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ROBOTIC ENGINEERING SELECTION BASED ON POSSIBILITY PYTHAGOREAN NEUTROSOPHIC VAGUE SOFT SETS AND ITS APPLICATION

M. Palanikumar[1], K. Arulmozhi[2], Aiyared Iampan[3,*] and G. Shanmugam[1]

[1]Department of Mathematics
Saveetha School of Engineering
Saveetha University
Saveetha Institute of Medical and Technical Sciences
Tamil Nadu, Chennai 602105, India
{ palanimaths86; gsm.maths }@gmail.com

[2]Department of Mathematics
Bharath Institute of Higher Education and Research
Tamil Nadu, Chennai 600073, India
arulmozhiems@gmail.com

[3]Fuzzy Algebras and Decision-Making Problems Research Unit
Department of Mathematics
School of Science
University of Phayao
19 Moo 2, Tambon Mae Ka, Amphur Mueang, Phayao 56000, Thailand
*Corresponding author: aiyared.ia@up.ac.th

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Abstract. *We discuss the possibility Pythagorean neutrosophic vague soft set (PPyNS-VSS). In addition, for the case of PPyNSVSS, we define some related operations such as complement, union, intersection, AND, and OR, as well as commutative laws, De Morgan's laws, associative laws, and distributive laws of holds. Also, we discuss comparison between the PPyNSVSS and the Pythagorean neutrosophic vague soft set (PyNSVSS) for dealing with decision-making problems and finding a similarity measure. Practical examples are provided to strengthen our results.*
Keywords: PyNSVSS, PPyNSVSS, Vague soft set, Decision-making problem

1. **Introduction.** A fuzzy set (FS) is used to model the uncertainty using the membership grade [1], intuitionistic fuzzy set [2]. A neutrosophic set (NSS) [3] and a Pythagorean fuzzy set (PyFS) [4] are used to quantify uncertainty using the truth, indeterminacy, and falsity membership grades. Zadeh [1] introduced FS, which suggests that decision-makers solve uncertain problems by taking membership degree into account. The concept of an intuitionistic fuzzy set (IFS) is introduced by Atanassov and is characterized by a degree of membership and non-membership satisfying the condition that the sum of its membership degree and non-membership degree does not exceed one [2]. However, we may encounter a problem in decision-making events where the sum of the degree of membership and non-membership of a particular attribute exceeds one. So Yager [4] introduced the concept of PyFS. It has been extended to intuitionistic fuzzy sets and is distinguished by the requirement that the square sum of its degree of membership and non-membership does not exceed one. A few years ago, the NSS was introduced by Smarandache [5]. The

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term "neutosophy" means knowledge of neutrality, and neutrality means the contrast between FS and IFS theory. It is a grade of truth, a grade of indeterminacy, and a grade of falsity. The NSS is a set in which each universal element has a truth, indeterminacy, and falsity grade that ranges between 0 and 1. The NSS is a generalization of the classical set, FS, etc. Jansi et al. introduced the PyNSS with many applications elaborated [6].

The theory of soft sets was proposed by Molodtsov [7]. In comparison with other uncertain theories, soft sets more accurately reflect the objectivity and complexity of decision-making in actual situations. Moreover, the combination of soft sets with other mathematical models is also a critical research area. For example, Maji et al. proposed the concept of fuzzy soft set (FSS) [8] and intuitionistic fuzzy soft set (IFSS) [9]. These two theories are applied to solving various decision-making problems. Alkhazaleh et al. [10] defined the concept of possibility fuzzy soft sets (PFSS), where a possibility of each element in the universe is attached with the parameterization of fuzzy sets while defining a fuzzy soft set. Karaaslan discussed the notion of possibility in neutrosophic soft sets using decision-making [11]. Palanikumar et al. discussed the notion of an possibility Pythagorean neutrosophic soft sets and its application of decision-making [12]. Palanikumar et al. discussed various applications based on decision-making approach [13, 14, 15]. Broumi et al. interacted with the concept of the neutrosophic soft set and its application [16]. An application of single-valued NSS is based on medical diagnosis [17] and context analysis [18]. Ejegwa [19] extended the distances for IFSs, such as Hamming and Euclidean with normalized distances, and various similarities to PyFSs, and applied them to multi criteria decision-making (MCDM) problems and the same to multi attribute decision-making (MADM) problems. According to [20], Biswas et al. launched the vague set (VS). A VS is defined by two functions, say truth-membership t_v and false-membership f_v, where $t_v(x)$ denotes the lower bound on the grade of membership of x derived from the evidence for x, and $f_v(x)$ denotes the upper bound on the grade of membership of x derived from the evidence for x and $t_v(x)$ and $f_v(x)$ belonging to $[0, 1]$, where the sum of its $t_v(x)$ and $f_v(x)$ does not exceed 1. The VS is an extension of FS and IFS and some applications established [21, 22, 23].

In recent years, Peng et al. [24] have extended the FSS to the Pythagorean fuzzy soft set. This model solves a class of MADM consisting of the sum of the degree of membership and non-membership values exceeding one, but the sum of the squares is equal to or not exceeding one. In general, the possibility of belongingness of the elements should be considered in multi attribute decision-making problems. However, Peng et al. [24] failed to do it. As for the problem, the purpose of this paper is to extend the concept of PPyNSVSS to parameterization of possibility Pythagorean neutrosophic vague set. We obtain a PPyNSVSS model with algorithm. We shall further establish a similarity measure method using this model and apply it to decision-making problems with suitable examples.

2. Preliminaries.

We will go over the ideas of neutrosophic set in this section to make the presentation as full as possible and to make the next talks more convenient.

Definition 2.1. [16] *A neutrosophic set A in the universe $\mathscr{U}$ is of the following form:* $\overline{A} = \{(u, \varepsilon_A^T(u), \varepsilon_A^I(u), \varepsilon_A^F(u)) \mid u \in \mathscr{U}\}$, *where $\varepsilon_A^T(u)$, $\varepsilon_A^I(u)$, $\varepsilon_A^F(u)$ represent the degree of truth-membership, degree of indeterminacy membership and degree of falsity-membership of A, respectively. The mapping $\varepsilon_A^T, \varepsilon_A^I, \varepsilon_A^F : \mathscr{U} \to [0, 1]$ and $0 \preceq \sup \varepsilon_A^T(u) + \sup \varepsilon_A^I(u) + \sup \varepsilon_A^F(u) \preceq 3$.*

Definition 2.2. [6] *A Pythagorean neutrosophic set (PyNSS) A in $\mathscr{U}$ is of the form:* $\overline{A} = \{(u, \varepsilon_A^T(u), \varepsilon_A^I(u), \varepsilon_A^F(u)) \mid u \in \mathscr{U}\}$, *where $\varepsilon_A^T(u)$, $\varepsilon_A^I(u)$, $\varepsilon_A^F(u)$ represent the degree of* *truth-membership, degree of indeterminacy membership and degree of falsity-membership of* A*, respectively. The mapping* $\varepsilon_A^T, \varepsilon_A^I, \varepsilon_A^F : \mathscr{U} \to [0,1]$ *and* $0 \preceq \left(\varepsilon_A^T(u)\right)^2 + \left(\varepsilon_A^I(u)\right)^2 + \left(\varepsilon_A^F(u)\right)^2 \preceq 2$. $\overline{A} = \left\langle \varepsilon_A^T, \varepsilon_A^I, \varepsilon_A^F \right\rangle$ *is called a Pythagorean neutrosophic number (PyNSN).*

Definition 2.3. [16, 6] *Let* $\overline{\kappa_1} = \left\langle \varepsilon_{\kappa_1}^T, \varepsilon_{\kappa_1}^I, \varepsilon_{\kappa_1}^F \right\rangle$, $\overline{\kappa_2} = \left\langle \varepsilon_{\kappa_2}^T, \varepsilon_{\kappa_2}^I, \varepsilon_{\kappa_2}^F \right\rangle$ *and* $\overline{\kappa_3} = \left\langle \varepsilon_{\kappa_3}^T, \varepsilon_{\kappa_3}^I, \varepsilon_{\kappa_3}^F \right\rangle$ *be the three PyNSNs over* $(\mathscr{U}, \mathcal{E})$*. Then*
(i) $\overline{\kappa_1}^c = \left\langle \varepsilon_{\kappa_1}^F, \varepsilon_{\kappa_1}^I, \varepsilon_{\kappa_1}^T \right\rangle$,
(ii) $\overline{\kappa_2} \bigvee \overline{\kappa_3} = \left\langle \max\left(\varepsilon_{\kappa_2}^T, \varepsilon_{\kappa_3}^T\right), \min\left(\varepsilon_{\kappa_2}^I, \varepsilon_{\kappa_3}^I\right), \min\left(\varepsilon_{\kappa_2}^F, \varepsilon_{\kappa_3}^F\right)\right\rangle$,
(iii) $\overline{\kappa_2} \bigwedge \overline{\kappa_3} = \left\langle \min\left(\varepsilon_{\kappa_2}^T, \varepsilon_{\kappa_3}^T\right), \max\left(\varepsilon_{\kappa_2}^I, \varepsilon_{\kappa_3}^I\right), \max\left(\varepsilon_{\kappa_2}^F, \varepsilon_{\kappa_3}^F\right)\right\rangle$,
(iv) $\overline{\kappa_2} \succeq \overline{\kappa_3}$ *if and only if* $\varepsilon_{\kappa_2}^T \succeq \varepsilon_{\kappa_3}^T$ *and* $\varepsilon_{\kappa_2}^I \preceq \varepsilon_{\kappa_3}^I$ *and* $\varepsilon_{\kappa_2}^F \preceq \varepsilon_{\kappa_3}^F$,
(v) $\overline{\kappa_2} = \overline{\kappa_3}$ *if and only if* $\varepsilon_{\kappa_2}^T = \varepsilon_{\kappa_3}^T$ *and* $\varepsilon_{\kappa_2}^I = \varepsilon_{\kappa_3}^I$ *and* $\varepsilon_{\kappa_2}^F = \varepsilon_{\kappa_3}^F$.

Definition 2.4. [24] *Let* $\mathscr{U}$ *be the universe and* $\mathcal{E}$ *be the set of parameters. The pair* $\left(\dot{\mathscr{O}}, A\right)$ *is called a Pythagorean fuzzy soft set (PyFSS) on* $\mathscr{U}$ *if* $A \sqsubseteq \mathcal{E}$ *and* $\dot{\mathscr{O}} : A \to P\dot{\mathscr{O}}(\mathscr{U})$*, where* $P\dot{\mathscr{O}}(\mathscr{U})$ *is the set of all Pythagorean fuzzy subsets of* $\mathscr{U}$*.*

Definition 2.5. [10] *Let* $\mathscr{U}$ *be the universe and* $\mathcal{E}$ *be the set of parameters. The pair* $(\mathscr{U}, \mathcal{E})$ *is a soft universe. Consider the mapping* $\dot{\mathscr{O}} : \mathcal{E} \to \dot{\mathscr{O}}(\mathscr{U})$ *and* ε *is a fuzzy subset of* $\mathcal{E}$*, i.e.,* $\varepsilon : \mathcal{E} \to \dot{\mathscr{O}}(\mathscr{U})$*. Let* $\dot{\mathscr{O}}_\varepsilon : \mathcal{E} \to \dot{\mathscr{O}}(\mathscr{U}) \times \dot{\mathscr{O}}(\mathscr{U})$ *be a function defined as* $\dot{\mathscr{O}}_\varepsilon(e) = \left(\dot{\mathscr{O}}(e)(u), \varepsilon(e)(u)\right)$, $\forall u \in \mathscr{U}$*. Then* $\dot{\mathscr{O}}_\varepsilon$ *is called a possibility fuzzy soft set (PFSS) on* $(\mathscr{U}, \mathcal{E})$*.*

Definition 2.6. [20] *(i) A vague set* L *in* $\mathscr{U}$ *is a pair* $\left(\mathscr{T}_L, \dot{\mathscr{O}}_L\right)$*, where* $\mathscr{T}_L : \mathscr{U} \to [0,1]$, $\dot{\mathscr{O}}_L : \mathscr{U} \to [0,1]$ *are mappings such that* $\mathscr{T}_L(u) + \dot{\mathscr{O}}_L(u) \preceq 1$, $\forall u \in \mathscr{U}$*. The functions* $\mathscr{T}_L$ *and* $\dot{\mathscr{O}}_L$ *are called true membership function and false membership function, respectively.*
(ii) The interval $\left[\mathscr{T}_L(u), 1 - \dot{\mathscr{O}}_L(u)\right]$ *is called the vague value of* u *in* L *and it is denoted by* $V_L(u)$*, i.e.,* $V_L(u) = \left[\mathscr{T}_L(u), 1 - \dot{\mathscr{O}}_L(u)\right]$.

Definition 2.7. [20] *(i) A vague set* L *is contained in the other vague set* M*,* $L \subseteq M$ *if and only if* $V_L(u) \preceq V_M(u)$*, i.e.,* $\mathscr{T}_L(u) \preceq \mathscr{T}_M(u)$ *and* $1 - \dot{\mathscr{O}}_L(u) \preceq 1 - \dot{\mathscr{O}}_M(u)$, $\forall u \in \mathscr{U}$*.*
(ii) The union of two vague sets L *and* M*, as* $N = L \cup M$*,* $\mathscr{T}_N = \max\{\mathscr{T}_L, \mathscr{T}_M\}$ *and* $1 - \dot{\mathscr{O}}_N = \max\left\{1 - \dot{\mathscr{O}}_L, 1 - \dot{\mathscr{O}}_M\right\} = 1 - \min\left\{\dot{\mathscr{O}}_L, \dot{\mathscr{O}}_M\right\}$.
(iii) The intersection of two vague sets L *and* M *as* $N = L \cap M$*,* $\mathscr{T}_N = \min\{\mathscr{T}_L, \mathscr{T}_M\}$ *and* $1 - \dot{\mathscr{O}}_N = \min\left\{1 - \dot{\mathscr{O}}_L, 1 - \dot{\mathscr{O}}_M\right\} = 1 - \max\left\{\dot{\mathscr{O}}_L, \dot{\mathscr{O}}_M\right\}$.

Definition 2.8. [20] *A vague set* L *of a set* $\mathscr{U}$*,* $\forall u \in \mathscr{U}$ *with*
(i) $\mathscr{T}_L(u) = 0$ *and* $\dot{\mathscr{O}}_L(u) = 1$ *is called zero vague set of* $\mathscr{U}$*,*
(ii) $\mathscr{T}_L(u) = 1$ *and* $\dot{\mathscr{O}}_L(u) = 0$ *is called unit vague set of* $\mathscr{U}$*.*

3. **Possibility Pythagorean Neutrosophic Vague Soft Set.** In this section, we introduced the basic concept of possibility Pythagorean neutrosophic vague soft set and some of its algebraic operations.

Definition 3.1. *Let* $\mathscr{U}$ *be the universe and* $\mathcal{E}$ *be the set of parameters. The pair* $(\mathscr{U}, \mathcal{E})$ *is called a soft universe. Suppose that* $\overline{\dot{\mathscr{O}}} : \mathcal{E} \to \overline{P\dot{\mathscr{O}}(\mathscr{U})}$*, and* $\overline{p}$ *is a possibility neutrosophic set of* $\mathcal{E}$*, i.e.,* $\overline{p} : \mathcal{E} \to \overline{P\dot{\mathscr{O}}(\mathscr{U})}$*, where* $\overline{P\dot{\mathscr{O}}(\mathscr{U})}$ *denotes the collection of all* *possibility Pythagorean neutrosophic vague set of* $\mathscr{U}$. *If* $\overline{\dot{\mathscr{O}}}_p : \mathcal{E} \to \overline{P\dot{\mathscr{O}}(\mathscr{U})} \times \overline{P\dot{\mathscr{O}}(\mathscr{U})}$ *is a function defined as* $\overline{\dot{\mathscr{O}}}_p(e) = \left(\overline{\dot{\mathscr{O}}(e)(u)}, \overline{p(e)(u)}\right)$, $u \in \mathscr{U}$, *then* $\overline{\dot{\mathscr{O}}}_p$ *is a PPyNVSS on* $(\mathscr{U}, \mathcal{E})$. *For each parameter* e, $\overline{\dot{\mathscr{O}}}_p(e) = \left\{\left\langle u, \left(\left[\varepsilon^{T^-}_{\dot{\mathscr{O}}}(e)(u), 1-\varepsilon^{F^-}_{\dot{\mathscr{O}}}(e)(u)\right], \left[\varepsilon^{I^-}_{\dot{\mathscr{O}}}(e)(u), \varepsilon^{I^+}_{\dot{\mathscr{O}}}(e)(u)\right], \left[\varepsilon^{F^-}_{\dot{\mathscr{O}}}(e)(u), 1-\varepsilon^{T^-}_{\dot{\mathscr{O}}}(e)(u)\right]\right), \left(\varepsilon^T_p(e)(u), \varepsilon^I_p(e)(u), \varepsilon^F_p(e)(u)\right)\right\rangle, u \in \mathscr{U}\right\}$.

Example 3.1. *Let* $R = \{R_1, R_2, R_3\}$ *be the set of three robotics of a decision-maker to* $\mathcal{E} = \{e_1 = \text{speed}, e_2 = \text{precision}, e_3 = \text{completion of work}\}$ *being a set of parameters. Suppose that* $\overline{\dot{\mathscr{O}}}_p : \mathcal{E} \to \overline{P\dot{\mathscr{O}}(\mathscr{U})} \times \overline{P\dot{\mathscr{O}}(\mathscr{U})}$ *is given by*

$$\overline{\dot{\mathscr{O}}_p(e_1)} = \begin{cases} \frac{u_1}{\langle([0.7,0.75],[0.8,0.85],[0.25,0.3]),(0.85,0.75,0.65)\rangle} \\ \frac{u_2}{\langle([0.6,0.65],[0.5,0.7],[0.35,0.4]),(0.55,0.45,0.15)\rangle} \\ \frac{u_3}{\langle([0.4,0.5],[0.6,0.8],[0.5,0.6]),(0.65,0.45,0.25)\rangle}; \end{cases}$$

$$\overline{\dot{\mathscr{O}}_p(e_2)} = \begin{cases} \frac{u_1}{\langle([0.4,0.8],[0.8,0.9],[0.2,0.6]),(0.75,0.55,0.45)\rangle} \\ \frac{u_2}{\langle([0.5,0.7],[0.6,0.7],[0.3,0.5]),(0.55,0.35,0.25)\rangle} \\ \frac{u_3}{\langle([0.5,0.8],[0.8,0.85],[0.2,0.5]),(0.85,0.65,0.55)\rangle}. \end{cases}$$

Definition 3.2. *Let* $\mathscr{U}$ *be a non-empty set of the universe and* $\mathcal{E}$ *be a set of parameters. Suppose that* $\overline{\dot{\mathscr{O}}}_p$ *and* $\overline{\ddot{\mathscr{O}}}_q$ *are two PPyNVSSs on* $(\mathscr{U}, \mathcal{E})$. *Now* $\overline{\ddot{\mathscr{O}}}_q$ *is a possibility Pythagorean neutrosophic vague soft subset of* $\overline{\dot{\mathscr{O}}}_p$ *(denoted by* $\overline{\ddot{\mathscr{O}}}_q \sqsubseteq \overline{\dot{\mathscr{O}}}_p$*) if and only if*

(i) $\overline{\ddot{\mathscr{O}}(e)(u)} \sqsubseteq \overline{\dot{\mathscr{O}}(e)(u)}$ *if* $\varepsilon^{T^-}_{\dot{\mathscr{O}}}(e)(u) \succeq \varepsilon^{T^-}_{\ddot{\mathscr{O}}}(e)(u)$, $1-\varepsilon^{F^-}_{\dot{\mathscr{O}}}(e)(u) \succeq 1-\varepsilon^{F^-}_{\ddot{\mathscr{O}}}(e)(u)$, $\varepsilon^{I^-}_{\dot{\mathscr{O}}}(e)(u) \succeq \varepsilon^{I^-}_{\ddot{\mathscr{O}}}(e)(u)$, $\varepsilon^{I^+}_{\dot{\mathscr{O}}}(e)(u) \succeq \varepsilon^{I^+}_{\ddot{\mathscr{O}}}(e)(u)$ *and* $\varepsilon^{F^-}_{\dot{\mathscr{O}}}(e)(u) \preceq \varepsilon^{F^-}_{\ddot{\mathscr{O}}}(e)(u)$, $1-\varepsilon^{T^-}_{\dot{\mathscr{O}}}(e)(u) \preceq 1-\varepsilon^{T^-}_{\ddot{\mathscr{O}}}(e)(u)$,

(ii) $\overline{q(e)(u)} \sqsubseteq \overline{p(e)(u)}$ *if* $\varepsilon^T_p(e)(u) \succeq \varepsilon^T_q(e)(u)$, $\varepsilon^I_p(e)(u) \succeq \varepsilon^I_q(e)(u)$, $\varepsilon^F_p(e)(u) \preceq \varepsilon^F_q(e)(u)$, $\forall e \in \mathcal{E}$ *and* $\forall u \in \mathscr{U}$.

Definition 3.3. *Let* $\mathscr{U}$ *be the universe and* $\mathcal{E}$ *be the set of parameters. Suppose that* $\overline{\dot{\mathscr{O}}}_p$ *and* $\overline{\ddot{\mathscr{O}}}_q$ *are the PPyNVSSs on* $(\mathscr{U}, \mathcal{E})$. *Now* $\overline{\dot{\mathscr{O}}}_p$ *and* $\overline{\ddot{\mathscr{O}}}_q$ *are equal (denoted by* $\overline{\dot{\mathscr{O}}}_p = \overline{\ddot{\mathscr{O}}}_q$*) if and only if* $\overline{\dot{\mathscr{O}}}_p \sqsubseteq \overline{\ddot{\mathscr{O}}}_q$ *and* $\overline{\dot{\mathscr{O}}}_p \sqsupseteq \overline{\ddot{\mathscr{O}}}_q$.

Definition 3.4. *Let* $\mathscr{U}$ *be the universe and* $\mathcal{E}$ *be the set of parameters. Let* $\overline{\dot{\mathscr{O}}}_p$ *be a PPyNVSS on* $(\mathscr{U}, \mathcal{E})$. *The complement of* $\overline{\dot{\mathscr{O}}}_p$ *is defined by* $\overline{\dot{\mathscr{O}}}^c_p = \left\langle \overline{\dot{\mathscr{O}}^c(e)(u)}, \overline{p^c(e)(u)}\right\rangle$, *where* $\overline{\dot{\mathscr{O}}^c(e)(u)} = \left\langle\left[1-\left(1-\varepsilon^{F^-}_{\dot{\mathscr{O}}}(e)(u)\right), 1-\varepsilon^{T^-}_{\dot{\mathscr{O}}}(e)(u)\right], \left[1-\varepsilon^{I^+}_{\dot{\mathscr{O}}}(e)(u), 1-\varepsilon^{I^-}_{\dot{\mathscr{O}}}(e)(u)\right], \left[1-\left(1-\varepsilon^{T^-}_{\dot{\mathscr{O}}}(e)(u)\right), 1-\varepsilon^{F^-}_{\dot{\mathscr{O}}}(e)(u)\right]\right\rangle$, $\overline{p^c(e)(u)} = \left\langle 1-\varepsilon^T_p(e)(u), 1-\varepsilon^I_p(e)(u), 1-\varepsilon^F_p(e)(u)\right\rangle$. *Clearly,* $\left(\overline{\dot{\mathscr{O}}}^c_p\right)^c = \overline{\dot{\mathscr{O}}}_p$.

Definition 3.5. *Let* $\mathscr{U}$ *be the universe and* $\mathcal{E}$ *be the set of parameters. Let* $\overline{\dot{\mathscr{O}}}_p$ *and* $\overline{\ddot{\mathscr{O}}}_q$ *be the PPyNVSSs on* $(\mathscr{U}, \mathcal{E})$. *The union and intersection of* $\overline{\dot{\mathscr{O}}}_p$ *and* $\overline{\ddot{\mathscr{O}}}_q$ *over* $(\mathscr{U}, \mathcal{E})$ *are denoted by* $\overline{\dot{\mathscr{O}}}_p \bigvee \overline{\ddot{\mathscr{O}}}_q$ *and* $\overline{\dot{\mathscr{O}}}_p \bigwedge \overline{\ddot{\mathscr{O}}}_q$ *respectively and are defined by* $\overline{J}_j : \mathcal{E} \to \overline{P\dot{\mathscr{O}}(\mathscr{U})} \times \overline{P\dot{\mathscr{O}}(\mathscr{U})}$, $\overline{I}_i : \mathcal{E} \to \overline{P\dot{\mathscr{O}}(\mathscr{U})} \times \overline{P\dot{\mathscr{O}}(\mathscr{U})}$ *such that* $\overline{J}_j(e)(u) = \left\langle \overline{J(e)(u)}, \overline{j(e)(u)}\right\rangle$, $\overline{I}_i(e)(u) =$

$\left\langle\overline{I(e)(u)}, \overline{i(e)(u)}\right\rangle$, *where* $\overline{J(e)(u)} = \overline{\dot{\mathscr{O}}(e)(u)} \bigvee \overline{\ddot{\mathscr{O}}(e)(u)}$, $\overline{j(e)(u)} = \overline{p(e)(u)} \bigvee \overline{q(e)(u)}$, $\overline{I(e)(u)} = \overline{\dot{\mathscr{O}}(e)(u)} \bigwedge \overline{\ddot{\mathscr{O}}(e)(u)}$ *and* $\overline{i(e)(u)} = \overline{p(e)(u)} \bigwedge \overline{q(e)(u)}$, $\forall u \in \mathscr{U}$.

Example 3.2. *Let* $\overline{\dot{\mathscr{O}}_p}$ *and* $\overline{\ddot{\mathscr{O}}_q}$ *be the two PPyNVSSs on* $(\mathscr{U}, \mathcal{E})$ *defined by*

$$\overline{\dot{\mathscr{O}}_p(e_1)} = \begin{cases} \frac{u_1}{\langle([0.5,0.75],[0.7,0.8],[0.25,0.5]),(0.45,0.35,0.75)\rangle} \\ \frac{u_2}{\langle([0.5,0.85],[0.6,0.85],[0.15,0.5]),(0.65,0.75,0.55)\rangle} \\ \frac{u_3}{\langle([0.7,0.85],[0.7,0.9],[0.15,0.3]),(0.85,0.45,0.35)\rangle}; \end{cases}$$

$$\overline{\dot{\mathscr{O}}_p(e_2)} = \begin{cases} \frac{u_1}{\langle([0.6,0.9],[0.7,0.85],[0.1,0.4]),(0.75,0.55,0.65)\rangle} \\ \frac{u_2}{\langle([0.6,0.8],[0.6,0.8],[0.2,0.4]),(0.65,0.95,0.85)\rangle} \\ \frac{u_3}{\langle([0.4,0.75],[0.5,0.75],[0.25,0.6]),(0.55,0.45,0.35)\rangle}; \end{cases}$$

$$\overline{\ddot{\mathscr{O}}_q(e_1)} = \begin{cases} \frac{u_1}{\langle([0.45,0.7],[0.65,0.75],[0.3,0.55]),(0.25,0.65,0.85)\rangle} \\ \frac{u_2}{\langle([0.4,0.8],[0.65,0.8],[0.2,0.6]),(0.35,0.75,0.45)\rangle} \\ \frac{u_3}{\langle([0.4,0.65],[0.75,0.85],[0.35,0.6]),(0.75,0.55,0.85)\rangle}; \end{cases}$$

$$\overline{\ddot{\mathscr{O}}_q(e_2)} = \begin{cases} \frac{u_1}{\langle([0.65,0.8],[0.6,0.85],[0.2,0.35]),(0.85,0.45,0.65)\rangle} \\ \frac{u_2}{\langle([0.65,0.85],[0.65,0.85],[0.15,0.35]),(0.75,0.65,0.35)\rangle} \\ \frac{u_3}{\langle([0.5,0.8],[0.55,0.7],[0.2,0.5]),(0.35,0.55,0.45)\rangle}. \end{cases}$$

$\overline{\dot{\mathscr{O}}_p} \bigvee \overline{\ddot{\mathscr{O}}_q}$ *is determined by*

$$\overline{\dot{\mathscr{O}}_p} \bigvee \overline{\ddot{\mathscr{O}}_q}(e_1) = \begin{cases} \frac{u_1}{\langle([0.5,0.75],[0.65,0.75],[0.25,0.5]),(0.45,0.35,0.75)\rangle} \\ \frac{u_2}{\langle([0.5,0.85],[0.6,0.8],[0.15,0.5]),(0.65,0.75,0.45)\rangle} \\ \frac{u_3}{\langle([0.7,0.85],[0.7,0.85],[0.15,0.3]),(0.85,0.45,0.35)\rangle}; \end{cases}$$

$$\overline{\dot{\mathscr{O}}_p} \bigvee \overline{\ddot{\mathscr{O}}_q}(e_2) = \begin{cases} \frac{u_1}{\langle([0.65,0.9],[0.6,0.85],[0.1,0.35]),(0.85,0.45,0.65)\rangle} \\ \frac{u_2}{\langle([0.65,0.85],[0.6,0.8],[0.15,0.35]),(0.75,0.65,0.35)\rangle} \\ \frac{u_3}{\langle([0.5,0.8],[0.5,0.7],[0.2,0.5]),(0.55,0.45,0.35)\rangle}. \end{cases}$$

$\overline{\dot{\mathscr{O}}_p} \bigwedge \overline{\ddot{\mathscr{O}}_q}$ *is determined by*

$$\overline{\dot{\mathscr{O}}_p} \bigwedge \overline{\ddot{\mathscr{O}}_q}(e_1) = \begin{cases} \frac{u_1}{\langle([0.45,0.7],[0.7,0.8],[0.3,0.55]),(0.25,0.65,0.85)\rangle} \\ \frac{u_2}{\langle([0.4,0.8],[0.65,0.85],[0.2,0.6]),(0.35,0.75,0.55)\rangle} \\ \frac{u_3}{\langle([0.4,0.65],[0.75,0.9],[0.35,0.6]),(0.75,0.55,0.85)\rangle}; \end{cases}$$

$$\overline{\dot{\mathscr{O}}_p} \bigwedge \overline{\ddot{\mathscr{O}}_q}(e_2) = \begin{cases} \frac{u_1}{\langle([0.6,0.8],[0.7,0.85],[0.2,0.4]),(0.75,0.55,0.65)\rangle} \\ \frac{u_2}{\langle([0.6,0.8],[0.65,0.85],[0.2,0.4]),(0.65,0.95,0.85)\rangle} \\ \frac{u_3}{\langle([0.4,0.75],[0.55,0.75],[0.25,0.6]),(0.35,0.55,0.45)\rangle}. \end{cases}$$

Definition 3.6. *A PPyNVSS* $\overline{\emptyset_\theta(e)(u)} = \left\langle \overline{\emptyset(e)(u)}, \overline{\theta(e)(u)} \right\rangle$ *is said to be a possibility null Pythagorean neutrosophic vague soft set* $\overline{\emptyset_\theta} : \mathcal{E} \to \overline{P\dot{\mathscr{O}}(\mathscr{U})} \times \overline{P\dot{\mathscr{O}}(\mathscr{U})}$, *where* $\overline{\emptyset(e)(u)} = ([0,0],[1,1],[1,1])$ *and* $\overline{\theta(e)(u)} = ([0,0],[1,1],[1,1])$, $\forall u \in \mathscr{U}$.

Definition 3.7. *A PPyNVSS* $\overline{\Omega_\Lambda(e)(u)} = \left\langle \overline{\Omega(e)(u)}, \overline{\Lambda(e)(u)} \right\rangle$ *is said to be a possibility absolute Pythagorean neutrosophic vague soft set* $\overline{\Omega_\Lambda} : \mathcal{E} \to \overline{P\dot{\mathscr{O}}(\mathscr{U})} \times \overline{P\dot{\mathscr{O}}(\mathscr{U})}$, *where* $\overline{\Omega(e)(u)} = ([1,1],[0,0],[0,0])$ *and* $\overline{\Lambda(e)(u)} = ([1,1],[0,0],[0,0])$, $\forall u \in \mathscr{U}$.

Theorem 3.1. *Let* $\overline{\dot{\mathscr{O}}_p}$ *be the PPyNVSS on* $(\mathscr{U}, \mathcal{E})$. *Then*

(i) $\overline{\dot{\mathscr{O}}_p} = \overline{\dot{\mathscr{O}}_p} \bigvee \overline{\dot{\mathscr{O}}_p}$, $\overline{\dot{\mathscr{O}}_p} = \overline{\dot{\mathscr{O}}_p} \bigwedge \overline{\dot{\mathscr{O}}_p}$,

(ii) $\overline{\dot{\mathscr{O}}_p} \sqsubseteq \overline{\dot{\mathscr{O}}_p} \bigvee \overline{\dot{\mathscr{O}}_p}$, $\overline{\dot{\mathscr{O}}_p} \sqsubseteq \overline{\dot{\mathscr{O}}_p} \bigwedge \overline{\dot{\mathscr{O}}_p}$,

(iii) $\overline{\dot{\mathscr{O}}_p} \bigvee \overline{\emptyset_\theta} = \overline{\dot{\mathscr{O}}_p}$, $\overline{\dot{\mathscr{O}}_p} \bigwedge \overline{\emptyset_\theta} = \overline{\emptyset_\theta}$,

(iv) $\overline{\dot{\mathscr{O}}_p} \bigvee \overline{\Omega_\Lambda} = \overline{\Omega_\Lambda}$, $\overline{\dot{\mathscr{O}}_p} \bigwedge \overline{\Omega_\Lambda} = \overline{\dot{\mathscr{O}}_p}$.

Proof: The proof of Theorem 3.1 is handed over to Definitions 3.6 and 3.7.

Remark 3.1. *Let* $\overline{\dot{\mathscr{O}}_p}$ *be the PPyNVSS on* $(\mathscr{U}, \mathcal{E})$. *If* $\overline{\dot{\mathscr{O}}_p} \neq \overline{\Omega_\Lambda}$ *or* $\overline{\dot{\mathscr{O}}_p} \neq \overline{\emptyset_\theta}$, *then* $\overline{\dot{\mathscr{O}}_p} \bigvee \overline{\dot{\mathscr{O}}^c_p} \neq \overline{\Omega_\Lambda}$ *and* $\overline{\dot{\mathscr{O}}_p} \bigwedge \overline{\dot{\mathscr{O}}^c_p} \neq \overline{\emptyset_\theta}$.

Theorem 3.2. *Let* $\overline{\dot{\mathscr{O}}_p}$, $\overline{\ddot{\mathscr{O}}_q}$ *and* $\overline{\dddot{\mathscr{O}}_r}$ *be the PPyNVSSs over* $(\mathscr{U}, \mathcal{E})$. *Then the commutative laws, De Morgan's laws, associative laws, and distributive laws hold:*

(1) $\overline{\dot{\mathscr{O}}_p} \bigvee \overline{\ddot{\mathscr{O}}_q} = \overline{\ddot{\mathscr{O}}_q} \bigvee \overline{\dot{\mathscr{O}}_p}$,

(2) $\overline{\dot{\mathscr{O}}_p} \bigwedge \overline{\ddot{\mathscr{O}}_q} = \overline{\ddot{\mathscr{O}}_q} \bigwedge \overline{\dot{\mathscr{O}}_p}$,

(3) $\overline{\dot{\mathscr{O}}_p} \bigvee \left(\overline{\ddot{\mathscr{O}}_q} \bigvee \overline{\dddot{\mathscr{O}}_r} \right) = \left(\overline{\dot{\mathscr{O}}_p} \bigvee \overline{\ddot{\mathscr{O}}_q} \right) \bigvee \overline{\dddot{\mathscr{O}}_r}$,

(4) $\overline{\dot{\mathscr{O}}_p} \bigwedge \left(\overline{\ddot{\mathscr{O}}_q} \bigwedge \overline{\dddot{\mathscr{O}}_r} \right) = \left(\overline{\dot{\mathscr{O}}_p} \bigwedge \overline{\ddot{\mathscr{O}}_q} \right) \bigwedge \overline{\dddot{\mathscr{O}}_r}$,

(5) $\left(\overline{\dot{\mathscr{O}}_p} \bigvee \overline{\ddot{\mathscr{O}}_q} \right)^c = \overline{\dot{\mathscr{O}}^c_p} \bigwedge \overline{\ddot{\mathscr{O}}^c_q}$,

(6) $\left(\overline{\dot{\mathscr{O}}_p} \bigwedge \overline{\ddot{\mathscr{O}}_q} \right)^c = \overline{\dot{\mathscr{O}}^c_p} \bigvee \overline{\ddot{\mathscr{O}}^c_q}$,

(7) $\left(\overline{\dot{\mathscr{O}}_p} \bigvee \overline{\ddot{\mathscr{O}}_q} \right) \bigwedge \overline{\dot{\mathscr{O}}_p} = \overline{\dot{\mathscr{O}}_p}$,

(8) $\left(\overline{\dot{\mathscr{O}}_p} \bigwedge \overline{\ddot{\mathscr{O}}_q} \right) \bigvee \overline{\dot{\mathscr{O}}_p} = \overline{\dot{\mathscr{O}}_p}$,

(9) $\overline{\dot{\mathscr{O}}_p} \bigvee \left(\overline{\ddot{\mathscr{O}}_q} \bigwedge \overline{\dddot{\mathscr{O}}_r} \right) = \left(\overline{\dot{\mathscr{O}}_p} \bigvee \overline{\ddot{\mathscr{O}}_q} \right) \bigwedge \left(\overline{\dot{\mathscr{O}}_p} \bigvee \overline{\dddot{\mathscr{O}}_r} \right)$,

(10) $\overline{\dot{\mathscr{O}}_p} \bigwedge \left(\overline{\ddot{\mathscr{O}}_q} \bigvee \overline{\dddot{\mathscr{O}}_r} \right) = \left(\overline{\dot{\mathscr{O}}_p} \bigwedge \overline{\ddot{\mathscr{O}}_q} \right) \bigvee \left(\overline{\dot{\mathscr{O}}_p} \bigwedge \overline{\dddot{\mathscr{O}}_r} \right)$.

Proof: The proof of Theorem 3.2 is handed over to Definitions 3.4 and 3.5.

Definition 3.8. *Let* $\left(\overline{\dot{\mathscr{O}}_p}, A \right)$ *and* $\left(\overline{\ddot{\mathscr{O}}_q}, B \right)$ *be the PPyNVSSs on* $(\mathscr{U}, \mathcal{E})$. *Then the operation "*$\left(\overline{\dot{\mathscr{O}}_p}, A \right)$ *AND* $\left(\overline{\ddot{\mathscr{O}}_q}, B \right)$*" is defined by* $\left(\overline{\dot{\mathscr{O}}_p}, A \right) \bigwedge \left(\overline{\ddot{\mathscr{O}}_q}, B \right) = \left(\overline{\dddot{\mathscr{O}}_r}, A \times B \right)$, *where* $\overline{\dddot{\mathscr{O}}_r(\theta, \kappa)} = \left\langle \overline{\dddot{\mathscr{O}}(\theta, \kappa)(u)}, \overline{r(\theta, \kappa)(u)} \right\rangle$ *such that* $\overline{\dddot{\mathscr{O}}(\theta, \kappa)} = \overline{\dot{\mathscr{O}}(\theta)} \bigwedge \overline{\ddot{\mathscr{O}}(\kappa)}$ *and* $\overline{r(\theta, \kappa)} = \overline{p(\theta)} \bigwedge \overline{q(\kappa)}$, $\forall (\theta, \kappa) \in A \times B$.

Definition 3.9. *Let* $\left(\overline{\dot{\mathscr{O}}_p}, A\right)$ *and* $\left(\overline{\ddot{\mathscr{O}}_q}, B\right)$ *be the PPyNVSSs on* $(\mathscr{U}, \mathcal{E})$*. Then the operation "*$\left(\overline{\dot{\mathscr{O}}_p}, A\right)$ *OR* $\left(\overline{\ddot{\mathscr{O}}_q}, B\right)$*" is defined by* $\left(\overline{\dot{\mathscr{O}}_p}, A\right) \bigvee \left(\overline{\ddot{\mathscr{O}}_q}, B\right) = \left(\overline{\dddot{\mathscr{O}}_r}, A \times B\right)$*, where* $\overline{\dddot{\mathscr{O}}_r(\theta,\kappa)} = \left\langle \overline{\dddot{\mathscr{O}}(\theta,\kappa)(u)}, \overline{r(\theta,\kappa)(u)} \right\rangle$ *such that* $\overline{\dddot{\mathscr{O}}(\theta,\kappa)} = \overline{\dot{\mathscr{O}}(\theta)} \bigvee \overline{\ddot{\mathscr{O}}(\kappa)}$ *and* $\overline{r(\theta,\kappa)} = \overline{p(\theta)} \bigvee \overline{q(\kappa)}$, $\forall (\theta,\kappa) \in A \times B$.

Theorem 3.3. *Let* $\left(\overline{\dot{\mathscr{O}}_p}, A\right)$ *and* $\left(\overline{\ddot{\mathscr{O}}_q}, B\right)$ *be the PPyNVSSs on* $(\mathscr{U}, \mathcal{E})$*. Then*
(i) $\left(\left(\overline{\dot{\mathscr{O}}_p}, A\right) \bigwedge \left(\overline{\ddot{\mathscr{O}}_q}, B\right)\right)^c = \left(\overline{\dot{\mathscr{O}}_p}, A\right)^c \bigvee \left(\overline{\ddot{\mathscr{O}}_q}, B\right)^c$,
(ii) $\left(\left(\overline{\dot{\mathscr{O}}_p}, A\right) \bigvee \left(\overline{\ddot{\mathscr{O}}_q}, B\right)\right)^c = \left(\overline{\dot{\mathscr{O}}_p}, A\right)^c \bigwedge \left(\overline{\ddot{\mathscr{O}}_q}, B\right)^c$.

Proof: The proof of Theorem 3.3 is handed over to Definitions 3.8 and 3.9.

4. **Similarity Measure between Two PPyNVSSs.** In this section, we introduced the distance between the Possibility Pythagorean neutrosophic vague soft sets.

Definition 4.1. *Let* $\mathscr{U}$ *be the universe and* $\mathcal{E}$ *be the set of parameters. Suppose that* $\overline{\dot{\mathscr{O}}_p}$ *and* $\overline{\ddot{\mathscr{O}}_q}$ *be the PPyNVSSs on* $(\mathscr{U}, \mathcal{E})$*. The similarity measure between two PPyNVSSs* $\overline{\dot{\mathscr{O}}_p}$ *and* $\overline{\ddot{\mathscr{O}}_q}$ *is defined by* $Sim\left(\overline{\dot{\mathscr{O}}_p}, \overline{\ddot{\mathscr{O}}_q}\right) = \Phi\left(\overline{\dot{\mathscr{O}}}, \overline{\ddot{\mathscr{O}}}\right) \cdot \Psi(\overline{p}, \overline{q})$ *such that* $\Phi\left(\overline{\dot{\mathscr{O}}}, \overline{\ddot{\mathscr{O}}}\right) = \frac{X_1+X_2+Y_1+Y_2+Z_1+Z_2}{6}$ *and* $\Psi(\overline{p}, \overline{q}) = 1 - \frac{\sum |(\theta_{1i}+\theta_{2i})-(\kappa_{1i}+\kappa_{2i})|}{\sum |(\theta_{1i}+\theta_{2i})+(\kappa_{1i}+\kappa_{2i})|}$,
where

$$X_1 = T^-\left(\overline{\dot{\mathscr{O}}(e)(u)}, \overline{\ddot{\mathscr{O}}(e)(u)}\right) = \frac{\sum_{i=1}^n \left(\varepsilon^{T-}_{\dot{\mathscr{O}}(e_i)}(u) \cdot \varepsilon^{T-}_{\ddot{\mathscr{O}}(e_i)}(u)\right)}{\sum_{i=1}^n \left(1 - \sqrt{\left(1 - \varepsilon^{2T}_{\dot{\mathscr{O}}(e_i)}(u)\right) \cdot \left(1 - \varepsilon^{2T}_{\ddot{\mathscr{O}}(e_i)}(u)\right)}\right)},$$

$$X_2 = 1 - F^-\left(\overline{\dot{\mathscr{O}}(e)(u)}, \overline{\ddot{\mathscr{O}}(e)(u)}\right)$$
$$= \frac{\sum_{i=1}^n \left(\left(1 - \varepsilon^{F-}_{\dot{\mathscr{O}}(e_i)}(u)\right) \cdot \left(1 - \varepsilon^{F-}_{\ddot{\mathscr{O}}(e_i)}(u)\right)\right)}{\sum_{i=1}^n 1 + \sqrt{\left(1 + \left(1 - \varepsilon^{2F-}_{\dot{\mathscr{O}}(e_i)}(u)\right)\right) \cdot \left(1 + \left(1 - \varepsilon^{2F-}_{\ddot{\mathscr{O}}(e_i)}(u)\right)\right)}},$$

$$Y_1 = I^-\left(\overline{\dot{\mathscr{O}}(e)(u)}, \overline{\ddot{\mathscr{O}}(e)(u)}\right) = \frac{\sum_{i=1}^n \left(\varepsilon^{I-}_{\dot{\mathscr{O}}(e_i)}(u) \cdot \varepsilon^{I-}_{\ddot{\mathscr{O}}(e_i)}(u)\right)}{\sum_{i=1}^n \left(1 - \sqrt{\left(1 - \varepsilon^{2I-}_{\dot{\mathscr{O}}(e_i)}(u)\right) \cdot \left(1 - \varepsilon^{2I-}_{\ddot{\mathscr{O}}(e_i)}(u)\right)}\right)},$$

$$Y_2 = I^+\left(\overline{\dot{\mathscr{O}}(e)(u)}, \overline{\ddot{\mathscr{O}}(e)(u)}\right) = \frac{\sum_{i=1}^n \left(\varepsilon^{I+}_{\dot{\mathscr{O}}(e_i)}(u) \cdot \varepsilon^{I+}_{\ddot{\mathscr{O}}(e_i)}(u)\right)}{\sum_{i=1}^n \left(1 + \sqrt{\left(1 + \varepsilon^{2I+}_{\dot{\mathscr{O}}(e_i)}(u)\right) \cdot \left(1 + \varepsilon^{2I+}_{\ddot{\mathscr{O}}(e_i)}(u)\right)}\right)},$$

$$Z_1 = F^-\left(\overline{\dot{\mathscr{O}}(e)(u)}, \overline{\ddot{\mathscr{O}}(e)(u)}\right) = \sqrt{1 - \frac{\sum_{i=1}^n \left|\varepsilon^{2F-}_{\dot{\mathscr{O}}(e_i)}(u) - \varepsilon^{2F-}_{\ddot{\mathscr{O}}(e_i)}(u)\right|}{\sum_{i=1}^n 1 + \left(\left(\varepsilon^{2F-}_{\dot{\mathscr{O}}(e_i)}(u)\right) \cdot \left(\varepsilon^{2F-}_{\ddot{\mathscr{O}}(e_i)}(u)\right)\right)}},$$

$$Z_2 = T^-\left(\overline{\dot{\mathscr{O}}(e)(u)}, \overline{\ddot{\mathscr{O}}(e)(u)}\right) = \sqrt{1 + \frac{\sum_{i=1}^n \left|\left(1 - \varepsilon^{2T-}_{\dot{\mathscr{O}}(e_i)}(u)\right) + \left(1 - \varepsilon^{2T-}_{\ddot{\mathscr{O}}(e_i)}(u)\right)\right|}{\sum_{i=1}^n 1 + \left(\left(1 - \varepsilon^{2T-}_{\dot{\mathscr{O}}(e_i)}(u)\right) \cdot \left(1 - \varepsilon^{2T-}_{\ddot{\mathscr{O}}(e_i)}(u)\right)\right)}}$$

and $\theta_{1i} = \frac{\varepsilon^{2T}_{p(e_i)}(u)}{\varepsilon^{2T}_{p(e_i)}(u)+\varepsilon^{2F}_{p(e_i)}(u)}$, $\theta_{2i} = \frac{\varepsilon^{2T}_{p(e_i)}(u)}{\varepsilon^{2T}_{p(e_i)}(u)+\varepsilon^{2I}_{p(e_i)}(u)}$, $\kappa_{1i} = \frac{\varepsilon^{2T}_{q(e_i)}(u)}{\varepsilon^{2T}_{q(e_i)}(u)+\varepsilon^{2F}_{q(e_i)}(u)}$, $\kappa_{2i} = \frac{\varepsilon^{2T}_{q(e_i)}(u)}{\varepsilon^{2T}_{q(e_i)}(u)+\varepsilon^{2I}_{q(e_i)}(u)}$.

Theorem 4.1. *Let $\overline{\dot{\mathscr{O}}}_p$, $\overline{\ddot{\mathscr{O}}}_q$ and $\overline{\dddot{\mathscr{O}}}_r$ be the PPyNVSSs over $(\mathscr{U}, \mathcal{E})$. Then*
(i) $Sim\left(\overline{\dot{\mathscr{O}}}_p, \overline{\ddot{\mathscr{O}}}_q\right) = Sim\left(\overline{\ddot{\mathscr{O}}}_q, \overline{\dot{\mathscr{O}}}_p\right)$,
(ii) $0 \preceq Sim\left(\overline{\dot{\mathscr{O}}}_p, \overline{\ddot{\mathscr{O}}}_q\right) \preceq 1$,
(iii) $\overline{\dot{\mathscr{O}}}_p = \overline{\ddot{\mathscr{O}}}_q \implies Sim\left(\overline{\dot{\mathscr{O}}}_p, \overline{\ddot{\mathscr{O}}}_q\right) = 1$,
(iv) $\overline{\dot{\mathscr{O}}}_p \sqsubseteq \overline{\ddot{\mathscr{O}}}_q \sqsubseteq \overline{\dddot{\mathscr{O}}}_r \implies Sim\left(\overline{\dot{\mathscr{O}}}_p, \overline{\dddot{\mathscr{O}}}_r\right) \preceq Sim\left(\overline{\ddot{\mathscr{O}}}_q, \overline{\dddot{\mathscr{O}}}_r\right)$,
(v) $\overline{\dot{\mathscr{O}}}_p \bigwedge \overline{\ddot{\mathscr{O}}}_q = \{\phi\} \Leftrightarrow Sim\left(\overline{\dot{\mathscr{O}}}_p, \overline{\ddot{\mathscr{O}}}_q\right) = 0$.

Proof: The proofs (i), (ii), (iii) and (v) are trivial. (iv) Clearly, $\varepsilon^{T-}_{\dot{\mathscr{O}}(e)}(u) \cdot \varepsilon^{T-}_{\dddot{\mathscr{O}}(e)}(u) \preceq \varepsilon^{T-}_{\ddot{\mathscr{O}}(e)}(u) \cdot \varepsilon^{T-}_{\dddot{\mathscr{O}}(e)}(u)$ implies that

$$\sum_{i=1}^{n} \left(\varepsilon^{T-}_{\dot{\mathscr{O}}(e_i)}(u) \cdot \varepsilon^{T-}_{\dddot{\mathscr{O}}(e_i)}(u)\right) \preceq \sum_{i=1}^{n} \left(\varepsilon^{T-}_{\ddot{\mathscr{O}}(e_i)}(u) \cdot \varepsilon^{T-}_{\dddot{\mathscr{O}}(e_i)}(u)\right). \tag{1}$$

Clearly, $\left(\varepsilon^{2T-}_{\dot{\mathscr{O}}(e)}(u)\right) \preceq \left(\varepsilon^{2T-}_{\ddot{\mathscr{O}}(e)}(u)\right)$ implies that $-\left(\varepsilon^{2T-}_{\dot{\mathscr{O}}(e)}(u)\right) \succeq -\left(\varepsilon^{2T-}_{\ddot{\mathscr{O}}(e)}(u)\right)$ and

$$\begin{aligned}&\sum_{i=1}^{n} 1 - \sqrt{\left(1-\left(\varepsilon^{2T-}_{\dot{\mathscr{O}}(e_i)}(u)\right)\right) \cdot \left(1-\left(\varepsilon^{2T-}_{\dddot{\mathscr{O}}(e_i)}(u)\right)\right)} \\ &\preceq \sum_{i=1}^{n} 1 - \sqrt{\left(1-\left(\varepsilon^{2T-}_{\ddot{\mathscr{O}}(e_i)}(u)\right)\right) \cdot \left(1-\left(\varepsilon^{2T-}_{\dddot{\mathscr{O}}(e_i)}(u)\right)\right)}.\end{aligned} \tag{2}$$

Equation (1) is divided by Equation (2),

$$\begin{aligned}&\frac{\sum_{i=1}^{n}\left(\varepsilon^{T-}_{\dot{\mathscr{O}}(e_i)}(u) \cdot \varepsilon^{T-}_{\dddot{\mathscr{O}}(e_i)}(u)\right)}{\sum_{i=1}^{n} 1 - \sqrt{\left(1-\left(\varepsilon^{2T}_{\dot{\mathscr{O}}(e_i)}(u)\right)\right) \cdot \left(1-\left(\varepsilon^{2T}_{\dddot{\mathscr{O}}(e_i)}(u)\right)\right)}} \\ &\preceq \frac{\sum_{i=1}^{n}\left(\varepsilon^{T-}_{\ddot{\mathscr{O}}(e_i)}(u) \cdot \varepsilon^{T-}_{\dddot{\mathscr{O}}(e_i)}(u)\right)}{\sum_{i=1}^{n} 1 - \sqrt{\left(1-\left(\varepsilon^{2T}_{\ddot{\mathscr{O}}(e_i)}(u)\right)\right) \cdot \left(1-\left(\varepsilon^{2T}_{\dddot{\mathscr{O}}(e_i)}(u)\right)\right)}}.\end{aligned} \tag{3}$$

Clearly, $\left(1-\varepsilon^{F-}_{\dot{\mathscr{O}}(e)}(u)\right) \cdot \left(1-\varepsilon^{F-}_{\dddot{\mathscr{O}}(e)}(u)\right) \preceq \left(1-\varepsilon^{F-}_{\ddot{\mathscr{O}}(e)}(u)\right) \cdot \left(1-\varepsilon^{F-}_{\dddot{\mathscr{O}}(e)}(u)\right)$ implies that

$$\sum_{i=1}^{n} \left(1-\varepsilon^{F-}_{\dot{\mathscr{O}}(e)}(u)\right) \cdot \left(1-\varepsilon^{F-}_{\dddot{\mathscr{O}}(e)}(u)\right) \preceq \sum_{i=1}^{n} \left(1-\varepsilon^{F-}_{\ddot{\mathscr{O}}(e)}(u)\right) \cdot \left(1-\varepsilon^{F-}_{\dddot{\mathscr{O}}(e)}(u)\right). \tag{4}$$

Clearly, $\left(\varepsilon^{2F-}_{\dot{\mathscr{O}}(e)}(u)\right) \preceq \left(\varepsilon^{2F-}_{\ddot{\mathscr{O}}(e)}(u)\right)$ implies that $-\left(\varepsilon^{2F-}_{\dot{\mathscr{O}}(e)}(u)\right) \succeq -\left(\varepsilon^{2F-}_{\ddot{\mathscr{O}}(e)}(u)\right)$ and

$$\begin{aligned}&\sum_{i=1}^{n} 1 - \sqrt{\left(1-\left(1-\varepsilon^{2F-}_{\dot{\mathscr{O}}(e_i)}(u)\right)\right) \cdot \left(1-\left(1-\varepsilon^{2F-}_{\dddot{\mathscr{O}}(e_i)}(u)\right)\right)} \\ &\preceq \sum_{i=1}^{n} 1 - \sqrt{\left(1-\left(1-\varepsilon^{2F-}_{\ddot{\mathscr{O}}(e_i)}(u)\right)\right) \cdot \left(1-\left(1-\varepsilon^{2F-}_{\dddot{\mathscr{O}}(e_i)}(u)\right)\right)}.\end{aligned} \tag{5}$$

Equation (4) is divided by Equation (5),

$$\frac{\sum_{i=1}^{n}\left(1-\varepsilon_{\dot{\mathscr{O}}(e_i)}^{F-}(u)\cdot 1-\varepsilon_{\dddot{\mathscr{O}}(e_i)}^{F-}(u)\right)}{\sum_{i=1}^{n} 1-\sqrt{\left(1-\left(1-\varepsilon_{\dot{\mathscr{O}}(e_i)}^{2T}(u)\right)\right)\cdot\left(1-\left(1-\varepsilon_{\dddot{\mathscr{O}}(e_i)}^{2T}(u)\right)\right)}} \preceq \frac{\sum_{i=1}^{n}\left(1-\varepsilon_{\ddot{\mathscr{O}}(e_i)}^{F-}(u)\cdot 1-\varepsilon_{\dddot{\mathscr{O}}(e_i)}^{F-}(u)\right)}{\sum_{i=1}^{n} 1-\sqrt{\left(1-\left(1-\varepsilon_{\ddot{\mathscr{O}}(e_i)}^{2T}(u)\right)\right)\cdot\left(1-\left(1-\varepsilon_{\dddot{\mathscr{O}}(e_i)}^{2T}(u)\right)\right)}}. \tag{6}$$

Clearly, $\varepsilon_{\dot{\mathscr{O}}(e)}^{I-}(u)\cdot\varepsilon_{\dddot{\mathscr{O}}(e)}^{I-}(u) \preceq \varepsilon_{\ddot{\mathscr{O}}(e)}^{I-}(u)\cdot\varepsilon_{\dddot{\mathscr{O}}(e)}^{I-}(u)$ implies that

$$\sum_{i=1}^{n}\left(\varepsilon_{\dot{\mathscr{O}}(e_i)}^{I-}(u)\cdot\varepsilon_{\dddot{\mathscr{O}}(e_i)}^{I-}(u)\right) \preceq \sum_{i=1}^{n}\left(\varepsilon_{\ddot{\mathscr{O}}(e_i)}^{I-}(u)\cdot\varepsilon_{\dddot{\mathscr{O}}(e_i)}^{I-}(u)\right). \tag{7}$$

Clearly, $\left(\varepsilon_{\dot{\mathscr{O}}(e)}^{2I-}(u)\right) \preceq \left(\varepsilon_{\ddot{\mathscr{O}}(e)}^{2I-}(u)\right)$ implies that $-\left(\varepsilon_{\dot{\mathscr{O}}(e)}^{2I-}(u)\right) \succeq -\left(\varepsilon_{\ddot{\mathscr{O}}(e)}^{2I-}(u)\right)$ and

$$\sum_{i=1}^{n} 1-\sqrt{\left(1-\left(\varepsilon_{\dot{\mathscr{O}}(e_i)}^{2I-}(u)\right)\right)\cdot\left(1-\left(\varepsilon_{\dddot{\mathscr{O}}(e_i)}^{2I-}(u)\right)\right)} \preceq \sum_{i=1}^{n} 1-\sqrt{\left(1-\left(\varepsilon_{\ddot{\mathscr{O}}(e_i)}^{2I-}(u)\right)\right)\cdot\left(1-\left(\varepsilon_{\dddot{\mathscr{O}}(e_i)}^{2I-}(u)\right)\right)}. \tag{8}$$

Equation (7) is divided by Equation (8),

$$\frac{\sum_{i=1}^{n}\left(\varepsilon_{\dot{\mathscr{O}}(e_i)}^{I-}(u)\cdot\varepsilon_{\dddot{\mathscr{O}}(e_i)}^{I-}(u)\right)}{\sum_{i=1}^{n} 1-\sqrt{\left(1-\left(\varepsilon_{\dot{\mathscr{O}}(e_i)}^{2I-}(u)\right)\right)\cdot\left(1-\left(\varepsilon_{\dddot{\mathscr{O}}(e_i)}^{2I-}(u)\right)\right)}} \preceq \frac{\sum_{i=1}^{n}\left(\varepsilon_{\ddot{\mathscr{O}}(e_i)}^{I-}(u)\cdot\varepsilon_{\dddot{\mathscr{O}}(e_i)}^{I-}(u)\right)}{\sum_{i=1}^{n} 1-\sqrt{\left(1-\left(\varepsilon_{\ddot{\mathscr{O}}(e_i)}^{2I-}(u)\right)\right)\cdot\left(1-\left(\varepsilon_{\dddot{\mathscr{O}}(e_i)}^{2I-}(u)\right)\right)}}. \tag{9}$$

Similarly,

$$\frac{\sum_{i=1}^{n}\left(\varepsilon_{\dot{\mathscr{O}}(e_i)}^{I+}(u)\cdot\varepsilon_{\dddot{\mathscr{O}}(e_i)}^{I+}(u)\right)}{\sum_{i=1}^{n} 1-\sqrt{\left(1-\left(\varepsilon_{\dot{\mathscr{O}}(e_i)}^{2I+}(u)\right)\right)\cdot\left(1-\left(\varepsilon_{\dddot{\mathscr{O}}(e_i)}^{2I+}(u)\right)\right)}} \preceq \frac{\sum_{i=1}^{n}\left(\varepsilon_{\ddot{\mathscr{O}}(e_i)}^{I+}(u)\cdot\varepsilon_{\dddot{\mathscr{O}}(e_i)}^{I+}(u)\right)}{\sum_{i=1}^{n} 1-\sqrt{\left(1-\left(\varepsilon_{\ddot{\mathscr{O}}(e_i)}^{2I+}(u)\right)\right)\cdot\left(1-\left(\varepsilon_{\dddot{\mathscr{O}}(e_i)}^{2I+}(u)\right)\right)}}. \tag{10}$$

Clearly, $\varepsilon_{\dot{\mathscr{O}}(e)}^{2F-}(u) \succeq \varepsilon_{\ddot{\mathscr{O}}(e)}^{2F-}(u)$ and $\varepsilon_{\dot{\mathscr{O}}(e)}^{2F-}(u)-\varepsilon_{\dddot{\mathscr{O}}(e)}^{2F-}(u) \succeq \varepsilon_{\ddot{\mathscr{O}}(e)}^{2F-}(u)-\varepsilon_{\dddot{\mathscr{O}}(e)}^{2F-}(u)$. Hence

$$\sum_{i=1}^{n}\left|\varepsilon_{\dot{\mathscr{O}}(e_i)}^{2F-}(u)-\varepsilon_{\dddot{\mathscr{O}}(e_i)}^{2F-}(u)\right| \succeq \sum_{i=1}^{n}\left|\varepsilon_{\ddot{\mathscr{O}}(e_i)}^{2F-}(u)-\varepsilon_{\dddot{\mathscr{O}}(e_i)}^{2F-}(u)\right|. \tag{11}$$

Also, $\left(\varepsilon_{\dot{\mathscr{O}}(e)}^{2F-}(u)\cdot\varepsilon_{\dddot{\mathscr{O}}(e)}^{2F-}(u)\right) \succeq \left(\varepsilon_{\ddot{\mathscr{O}}(e)}^{2F-}(u)\cdot\varepsilon_{\dddot{\mathscr{O}}(e)}^{2F-}(u)\right)$ implies that

$$\sum_{i=1}^{n} 1+\left(\varepsilon^{2F-}_{\dot{\mathscr{O}}(e_i)}(u)\cdot\varepsilon^{2F-}_{\dddot{\mathscr{O}}(e_i)}(u)\right)\succeq\sum_{i=1}^{n} 1+\left(\varepsilon^{2F-}_{\ddot{\mathscr{O}}(e_i)}(u)\cdot\varepsilon^{2F-}_{\dddot{\mathscr{O}}(e_i)}(u)\right). \tag{12}$$

Equation (11) is divided by Equation (12), and we get

$$\sqrt{1-\frac{\sum_{i=1}^{n}\left|\varepsilon^{2F-}_{\dot{\mathscr{O}}(e_i)}(u)-\varepsilon^{2F-}_{\dddot{\mathscr{O}}(e_i)}(u)\right|}{\sum_{i=1}^{n}1+\left(\varepsilon^{2F-}_{\dot{\mathscr{O}}(e_i)}(u)\cdot\varepsilon^{2F-}_{\dddot{\mathscr{O}}(e_i)}(u)\right)}}\preceq\sqrt{1-\frac{\sum_{i=1}^{n}\left|\varepsilon^{2F-}_{\ddot{\mathscr{O}}(e_i)}(u)-\varepsilon^{2F-}_{\dddot{\mathscr{O}}(e_i)}(u)\right|}{\sum_{i=1}^{n}1+\left(\varepsilon^{2F-}_{\ddot{\mathscr{O}}(e_i)}(u)\cdot\varepsilon^{2F-}_{\dddot{\mathscr{O}}(e_i)}(u)\right)}}. \tag{13}$$

Clearly, $\left(1-\varepsilon^{2T^-}_{\dot{\mathscr{O}}(e)}(u)\right)\succeq\left(1-\varepsilon^{2T^-}_{\ddot{\mathscr{O}}(e)}(u)\right)$. Hence,

$$\sum_{i=1}^{n}\left|\left(1-\varepsilon^{2T^-}_{\dot{\mathscr{O}}(e_i)}(u)\right)-\left(1-\varepsilon^{2T^-}_{\dddot{\mathscr{O}}(e_i)}(u)\right)\right|\succeq\sum_{i=1}^{n}\left|\left(1-\varepsilon^{2T^-}_{\ddot{\mathscr{O}}(e_i)}(u)\right)-\left(1-\varepsilon^{2T^-}_{\dddot{\mathscr{O}}(e_i)}(u)\right)\right|, \tag{14}$$

$$\sum_{i=1}^{n}1+\left(1-\varepsilon^{2T^-}_{\dot{\mathscr{O}}(e_i)}(u)\right)\cdot\left(1-\varepsilon^{2T^-}_{\dddot{\mathscr{O}}(e_i)}(u)\right)\succeq\sum_{i=1}^{n}1+\left(1-\varepsilon^{2T^-}_{\ddot{\mathscr{O}}(e_i)}(u)\right)\cdot\left(1-\varepsilon^{2T^-}_{\dddot{\mathscr{O}}(e_i)}(u)\right). \tag{15}$$

Equation (14) is divided by Equation (15), and we get

$$\sqrt{1-\frac{\sum_{i=1}^{n}\left|\left(1-\varepsilon^{2T^-}_{\dot{\mathscr{O}}(e_i)}(u)\right)-\left(1-\varepsilon^{2T^-}_{\dddot{\mathscr{O}}(e_i)}(u)\right)\right|}{\sum_{i=1}^{n}1+\left(\left(1-\varepsilon^{2T^-}_{\dot{\mathscr{O}}(e_i)}(u)\right)\cdot\left(1-\varepsilon^{2T^-}_{\dddot{\mathscr{O}}(e_i)}(u)\right)\right)}}$$

$$\preceq\sqrt{1-\frac{\sum_{i=1}^{n}\left|\left(1-\varepsilon^{2T^-}_{\ddot{\mathscr{O}}(e_i)}(u)\right)-\left(1-\varepsilon^{2T^-}_{\dddot{\mathscr{O}}(e_i)}(u)\right)\right|}{\sum_{i=1}^{n}1+\left(\left(1-\varepsilon^{2T^-}_{\ddot{\mathscr{O}}(e_i)}(u)\right)\cdot\left(1-\varepsilon^{2T^-}_{\dddot{\mathscr{O}}(e_i)}(u)\right)\right)}}. \tag{16}$$

Hence,

$$\Phi\left(\overline{\dot{\mathscr{O}}},\overline{\dddot{\mathscr{O}}}\right)\preceq\Phi\left(\overline{\ddot{\mathscr{O}}},\overline{\dddot{\mathscr{O}}}\right). \tag{17}$$

Clearly, $\theta_{1i}\preceq\kappa_{1i}\preceq\tau_{1i}$ and $\theta_{2i}\preceq\kappa_{2i}\preceq\tau_{2i}$, where

$$\theta_{1i}=\frac{\varepsilon^{2T}_{p(e_i)}(u)}{\varepsilon^{2T}_{p(e_i)}(u)+\varepsilon^{2F}_{p(e_i)}(u)},\quad\theta_{2i}=\frac{\varepsilon^{2T}_{p(e_i)}(u)}{\varepsilon^{2T}_{p(e_i)}(u)+\varepsilon^{2I}_{p(e_i)}(u)}$$

$$\kappa_{1i}=\frac{\varepsilon^{2T}_{q(e_i)}(u)}{\varepsilon^{2T}_{q(e_i)}(u)+\varepsilon^{2F}_{q(e_i)}(u)},\quad\kappa_{2i}=\frac{\varepsilon^{2T}_{q(e_i)}(u)}{\varepsilon^{2T}_{q(e_i)}(u)+\varepsilon^{2I}_{q(e_i)}(u)}$$

$$\tau_{1i}=\frac{\varepsilon^{2T}_{r(e_i)}(u)}{\varepsilon^{2T}_{r(e_i)}(u)+\varepsilon^{2F}_{r(e_i)}(u)},\quad\tau_{2i}=\frac{\varepsilon^{2T}_{r(e_i)}(u)}{\varepsilon^{2T}_{r(e_i)}(u)+\varepsilon^{2I}_{r(e_i)}(u)}.$$

Clearly, $(\theta_{1i}+\theta_{2i})\preceq(\kappa_{1i}+\kappa_{2i})\preceq(\tau_{1i}+\tau_{2i})$ and $(\theta_{1i}+\theta_{2i})-(\tau_{1i}+\tau_{2i})\preceq(\kappa_{1i}+\kappa_{2i})-(\tau_{1i}+\tau_{2i})$. Hence, $|(\kappa_{1i}+\kappa_{2i})-(\tau_{1i}+\tau_{2i})|\preceq|(\theta_{1i}+\theta_{2i})-(\kappa_{1i}+\kappa_{2i})|$ and

$$-\left|(\theta_{1i}+\theta_{2i})-(\tau_{1i}+\tau_{2i})\right|\preceq-\left|(\kappa_{1i}+\kappa_{2i})-(\tau_{1i}+\tau_{2i})\right|, \tag{18}$$

$$\left|(\theta_{1i}+\theta_{2i})+(\tau_{1i}+\tau_{2i})\right|\preceq\left|(\kappa_{1i}+\kappa_{2i})+(\tau_{1i}+\tau_{2i})\right|. \tag{19}$$

Equation (18) is divided by Equation (19), and we get

$$1-\frac{\sum_{i=1}^{n}\left|(\theta_{1i}+\theta_{2i})-(\tau_{1i}+\tau_{2i})\right|}{\sum_{i=1}^{n}\left|(\theta_{1i}+\theta_{2i})+(\tau_{1i}+\tau_{2i})\right|}\preceq1-\frac{\sum_{i=1}^{n}\left|(\kappa_{1i}+\kappa_{2i})-(\tau_{1i}+\tau_{2i})\right|}{\sum_{i=1}^{n}\left|(\kappa_{1i}+\kappa_{2i})+(\tau_{1i}+\tau_{2i})\right|}.$$

Hence,

$$\Psi\left(\overline{p},\overline{r}\right)\preceq\Psi\left(\overline{q},\overline{r}\right). \tag{20}$$

From Equations (17) and (20),

$$\Phi\left(\overline{\dot{\mathscr{O}}},\overline{\dddot{\mathscr{O}}}\right)\cdot\Psi\left(\overline{p},\overline{r}\right)\preceq\Phi\left(\overline{\ddot{\mathscr{O}}},\overline{\dddot{\mathscr{O}}}\right)\cdot\Psi\left(\overline{q},\overline{r}\right).$$

Hence, $Sim\left(\overline{\dot{\mathscr{O}}_p},\overline{\dddot{\mathscr{O}}_r}\right)\preceq Sim\left(\overline{\ddot{\mathscr{O}}_q},\overline{\dddot{\mathscr{O}}_r}\right)$. This proves (iv).

5. **Robotic Engineering Selection Based on Similarity.** In our daily life, we face problems in decision-making in areas such as education, the economy, management, politics, and technology. As you embark on the robotic engineering selection process, the following five items are important to consider before making your final decision. Our goal is to select the optimal one out of a great number of alternatives based on the assessment of experts against the criteria.

5.1. **Robotic entries.** Robotics is an applied engineering science that has been referred to as a combination of machine tool technology and computer science. It includes machine design, production theory, microelectronics, computer programming, and artificial intelligence. Now, we have randomized five types of robotics, namely manipulator robotic, legged robotic, wheeled robotic, autonomous underwater vehicle robotic, and unmanned aerial vehicle robotic. There are five types of criteria for choosing a robotics system by $E=\{e_1$: robot controller features, e_2: affordable off line programming software, e_3:

Table 1. PPyNVSS for the ideal robotic data

$\overline{\mathscr{L}_{p(e)}}$	e_1	e_2	e_3
$\overline{\mathscr{L}(e)}$	$\langle[0.81,0.87],[0.7,0.75],[0.13,0.19]\rangle$	$\langle[0.82,0.89],[0.8,0.85],[0.11,0.18]\rangle$	$\langle[0.83,0.86],[0.75,0.8],[0.14,0.17]\rangle$
$\overline{p(e)}$	$\langle 1,0,0\rangle$	$\langle 1,0,0\rangle$	$\langle 1,0,0\rangle$

$\overline{\mathscr{L}_{p(e)}}$	e_4	e_5
$\overline{\mathscr{L}(e)}$	$\langle[0.75,0.82],[0.8,0.85],[0.18,0.25]\rangle$	$\langle[0.82,0.85],[0.7,0.75],[0.15,0.18]\rangle$
$\overline{p(e)}$	$\langle 1,0,0\rangle$	$\langle 1,0,0\rangle$

Table 2. PPyNVSS for the manipulator robotic data

$\overline{\mathscr{A}_{p_1(e)}}$	e_1	e_2	e_3
$\overline{\mathscr{A}(e)}$	$\langle[0.35,0.36],[0.75,0.8],[0.64,0.65]\rangle$	$\langle[0.41,0.62],[0.85,0.9],[0.38,0.59]\rangle$	$\langle[0.56,0.57],[0.82,0.85],[0.43,0.44]\rangle$
$\overline{p_1(e)}$	$\langle 0.95,0.65,0.75\rangle$	$\langle 0.65,0.55,0.45\rangle$	$\langle 0.65,0.75,0.55\rangle$

$\overline{\mathscr{A}_{p_1(e)}}$	e_4	e_5
$\overline{\mathscr{A}(e)}$	$\langle[0.72,0.73],[0.88,0.9],[0.27,0.28]\rangle$	$\langle[0.65,0.66],[0.75,0.8],[0.34,0.35]\rangle$
$\overline{p_1(e)}$	$\langle 0.75,0.65,0.85\rangle$	$\langle 0.85,0.65,0.45\rangle$

Table 3. PPyNVSS for the legged robotic data

$\overline{\mathscr{B}_{p_2(e)}}$	e_1	e_2	e_3
$\overline{\mathscr{B}(e)}$	$\langle[0.55,0.64],[0.75,0.8],[0.36,0.45]\rangle$	$\langle[0.5,0.52],[0.82,0.87],[0.48,0.5]\rangle$	$\langle[0.4,0.52],[0.8,0.83],[0.48,0.6]\rangle$
$\overline{p_2(e)}$	$\langle 0.75,0.65,0.35\rangle$	$\langle 0.65,0.75,0.55\rangle$	$\langle 0.7,0.45,0.35\rangle$

$\overline{\mathscr{B}_{p_2(e)}}$	e_4	e_5
$\overline{\mathscr{B}(e)}$	$\langle[0.35,0.36],[0.84,0.88],[0.64,0.65]\rangle$	$\langle[0.45,0.47],[0.73,0.8],[0.53,0.55]\rangle$
$\overline{p_2(e)}$	$\langle 0.55,0.35,0.45\rangle$	$\langle 0.8,0.65,0.55\rangle$

TABLE 4. PPyNVSS for the wheeled robotic data

$\overline{\mathscr{C}_{p_3(e)}}$	e_1	e_2	e_3
$\overline{\mathscr{C}(e)}$	$\langle[0.5,0.65],[0.71,0.8],[0.35,0.5]\rangle$	$\langle[0.55,0.6],[0.81,0.87],[0.4,0.45]\rangle$	$\langle[0.65,0.75],[0.8,0.85],[0.25,0.35]\rangle$
$\overline{p_3(e)}$	$\langle 0.8,0.4,0.5\rangle$	$\langle 0.75,0.55,0.6\rangle$	$\langle 0.6,0.7,0.55\rangle$

$\overline{\mathscr{C}_{p_3(e)}}$	e_4	e_5
$\overline{\mathscr{C}(e)}$	$\langle[0.45,0.6],[0.83,0.88],[0.4,0.55]\rangle$	$\langle[0.35,0.55],[0.75,0.8],[0.45,0.65]\rangle$
$\overline{p_3(e)}$	$\langle 0.8,0.65,0.55\rangle$	$\langle 0.65,0.5,0.65\rangle$

TABLE 5. PPyNVSS for the autonomous underwater vehicle robotic data

$\overline{\mathscr{D}_{p_4(e)}}$	e_1	e_2	e_3
$\overline{\mathscr{D}(e)}$	$\langle[0.5,0.65],[0.75,0.8],[0.35,0.5]\rangle$	$\langle[0.45,0.5],[0.82,0.86],[0.5,0.55]\rangle$	$\langle[0.45,0.5],[0.76,0.82],[0.5,0.55]\rangle$
$\overline{p_4(e)}$	$\langle 0.8,0.65,0.55\rangle$	$\langle 0.6,0.55,0.5\rangle$	$\langle 0.65,0.55,0.75\rangle$

$\overline{\mathscr{D}_{p_4(e)}}$	e_4	e_5
$\overline{\mathscr{D}(e)}$	$\langle[0.6,0.7],[0.83,0.88],[0.3,0.4]\rangle$	$\langle[0.5,0.65],[0.74,0.77],[0.35,0.5]\rangle$
$\overline{p_4(e)}$	$\langle 0.7,0.55,0.65\rangle$	$\langle 0.75,0.35,0.55\rangle$

TABLE 6. PPyNVSS for the unwanned aerial vehicle robotic data

$\overline{\mathscr{E}_{p_5(e)}}$	e_1	e_2	e_3
$\overline{\mathscr{E}(e)}$	$\langle[0.4,0.85],[0.72,0.8],[0.15,0.6]\rangle$	$\langle[0.46,0.8],[0.81,0.86],[0.2,0.54]\rangle$	$\langle[0.4,0.75],[0.78,0.84],[0.25,0.6]\rangle$
$\overline{p_5(e)}$	$\langle 0.55,0.5,0.6\rangle$	$\langle 0.85,0.45,0.35\rangle$	$\langle 0.8,0.55,0.55\rangle$

$\overline{\mathscr{E}_{p_5(e)}}$	e_4	e_5
$\overline{\mathscr{E}(e)}$	$\langle[0.5,0.65],[0.82,0.87],[0.35,0.5]\rangle$	$\langle[0.4,0.7],[0.75,0.8],[0.3,0.6]\rangle$
$\overline{p_5(e)}$	$\langle 0.7,0.55,0.45\rangle$	$\langle 0.8,0.35,0.55\rangle$

safety codes, e_4: continuous-duty cycle time, e_5: experience and reputation of the robot manufacturer}.

In order to discover the robotic data that is closest to the ideal robotic data, we need to use Definition 4.1 to calculate the similarity measure of PPyNVSSs in Table 2 to Table 6 with the one in Table 1. The robotic data should be used as the similarity threshold. Below the table is a formula for calculating the similarity measure for the five categories of robotics.

	T	I	F	Φ	Ψ	Similarity
$\overline{(\mathscr{L},\mathscr{A})}$	0.841841	0.992614	0.906279	0.913578	0.749102	0.684363
$\overline{(\mathscr{L},\mathscr{B})}$	0.769749	0.996851	0.866017	0.877539	0.788149	0.691632
$\overline{(\mathscr{L},\mathscr{C})}$	0.856917	0.996861	0.910719	0.921499	0.762049	0.702227
$\overline{(\mathscr{L},\mathscr{D})}$	0.839134	0.997992	0.905781	0.914302	0.754157	0.689528
$\overline{(\mathscr{L},\mathscr{E})}$	0.855988	0.997727	0.911717	0.921811	0.812534	0.749002

According to the preceding findings, the unwanted aerial vehicle robotic data is the most similar to the ideal robotic data, with a similarity measure of $\mathbf{0.749002}$. Therefore, the optimal one is unwanted aerial vehicle robotic.

5.2. **PyNVSS approach without the generalization parameter.** To explore the effect of the possibility parameter, we investigate the above mentioned robotic entries using the PyNVSS technique. The similarity measure for the five categories of robotic data mentioned above is calculated as follows. We have got

	T	I	F	Similarity
$\overline{(\mathscr{L},\mathscr{A})}$	0.841841	0.992614	0.906279	0.913578
$\overline{(\mathscr{L},\mathscr{B})}$	0.769749	0.996851	0.866017	0.877539
$\overline{(\mathscr{L},\mathscr{C})}$	0.856917	0.996861	0.910719	0.921499
$\overline{(\mathscr{L},\mathscr{D})}$	0.839134	0.997992	0.905781	0.914302
$\overline{(\mathscr{L},\mathscr{E})}$	0.855988	0.997727	0.911717	0.921811

According to the preceding findings, the parameter has a considerable impact on the calculation of the PPyNVSSs similarity measure. From the standpoint of similarity measure, the first fourth robotic data are considerably different from the ideal robotic data. If the robotic data one unit chooses the threshold $\langle[0.5, 0.65], [0.82, 0.87], [0.35, 0.5]\rangle$, the unwanted aerial vehicle data should be chosen as the finest in the world for real-world applications.

5.3. **Comparison for the PPyNVSS and PyNVSS.** Using the PyNVSS technique without the generalization parameter, on the other hand, we are unable to determine which robotic data is the best. As a result, the possibility parameter has a significant impact on the unwanted aerial vehicle data similarity measure. As a result, the PPyNVSS method is more scientific and rational.

5.4. **Advantage.** The main goal of this work is to present a possibility Pythagorean neutrosophic vague soft set to solve the phenomena related to decision-making. From the above discussion, the possibility parameter has an important influence on the similarity measure. Therefore, the PPyNVSS approach is more scientific and reasonable than the PyNVSS approach in the process of decision-making. To illustrate the validity of this similarity measure, the possibility of a Pythagorean neutrosophic vague soft set is applied to decision-making problems.

6. **Conclusion.** We discussed the logical consistency of PPyNSVSS and PyNSVSS. Some of the algebraic operations we established are complement, union, intersection, AND, OR, commutative laws, De Morgan's laws, associative laws, and distributive laws, all of which hold. We contrasted PPyNSVSS with PyNSVSS in order to address decision-making concerns and construct a similarity distance.

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Author Biography

M. Palanikumar works at Department of Mathematics, Saveetha School of Engineering, Saveetha University, Saveetha Institute of Medical and Technical Sciences, Tamil Nadu, Chennai, India. He received his B.Sc., M.Sc., and M.Phil. degrees in Mathematics from affiliated to Madurai Kamaraj University, Madurai, India and Ph.D. degree from Department of Mathematics, Annamalai University, India. He has published more than 45 research papers in various international journals and has one international book published. His areas of interest include algebraic theory, such as semigroup theory, ring theory, semiring theory, fuzzy algebraic theory, and decision-making with applications.

K. Arulmozhi works at Department of Mathematics, Bharath Institute of Higher Education and Research, Tamil Nadu, Chennai, India. She received his B.Sc. and M.Sc. degrees in Mathematics from affiliated to Thiruvalluvar University, India and M.Phil. and Ph.D. degrees in Mathematics from Annamalai University, India. She has published more than 55 research papers in various international and national journals and has two international books published. She has received the Best Research Scholar Award (DKIRF Faculty Awards) in 2018. Her areas of interest include algebraic theory, such as semigroup theory, ring theory, semiring theory, and fuzzy algebraic theory and applications.

Aiyared Iampan is an Associate Professor at Department of Mathematics, School of Science, University of Phayao, Phayao, Thailand. He received his B.S., M.S., and Ph.D. degrees in Mathematics from Naresuan University, Phitsanulok, Thailand, under the thesis advisor of Professor Dr. Manoj Siripitukdet. His areas of interest include algebraic theory of semigroups, ternary semigroups, and Γ-semigroups, lattices and ordered algebraic structures, fuzzy algebraic structures, and logical algebras. He was the founder of the Group for Young Algebraists in University of Phayao in 2012 and one of the co-founders of the Fuzzy Algebras and Decision-Making Problems Research Unit in University of Phayao in 2021.

G. Shanmugam works at Department of Mathematics, Saveetha School of Engineering, Saveetha University, Saveetha Institute of Medical and Technical Sciences, Chennai, Tamilnadu, India. He received his B.Sc., M.Sc., M.Phil., and Ph.D. degrees in Mathematics from Bharathidasan University, Tiruchirapalli, India. He has published more than 12 research papers in various international journals. His areas of interest include topology, algebraic theory, and decision-making with applications.