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MULTIPLE ATTRIBUTE DECISION-MAKING BASED ON SINE TRIGONOMETRIC FERMATEAN NORMAL FUZZY AGGREGATION OPERATOR

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Abstract. *In this paper, we introduce the concept of multiple attribute decision-making (MADM) problems based on the sine trigonometric Fermatean normal fuzzy set (ST-FNFS). The sine trigonometric Fermatean normal fuzzy sets are new generalizations of Fermatean fuzzy sets. The present article has discussed a concept of some trigonometric functions such as sine trigonometric Fermatean normal fuzzy weighted averaging (ST-FNFWA), sine trigonometric Fermatean normal fuzzy weighted geometric (ST-FNFWG), sine trigonometric generalized Fermatean normal fuzzy weighted averaging (ST-GFNFWA), and sine trigonometric generalized Fermatean normal fuzzy weighted geometric (ST-GFNFWG). We obtained an algorithm that deals with the MADM problems based on these operators. We discuss the applicability of the Hamming distance, which is further extended in real-life illustrative problems. Also, some intersecting properties of these sets under the different algebraic operations are to be elaborated in this communication.*

Keywords: Sine trigonometric Fermatean normal fuzzy weighted averaging, Sine trigonometric Fermatean normal fuzzy weighted geometric, Sine trigonometric generalized Fermatean normal fuzzy weighted averaging, Sine trigonometric generalized Fermatean normal fuzzy weighted geometric

1. Introduction.

Uncertainty can be seen everywhere in most real problems. Many uncertain theories are proposed, including fuzzy set [1], intuitionistic fuzzy set [2], and Pythagorean fuzzy set [3, 4], and Fermatean fuzzy set [5] are put forward. A fuzzy set is a set with a grade of belongingness that comes between zero and one, and such grades are

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called the membership value of an element in that given set. Later, the notion of an intuitionistic fuzzy set logic is launched by Atanassov and by the condition that the sum of its membership grade and non-membership grade does not exceed 1 [2]. In some cases, we may face one problem in the decision-making (DM) approach, the sum of the membership grade and non-membership grade exceeds 1. Therefore, Yager [3] introduced the logic for Pythagorean fuzzy set logic, which is a new generalization of IFS and characterized by the condition that the square sum of its membership grade and non-membership grade does not exceed 1. However, we may interact with a problem in DM events, where the square sum of the degree of membership and non-membership of a particular attribute exceeds unity. In 2020, Senapati and Yager proposed the notion of a Fermatean fuzzy set [5] by the condition that the cubic sum of its degrees of membership and non-membership does not exceed unity. Yang and Chang proposed the notion of interval-valued Pythagorean normal fuzzy information aggregation operators for MADM [6]. Zhang and Xu proposed the extension of Pythagorean fuzzy sets based on the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) to MCDM [7]. Hwang and Yoon discussed various real-life applications of MADM [8]. Jana and Pal were studied by the concept of bipolar intuitionistic fuzzy soft sets with applications [9]. Jana and Pal introduced the area for robust single-valued neutrosophic soft aggregation operators based on MCDM [10] with bipolar fuzzy soft in 2019 [11]. Jana et al. introduced the concept of Pythagorean fuzzy Dombi aggregation operators [12]. In 2020, Jana et al. studied trapezoidal neutrosophic aggregation operators and their application to the MADM process [13]. In 2021, Jana and Pal interacted with the application for the MCDM process based on single-valued neutrosophic Dombi power aggregation operators [14]. In recent work, Jana et al. studied introduced the concept of an MCDM approach based on SVTrN Dombi aggregation functions [15]. TOPSIS extends to interval-valued intuitionistic fuzzy soft set (IFSS) which was discussed by Zulqarnain et al. in 2021. They also discussed a new type of correlation coefficient under IFSS's [16]. Samatha et al. discussed the notion of clustering Indonesian patients with personality disorders using fuzzy C-means [17].

The purpose of this paper is to choose a problem as the selection of a college for undergoing teaching education. The evaluation of teacher education is carried out according to various standards of experts. In this research paper, we extend the concept of the Fermatean fuzzy set. We obtained Fermatean fuzzy set information with aggregation operators. The paper is organized into seven sections as follows. Section 1 is called an introduction. In Section 2 brief description of the Fermatean fuzzy set is given. Section 3 discusses MADM based on Fermatean normal fuzzy number (FNFN) and its operations. Section 4 deals with Hamming distance approach for FNFN. Section 5 discusses aggregation operators for FNFN. Section 6 deals with FNF information, an algorithm with an illustrative example and comparison for the proposed and existing. Finally, a conclusion is needed in Section 7. Throughout this article, we deal with the conditions that $0 \leq \left(\varsigma^{\mathcal{M}}(u)\right)^2 + \left(\varsigma^{\mathcal{NM}}(u)\right)^2 > 1$ but $0 \leq \left(\varsigma^{\mathcal{M}}(u)\right)^3 + \left(\varsigma^{\mathcal{NM}}(u)\right)^3 \leq 1$.

2. Background.

In this section, we review some basic definitions.

Definition 2.1. *A fuzzy set $\mathcal{L}$ in the universe $\mathcal{U}$ is of the form: $\mathcal{L} = \left\{u, \left\langle \varsigma_{\mathcal{L}}^{\mathcal{M}}(u), \varsigma_{\mathcal{L}}^{\mathcal{NM}}(u) \right\rangle \middle| u \in \mathcal{U}\right\}$, where $\varsigma_{\mathcal{L}}^{\mathcal{M}}(u)$ and $\varsigma_{\mathcal{L}}^{\mathcal{NM}}(u)$ are called as degree of membership and non membership of $\mathcal{L}$, respectively. The mapping $\varsigma_{\mathcal{L}}^{\mathcal{M}} : \mathcal{U} \to [0,1]$ and $\varsigma_{\mathcal{L}}^{\mathcal{NM}} : \mathcal{U} \to [0,1]$ and $0 \leq \left(\varsigma_{\mathcal{L}}^{\mathcal{M}}(u)\right)^3 + \left(\varsigma_{\mathcal{L}}^{\mathcal{NM}}(u)\right)^3 \leq 1$. Here $\mathcal{L} = \left\langle \varsigma_{\mathcal{L}}^{\mathcal{M}}, \varsigma_{\mathcal{L}}^{\mathcal{NM}} \right\rangle$ is called a Fermatean fuzzy number (FFN).*

Definition 2.2. *For any FFNs, $\mathcal{L} = \left\langle \varsigma^{\mathcal{M}}, \varsigma^{\mathcal{NM}} \right\rangle$, $\mathcal{L}_1 = \left\langle \varsigma_1^{\mathcal{M}}, \varsigma_1^{\mathcal{NM}} \right\rangle$ and $\mathcal{L}_2 = \left\langle \varsigma_2^{\mathcal{M}}, \varsigma_2^{\mathcal{NM}} \right\rangle$, $\varsigma^{\mathcal{M}}$, $\varsigma^{\mathcal{NM}}$ are called membership and non membership of $\mathcal{L}$, respectively and Ξ is a positive integer. Then the following statements are valid.*

1) $\mathcal{L}_1 \boxplus \mathcal{L}_2 = \left(\sqrt[3]{\left(\varsigma_1^{\mathcal{M}}\right)^3 + \left(\varsigma_2^{\mathcal{M}}\right)^3 - \left(\varsigma_1^{\mathcal{M}}\right)^3 \cdot \left(\varsigma_2^{\mathcal{M}}\right)^3}, \left(\varsigma_1^{\mathcal{NM}} \cdot \varsigma_2^{\mathcal{NM}}\right) \right)$

2) $\mathcal{L}_1 \boxtimes \mathcal{L}_2 = \left(\left(\varsigma_1^{\mathcal{M}} \cdot \varsigma_2^{\mathcal{M}}\right), \sqrt[3]{\left(\varsigma_1^{\mathcal{NM}}\right)^3 + \left(\varsigma_2^{\mathcal{NM}}\right)^3 - \left(\varsigma_1^{\mathcal{NM}}\right)^3 \cdot \left(\varsigma_2^{\mathcal{NM}}\right)^3} \right)$

3) $\Xi \cdot \mathcal{L} = \left(\sqrt[3]{1 - \left(1 - \left(\varsigma^{\mathcal{M}}\right)^3\right)^{\Xi}}, \left(\varsigma^{\mathcal{NM}}\right)^{\Xi} \right)$

4) $\mathcal{L}^{\Xi} = \left(\left(\varsigma^{\mathcal{M}}\right)^{\Xi}, \sqrt[3]{1 - \left(1 - \left(\varsigma^{\mathcal{NM}}\right)^3\right)^{\Xi}} \right)$

Definition 2.3. *Let $\mathcal{L} = \left\langle \varsigma^{\mathcal{M}}, \varsigma^{\mathcal{NM}} \right\rangle$ be FFN, its score function and accuracy function are defined as $S(\mathcal{L}) = \frac{1}{2}\left(\left(\varsigma^{\mathcal{M}}\right)^3 - \left(\varsigma^{\mathcal{NM}}\right)^3\right)$, $H(\mathcal{L}) = \frac{1}{2}\left(\left(\varsigma^{\mathcal{M}}\right)^3 + \left(\varsigma^{\mathcal{NM}}\right)^3\right)$, respectively, where $S(\mathcal{L}) \in [-1, 1]$ and $H(\mathcal{L}) \in [0, 1]$.*

Definition 2.4.

1) *Let R be a set of real number, the membership of fuzzy number $M(x) = e^{-\left(\frac{x-\tau}{\vartheta}\right)^2}$, $(\vartheta > 0)$ is called a normal fuzzy number (NFN) $M = (\tau, \vartheta)$, where $\mathbb{N}$ is an NFN set.*
2) *Let $M_1 = (\tau_1, \vartheta_1) \in \mathbb{N}$ and $M_2 = (\tau_2, \vartheta_2) \in \mathbb{N}$, and then the distance between M_1 and M_2 can be defined as $\mathcal{D}(M_1, M_2) = \sqrt{(\tau_1 - \tau_2)^2 + \frac{1}{2}(\vartheta_1 - \vartheta_2)^2}$.*

The sine trigonometric Fermatean normal fuzzy set model has become a useful and remarkable generalized version of Fermatean fuzzy set.

3. New Operations for ST-FNFN.

Definition 3.1. *Let $(\tau, \vartheta) \in \mathbb{N}$, $\mathcal{L} = \left\langle (\tau, \vartheta); \varsigma^{\mathcal{M}}, \varsigma^{\mathcal{NM}} \right\rangle$ be a Fermatean normal fuzzy number (FNFN), when its grades of membership, non-membership are defined as $\varsigma_{\mathcal{L}}^{\mathcal{M}} = \varsigma_{\mathcal{L}}^{\mathcal{M}} e^{-\left(\frac{x-\tau}{\vartheta}\right)^3}$ and $\varsigma_{\mathcal{L}}^{\mathcal{NM}} = 1 - \left(1 - \varsigma_{\mathcal{L}}^{\mathcal{NM}}\right) e^{-\left(\frac{x-\tau}{\vartheta}\right)^3}$, $x \in X$ respectively, where X is a non-empty set and $\varsigma_{\mathcal{L}}^{\mathcal{M}}, \varsigma_{\mathcal{L}}^{\mathcal{NM}} \in [0, 1]$ and $0 \le \left(\varsigma_{\mathcal{L}}^{\mathcal{M}}(x)\right)^3 + \left(\varsigma_{\mathcal{L}}^{\mathcal{NM}}(x)\right)^3 \le 1$.*

Definition 3.2. *Let $(\tau, \vartheta) \in \mathbb{N}$, $\mathcal{L} = \left\langle (\tau, \vartheta); \varsigma^{\mathcal{M}}, \varsigma^{\mathcal{NM}} \right\rangle$ be an FNFN. Then we define an ST-FNFN set as $\sin \mathcal{L} = \left\{ \sin\left(\pi/2 \cdot \left(\varsigma_{\mathcal{L}}^{\mathcal{M}}(u)\right)^3\right), 1 - \sin\left(\pi/2 \cdot \left(1 - \varsigma_{\mathcal{L}}^{\mathcal{NM}}(u)\right)^3\right) \right\}$. Clearly, $\sin \mathcal{L}$ is also FNFN, and also satisfied the following condition that the FNFN as, the membership, non membership grade of FNFN, respectively, $\sin\left(\pi/2 \cdot \varsigma_{\mathcal{L}}^{\mathcal{M}}(u)\right) : \mathcal{U} \to [0, 1]$ such that $0 \le \sin\left(\pi/2 \cdot \varsigma_{\mathcal{L}}^{\mathcal{M}}(u)\right) \le 1$ and $1 - \sin\left(\pi/2 \cdot \left(1 - \varsigma_{\mathcal{L}}^{\mathcal{NM}}(u)\right)\right) : \mathcal{U} \to [0, 1]$ such that $0 \le 1 - \sin\left(\pi/2 \cdot \left(1 - \varsigma_{\mathcal{L}}^{\mathcal{NM}}(u)\right)\right) \le 1$. Therefore, $\sin \mathcal{L} = \left\{ \sin\left(\pi/2 \cdot \left(\varsigma_{\mathcal{L}}^{\mathcal{M}}(u)\right)^3\right), 1 - \sin\left(\pi/2 \cdot \left(1 - \varsigma_{\mathcal{L}}^{\mathcal{NM}}(u)\right)^3\right) \right\}$ is an FNFN, where $\varsigma_{\mathcal{L}}^{\mathcal{M}} = \varsigma_{\mathcal{L}}^{\mathcal{M}} e^{-\left(\frac{x-\tau}{\vartheta}\right)^3}$ and $\varsigma_{\mathcal{L}}^{\mathcal{NM}} = \varsigma_{\mathcal{L}}^{\mathcal{NM}} e^{-\left(\frac{x-\tau}{\vartheta}\right)^3}$.*

Definition 3.3. *Let $(\tau, \vartheta) \in \mathbb{N}$, $\mathcal{L} = \left\langle (\tau, \vartheta); \varsigma^{\mathcal{M}}, \varsigma^{\mathcal{NM}} \right\rangle$ be an FNFN. Then $\sin \mathcal{L} = \left\{ \sin\left(\pi/2 \cdot \left(\varsigma_{\mathcal{L}}^{\mathcal{M}}\right)^3\right), 1 - \sin\left(\pi/2 \cdot \left(1 - \varsigma_{\mathcal{L}}^{\mathcal{NM}}\right)^3\right) \right\}$ is called a sine trigonometric FNFN, where $\varsigma_{\mathcal{L}}^{\mathcal{M}} = \varsigma_{\mathcal{L}}^{\mathcal{M}} e^{-\left(\frac{x-\tau}{\vartheta}\right)^3}$ and $\varsigma_{\mathcal{L}}^{\mathcal{NM}} = \varsigma_{\mathcal{L}}^{\mathcal{NM}} e^{-\left(\frac{x-\tau}{\vartheta}\right)^3}$.*

Definition 3.4. *Any three ST-FNFNs $\mathcal{L} = \left\langle (\tau, \vartheta); \varsigma^{\mathcal{M}}, \varsigma^{\mathcal{NM}} \right\rangle$, $\mathcal{L}_1 = \left\langle (\tau_1, \vartheta_1); \varsigma_1^{\mathcal{M}}, \varsigma_1^{\mathcal{NM}} \right\rangle$, $\mathcal{L}_2 = \left\langle (\tau_2, \vartheta_2); \varsigma_2^{\mathcal{M}}, \varsigma_2^{\mathcal{NM}} \right\rangle$ and Ξ is a positive integer. Then*

1) $\sin\mathcal{L}_1 \boxplus \sin\mathcal{L}_2$

$$= \left(\begin{array}{c} (\tau_1+\tau_2, \vartheta_1+\vartheta_2); \\ \sqrt[3\Xi]{\sin^2\left(\pi/2\cdot\left(\varsigma_1^{\mathcal{M}}\right)^{3\Xi}\right)+\sin^2\left(\pi/2\cdot\left(\varsigma_2^{\mathcal{M}}\right)^{3\Xi}\right)-\sin^2\left(\pi/2\cdot\left(\varsigma_1^{\mathcal{M}}\right)^{3\Xi}\right)\cdot\sin^2\left(\pi/2\cdot\left(\varsigma_2^{\mathcal{M}}\right)^{3\Xi}\right)}, \\ \sin^2\left(\pi/2\cdot\varsigma_1^{\mathcal{NM}}\right)\cdot\sin^2\left(\pi/2\cdot\varsigma_2^{\mathcal{NM}}\right) \end{array} \right),$$

2) $\sin\mathcal{L}_1 \boxtimes \sin\mathcal{L}_2$

$$= \left(\begin{array}{c} (\tau_1\cdot\tau_2, \vartheta_1\cdot\vartheta_2); \sin^2\left(\pi/2\cdot\varsigma_1^{\mathcal{M}}\right)\cdot\sin^2\left(\pi/2\cdot\varsigma_2^{\mathcal{M}}\right), \\ \sqrt[3\Xi]{\sin^2\left(\pi/2\cdot\left(\varsigma_1^{\mathcal{NM}}\right)^{3\Xi}\right)+\sin^2\left(\pi/2\cdot\left(\varsigma_2^{\mathcal{NM}}\right)^{3\Xi}\right)-\sin^2\left(\pi/2\cdot\left(\varsigma_1^{\mathcal{NM}}\right)^{3\Xi}\right)\cdot\sin^2\left(\pi/2\cdot\left(\varsigma_2^{\mathcal{NM}}\right)^{3\Xi}\right)} \end{array} \right),$$

3) $\Xi\cdot\sin\mathcal{L} = \left((\Xi\cdot\tau, \Xi\cdot\vartheta);\ \sqrt[3\Xi]{1-\left(1-\sin^2\left(\pi/2\cdot\left(\varsigma^{\mathcal{M}}\right)^{3\Xi}\right)\right)^{\Xi}}, \left(\sin^2\left(\pi/2\cdot\varsigma^{\mathcal{NM}}\right)\right)^{\Xi} \right),$

4) $(\sin\mathcal{L})^{\Xi} = \left(\left(\tau^{\Xi}, \vartheta^{\Xi}\right); \left(\sin^2\left(\pi/2\cdot\varsigma^{\mathcal{M}}\right)\right)^{\Xi},\ \sqrt[3\Xi]{1-\left(1-\sin^2\left(\pi/2\cdot\left(\varsigma^{\mathcal{NM}}\right)^{3\Xi}\right)\right)^{\Xi}} \right).$

We introduce a new distance for sine trigonometric Fermatean normal fuzzy numbers and its properties.

4. **Distance between ST-FNFNs.** In this section, we discuss the concept of distance between ST-FNFNs.

Definition 4.1. *Let* $\mathcal{L}_1 = \left\langle(\tau_1, \vartheta_1); \varsigma_1^{\mathcal{M}}, \varsigma_1^{\mathcal{NM}}\right\rangle$ *and* $\mathcal{L}_2 = \left\langle(\tau_2, \vartheta_2); \varsigma_2^{\mathcal{M}}, \varsigma_2^{\mathcal{NM}}\right\rangle$ *be the any two ST-FNFNs. Then*

(i) Euclidean distance between $\mathcal{L}_1$ *and* $\mathcal{L}_2$ *is defined as follows*

$$\mathcal{D}_E(\mathcal{L}_1, \mathcal{L}_2) = \frac{1}{3}\sqrt[3]{\begin{array}{l} \left(\frac{1+\sin^2\left(\pi/2\cdot\left(\varsigma_1^{\mathcal{M}}\right)^3\right)-\sin^2\left(\pi/2\cdot\left(\varsigma_1^{\mathcal{NM}}\right)^3\right)}{3}\tau_1 - \frac{1+\sin^2\left(\pi/2\cdot\left(\varsigma_2^{\mathcal{M}}\right)^3\right)-\sin^2\left(\pi/2\cdot\left(\varsigma_2^{\mathcal{NM}}\right)^3\right)}{3}\tau_2\right)^3 \\ +\frac{1}{3}\left(\frac{1+\sin^2\left(\pi/2\cdot\left(\varsigma_1^{\mathcal{M}}\right)^3\right)-\sin^2\left(\pi/2\cdot\left(\varsigma_1^{\mathcal{NM}}\right)^3\right)}{3}\vartheta_1 - \frac{1+\sin^2\left(\pi/2\cdot\left(\varsigma_2^{\mathcal{M}}\right)^3\right)-\sin^2\left(\pi/2\cdot\left(\varsigma_2^{\mathcal{NM}}\right)^3\right)}{3}\vartheta_2\right)^3 \end{array}}$$

(ii) Hamming distance between $\mathcal{L}_1$ *and* $\mathcal{L}_2$ *is defined as follows*

$$\mathcal{D}_H(\mathcal{L}_1, \mathcal{L}_2) = \frac{1}{3}\left(\begin{array}{l} \left|\frac{1+\sin^2\left(\pi/2\cdot\left(\varsigma_1^{\mathcal{M}}\right)^3\right)-\sin^2\left(\pi/2\cdot\left(\varsigma_1^{\mathcal{NM}}\right)^3\right)}{3}\tau_1 - \frac{1+\sin^2\left(\pi/2\cdot\left(\varsigma_2^{\mathcal{M}}\right)^3\right)-\sin^2\left(\pi/2\cdot\left(\varsigma_2^{\mathcal{NM}}\right)^3\right)}{3}\tau_2\right| \\ +\frac{1}{3}\left|\frac{1+\sin^2\left(\pi/2\cdot\left(\varsigma_1^{\mathcal{M}}\right)^3\right)-\sin^2\left(\pi/2\cdot\left(\varsigma_1^{\mathcal{NM}}\right)^3\right)}{3}\vartheta_1 - \frac{1+\sin^2\left(\pi/2\cdot\left(\varsigma_2^{\mathcal{M}}\right)^3\right)-\sin^2\left(\pi/2\cdot\left(\varsigma_2^{\mathcal{NM}}\right)^3\right)}{3}\vartheta_2\right| \end{array}\right)$$

Theorem 4.1. *For any three ST-FNFNs,* $\mathcal{L}_1 = \left\langle(\tau_1, \vartheta_1); \varsigma_1^{\mathcal{M}}, \varsigma_1^{\mathcal{NM}}\right\rangle$, $\mathcal{L}_2 = \left\langle(\tau_2, \vartheta_2); \varsigma_2^{\mathcal{M}}, \varsigma_2^{\mathcal{NM}}\right\rangle$ *and* $\mathcal{L}_3 = \left\langle(\tau_3, \vartheta_3); \varsigma_3^{\mathcal{M}}, \varsigma_3^{\mathcal{NM}}\right\rangle$. *Then*

1) $\mathcal{D}_E(\mathcal{L}_1, \mathcal{L}_2) = 0$ *if and only if* $\mathcal{L}_1 = \mathcal{L}_2$

2) $\mathcal{D}_E(\mathcal{L}_1, \mathcal{L}_2) = \mathcal{D}_E(\mathcal{L}_2, \mathcal{L}_1)$

3) $\mathcal{D}_E(\mathcal{L}_1, \mathcal{L}_3) \leq \mathcal{D}_E(\mathcal{L}_1, \mathcal{L}_2) + \mathcal{D}_E(\mathcal{L}_2, \mathcal{L}_3)$.

Proof: The proof follows from Definition 4.1(i).

Theorem 4.2. *For any three ST-FNFNs,* $\mathcal{L}_1 = \left\langle(\tau_1, \vartheta_1); \varsigma_1^{\mathcal{M}}, \varsigma_1^{\mathcal{NM}}\right\rangle$, $\mathcal{L}_2 = \left\langle(\tau_2, \vartheta_2); \varsigma_2^{\mathcal{M}}, \varsigma_2^{\mathcal{NM}}\right\rangle$ *and* $\mathcal{L}_3 = \left\langle(\tau_3, \vartheta_3); \varsigma_3^{\mathcal{M}}, \varsigma_3^{\mathcal{NM}}\right\rangle$. *Then*

1) $\mathcal{D}_H(\mathcal{L}_1, \mathcal{L}_2) = 0$ *if and only if* $\mathcal{L}_1 = \mathcal{L}_2$

2) $\mathcal{D}_H(\mathcal{L}_1, \mathcal{L}_2) = \mathcal{D}_H(\mathcal{L}_2, \mathcal{L}_1)$

3) $\mathcal{D}_H(\mathcal{L}_1,\mathcal{L}_3) \le \mathcal{D}_H(\mathcal{L}_1,\mathcal{L}_2) + \mathcal{D}_H(\mathcal{L}_2,\mathcal{L}_3)$.

Proof: The proof follows from Definition 4.1(ii).

We introduce the sine trigonometric Fermatean normal fuzzy aggregation operators.

5. ST-FNFN-Aggregation Operators.

5.1. ST-FNF weighted averaging (ST-FNFWA) operator.

Definition 5.1. *Let* $\mathcal{L}_i = \left\langle(\tau_i,\vartheta_i);\varsigma_i^{\mathcal{M}},\varsigma_i^{\mathcal{NM}}\right\rangle$ *be the collection of ST-FNFNs,* $W = (\varpi_1,\varpi_2,\dots,\varpi_n)$ *be a weight of* $\mathcal{L}_i$ *and* $\varpi_i \ge 0$, $\biguplus_{i=1}^n \varpi_i = 1$. *Then the ST-FNFWA operator can be defined as* $\text{ST-FNFWA}(\mathcal{L}_1,\mathcal{L}_2,\dots,\mathcal{L}_n) = \biguplus_{i=1}^n \varpi_i \sin\mathcal{L}_i$, $(i=1,2,\dots,n)$.

Theorem 5.1. *Let* $\mathcal{L}_i = \left\langle(\tau_i,\vartheta_i);\varsigma_i^{\mathcal{M}},\varsigma_i^{\mathcal{NM}}\right\rangle$ *be the collection of ST-FNFNs, and then ST-FNFWA operator is defined as*

$$\begin{aligned}&\text{ST-FNFWA}(\mathcal{L}_1,\mathcal{L}_2,\dots,\mathcal{L}_n)\\&=\left(\left(\biguplus_{i=1}^n \varpi_i\tau_i,\biguplus_{i=1}^n \varpi_i\vartheta_i\right);\sqrt[3\Xi]{1-\bigotimes_{i=1}^n\left(1-\sin^2\left(\pi/2\cdot\left(\varsigma_i^{\mathcal{M}}\right)^{3\Xi}\right)\right)^{\varpi_i}},\bigotimes_{i=1}^n\left(\sin^2\left(\pi/2\cdot\varsigma_i^{\mathcal{NM}}\right)\right)^{\varpi_i}\right).\end{aligned}$$

Proof: The proof follows from mathematical induction approach.

If $n=2$, then $\text{ST-FNFWA}(\mathcal{L}_1,\mathcal{L}_2) = \varpi_1\cdot\sin\mathcal{L}_1 \boxplus \varpi_2\cdot\sin\mathcal{L}_2$, where

$$\varpi_1\cdot\sin\mathcal{L}_1 = \left((\varpi_1\tau_1,\varpi_1\vartheta_1);\sqrt[3\Xi]{1-\left(1-\sin^2\left(\pi/2\cdot\left(\varsigma_1^{\mathcal{M}}\right)^{3\Xi}\right)\right)^{\varpi_1}},\left(\sin^2\left(\pi/2\cdot\varsigma_1^{\mathcal{NM}}\right)\right)^{\varpi_1}\right)$$

and

$$\varpi_2\cdot\sin\mathcal{L}_2 = \left((\varpi_2\tau_2,\varpi_2\vartheta_2);\sqrt[3\Xi]{1-\left(1-\sin^2\left(\pi/2\cdot\left(\varsigma_2^{\mathcal{M}}\right)^{3\Xi}\right)\right)^{\varpi_2}},\left(\sin^2\left(\pi/2\cdot\varsigma_2^{\mathcal{NM}}\right)\right)^{\varpi_2}\right).$$

We hand over to Definition 3.4, and get

$$\begin{aligned}&\varpi_1\cdot\sin\mathcal{L}_1 \boxplus \varpi_2\cdot\sin\mathcal{L}_2\\&=\left(\begin{array}{c}(\varpi_1\tau_1+\varpi_2\tau_2,\varpi_1\vartheta_1+\varpi_2\vartheta_2);\\ \sqrt[3\Xi]{\begin{array}{c}\left(1-\left(1-\sin^2\left(\pi/2\cdot\left(\varsigma_1^{\mathcal{M}}\right)^{3\Xi}\right)\right)^{\varpi_1}\right)+\left(1-\left(1-\sin^2\left(\pi/2\cdot\left(\varsigma_2^{\mathcal{M}}\right)^{3\Xi}\right)\right)^{\varpi_2}\right)\\ -\left(1-\left(1-\sin^2\left(\pi/2\cdot\left(\varsigma_1^{\mathcal{M}}\right)^{3\Xi}\right)\right)^{\varpi_1}\right)\cdot\left(1-\left(1-\sin^2\left(\pi/2\cdot\left(\varsigma_2^{\mathcal{M}}\right)^{3\Xi}\right)\right)^{\varpi_2}\right)\end{array}},\\ \left(\sin^2\left(\pi/2\cdot\varsigma_1^{\mathcal{NM}}\right)\right)^{\varpi_1}\cdot\left(\sin^2\left(\pi/2\cdot\varsigma_2^{\mathcal{NM}}\right)\right)^{\varpi_2}\end{array}\right)\\&=\left(\begin{array}{c}\left(\biguplus_{i=1}^2 \varpi_i\tau_i,\biguplus_{i=1}^2 \varpi_i\vartheta_i\right);\sqrt[3\Xi]{1-\bigotimes_{i=1}^2\left(1-\sin^2\left(\pi/2\cdot\left(\varsigma_i^{\mathcal{M}}\right)^{3\Xi}\right)\right)^{\varpi_i}},\\ \bigotimes_{i=1}^2\left(\sin^2\left(\pi/2\cdot\varsigma_i^{\mathcal{NM}}\right)\right)^{\varpi_i}\end{array}\right).\end{aligned}$$

Also hold for $n \ge 3$,

$$\begin{aligned}&\text{ST-FNFWA}(\mathcal{L}_1,\mathcal{L}_2,\dots,\mathcal{L}_k)\\&=\left(\left(\biguplus_{i=1}^k \varpi_i\tau_i,\biguplus_{i=1}^k \varpi_i\vartheta_i\right);\sqrt[3\Xi]{1-\bigotimes_{i=1}^k\left(1-\sin^2\left(\pi/2\cdot\left(\varsigma_i^{\mathcal{M}}\right)^{3\Xi}\right)\right)^{\varpi_i}},\bigotimes_{i=1}^k\left(\sin^2\left(\pi/2\cdot\varsigma_i^{\mathcal{NM}}\right)\right)^{\varpi_i}\right).\end{aligned}$$

If $n = k+1$, then

$$\textit{ST-FNFWA}\left(\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_k, \mathcal{L}_{k+1}\right)$$

$$= \left(\begin{array}{c} \left(\biguplus_{i=1}^{k} \varpi_i \tau_i + \varpi_{k+1}\tau_{k+1}, \biguplus_{i=1}^{k} \varpi_i \vartheta_i + \varpi_{k+1}\vartheta_{k+1} \right); \\ \sqrt[3\Xi]{\begin{array}{c} \biguplus_{i=1}^{k} \left(1 - \left(1 - \sin^2\left(\pi/2 \cdot \left(\varsigma_i^{\mathcal{M}}\right)^{3\Xi}\right)\right)^{\varpi_i}\right) + \left(1 - \left(1 - \sin^2\left(\pi/2 \cdot \left(\varsigma_{k+1}^{\mathcal{M}}\right)^{3\Xi}\right)\right)^{\varpi_{k+1}}\right) \\ - \bigotimes_{i=1}^{k} \left(1 - \left(1 - \sin^2\left(\pi/2 \cdot \left(\varsigma_i^{\mathcal{M}}\right)^{3\Xi}\right)\right)^{\varpi_i}\right) \cdot \left(1 - \left(1 - \sin^2\left(\pi/2 \cdot \left(\varsigma_{k+1}^{\mathcal{M}}\right)^{3\Xi}\right)\right)^{\varpi_{k+1}}\right) \end{array}}, \\ \bigotimes_{i=1}^{k} \left(\sin^2\left(\pi/2 \cdot \varsigma_i^{\mathcal{NM}}\right)\right)^{\varpi_i} \cdot \left(\sin^2\left(\pi/2 \cdot \varsigma_{k+1}^{\mathcal{NM}}\right)\right)^{\varpi_{k+1}} \end{array} \right)$$

$$= \left(\begin{array}{c} \left(\biguplus_{i=1}^{k+1} \varpi_i \tau_i, \biguplus_{i=1}^{k+1} \varpi_i \vartheta_i \right); \\ \sqrt[3\Xi]{1 - \bigotimes_{i=1}^{k} \left(1 - \sin^2\left(\pi/2 \cdot \left(\varsigma_i^{\mathcal{M}}\right)^{3\Xi}\right)\right)^{\varpi_i} \cdot \left(1 - \sin^2\left(\pi/2 \cdot \left(\varsigma_{k+1}^{\mathcal{M}}\right)^{3\Xi}\right)\right)^{\varpi_{k+1}}}, \\ \bigotimes_{i=1}^{k+1} \left(\sin^2\left(\pi/2 \cdot \varsigma_i^{\mathcal{NM}}\right)\right)^{\varpi_i} \end{array} \right)$$

$$= \left(\begin{array}{c} \left(\biguplus_{i=1}^{k+1} \varpi_i \tau_i, \biguplus_{i=1}^{k+1} \varpi_i \vartheta_i \right); \sqrt[3\Xi]{1 - \bigotimes_{i=1}^{k+1} \left(1 - \sin^2\left(\pi/2 \cdot \left(\varsigma_i^{\mathcal{M}}\right)^{3\Xi}\right)\right)^{\varpi_i}}, \\ \bigotimes_{i=1}^{k+1} \left(\sin^2\left(\pi/2 \cdot \varsigma_i^{\mathcal{NM}}\right)\right)^{\varpi_i} \end{array} \right)$$

It holds for any k.

Theorem 5.2. *Let* $\mathcal{L}_i = \left\langle (\tau_i, \vartheta_i); \sin^2\left(\pi/2 \cdot \varsigma_i^{\mathcal{M}}\right), \sin^2\left(\pi/2 \cdot \varsigma_i^{\mathcal{NM}}\right) \right\rangle$ $(i = 1, 2, \ldots, n)$ *be the collection of ST-FNFNs and all are equal with* $\sin^2\left(\pi/2 \cdot \left(\varsigma_i^{\mathcal{M}}\right)^{3\Xi}\right) = \left(\sin^2\left(\pi/2 \cdot \varsigma_i^{\mathcal{M}}\right)\right)^{3\Xi}$ *and* $\mathcal{L}_i = \mathcal{L}$*, then* $\textit{ST-FNFWA}(\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_n) = \sin \mathcal{L}$.

Proof: Given that, $(\tau_i, \vartheta_i) = (\tau, \vartheta)$, $\left(\sin^2\left(\pi/2 \cdot \left(\varsigma_i^{\mathcal{M}}\right), \sin^2\left(\pi/2 \cdot \left(\varsigma_i^{\mathcal{NM}}\right)\right)\right)\right) = \left(\sin^2\left(\pi/2 \cdot \varsigma^{\mathcal{M}}\right), \sin^2\left(\pi/2 \cdot \varsigma^{\mathcal{NM}}\right)\right)$, for $i = 1, 2, \ldots, n$ and $\biguplus_{i=1}^{n} \varpi_i = 1$. We hand over to Definition 3.4, and get,

$$\textit{ST-FNFWA}\left(\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_n\right)$$

$$= \left(\begin{array}{c} \left(\biguplus_{i=1}^{n} \varpi_i \tau_i, \biguplus_{i=1}^{n} \varpi_i \vartheta_i \right); \sqrt[3\Xi]{1 - \bigotimes_{i=1}^{n} \left(1 - \sin^2\left(\pi/2 \cdot \left(\varsigma^{\mathcal{M}}\right)^{3\Xi}\right)\right)^{\varpi_i}}, \\ \bigotimes_{i=1}^{n} \left(\sin^2\left(\pi/2 \cdot \varsigma_i^{\mathcal{NM}}\right)\right)^{\varpi_i} \end{array} \right)$$

$$= \left(\begin{array}{c} \left(\tau \biguplus_{i=1}^{n} \varpi_i, \vartheta \biguplus_{i=1}^{n} \varpi_i \right); \sqrt[3\Xi]{1 - \left(1 - \sin^2\left(\pi/2 \cdot \left(\varsigma^{\mathcal{M}}\right)^{3\Xi}\right)\right)^{\biguplus_{i=1}^{n} \varpi_i}}, \\ \left(\sin^2\left(\pi/2 \cdot \varsigma^{\mathcal{NM}}\right)\right)^{\biguplus_{i=1}^{n} \varpi_i} \end{array} \right)$$

$$= \left((\tau, \vartheta);\ \sqrt[3\Xi]{1 - \left(1 - \sin^2\left(\pi/2 \cdot \left(\varsigma^{\mathcal{M}}\right)^{3\Xi}\right)\right)}, \sin^2\left(\pi/2 \cdot \varsigma^{\mathcal{NM}}\right) \right)$$
$$= \left((\tau, \vartheta); \sin^2\left(\pi/2 \cdot \varsigma^{\mathcal{M}}\right), \sin^2\left(\pi/2 \cdot \varsigma^{\mathcal{NM}}\right) \right) = \sin \mathcal{L}$$

5.2. **ST-FNF weighted geometric (ST-FNFWG) operator.**

Definition 5.2. *Let* $\mathcal{L}_i = \left\langle (\tau_i, \vartheta_i); \varsigma_i^{\mathcal{M}}, \varsigma_i^{\mathcal{NM}} \right\rangle$ *be the collection of ST-FNFNs,* $W = (\varpi_1, \varpi_2, \ldots, \varpi_n)$ *be a weight of* $\mathcal{L}_i$*. Then the ST-FNFWG operator can be defined as ST-FNFWG*$(\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_n) = \bigotimes_{i=1}^{n} (\sin \mathcal{L}_i)^{\varpi_i}$, $(i = 1, 2, \ldots, n)$.

Theorem 5.3. *Let* $\mathcal{L}_i = \left\langle (\tau_i, \vartheta_i); \varsigma_i^{\mathcal{M}}, \varsigma_i^{\mathcal{NM}} \right\rangle$ *be the collection of ST-FNFNs, and then the ST-FNFWG operator can be defined as*

$$ST\text{-}FNFWG(\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_n)$$
$$= \left(\left(\bigotimes_{i=1}^{n} \tau_i^{\varpi_i}, \bigotimes_{i=1}^{n} \vartheta_i^{\varpi_i} \right); \bigotimes_{i=1}^{n} \left(\sin^2\left(\pi/2 \cdot \varsigma_i^{\mathcal{M}}\right)\right)^{\varpi_i}, \sqrt[3\Xi]{1 - \bigotimes_{i=1}^{n} \left(1 - \sin^2\left(\pi/2 \cdot \left(\varsigma_i^{\mathcal{NM}}\right)^{3\Xi}\right)\right)^{\varpi_i}} \right)$$

Proof: The proof follows from Theorem 5.1.

Theorem 5.4. *Let* $\mathcal{L}_i = \left\langle (\tau_i, \vartheta_i); \varsigma_i^{\mathcal{M}}, \varsigma_i^{\mathcal{NM}} \right\rangle$, $(i = 1, 2, \ldots, n)$ *be the collection of ST-FNFNs and all are equal with* $\sin^2\left(\pi/2 \cdot \left(\varsigma_i^{\mathcal{NM}}\right)^{3\Xi}\right) = \left(\sin^2\left(\pi/2 \cdot \varsigma_i^{\mathcal{NM}}\right)\right)^{3\Xi}$ *and* $\mathcal{L}_i = \mathcal{L}$, *then ST-FNFWG*$(\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_n) = \sin \mathcal{L}$.

Proof: The proof follows from Theorem 5.2.

5.3. **Generalized ST-FNFWA (ST-GFNFWA) operator.**

Definition 5.3. *Let* $\mathcal{L}_i = \left\langle (\tau_i, \vartheta_i); \varsigma_i^{\mathcal{M}}, \varsigma_i^{\mathcal{NM}} \right\rangle$ *be the collection of ST-FNFNs,* $W = (\varpi_1, \varpi_2, \ldots, \varpi_n)$ *be a weight of* $\mathcal{L}_i$*. Then ST-GFNFWA*$(\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_n) = \left(\biguplus_{i=1}^{n} \varpi_i (\sin \mathcal{L}_i)^{\Xi} \right)^{1/\Xi}$ $(i = 1, 2, \ldots, n)$ *is called an ST-GFNFWA operator.*

Theorem 5.5. *Let* $\mathcal{L}_i = \left\langle (\tau_i, \vartheta_i); \varsigma_i^{\mathcal{M}}, \varsigma_i^{\mathcal{NM}} \right\rangle$ *be the collection of ST-FNFNs. Then ST-GFNFWA operator can be defined as*

$$ST\text{-}GFNFWA(\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_n)$$
$$= \left(\begin{array}{c} \left(\left(\biguplus_{i=1}^{n} \varpi_i \tau_i^{\Xi} \right)^{1/\Xi}, \left(\biguplus_{i=1}^{n} \varpi_i \vartheta_i^{\Xi} \right)^{1/\Xi} \right); \left(\sqrt[3\Xi]{1 - \bigotimes_{i=1}^{n} \left(1 - \left(\sin^2\left(\pi/2 \cdot \left(\varsigma_i^{\mathcal{M}}\right)^{3\Xi}\right)\right)^{\Xi}\right)^{\varpi_i}} \right)^{1/\Xi}, \\ \sqrt[3\Xi]{1 - \left(1 - \left(\bigotimes_{i=1}^{n} \left(\sqrt[3\Xi]{1 - \left(1 - \left(\sin^2\left(\pi/2 \cdot \varsigma_i^{\mathcal{NM}}\right)\right)^{3\Xi}\right)^{\Xi}} \right)^{\varpi_i} \right)^{3\Xi}\right)^{1/\Xi}} \end{array} \right).$$

Proof: The proof follows from mathematical induction. First let us show that,

$$\biguplus_{i=1}^{n} \varpi_i (\sin \mathcal{L}_i)^{\Xi}$$
$$= \left(\begin{array}{c} \left(\left(\biguplus_{i=1}^{n} \varpi_i \tau_i^{\Xi} \right), \left(\biguplus_{i=1}^{n} \varpi_i \vartheta_i^{\Xi} \right) \right); \sqrt[3\Xi]{1 - \bigotimes_{i=1}^{n} \left(1 - \left(\sin^2\left(\pi/2 \cdot \left(\varsigma_i^{\mathcal{M}}\right)^{3\Xi}\right)\right)^{\Xi}\right)^{\varpi_i}}, \\ \bigotimes_{i=1}^{n} \left(\sqrt[3\Xi]{1 - \left(1 - \left(\sin^2\left(\pi/2 \cdot \varsigma_i^{\mathcal{NM}}\right)\right)^{3\Xi}\right)^{\Xi}} \right)^{\varpi_i} \end{array} \right).$$

If $n = 2$, then we hand over to Definition 3.4, and get

$$\varpi_1 (\sin \mathcal{L}_1) \boxplus \varpi_2 (\sin \mathcal{L}_2)$$

$$= \left(\begin{array}{c} \left(\varpi_1 \tau_1^{\Xi} + \varpi_2 \tau_2^{\Xi}, \varpi_1 \vartheta_1^{\Xi} + \varpi_2 \vartheta_2^{\Xi} \right), \\ \sqrt[3\Xi]{\begin{array}{c} \left(\sqrt[3\Xi]{1 - \left(1 - \left(\sin^2\left(\pi/2 \cdot \left(\varsigma_1^{\mathcal{M}}\right)^{3\Xi}\right)\right)\right)^{\varpi_1}} \right)^{3\Xi} + \left(\sqrt[3\Xi]{1 - \left(1 - \left(\sin^2\left(\pi/2 \cdot \left(\varsigma_2^{\mathcal{M}}\right)^{3\Xi}\right)\right)\right)^{\varpi_1}} \right)^{3\Xi}, \\ - \left(\sqrt[3\Xi]{1 - \left(1 - \left(\sin^2\left(\pi/2 \cdot \left(\varsigma_1^{\mathcal{M}}\right)^{3\Xi}\right)\right)\right)^{\varpi_1}} \right)^{3\Xi} \cdot \left(\sqrt[3\Xi]{1 - \left(1 - \left(\sin^2\left(\pi/2 \cdot \left(\varsigma_2^{\mathcal{M}}\right)^{3\Xi}\right)\right)\right)^{\varpi_1}} \right)^{3\Xi} \end{array}} \\ \left(\sqrt[3\Xi]{1 - \left(1 - \left(\sin^2\left(\pi/2 \cdot \varsigma_1^{\mathcal{NM}}\right)\right)^{3\Xi}\right)^{\Xi}} \right)^{\varpi_1} \cdot \left(\sqrt[3\Xi]{1 - \left(1 - \left(\sin^2\left(\pi/2 \cdot \varsigma_2^{\mathcal{NM}}\right)\right)^{3\Xi}\right)^{\Xi}} \right)^{\varpi_1} \end{array} \right)$$

$$= \left(\begin{array}{c} \left(\biguplus_{i=1}^{2} \varpi_i \tau_i^{\Xi}, \biguplus_{i=1}^{2} \varpi_i \vartheta_i^{\Xi} \right), \sqrt[3\Xi]{1 - \bigotimes_{i=1}^{2} \left(1 - \left(\sin^2\left(\pi/2 \cdot \left(\varsigma_1^{\mathcal{M}}\right)^{3\Xi}\right)\right)\right)^{\Xi}\right)^{\varpi_i}}, \\ \bigotimes_{i=1}^{2} \left(\sqrt[3\Xi]{1 - \left(1 - \left(\sin^2\left(\pi/2 \cdot \varsigma_i^{\mathcal{NM}}\right)\right)^{3\Xi}\right)^{\Xi}} \right)^{\varpi_i} \end{array} \right)$$

In general, $\left(\begin{array}{c} \left(\biguplus_{i=1}^{k} \varpi_i \tau_i^{\Xi}, \biguplus_{i=1}^{k} \varpi_i \vartheta_i^{\Xi} \right); \sqrt[3\Xi]{1 - \bigotimes_{i=1}^{k} \left(1 - \left(\sin^2\left(\pi/2 \cdot \left(\varsigma_1^{\mathcal{M}}\right)^{3\Xi}\right)\right)\right)^{\Xi}\right)^{\varpi_i}}, \\ \bigotimes_{i=1}^{k} \left(\sqrt[3\Xi]{1 - \left(1 - \left(\sin^2\left(\pi/2 \cdot \varsigma_i^{\mathcal{NM}}\right)\right)^{3\Xi}\right)^{\Xi}} \right)^{\varpi_i} \end{array} \right)$.

Now,

$$\biguplus_{i=1}^{k} \varpi_i (\sin \mathcal{L}_i)^{\Xi} + \varpi_{k+1} (\sin \mathcal{L}_{k+1})^{\Xi}$$

$$= \varpi_1 (\sin \mathcal{L}_1)^{\Xi} \boxplus \varpi_2 (\sin \mathcal{L}_2)^{\Xi} \boxplus \cdots \boxplus \varpi_k (\sin \mathcal{L}_k)^{\Xi} \boxplus \varpi_{k+1} (\sin \mathcal{L}_{k+1})^{\Xi}$$

$$= \left(\begin{array}{c} \left(\biguplus_{i=1}^{k} \varpi_i \tau_i^{\Xi} + \varpi_{k+1} \tau_{k+1}^{\Xi}, \biguplus_{i=1}^{k} \varpi_i \vartheta_i^{\Xi} + \varpi_{k+1} \vartheta_{k+1}^{\Xi} \right); \\ \sqrt[3\Xi]{\begin{array}{c} \left(\sqrt[3\Xi]{1 - \bigotimes_{i=1}^{k} \left(1 - \left(\sin^2\left(\pi/2 \cdot \left(\varsigma_i^{\mathcal{M}}\right)^{3\Xi}\right)\right)\right)^{\Xi}\right)^{\varpi_i}} \right)^{3\Xi} + \left(\sqrt[3\Xi]{1 - \left(1 - \left(\sin^2\left(\pi/2 \cdot \left(\varsigma_{k+1}^{\mathcal{M}}\right)^{3\Xi}\right)\right)\right)^{\Xi}\right)^{\varpi_1}} \right)^{3\Xi}, \\ - \left(\sqrt[3\Xi]{1 - \bigotimes_{i=1}^{k} \left(1 - \left(\sin^2\left(\pi/2 \cdot \left(\varsigma_i^{\mathcal{M}}\right)^{3\Xi}\right)\right)\right)^{\Xi}\right)^{\varpi_i}} \right)^{3\Xi} \cdot \left(\sqrt[3\Xi]{1 - \left(1 - \left(\sin^2\left(\pi/2 \cdot \left(\varsigma_{k+1}^{\mathcal{M}}\right)^{3\Xi}\right)\right)\right)^{\Xi}\right)^{\varpi_1}} \right)^{3\Xi} \end{array}} \\ \bigotimes_{i=1}^{k} \left(\sqrt[3\Xi]{1 - \left(1 - \left(\sin^2\left(\pi/2 \cdot \varsigma_i^{\mathcal{NM}}\right)\right)^{3\Xi}\right)^{\Xi}} \right)^{\varpi_i} \cdot \left(\sqrt[3\Xi]{1 - \left(1 - \left(\sin^2\left(\pi/2 \cdot \varsigma_{k+1}^{\mathcal{NM}}\right)\right)^{3\Xi}\right)^{\Xi}} \right)^{\varpi_1} \end{array} \right)$$

$$= \left(\begin{array}{c} \left(\biguplus_{i=1}^{k+1} \varpi_i \tau_i^{\Xi}, \biguplus_{i=1}^{k+1} \varpi_i \vartheta_i^{\Xi} \right); \sqrt[3\Xi]{1 - \bigotimes_{i=1}^{k+1} \left(1 - \left(\sin^2\left(\pi/2 \cdot \left(\varsigma_1^{\mathcal{M}}\right)^{3\Xi}\right)\right)\right)^{\Xi}\right)^{\varpi_i}}, \\ \bigotimes_{i=1}^{k+1} \left(\sqrt[3\Xi]{1 - \left(1 - \left(\sin^2\left(\pi/2 \cdot \varsigma_i^{\mathcal{NM}}\right)\right)^{3\Xi}\right)^{\Xi}} \right)^{\varpi_i} \end{array} \right)$$

and

$$\biguplus_{i=1}^{k+1} \left(\varpi_i (\sin \mathcal{L}_i)^{\Xi} \right)^{1/\Xi}$$

$$= \left(\begin{array}{c} \left(\left(\biguplus_{i=1}^{k+1} \varpi_i \tau_i^{\Xi} \right)^{1/\Xi}, \left(\biguplus_{i=1}^{k+1} \varpi_i \vartheta_i^{\Xi} \right)^{1/\Xi} \right); \\ \sqrt[3\Xi]{1 - \bigotimes_{i=1}^{k+1} \left(1 - \left(\sin^2 \left(\pi/2 \cdot \left(\varsigma_i^{\mathcal{M}} \right)^{3\Xi} \right)^{\Xi} \right)^{\varpi_i} \right)}^{1/\Xi}, \\ \sqrt[3\Xi]{1 - \left(1 - \left(\bigotimes_{i=1}^{n} \left(\sqrt[3\Xi]{1 - \left(1 - \left(\sin^2 \left(\pi/2 \cdot \varsigma_i^{\mathcal{NM}} \right) \right)^{3\Xi} \right)^{\Xi}} \right)^{\varpi_i} \right)^{3\Xi} \right)^{1/\Xi}} \end{array} \right).$$

It holds for any k.

Remark 5.1. *If $\Xi = 1$, then ST-GFNFWA operator is reduced to the ST-FNFWA operator.*

Theorem 5.6. *Let $\mathcal{L}_i = \left\langle (\tau_i, \vartheta_i); \varsigma_i^{\mathcal{M}}, \varsigma_i^{\mathcal{NM}} \right\rangle$, $(i = 1, 2, \ldots, n)$ be the collection of ST-FNFNs and all are equal with $\mathcal{L}_i = \mathcal{L}$, then ST-GFNFWA$(\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_n) = \sin \mathcal{L}$.*

Proof: The proof follows from Theorem 5.2.

5.4. Generalized ST-FNFWG (ST-GFNFWG) operator.

Definition 5.4. *Let $\mathcal{L}_i = \left\langle (\tau_i, \vartheta_i); \varsigma_i^{\mathcal{M}}, \varsigma_i^{\mathcal{NM}} \right\rangle$ be the collection of ST-FNFNs, $W = (\varpi_1, \varpi_2, \ldots, \varpi_n)$ be a weight of $\mathcal{L}_i$. Then ST-GFNFWG$(\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_n) = \frac{1}{\Xi} \left(\bigotimes_{i=1}^{n} (\Xi \sin \mathcal{L}_i)^{\varpi_i} \right)$, $(i = 1, 2, \ldots, n)$ is called an ST-GFNFWG operator.*

Theorem 5.7. *Let $\mathcal{L}_i = \left\langle (\tau_i, \vartheta_i); \varsigma_i^{\mathcal{M}}, \varsigma_i^{\mathcal{NM}} \right\rangle$ be the collection of ST-FNFNs. Then ST-GFNFWG operator can be defined as*

$$ST\text{-}GFNFWG(\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_n)$$

$$= \left(\begin{array}{c} \left(\frac{1}{\Xi} \bigotimes_{i=1}^{n} (\Xi \tau_i)^{\varpi_i}, \frac{1}{\Xi} \bigotimes_{i=1}^{n} (\Xi \vartheta_i)^{\varpi_i} \right); \\ \sqrt[3\Xi]{1 - \left(1 - \left(\bigotimes_{i=1}^{n} \left(\sqrt[3\Xi]{1 - \left(1 - \left(\sin^2 \left(\pi/2 \cdot \varsigma_i^{\mathcal{M}} \right) \right)^{3\Xi} \right)^{\Xi}} \right)^{\varpi_i} \right)^{3\Xi} \right)^{1/\Xi}} \\ \left(\sqrt[3\Xi]{1 - \bigotimes_{i=1}^{n} \left(1 - \left(\sin^2 \left(\pi/2 \cdot \left(\varsigma_i^{\mathcal{NM}} \right)^{3\Xi} \right) \right)^{\Xi} \right)^{\varpi_i}} \right)^{1/\Xi} \end{array} \right)$$

Proof: The proof follows from Theorem 5.5.

Remark 5.2. *If $\Xi = 1$, then ST-GFNFWG operator is reduced to the ST-FNFWG operator.*

Theorem 5.8. *Let $\mathcal{L}_i = \left\langle (\tau_i, \vartheta_i); \varsigma_i^{\mathcal{M}}, \varsigma_i^{\mathcal{NM}} \right\rangle$, $(i = 1, 2, \ldots, n)$ be the collection of ST-FNFNs and all are equal with $\mathcal{L}_i = \mathcal{L}$, then ST-GFNFWG$(\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_n) = \sin \mathcal{L}$.*

Proof: The proof follows from Theorem 5.2.

We discuss the MADM approach for ST-FNF and the ST-FNF algorithm with real-life application.

6. **ST-FNF MADM Approach.** Let $\mathcal{E} = \{\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_n\}$ represent the set of n-alternatives, $\mathcal{E} = \{e_1, e_2, \ldots, e_m\}$ represent set of m-attributes, $w = \{\varpi_1, \varpi_2, \ldots, \varpi_m\}$ be the weights of attributes, for $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$. $\mathcal{E}_{ij} = \left\langle (\tau_{ij}, \vartheta_{ij}); \varsigma_{ij}^{\mathcal{M}}, \varsigma_{ij}^{\mathcal{NM}} \right\rangle$ is an ST-FNFN of alternative $\mathcal{E}_i$ in attribute e_j. Since $\varsigma_{ij}^{\mathcal{M}}, \varsigma_{ij}^{\mathcal{NM}} \in [0,1]$ and $0 \leq \left(\varsigma_{ij}^{\mathcal{M}}(u)\right)^3 + \left(\varsigma_{ij}^{\mathcal{NM}}(u)\right)^3 \leq 1$. Here n-alternative sets and m-attribute sets give $n \times m$ decision matrix denoted by $\mathcal{D} = (\mathcal{E}_{ij})_{n \times m}$.

6.1. Algorithm for ST-FNF.

Step-1: Input the decision values for each alternative.

Step-2: Determine the normalized decision values for each alternative. The decision matrix $\mathcal{D} = (\mathcal{E}_{ij})_{n \times m}$ is normalized into $\overline{\mathcal{D}} = \left(\widehat{\mathcal{E}_{ij}}\right)_{n \times m}$; where $\widehat{\mathcal{E}_{ij}} = \left\langle \left(\overline{\tau_{ij}}, \overline{\vartheta_{ij}}\right); \overline{\varsigma_{ij}^{\mathcal{M}}}, \overline{\varsigma_{ij}^{\mathcal{NM}}} \right\rangle$ and $\overline{\tau_{ij}} = \frac{\tau_{ij}}{\max_i(\tau_{ij})}$, $\overline{\vartheta_{ij}} = \frac{\vartheta_{ij}}{\max_i(\vartheta_{ij})} \cdot \frac{\vartheta_{ij}}{\tau_{ij}}$, $\overline{\varsigma_{ij}^{\mathcal{M}}} = \varsigma_{ij}^{\mathcal{M}}$, $\overline{\varsigma_{ij}^{\mathcal{NM}}} = \varsigma_{ij}^{\mathcal{NM}}$.

Step-3: Aggregate the values of each alternative. On the basis of ST-FNF aggregation operators, attribute e_j in $\mathcal{E}_i$, $\widehat{\mathcal{E}_{ij}} = \left\langle \left(\overline{\tau_{ij}}, \overline{\vartheta_{ij}}\right); \overline{\varsigma_{ij}^{\mathcal{M}}}, \overline{\varsigma_{ij}^{\mathcal{NM}}} \right\rangle$ is aggregated into $\widehat{\mathcal{E}_i} = \left\langle \left(\overline{\tau_i}, \overline{\vartheta_i}\right); \overline{\varsigma_i^{\mathcal{M}}}, \overline{\varsigma_i^{\mathcal{NM}}} \right\rangle$.

Step-4: Compute the positive and negative ideal values of each alternative, where positive ideal value $\widehat{\mathcal{E}^+} = \left\langle \left(\max_{1 \leq i \leq n}\left(\overline{\tau_{ij}}\right), \min_{1 \leq i \leq n}\left(\overline{\vartheta_{ij}}\right)\right); 1, 0 \right\rangle$ and negative ideal value $\widehat{\mathcal{E}^-} = \left\langle \left(\min_{1 \leq i \leq n}\left(\overline{\tau_{ij}}\right), \max_{1 \leq i \leq n}\left(\overline{\vartheta_{ij}}\right)\right); 0, 1 \right\rangle$.

Step-5: Compute the Hamming distances between each alternative with two ideal values, where $\mathcal{D}_i^+ = \mathcal{D}_H\left(\widehat{\mathcal{E}_i} \text{ and } \widehat{\mathcal{E}^+}\right)$; $\mathcal{D}_i^- = \mathcal{D}_H\left(\widehat{\mathcal{E}_i}, \widehat{\mathcal{E}^-}\right)$.

Step-6: Find the relative closeness values and find the ranking of alternatives $\mathcal{D}_i^* = \frac{\mathcal{D}_i^-}{\mathcal{D}_i^+ + \mathcal{D}_i^-}$.

Step-7: Output yield for the optimal value is $\max \mathcal{D}_i^*$, and hence decision is to choose as the optimal solution to the problem.

6.2. Real-life applications.

In the selection of a college for undergoing teaching education, the evaluation of teacher education is carried out according to various standards of experts. There are various primary studies that have been conducted that have investigated the reasons why parents select a particular college that they think best suits their college students needs and parental aspirations. We identify a factor regarded as parental decision making: academic factor, which is divided into five identified elements: campus environment, overall cost, academic quality, student/faculty relationship, and career development. Our goal is to select the optimal one out of a great number of alternatives based on the assessment of experts against the criteria. A parent intends to choose the best college education. Here we intend to choose ten colleges that are nominated. The score of the college education evaluated by the experts is represented by $E = \{e_1$: campus environment, e_2: overall cost, e_3: academic quality, e_4: career development$\}$ and their corresponding weights are $w = \{0.35, 0.3, 0.25, 0.1\}$.

Step 1: According to the decision information as shown below as constructed.

	e_1	e_2	e_3
$\mathcal{E}_1$	$\langle(0.8, 0.45); 0.66, 0.88\rangle$	$\langle(0.65, 0.6); 0.85, 0.65\rangle$	$\langle(0.6, 0.55); 0.64, 0.79\rangle$
$\mathcal{E}_2$	$\langle(0.65, 0.6); 0.8, 0.65\rangle$	$\langle(0.6, 0.5); 0.84, 0.7\rangle$	$\langle(0.55, 0.45); 0.71, 0.85\rangle$
$\mathcal{E}_3$	$\langle(0.7, 0.5); 0.73, 0.69\rangle$	$\langle(0.7, 0.65); 0.91, 0.5\rangle$	$\langle(0.75, 0.55); 0.7, 0.85\rangle$
$\mathcal{E}_4$	$\langle(0.45, 0.3); 0.7, 0.85\rangle$	$\langle(0.75, 0.5); 0.78, 0.64\rangle$	$\langle(0.7, 0.65); 0.64, 0.78\rangle$
$\mathcal{E}_5$	$\langle(0.5, 0.45); 0.78, 0.7\rangle$	$\langle(0.7, 0.6); 0.77, 0.72\rangle$	$\langle(0.65, 0.5); 0.7, 0.86\rangle$

	e_4
$\mathcal{E}_1$	$\langle(0.55, 0.5); 0.85, 0.64\rangle$
$\mathcal{E}_2$	$\langle(0.8, 0.6); 0.91, 0.5\rangle$
$\mathcal{E}_3$	$\langle(0.75, 0.7); 0.85, 0.65\rangle$
$\mathcal{E}_4$	$\langle(0.7, 0.6); 0.84, 0.64\rangle$
$\mathcal{E}_5$	$\langle(0.6, 0.55); 0.7, 0.85\rangle$

Step 2: Normalized decision matrix constructed as follows.

	e_1	e_2	e_3	e_4
$\mathcal{E}_1$	$\langle(1, 0.4219); 0.66, 0.88\rangle$	$\langle(0.8667, 0.8521); 0.85, 0.65\rangle$	$\langle(0.8, 0.7756); 0.64, 0.79\rangle$	$\langle(0.6875, 0.6494); 0.85, 0.64\rangle$
$\mathcal{E}_2$	$\langle(0.8125, 0.9231); 0.8, 0.65\rangle$	$\langle(0.8, 0.641); 0.84, 0.7\rangle$	$\langle(0.7333, 0.5664); 0.71, 0.85\rangle$	$\langle(1, 0.6429); 0.91, 0.5\rangle$
$\mathcal{E}_3$	$\langle(0.875, 0.5952); 0.73, 0.69\rangle$	$\langle(0.9333, 0.9286); 0.91, 0.5\rangle$	$\langle(1, 0.6205); 0.7, 0.85\rangle$	$\langle(0.9375, 0.9333); 0.85, 0.65\rangle$
$\mathcal{E}_4$	$\langle(0.5625, 0.3333); 0.7, 0.85\rangle$	$\langle(1, 0.5128); 0.78, 0.64\rangle$	$\langle(0.9333, 0.9286); 0.64, 0.78\rangle$	$\langle(0.875, 0.7347); 0.84, 0.64\rangle$
$\mathcal{E}_5$	$\langle(0.625, 0.675); 0.78, 0.7\rangle$	$\langle(0.9333, 0.7912); 0.77, 0.72\rangle$	$\langle(0.8667, 0.5917); 0.7, 0.86\rangle$	$\langle(0.75, 0.7202); 0.7, 0.85\rangle$

Step 3: Aggregating the information with ST-FNFWA operator of each alternative can be founded as follows.

	ST-FNFWA operator ($\Xi = 1$)
$\widehat{\mathcal{E}_1}$	$\langle(0.8788, 0.6621); 0.9821, 0.6317\rangle$
$\widehat{\mathcal{E}_2}$	$\langle(0.8077, 0.7213); 0.8442, 0.297\rangle$
$\widehat{\mathcal{E}_3}$	$\langle(0.93, 0.7354); 0.8847, 0.637\rangle$
$\widehat{\mathcal{E}_4}$	$\langle(0.8177, 0.5761); 0.9741, 0.7752\rangle$
$\widehat{\mathcal{E}_5}$	$\langle(0.7904, 0.6936); 0.8559, 0.2539\rangle$

Step 4: Determine the positive and negative ideal values of the alternatives as $\widehat{\mathcal{E}^+} = \langle(0.93, 0.5761), 1, 0\rangle$ and $\widehat{\mathcal{E}^-}\langle(0.7904, 0.7354), 0, 1\rangle$.

Step 5: The Hamming distance between each alternative as follows:

The positive ideal values are $\mathcal{D}_1^+ = 0.0863$, $\mathcal{D}_2^+ = 0.2801$, $\mathcal{D}_3^+ = 0.045$, $\mathcal{D}_4^+ = 0.0872$ and $\mathcal{D}_5^+ = 0.25$. The negative ideal values are $\mathcal{D}_1^- = 0.1376$, $\mathcal{D}_2^- = 0.0562$, $\mathcal{D}_3^- = 0.179$, $\mathcal{D}_4^- = 0.3111$ and $\mathcal{D}_5^- = 0.026$.

Step 6: Calculate the relative closeness values of each alternative as $\mathcal{D}_1^* = 0.6144$, $\mathcal{D}_2^* = 0.1671$, $\mathcal{D}_3^* = 0.7992$, $\mathcal{D}_4^* = 0.7811$ and $\mathcal{D}_5^* = 0.0944$.

Step 7: Ranking of alternatives is $\mathcal{E}_3 \geq \mathcal{E}_4 \geq \mathcal{E}_1 \geq \mathcal{E}_2 \geq \mathcal{E}_5$.

It is observed that the third college education is the best for students in effective manner. Finally parents select the third college because of the following reasons.

1) Campus environment is found to be the best among the remaining colleges.
2) Overall cost is better than the any other colleges.
3) Academic quality is evaluated to be falling in line with the expectation of parents.
4) Career development is the best than any other college.

6.3. **Applicability for the proposed and existing methods.** In this subsection, discuss the comparison between the existing models and proposed models, which demonstrates its applicability and advantages. Use ST-FNFWA, ST-FNFWG, ST-GFNFWA, and ST-GFNFWG approaches based on Hamming distance. The various distances are as shown below.

$\Xi = 1$	ST-FNFWA	ST-FNFWG	ST-GFNFWA	ST-GFNFWG
TOPSIS Hamming distance [6]	$\mathcal{E}_3 \geq \mathcal{E}_2 \geq \mathcal{E}_1$ $\mathcal{E}_5 \geq \mathcal{E}_4$	$\mathcal{E}_3 \geq \mathcal{E}_2 \geq \mathcal{E}_1$ $\mathcal{E}_5 \geq \mathcal{E}_4$	$\mathcal{E}_3 \geq \mathcal{E}_2 \geq \mathcal{E}_1$ $\mathcal{E}_5 \geq \mathcal{E}_4$	$\mathcal{E}_3 \geq \mathcal{E}_2 \geq \mathcal{E}_1$ $\mathcal{E}_5 \geq \mathcal{E}_4$
TOPSIS Hamming distance (**proposed**)	$\mathcal{E}_3 \geq \mathcal{E}_4 \geq \mathcal{E}_1$ $\mathcal{E}_2 \geq \mathcal{E}_5$	$\mathcal{E}_3 \geq \mathcal{E}_4 \geq \mathcal{E}_5$ $\mathcal{E}_1 \geq \mathcal{E}_2$	$\mathcal{E}_3 \geq \mathcal{E}_4 \geq \mathcal{E}_1$ $\mathcal{E}_2 \geq \mathcal{E}_5$	$\mathcal{E}_3 \geq \mathcal{E}_4 \geq \mathcal{E}_5$ $\mathcal{E}_1 \geq \mathcal{E}_2$

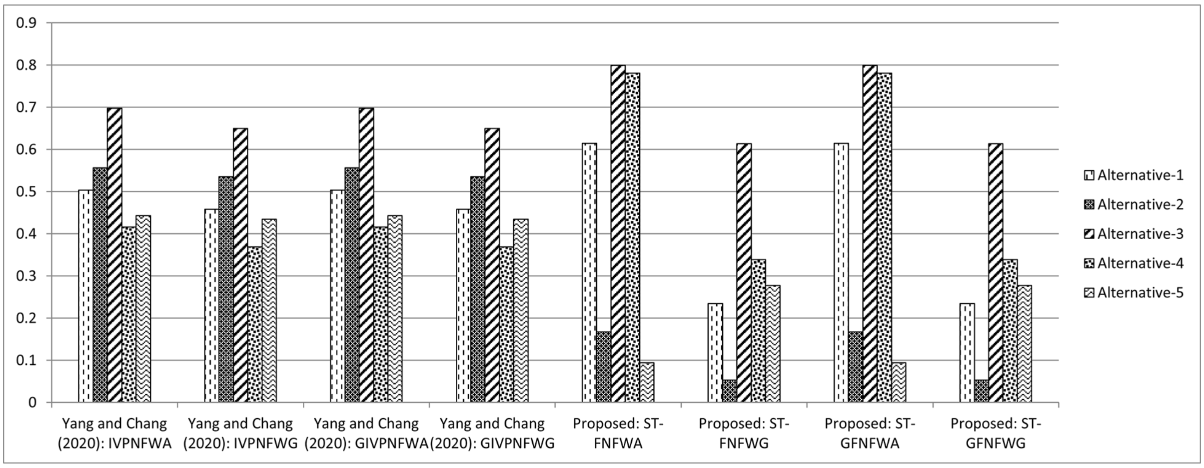

FIGURE 1. Hamming distance based on the existing and proposed methods

Suppose that $\Xi = 2$ values from ST-FNFWA method.

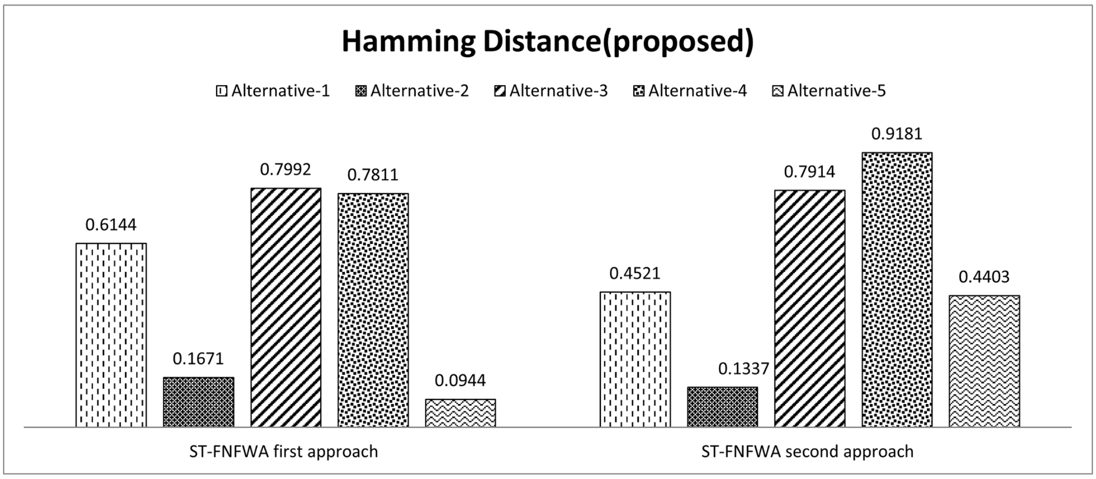

FIGURE 2. Hamming distance based on ST-FNFWA method

We note that the alternative ranking is based on the ST-FNFWA operator. If $\Xi = 1$, then ranking of alternative is $\mathcal{E}_3 \geq \mathcal{E}_4 \geq \mathcal{E}_1 \geq \mathcal{E}_2 \geq \mathcal{E}_5$; If $\Xi = 2$, then ranking of alternative in a new order is $\mathcal{E}_4 \geq \mathcal{E}_3 \geq \mathcal{E}_1 \geq \mathcal{E}_5 \geq \mathcal{E}_2$. As a result, the best alternative is to change $\mathcal{E}_3$ into $\mathcal{E}_4$. Similarly, the alternative ranking is found based on ST-FNFWG, ST-GFNFWA, and ST-GFNFWG operators with Ξ.

7. **Conclusion.** In this article, we construct a Hamming distance for ST-FNFNs. The applicability of the Hamming distance measure is established in a real-life example. We have proposed the improved sine trigonometric aggregation operation rules for ST-FNFWA, ST-FNFWG, ST-GFNFWA, and ST-GFNFWG. The application of the ST-FNFS MADM can help people make the correct decision out of available alternatives in indeterminate and inconsistent information environments. We have applied the ST-FNFWA, ST-FNFWG, ST-GFNFWA, and ST-GFNFWG operators to MADM based on Ξ. The distinct ranking of alternatives can be obtained with ST-FNFWA, ST-FNFWG, ST-GFNFWA, and ST-GFNFWG operators based on Ξ. Lastly, the above analysis shows that the generalized values of Ξ have the most impressive ranking of alternatives. The decision-makers may set the values of Ξ according to the actual situation for the best reasonable ranking and then make appropriate decisions. As a result, the decision-maker may make a decision based on Ξ to arrive at the result. This is an emerging field of study, and the authors are confident that the discussions in this article will be helpful to future researchers interested in this area of research.

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