\ &((0.8, 0.2, 0.8)/a_4, (\{0.4/S_1, 0.6/S_2, 0.8/S_3\},\\ &\{0.7/S_4, 0.5/S_5, 0.6/S_6\}, \{0.3/S_7, 0.5/S_8, 0.4/S_9\})),\\ &((0.8, 0.7, 0.8)/a_5, (\{0.6/S_1, 0.3/S_2, 0.8/S_3\},\\ &\{0.3/S_4, 0.2/S_5, 0.6/S_6\}, \{0.3/S_7, 0.5/S_8, 0.4/S_9\}))\}.\end{aligned}$$

We can write the fprf-soft set Γ_X^C in Table 9 as follows.

Table 9. The tabular of the *fprf*-soft set Γ_X^C

U	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9
$(0.6, 0.3, 0.5)/a_1$	0.6	0.3	0.1	0.2	0.5	0.4	0.8	0.3	0.5
$(0.5, 0.7, 0.3)/a_2$	0.3	0.5	0.7	0.3	0.2	0.6	0.3	0.7	0.8
$(0.3, 0.1, 0.7)/a_3$	0.1	0.6	0.7	0.2	0.3	0.5	0.4	0.6	0.6
$(0.8, 0.2, 0.8)/a_4$	0.4	0.6	0.8	0.7	0.5	0.6	0.3	0.5	0.4
$(0.8, 0.7, 0.8)/a_5$	0.6	0.3	0.8	0.3	0.2	0.6	0.3	0.5	0.4

Step 3. *The multiply weight $\Gamma_X^{C^*}$ can be found as seen in Table 10.*

Table 10. The tabular of the multiply weight $\Gamma_X^{C^*}$

U	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9
a_1	0.36	0.18	0.06	0.06	0.15	0.12	0.40	0.15	0.25
a_2	0.15	0.25	0.35	0.21	0.14	0.42	0.09	0.21	0.24
a_3	0.03	0.18	0.21	0.02	0.03	0.05	0.28	0.42	0.42
a_4	0.32	0.24	0.64	0.14	0.10	0.12	0.24	0.40	0.32
a_5	0.48	0.24	0.64	0.21	0.14	0.42	0.24	0.40	0.32
Choice value	1.34	1.09	1.90	0.53	0.56	1.13	1.25	1.58	1.55

Step 4. *We construct the comparison table (see Table 11).*

Step 5. *We compute the row sum (r_i) and the column sum (c_i) and the score value ($S_i = r_i - c_i$) of o_i for all i (see Table 12).*

Table 11. The tabular form of the comparison table

	S_1	S_2	S_3		S_4	S_5	S_6		S_7	S_8	S_9
S_1	5	3	1	S_4	5	3	1	S_7	5	1	1
S_2	2	5	1	S_5	2	5	1	S_8	4	5	3
S_3	4	4	5	S_6	4	4	5	S_9	4	3	5

Table 12. The tabular form of row sum, column sum and score value

U	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9
Row sum (r_i)	9	8	13	9	8	13	7	12	12
Column sum (c_i)	11	12	7	11	12	7	13	9	9
Score value ($S_i = r_i - c_i$)	-2	-4	6	-2	-4	6	-6	3	3

Step 6. *From the above Table 12, it is clear that the maximum score is 6, which was scored by S_3 of U_1, the maximum score is 6, which was scored by S_6 of U_2 and the maximum score is 3, which was scored by S_8 and S_9 of U_3.*

Step 7. *We compute the consistency test (CI) and the consistency ratio (CR) (see Table 13).*

Table 13. The tabular form of the consistency test (CI) and the consistency ratio (CR) of U

U	CI	CR
U_1	-0.375	-0.26
U_2	-0.375	-0.26
U_3	-0.75	-0.52

From the above Table 13, it is clear that the consistency ratio (CR) is acceptable because it does not exceed 0.10.

Step 8. *Hence by the decision Mr. X and Mr. Y select S_3 for the states with availability of land, and select S_6 for the states with availability of labour, and select S_8 for the states with availability of safety corresponding to the choice value ($S_8 > S_9$).*

Remark 5.1. *The advantage of the new algorithm is designed for multiple universes which is generalization of Roy and Maji's method. For the universe $\{U_i : i \in I\}$, if $i = 1$, then we have Roy and Maji's method. Moreover, the advantage of the algorithm is presenting the consistency ratio (CR) for checking the correctness of the algorithm.*

6. Conclusions.

In this paper, we presented the concept of fuzzy parameterized relative fuzzy soft sets (*fprf*-soft set) and their properties. The new algorithm for multiple universes in decision-making problems based on fuzzy parameterized relative fuzzy soft sets (*fprf*-soft set) is presented. Finally, a practical example was presented in detail which shows that the method in this paper can be used successfully for solving decision-making problem.

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