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NOVEL POSSIBILITY PYTHAGOREAN CUBIC FUZZY SOFT SETS AND THEIR APPLICATIONS

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Abstract. *We explain the idea of a possibility Pythagorean cubic fuzzy soft set and discuss how it may be used to solve practical issues. The generalization of the soft sets are possibility Pythagorean cubic fuzzy soft sets. Also, we discuss to acquire a laptop and the many steps a customer takes before making a purchase. We suggest an algorithm based on the fuzzy soft set approach to resolve the problem of decision-making. A similarity measure is obtained by comparing the possibility Pythagorean cubic fuzzy soft set and Pythagorean cubic fuzzy soft set for dealing with decision-making problems. A demonstrative example is then discussed to demonstrate that they can be utilized successfully to address problems with uncertainties.*

Keywords: Fuzzy soft set, Cubic fuzzy soft set, Pythagorean cubic fuzzy soft set, Possibility Pythagorean cubic fuzzy soft set, Decision-making problem

1. **Introduction.** Uncertainty is prevalent in the majority of real-world problems. Numerous unreliable theories have been proposed to deal with uncertainties, including fuzzy sets [1], interval-valued fuzzy soft sets [2], and Pythagorean fuzzy sets [3]. Fuzzy set was introduced by Zadeh and it suggests that decision-makers should take membership degree into account when resolving unclear problems. Yager [3] first encountered the concept of Pythagorean fuzzy sets. The theory of soft sets was proposed by Molodtsov [4]. It is a

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parameterization tool for dealing with uncertainty. Soft sets, when compared to other uncertain theories, more closely reflect the objectivity and complexity of decision-making in real-world circumstances. There have been significant advances in both theory and application. Furthermore, combining soft sets with other mathematical models is an important study field. Maji et al., for example, proposed the concept of fuzzy soft set [5]. These two ideas are used to tackle a variety of decision-making issues. The concept of possibility fuzzy soft sets was defined by Alkhazaleh et al. [6]. Fuzzy soft set that has been expanded to a Pythagorean fuzzy soft set was discussed by Peng et al. [7]. This model resolved a type of multi-attribute decision-making, when the total of the membership and non-membership degree is more than 1, but the sum of the squares is equal to or less than 1. In multi-attribute decision-making situations, the likelihood of the elements belongingness should generally be taken into account. Jana et al. [8] talked about Pythagorean fuzzy Dombi aggregation operators and their uses in multiple attribute decision-making. Peng and Yang [9] discussed the concept of fundamental properties of interval-valued Pythagorean fuzzy aggregation operators. In a decision-making, Jana and Pal interacted with bipolar intuitionistic fuzzy soft sets [10]. Palanikumar and Arulmozhi [11] presented the notion of possibility Pythagorean bipolar fuzzy soft sets and its application. The idea of possibility neutrosophic soft sets and the PNS-decision-making approach were first developed by Karaaslan [12]. Pythagorean cubic fuzzy aggregation operations were introduced by Khana et al. [13] and their use in multi-criteria decision-making issues. Regarding the issue, the aim of this study is to parameterize the possibility Pythagorean cubic fuzzy set using a soft set model, extending the idea of possibility Pythagorean fuzzy soft set. For a multiple criterion choice study of bridge building techniques, Chen [14] integrated the idea of an interval-valued Pythagorean fuzzy compromise approach with correlation-based proximity indices. Thammajitr et al. discussed the concept for fuzzy parameterized relative fuzzy soft sets in decision-making problems [15]. Palanikumar et al. discussed various applications based on decision-making approach [16, 17, 18, 19, 20, 21].

The paper is divided into the following seven sections. The introduction is in Section 1, and the preliminary discussions of the Pythagorean fuzzy soft sets and the possibility Pythagorean interval-valued fuzzy soft sets are in Section 2. The possibility Pythagorean cubic fuzzy soft sets are shown in Section 3 along with a soft model. With the use of numerical examples, Section 4 introduces the use of possibility Pythagorean cubic fuzzy soft sets in decision-making. Section 5 offers an additional Pythagorean cubic fuzzy soft set without a generalization parameter. In Section 6, we also contrast the possibility of the Pythagorean cubic fuzzy soft sets with the Pythagorean cubic fuzzy soft sets. Section 7 includes conclusion and discussion.

2. **Preliminaries.**

Definition 2.1. [3] *Let X be a non-empty set of the universe, a Pythagorean fuzzy set (PFS) A in X is an object having the following form: $A = \{x, \mu_A(x), \nu_A(x) | x \in X\}$, where $\mu_A(x)$ and $\nu_A(x)$ represent the degree of membership and degree of non-membership of A, respectively. Consider the mapping $\mu_A : X \to [0,1]$, $\nu_A : X \to [0,1]$ and $0 \le (\mu_A(x))^2 + (\nu_A(x))^2 \le 1$. The degree of indeterminacy is determined as*

$$\pi_A(x) = \left[\sqrt{1 - (\mu_A(x))^2 - (\nu_A(x))^2}\right].$$

$A = \langle \mu_A, \nu_A \rangle$ is called a Pythagorean fuzzy number.

Definition 2.2. [9, 14] *Let X be a non-empty set of the universe, a Pythagorean interval-valued fuzzy set (PIVFS) A in X is an object having the following form: $\widetilde{A} = \{x, \widetilde{\mu}_A(x), \widetilde{\nu}_A(x) | x \in X\}$, where $\widetilde{\mu}_A(x) = \left[\mu_A^L(x), \mu_A^U(x)\right]$ and $\widetilde{\nu}_A(x) = \left[\nu_A^L(x), \nu_A^U(x)\right]$ represent the* *degree of membership and degree of non-membership of* A*, respectively. Consider the mapping* $\widetilde{\mu}_A : X \to [0,1]$*,* $\widetilde{\nu}_A : X \to [0,1]$ *and* $0 \le (\widetilde{\mu}_A(x))^2 + (\widetilde{\nu}_A(x))^2 \le 1$ *means that* $0 \le \left(\mu_A^U(x)\right)^2 + \left(\nu_A^U(x)\right)^2 \le 1$*. The degree of indeterminacy is determined as* $\widetilde{\pi}_A(x) = \left[\pi_A^L(x), \pi_A^U(x)\right] = \left[\sqrt{1-\left(\mu_A^U(x)\right)^2-\left(\nu_A^U(x)\right)^2}, \sqrt{1-\left(\mu_A^L(x)\right)^2-\left(\nu_A^L(x)\right)^2}\right]$*.* $A = \left\langle\left[\mu_A^L, \mu_A^U\right], \left[\nu_A^L, \nu_A^U\right]\right\rangle$ *is called a Pythagorean interval-valued fuzzy number.*

Definition 2.3. [13] *Let* X *be a non-empty set of the universe, a Pythagorean cubic fuzzy set (PCFS)* A *in* X *is an object having the following form:* $A = \{x, \langle(\widetilde{\mu}_A(x), \widetilde{\nu}_A(x)), (\phi_A(x), \eta_A(x))\rangle | x \in X\}$*, where* $\widetilde{\mu}_A(x) = \left[\mu_A^L(x), \mu_A^U(x)\right]$ *and* $\widetilde{\nu}_A(x) = \left[\nu_A^L(x), \nu_A^U(x)\right]$ *represent the degree of membership and degree of non-membership of* A *respectively and* $\phi_A(x)$*,* $\eta_A(x)$ *represent the degree of membership and degree of non-membership of* A *respectively. Consider the mapping* $\widetilde{\mu}_A : X \to [0,1]$*,* $\widetilde{\nu}_A : X \to [0,1]$ *and* $0 \le (\widetilde{\mu}_A(x))^2 + (\widetilde{\nu}_A(x))^2 \le 1$ *means that* $0 \le \left(\mu_A^U(x)\right)^2 + \left(\nu_A^U(x)\right)^2 \le 1$ *and* $0 \le (\phi_A(x))^2 + (\eta_A(x))^2 \le 1$*. The degree of indeterminacy is determined as* $\pi_A(x) = \left\langle\left[\sqrt{1-(\mu_A^U(x))^2-(\nu_A^U(x))^2}, \sqrt{1-(\mu_A^L(x))^2-(\nu_A^L(x))^2}\right], \sqrt{1-(\phi_A(x))^2-(\eta_A(x))^2}\right\rangle$*.* $A = \left\langle\left(\left[\mu_A^L, \mu_A^U\right], \left[\nu_A^L, \nu_A^U\right]\right), (\phi_A(x), \eta_A(x))\right\rangle$ *is called a Pythagorean cubic fuzzy number.*

Definition 2.4. *Given that* $\beta_1 = \langle(\widetilde{\mu}_{\beta_1}, \widetilde{\nu}_{\beta_1}), (\phi_{\beta_1}, \eta_{\beta_1})\rangle$*,* $\beta_2 = \langle(\widetilde{\mu}_{\beta_2}, \widetilde{\nu}_{\beta_2}), (\phi_{\beta_2}, \eta_{\beta_2})\rangle$*, and* $\beta_3 = \langle(\widetilde{\mu}_{\beta_3}, \widetilde{\nu}_{\beta_3}), (\phi_{\beta_3}, \eta_{\beta_3})\rangle$ *are any three Pythagorean cubic fuzzy numbers over* (X,E)*, then the following properties hold:*

1) $\beta_1^c = \langle(\widetilde{\nu}_{\beta_1}, \widetilde{\mu}_{\beta_1}), (\eta_{\beta_1}, \phi_{\beta_1})\rangle$*.*
2) $\beta_2 \cup \beta_3 = \langle(\max(\widetilde{\mu}_{\beta_2}, \widetilde{\mu}_{\beta_3}), \min(\widetilde{\nu}_{\beta_2}, \widetilde{\nu}_{\beta_3})), (\max(\phi_{\beta_2}, \phi_{\beta_3}), \min(\eta_{\beta_2}, \eta_{\beta_3}))\rangle$*.*
3) $\beta_2 \cap \beta_3 = \langle(\min(\widetilde{\mu}_{\beta_2}, \widetilde{\mu}_{\beta_3}), \max(\widetilde{\nu}_{\beta_2}, \widetilde{\nu}_{\beta_3})), (\min(\phi_{\beta_2}, \phi_{\beta_3}), \max(\eta_{\beta_2}, \eta_{\beta_3}))\rangle$*.*
4) $\beta_2 \ge \beta_3$ *if and only if* $\widetilde{\mu}_{\beta_2} \ge \widetilde{\mu}_{\beta_3}$*,* $\widetilde{\nu}_{\beta_2} \le \widetilde{\nu}_{\beta_3}$*,* $\phi_{\beta_2} \ge \phi_{\beta_3}$*, and* $\eta_{\beta_2} \le \eta_{\beta_3}$*.*
5) $\beta_2 = \beta_3$ *if and only if* $\widetilde{\mu}_{\beta_2} = \widetilde{\mu}_{\beta_3}$*,* $\widetilde{\nu}_{\beta_2} = \widetilde{\nu}_{\beta_3}$*,* $\phi_{\beta_2} = \phi_{\beta_3}$*, and* $\eta_{\beta_2} = \eta_{\beta_3}$*.*

Definition 2.5. [7] *Let* X *be a non-empty set of the universe and* E *be a set of parameters. The pair* $(\mathscr{F}, A)$ *is called a Pythagorean fuzzy soft set on* X *if* $A \subseteq E$ *and* $\mathscr{F} : A \to \mathscr{PF}(X)$*, where* $\mathscr{PF}(X)$ *is the set of all Pythagorean fuzzy subsets of* X*.*

Definition 2.6. [6] *Let* X *be a non-empty set of the universe and* E *be a set of parameters. The pair* (X,E) *is a soft universe. Consider the mapping* $\mathscr{F} : E \to \mathscr{F}(X)$ *and* μ *is a fuzzy subset of* E*, i.e.,* $\mu : E \to \mathscr{F}(X)$*. Let* $\mathscr{F}_\mu : E \to \mathscr{F}(X) \times \mathscr{F}(X)$ *be a function defined as* $\mathscr{F}_\mu(e) = (\mathscr{F}(e)(x), \mu(e)(x))$*,* $\forall x \in X$*. Then* $\mathscr{F}_\mu$ *is called a possibility fuzzy soft set on* (X,E)*.*

3. **Similarity Measure between Two PPCFSSs.** In this section, we will look the notion of possibility Pythagorean cubic fuzzy soft set (PPCFSS) is a generalization of possibility Pythagorean fuzzy soft set and possibility Pythagorean interval-valued fuzzy soft set.

Definition 3.1. *Let* X *be a non-empty set of the universe and* E *be a set of parameters. The pair* (X,E) *is called a soft universe. Suppose that* $\widetilde{\mathscr{F}} : E \to \widetilde{\mathscr{PF}}(X)$*,* $\mathcal{F} : E \to \mathcal{PF}(X)$*,* $\widetilde{p}$ *and* p *are called the Pythagorean interval-valued fuzzy sets and Pythagorean fuzzy subsets of* E*, respectively. That is* $\widetilde{p} : E \to \widetilde{\mathscr{PF}}(X)$*,* $p : E \to \mathcal{PF}(X)$*, where* $\widetilde{\mathscr{PF}}(X)$ *and* $\mathcal{PF}(X)$ *denote the collection of all Pythagorean interval-valued fuzzy subsets and Pythagorean fuzzy subsets of* X*, respectively. If* $\widetilde{\mathscr{F}}_p : E \to \widetilde{\mathscr{PF}}(X) \times \widetilde{\mathscr{PF}}(X)$ *and*

$\mathcal{F}_p : E \to \mathcal{PF}(X)\times\mathcal{PF}(X)$ *are a function defined as* $\widetilde{\mathscr{F}}_p(e) = \left(\widetilde{\mathscr{F}}(e)(x), \widetilde{p}(e)(x)\right)$, $x \in X$ *and* $\mathcal{F}_p(e) = (\mathcal{F}(e)(x), p(e)(x))$, $x \in X$, *then* $\left\langle \widetilde{\mathscr{F}}_p, \mathcal{F}_p \right\rangle$ *is called a possibility Pythagorean cubic fuzzy soft set (PPCFSS) on* (X, E). *For each parameter* e,

$$\left\langle \widetilde{\mathscr{F}}_p(e), \mathcal{F}_p(e) \right\rangle = \Big\{x, \Big\langle \Big(\left(\eta_{\widetilde{\mathscr{F}}(e)}(x), \pi_{\mathcal{F}(e)}(x)\right), \left(\xi_{\widetilde{\mathscr{F}}(e)}(x), \varpi_{\mathcal{F}(e)}(x)\right)\Big), \left(\left(\eta_{\widetilde{p}(e)}(x), \pi_{p(e)}(x)\right), \left(\xi_{\widetilde{p}(e)}(x), \varpi_{p(e)}(x)\right)\right) \Big\rangle, x \in X\Big\}.$$

We described the procedures for calculating the similarity measures between PPCFSSs.

Similarity measure between two PPCFSSs such as $\left\langle \widetilde{\mathscr{F}}_p, \mathcal{F}_p \right\rangle$ and $\left\langle \widetilde{\mathscr{G}}_q, \mathcal{G}_q \right\rangle$ is defined as $\left\langle \mathrm{Sim}\left(\left\langle \widetilde{\mathscr{F}}_p, \mathcal{F}_p \right\rangle, \left\langle \widetilde{\mathscr{G}}_q, \mathcal{G}_q \right\rangle\right)\right\rangle = \left\langle \mathrm{Sim}\left(\widetilde{\mathscr{F}}_p, \widetilde{\mathscr{G}}_q\right), \mathrm{Sim}\left(\mathcal{F}_p, \mathcal{G}_q\right)\right\rangle = \left\langle \left(\varphi\left(\widetilde{\mathscr{F}}, \widetilde{\mathscr{G}}\right)\cdot\psi(\widetilde{p}, \widetilde{q})\right), (\varphi(\mathcal{F}, \mathcal{G})\cdot\psi(p, q))\right\rangle$ such that $\left\langle \varphi\left(\widetilde{\mathscr{F}}, \widetilde{\mathscr{G}}\right), \varphi(\mathcal{F}, \mathcal{G})\right\rangle = \left\langle \left[\varphi\left(\mathscr{F}^L, \mathscr{G}^L\right), \varphi\left(\mathscr{F}^U, \mathscr{G}^U\right)\right], \varphi(\mathcal{F}, \mathcal{G})\right\rangle = \left\langle \left[\frac{T\left(\mathscr{F}^L(e)(x), \mathscr{G}^L(e)(x)\right) + S\left(\mathscr{F}^L(e)(x), \mathscr{G}^L(e)(x)\right)}{2}, \frac{T\left(\mathscr{F}^U(e)(x), \mathscr{G}^U(e)(x)\right) + S\left(\mathscr{F}^U(e)(x), \mathscr{G}^U(e)(x)\right)}{2}\right], \frac{T(\mathcal{F}(e)(x), \mathcal{G}(e)(x)) + S(\mathcal{F}(e)(x), \mathcal{G}(e)(x))}{2}\right\rangle$ and $\left\langle \left[\psi\left(p^L, q^L\right), \psi\left(p^U, q^U\right)\right], \psi(p, q)\right\rangle = \left\langle \left[1 - \frac{\sum\left|\alpha_i^L - \beta_i^L\right|}{\sum\left|\alpha_i^L + \beta_i^L\right|}, 1 - \frac{\sum\left|\alpha_i^U - \beta_i^U\right|}{\sum\left|\alpha_i^U + \beta_i^U\right|}\right], 1 - \frac{\sum|\alpha_i - \beta_i|}{\sum|\alpha_i + \beta_i|}\right\rangle$, where $\left\langle \left[T\left(\mathscr{F}^L(e)(x), \mathscr{G}^L(e)(x)\right), T\left(\mathscr{F}^U(e)(x), \mathscr{G}^U(e)(x)\right)\right], T(\mathcal{F}(e)(x), \mathcal{G}(e)(x))\right\rangle = \left\langle \left[\frac{\sum_{i=1}^n\left(\eta^L_{\mathscr{F}(e_i)}(x)\cdot\eta^L_{\mathscr{G}(e_i)}(x)\right)}{\sum_{i=1}^n\left(1 - \sqrt{\left(1 - \eta^{2L}_{\mathscr{F}(e_i)}(x)\right)\cdot\left(1 - \eta^{2L}_{\mathscr{G}(e_i)}(x)\right)}\right)}, \frac{\sum_{i=1}^n\left(\eta^U_{\mathscr{F}(e_i)}(x)\cdot\eta^U_{\mathscr{G}(e_i)}(x)\right)}{\sum_{i=1}^n\left(1 - \sqrt{\left(1 - \eta^{2U}_{\mathscr{F}(e_i)}(x)\right)\cdot\left(1 - \eta^{2U}_{\mathscr{G}(e_i)}(x)\right)}\right)}\right], \frac{\sum_{i=1}^n\left(\pi_{\mathcal{F}(e_i)}(x)\cdot\pi_{\mathcal{G}(e_i)}(x)\right)}{\sum_{i=1}^n\left(1 - \sqrt{\left(1 - \pi^2_{\mathcal{F}(e_i)}(x)\right)\cdot\left(1 - \pi^2_{\mathcal{G}(e_i)}(x)\right)}\right)}\right\rangle$ and $\left\langle \left[S\left(\mathscr{F}^L(e)(x), \mathscr{G}^L(e)(x)\right), S\left(\mathscr{F}^U(e)(x), \mathscr{G}^U(e)(x)\right)\right], S(\mathcal{F}(e)(x), \mathcal{G}(e)(x))\right\rangle = \left\langle \left[\sqrt{1 - \frac{\sum_{i=1}^n\left|\xi^{2L}_{\mathscr{F}(e_i)}(x) - \xi^{2L}_{\mathscr{G}(e_i)}(x)\right|}{\sum_{i=1}^n 1 + \left(\left(\xi^{2L}_{\mathscr{F}(e_i)}(x)\right)\cdot\left(\xi^{2L}_{\mathscr{G}(e_i)}(x)\right)\right)}}, \sqrt{1 - \frac{\sum_{i=1}^n\left|\xi^{2U}_{\mathscr{F}(e_i)}(x) - \xi^{2U}_{\mathscr{G}(e_i)}(x)\right|}{\sum_{i=1}^n 1 + \left(\left(\xi^{2U}_{\mathscr{F}(e_i)}(x)\right)\cdot\left(\xi^{2U}_{\mathscr{G}(e_i)}(x)\right)\right)}}\right], \sqrt{1 - \frac{\sum_{i=1}^n\left|\varpi^2_{\mathcal{F}(e_i)}(x) - \varpi^2_{\mathcal{G}(e_i)}(x)\right|}{\sum_{i=1}^n 1 + \left(\left(\varpi^2_{\mathcal{F}(e_i)}(x)\right)\cdot\left(\varpi^2_{\mathcal{G}(e_i)}(x)\right)\right)}}\right\rangle$ and $\alpha_i^L = \frac{\eta^{2L}_{p(e_i)}(x)}{\eta^{2L}_{p(e_i)}(x) + \xi^{2L}_{p(e_i)}(x)}$, $\alpha_i^U = \frac{\eta^{2U}_{p(e_i)}(x)}{\eta^{2U}_{p(e_i)}(x) + \xi^{2U}_{p(e_i)}(x)}$, $\alpha_i = \frac{\pi^2_{p(e_i)}(x)}{\pi^2_{p(e_i)}(x) + \varpi^2_{p(e_i)}(x)}$, $\beta_i^L = \frac{\eta^{2L}_{q(e_i)}(x)}{\eta^{2L}_{q(e_i)}(x) + \xi^{2L}_{q(e_i)}(x)}$, $\beta_i^U = \frac{\eta^{2U}_{q(e_i)}(x)}{\eta^{2U}_{q(e_i)}(x) + \xi^{2U}_{q(e_i)}(x)}$, $\beta_i = \frac{\pi^2_{q(e_i)}(x)}{\pi^2_{q(e_i)}(x) + \varpi^2_{q(e_i)}(x)}$.

Theorem 3.1. *Let* $\left\langle \widetilde{\mathscr{F}}_p, \mathcal{F}_p \right\rangle$, $\left\langle \widetilde{\mathscr{G}}_q, \mathcal{G}_q \right\rangle$, *and* $\left\langle \widetilde{\mathscr{H}}_r, \mathcal{H}_r \right\rangle$ *be the any three PPCFSSs over* (X, E). *If* $\left\langle \widetilde{\mathscr{F}}_p, \mathcal{F}_p \right\rangle \subseteq \left\langle \widetilde{\mathscr{G}}_q, \mathcal{G}_q \right\rangle \subseteq \left\langle \widetilde{\mathscr{H}}_r, \mathcal{H}_r \right\rangle$, *then*

$$\mathrm{Sim}\left(\left\langle \widetilde{\mathscr{F}}_p, \mathcal{F}_p \right\rangle, \left\langle \widetilde{\mathscr{H}}_r, \mathcal{H}_r \right\rangle\right) \le \mathrm{Sim}\left(\left\langle \widetilde{\mathscr{G}}_q, \mathcal{G}_q \right\rangle, \left\langle \widetilde{\mathscr{H}}_r, \mathcal{H}_r \right\rangle\right).$$

Proof: Now, $\left\langle \widetilde{\mathscr{F}}_p, \mathcal{F}_p \right\rangle \subseteq \left\langle \widetilde{\mathscr{G}}_q, \mathcal{G}_q \right\rangle \implies \widetilde{\mathscr{F}}_p \subseteq \widetilde{\mathscr{G}}_q$ and $\mathcal{F}_p \subseteq \mathcal{G}_q$, $\left\langle \widetilde{\mathscr{F}}_p, \mathcal{F}_p \right\rangle \subseteq \left\langle \widetilde{\mathscr{H}}_r, \mathcal{H}_r \right\rangle \implies \widetilde{\mathscr{F}}_p \subseteq \widetilde{\mathscr{H}}_r$ and $\mathcal{F}_p \subseteq \mathcal{H}_r$ and $\left\langle \widetilde{\mathscr{G}}_q, \mathcal{G}_q \right\rangle \subseteq \left\langle \widetilde{\mathscr{H}}_r, \mathcal{H}_r \right\rangle \implies \widetilde{\mathscr{G}}_q \subseteq \widetilde{\mathscr{H}}_r$ and $\mathcal{G}_q \subseteq \mathcal{H}_r$. It is observed that

$$\begin{cases} \left[\eta^L_{\mathscr{F}(e)}(x), \eta^U_{\mathscr{F}(e)}(x)\right] = \eta_{\widetilde{\mathscr{F}}(e)}(x), & \left[\xi^L_{\mathscr{F}(e)}(x), \xi^U_{\mathscr{F}(e)}(x)\right] = \xi_{\widetilde{\mathscr{F}}(e)}(x) \\ \left[\eta^L_{p(e)}(x), \eta^U_{p(e)}(x)\right] = \eta_{\widetilde{p}(e)}(x), & \left[\xi^L_{p(e)}(x), \xi^U_{p(e)}(x)\right] = \xi_{\widetilde{p}(e)}(x) \\ \left[\eta^L_{\mathscr{G}(e)}(x), \eta^U_{\mathscr{G}(e)}(x)\right] = \eta_{\widetilde{\mathscr{G}}(e)}(x), & \left[\xi^L_{\mathscr{G}(e)}(x), \xi^U_{\mathscr{G}(e)}(x)\right] = \xi_{\widetilde{\mathscr{G}}(e)}(x) \\ \left[\eta^L_{q(e)}(x), \eta^U_{q(e)}(x)\right] = \eta_{\widetilde{q}(e)}(x), & \left[\xi^L_{q(e)}(x), \xi^U_{q(e)}(x)\right] = \xi_{\widetilde{q}(e)}(x) \\ \left[\eta^L_{\mathscr{H}(e)}(x), \eta^U_{\mathscr{H}(e)}(x)\right] = \eta_{\widetilde{\mathscr{H}}(e)}(x), & \left[\xi^L_{\mathscr{H}(e)}(x), \xi^U_{\mathscr{H}(e)}(x)\right] = \xi_{\widetilde{\mathscr{H}}(e)}(x) \\ \left[\eta^L_{r(e)}(x), \eta^U_{r(e)}(x)\right] = \eta_{\widetilde{r}(e)}(x), & \left[\xi^L_{r(e)}(x), \xi^U_{r(e)}(x)\right] = \xi_{\widetilde{r}(e)}(x). \end{cases}$$

That is,

$$\begin{cases} \widetilde{\mathscr{F}_p} \subseteq \widetilde{\mathscr{G}_q} \implies \eta_{\widetilde{\mathscr{F}}(e)}(x) \le \eta_{\widetilde{\mathscr{G}}(e)}(x), \;\; \xi_{\widetilde{\mathscr{F}}(e)}(x) \ge \xi_{\widetilde{\mathscr{G}}(e)}(x) \\ \eta_{\widetilde{p}(e)}(x) \le \eta_{\widetilde{q}(e)}(x), \;\; \xi_{\widetilde{p}(e)}(x) \ge \xi_{\widetilde{q}(e)}(x) \\ \widetilde{\mathscr{F}_p} \subseteq \widetilde{\mathscr{H}_r} \implies \eta_{\widetilde{\mathscr{F}}(e)}(x) \le \eta_{\widetilde{\mathscr{H}}(e)}(x), \;\; \xi_{\widetilde{\mathscr{F}}(e)}(x) \ge \xi_{\widetilde{\mathscr{H}}(e)}(x) \\ \eta_{\widetilde{p}(e)}(x) \le \eta_{\widetilde{r}(e)}(x), \;\; \xi_{\widetilde{p}(e)}(x) \ge \xi_{\widetilde{r}(e)}(x) \\ \widetilde{\mathscr{G}_q} \subseteq \widetilde{\mathscr{H}_r} \implies \eta_{\widetilde{\mathscr{G}}(e)}(x) \le \eta_{\widetilde{\mathscr{H}}(e)}(x), \;\; \xi_{\widetilde{\mathscr{G}}(e)}(x) \ge \xi_{\widetilde{\mathscr{H}}(e)}(x) \\ \eta_{\widetilde{q}(e)}(x) \le \eta_{\widetilde{r}(e)}(x), \;\; \xi_{\widetilde{q}(e)}(x) \ge \xi_{\widetilde{r}(e)}(x) \end{cases} \tag{1}$$

$$\begin{cases} \mathcal{F}_p \subseteq \mathcal{G}_q \implies \pi_{\mathcal{F}(e)}(x) \le \pi_{\mathcal{G}(e)}(x), \;\; \varpi_{\mathcal{F}(e)}(x) \ge \varpi_{\mathcal{G}(e)}(x) \\ \pi_{p(e)}(x) \le \pi_{q(e)}(x), \;\; \varpi_{p(e)}(x) \ge \varpi_{q(e)}(x) \\ \mathcal{F}_p \subseteq \mathcal{H}_r \implies \pi_{\mathcal{F}(e)}(x) \le \pi_{\mathcal{H}(e)}(x), \;\; \varpi_{\mathcal{F}(e)}(x) \ge \varpi_{\mathcal{H}(e)}(x) \\ \pi_{p(e)}(x) \le \pi_{r(e)}(x), \;\; \varpi_{p(e)}(x) \ge \varpi_{r(e)}(x) \\ \mathcal{G}_q \subseteq \mathcal{H}_r \implies \pi_{\mathcal{G}(e)}(x) \le \pi_{\mathcal{H}(e)}(x), \;\; \varpi_{\mathcal{G}(e)}(x) \ge \varpi_{\mathcal{H}(e)}(x) \\ \pi_{q(e)}(x) \le \pi_{r(e)}(x), \;\; \varpi_{q(e)}(x) \ge \varpi_{r(e)}(x). \end{cases} \tag{2}$$

From Equations (1) and (2), we get

$$\begin{cases} \eta_{\widetilde{\mathscr{F}}(e)}(x) \cdot \eta_{\widetilde{\mathscr{H}}(e)}(x) \le \eta_{\widetilde{\mathscr{G}}(e)}(x) \cdot \eta_{\widetilde{\mathscr{H}}(e)}(x) \\ \pi_{\mathcal{F}(e)}(x) \cdot \pi_{\mathcal{H}(e)}(x) \le \pi_{\mathcal{G}(e)}(x) \cdot \pi_{\mathcal{H}(e)}(x) \end{cases}$$

implies

$$\begin{cases} \displaystyle\sum_{i=1}^{n} \left(\eta_{\widetilde{\mathscr{F}}(e_i)}(x) \cdot \eta_{\widetilde{\mathscr{H}}(e_i)}(x)\right) \le \sum_{i=1}^{n} \left(\eta_{\widetilde{\mathscr{G}}(e_i)}(x) \cdot \eta_{\widetilde{\mathscr{H}}(e_i)}(x)\right) \\ \displaystyle\sum_{i=1}^{n} \left(\pi_{\mathcal{F}(e_i)}(x) \cdot \pi_{\mathcal{H}(e_i)}(x)\right) \le \sum_{i=1}^{n} \left(\pi_{\mathcal{G}(e_i)}(x) \cdot \pi_{\mathcal{H}(e_i)}(x)\right). \end{cases} \tag{3}$$

By Equations (1) and (2), $\left(\eta_{\widetilde{\mathscr{F}}(e)}(x)\right)^2 \le \left(\eta_{\widetilde{\mathscr{G}}(e)}(x)\right)^2$ and $\left(\pi_{\mathcal{F}(e)}(x)\right)^2 \le \left(\pi_{\mathcal{G}(e)}(x)\right)^2$. This implies

$$\begin{cases} \left(1 - \left(\eta_{\widetilde{\mathscr{F}}(e)}(x)\right)^2\right) \cdot \left(1 - \left(\eta_{\widetilde{\mathscr{H}}(e)}(x)\right)^2\right) \ge \left(1 - \left(\eta_{\widetilde{\mathscr{G}}(e)}(x)\right)^2\right) \cdot \left(1 - \left(\eta_{\widetilde{\mathscr{H}}(e)}(x)\right)^2\right) \\ \left(1 - \left(\pi_{\mathcal{F}(e)}(x)\right)^2\right) \cdot \left(1 - \left(\pi_{\mathcal{H}(e)}(x)\right)^2\right) \ge \left(1 - \left(\pi_{\mathcal{G}(e)}(x)\right)^2\right) \cdot \left(1 - \left(\pi_{\mathcal{H}(e)}(x)\right)^2\right) \end{cases}$$

and

$$\begin{cases} 1-\sqrt{\left(1-\left(\eta_{\widetilde{\mathscr{F}}(e)}(x)\right)^2\right)\cdot\left(1-\left(\eta_{\widetilde{\mathscr{H}}(e)}(x)\right)^2\right)} \\ \leq 1-\sqrt{\left(1-\left(\eta_{\widetilde{\mathscr{G}}(e)}(x)\right)^2\right)\cdot\left(1-\left(\eta_{\widetilde{\mathscr{H}}(e)}(x)\right)^2\right)} \\ 1-\sqrt{\left(1-\left(\pi_{\mathcal{F}(e)}(x)\right)^2\right)\cdot\left(1-\left(\pi_{\mathcal{H}(e)}(x)\right)^2\right)} \\ \leq 1-\sqrt{\left(1-\left(\pi_{\mathcal{G}(e)}(x)\right)^2\right)\cdot\left(1-\left(\pi_{\mathcal{H}(e)}(x)\right)^2\right)} \end{cases}$$

and

$$\begin{cases} \sum_{i=1}^{n} 1-\sqrt{\left(1-\left(\eta_{\widetilde{\mathscr{F}}(e_i)}(x)\right)^2\right)\cdot\left(1-\left(\eta_{\widetilde{\mathscr{H}}(e_i)}(x)\right)^2\right)} \\ \leq \sum_{i=1}^{n} 1-\sqrt{\left(1-\left(\eta_{\widetilde{\mathscr{G}}(e_i)}(x)\right)^2\right)\cdot\left(1-\left(\eta_{\widetilde{\mathscr{H}}(e_i)}(x)\right)^2\right)} \\ \sum_{i=1}^{n} 1-\sqrt{\left(1-\left(\pi_{\mathcal{F}(e_i)}(x)\right)^2\right)\cdot\left(1-\left(\pi_{\mathcal{H}(e_i)}(x)\right)^2\right)} \\ \leq \sum_{i=1}^{n} 1-\sqrt{\left(1-\left(\pi_{\mathcal{G}(e_i)}(x)\right)^2\right)\cdot\left(1-\left(\pi_{\mathcal{H}(e_i)}(x)\right)^2\right)}. \end{cases} \tag{4}$$

From Equations (3) and (4), we get

$$\begin{cases} \dfrac{\sum_{i=1}^{n}\left(\eta_{\widetilde{\mathscr{F}}(e_i)}(x)\cdot\eta_{\widetilde{\mathscr{H}}(e_i)}(x)\right)}{\sum_{i=1}^{n} 1-\sqrt{\left(1-\left(\eta_{\widetilde{\mathscr{F}}(e_i)}(x)\right)^2\right)\cdot\left(1-\left(\eta_{\widetilde{\mathscr{H}}(e_i)}(x)\right)^2\right)}} \\ \leq \dfrac{\sum_{i=1}^{n}\left(\eta_{\widetilde{\mathscr{G}}(e_i)}(x)\cdot\eta_{\widetilde{\mathscr{H}}(e_i)}(x)\right)}{\sum_{i=1}^{n} 1-\sqrt{\left(1-\left(\eta_{\widetilde{\mathscr{G}}(e_i)}(x)\right)^2\right)\cdot\left(1-\left(\eta_{\widetilde{\mathscr{H}}(e_i)}(x)\right)^2\right)}} \\ \dfrac{\sum_{i=1}^{n}\left(\pi_{\mathcal{F}(e_i)}(x)\cdot\pi_{\mathcal{H}(e_i)}(x)\right)}{\sum_{i=1}^{n} 1-\sqrt{\left(1-\left(\pi_{\mathcal{F}(e_i)}(x)\right)^2\right)\cdot\left(1-\left(\pi_{\mathcal{H}(e_i)}(x)\right)^2\right)}} \\ \leq \dfrac{\sum_{i=1}^{n}\left(\pi_{\mathcal{G}(e_i)}(x)\cdot\pi_{\mathcal{H}(e_i)}(x)\right)}{\sum_{i=1}^{n} 1-\sqrt{\left(1-\left(\pi_{\mathcal{G}(e_i)}(x)\right)^2\right)\cdot\left(1-\left(\pi_{\mathcal{H}(e_i)}(x)\right)^2\right)}}. \end{cases} \tag{5}$$

By Equations (1) and (2), we get

$$\begin{cases} \xi^2_{\widetilde{\mathscr{F}}(e)}(x) \geq \xi^2_{\widetilde{\mathscr{G}}(e)}(x) \text{ and } \xi^2_{\widetilde{\mathscr{F}}(e)}(x)-\xi^2_{\widetilde{\mathscr{H}}(e)}(x) \geq \xi^2_{\widetilde{\mathscr{G}}(e)}(x)-\xi^2_{\widetilde{\mathscr{H}}(e)}(x) \\ \varpi^2_{\mathcal{F}(e)}(x) \geq \varpi^2_{\mathcal{G}(e)}(x) \text{ and } \varpi^2_{\mathcal{F}(e)}(x)-\varpi^2_{\mathcal{H}(e)}(x) \geq \varpi^2_{\mathcal{G}(e)}(x)-\varpi^2_{\mathcal{H}(e)}(x). \end{cases}$$

Hence,

$$\begin{cases} \sum_{i=1}^{n}\left|\xi^2_{\widetilde{\mathscr{F}}(e_i)}(x)-\xi^2_{\widetilde{\mathscr{H}}(e_i)}(x)\right| \geq \sum_{i=1}^{n}\left|\xi^2_{\widetilde{\mathscr{G}}(e_i)}(x)-\xi^2_{\widetilde{\mathscr{H}}(e_i)}(x)\right| \\ \sum_{i=1}^{n}\left|\varpi^2_{\mathcal{F}(e_i)}(x)-\varpi^2_{\mathcal{H}(e_i)}(x)\right| \geq \sum_{i=1}^{n}\left|\varpi^2_{\mathcal{G}(e_i)}(x)-\varpi^2_{\mathcal{H}(e_i)}(x)\right|. \end{cases} \tag{6}$$

From Equations (1) and (2), we have

$$\begin{cases} \left(\xi^2_{\widetilde{\mathscr{F}}(e)}(x) \cdot \xi^2_{\widetilde{\mathscr{H}}(e)}(x)\right) \geq \left(\xi^2_{\widetilde{\mathscr{G}}(e)}(x) \cdot \xi^2_{\widetilde{\mathscr{H}}(e)}(x)\right) \\ \left(\varpi^2_{\mathcal{F}(e)}(x) \cdot \varpi^2_{\mathcal{H}(e)}(x)\right) \geq \left(\varpi^2_{\mathcal{G}(e)}(x) \cdot \varpi^2_{\mathcal{H}(e)}(x)\right) \end{cases}$$

implies

$$\begin{cases} \sum_{i=1}^{n} 1 + \left(\xi^2_{\widetilde{\mathscr{F}}(e_i)}(x) \cdot \xi^2_{\widetilde{\mathscr{H}}(e_i)}(x)\right) \geq \sum_{i=1}^{n} 1 + \left(\xi^2_{\widetilde{\mathscr{G}}(e_i)}(x) \cdot \xi^2_{\widetilde{\mathscr{H}}(e_i)}(x)\right) \\ \sum_{i=1}^{n} 1 + \left(\varpi^2_{\mathcal{F}(e_i)}(x) \cdot \varpi^2_{\mathcal{H}(e_i)}(x)\right) \geq \sum_{i=1}^{n} 1 + \left(\varpi^2_{\mathcal{G}(e_i)}(x) \cdot \varpi^2_{\mathcal{H}(e_i)}(x)\right). \end{cases} \tag{7}$$

From Equations (6) and (7), we get

$$\begin{cases} \dfrac{\sum_{i=1}^{n} \left|\xi^2_{\widetilde{\mathscr{F}}(e_i)}(x) - \xi^2_{\widetilde{\mathscr{H}}(e_i)}(x)\right|}{\sum_{i=1}^{n} 1 + \left(\xi^2_{\widetilde{\mathscr{F}}(e_i)}(x) \cdot \xi^2_{\widetilde{\mathscr{H}}(e_i)}(x)\right)} \geq \dfrac{\sum_{i=1}^{n} \left|\xi^2_{\widetilde{\mathscr{G}}(e_i)}(x) - \xi^2_{\widetilde{\mathscr{H}}(e_i)}(x)\right|}{\sum_{i=1}^{n} 1 + \left(\xi^2_{\widetilde{\mathscr{G}}(e_i)}(x) \cdot \xi^2_{\widetilde{\mathscr{H}}(e_i)}(x)\right)} \\ \dfrac{\sum_{i=1}^{n} \left|\varpi^2_{\mathcal{F}(e_i)}(x) - \varpi^2_{\mathcal{H}(e_i)}(x)\right|}{\sum_{i=1}^{n} 1 + \left(\varpi^2_{\mathcal{F}(e_i)}(x) \cdot \varpi^2_{\mathcal{H}(e_i)}(x)\right)} \geq \dfrac{\sum_{i=1}^{n} \left|\varpi^2_{\mathcal{G}(e_i)}(x) - \varpi^2_{\mathcal{H}(e_i)}(x)\right|}{\sum_{i=1}^{n} 1 + \left(\varpi^2_{\mathcal{G}(e_i)}(x) \cdot \varpi^2_{\mathcal{H}(e_i)}(x)\right)} \end{cases}$$

and

$$\begin{cases} 1 - \dfrac{\sum_{i=1}^{n} \left|\xi^2_{\widetilde{\mathscr{F}}(e_i)}(x) - \xi^2_{\widetilde{\mathscr{H}}(e_i)}(x)\right|}{\sum_{i=1}^{n} 1 + \left(\xi^2_{\widetilde{\mathscr{F}}(e_i)}(x) \cdot \xi^2_{\widetilde{\mathscr{H}}(e_i)}(x)\right)} \leq 1 - \dfrac{\sum_{i=1}^{n} \left|\xi^2_{\widetilde{\mathscr{G}}(e_i)}(x) - \xi^2_{\widetilde{\mathscr{H}}(e_i)}(x)\right|}{\sum_{i=1}^{n} 1 + \left(\xi^2_{\widetilde{\mathscr{G}}(e_i)}(x) \cdot \xi^2_{\widetilde{\mathscr{H}}(e_i)}(x)\right)} \\ 1 - \dfrac{\sum_{i=1}^{n} \left|\varpi^2_{\mathcal{F}(e_i)}(x) - \varpi^2_{\mathcal{H}(e_i)}(x)\right|}{\sum_{i=1}^{n} 1 + \left(\varpi^2_{\mathcal{F}(e_i)}(x) \cdot \varpi^2_{\mathcal{H}(e_i)}(x)\right)} \leq 1 - \dfrac{\sum_{i=1}^{n} \left|\varpi^2_{\mathcal{G}(e_i)}(x) - \varpi^2_{\mathcal{H}(e_i)}(x)\right|}{\sum_{i=1}^{n} 1 + \left(\varpi^2_{\mathcal{G}(e_i)}(x) \cdot \varpi^2_{\mathcal{H}(e_i)}(x)\right)} \end{cases}$$

and

$$\begin{cases} \sqrt{1 - \dfrac{\sum_{i=1}^{n} \left|\xi^2_{\widetilde{\mathscr{F}}(e_i)}(x) - \xi^2_{\widetilde{\mathscr{H}}(e_i)}(x)\right|}{\sum_{i=1}^{n} 1 + \left(\xi^2_{\widetilde{\mathscr{F}}(e_i)}(x) \cdot \xi^2_{\widetilde{\mathscr{H}}(e_i)}(x)\right)}} \leq \sqrt{1 - \dfrac{\sum_{i=1}^{n} \left|\xi^2_{\widetilde{\mathscr{G}}(e_i)}(x) - \xi^2_{\widetilde{\mathscr{H}}(e_i)}(x)\right|}{\sum_{i=1}^{n} 1 + \left(\xi^2_{\widetilde{\mathscr{G}}(e_i)}(x) \cdot \xi^2_{\widetilde{\mathscr{H}}(e_i)}(x)\right)}} \\ \sqrt{1 - \dfrac{\sum_{i=1}^{n} \left|\varpi^2_{\mathcal{F}(e_i)}(x) - \varpi^2_{\mathcal{H}(e_i)}(x)\right|}{\sum_{i=1}^{n} 1 + \left(\varpi^2_{\mathcal{F}(e_i)}(x) \cdot \varpi^2_{\mathcal{H}(e_i)}(x)\right)}} \leq \sqrt{1 - \dfrac{\sum_{i=1}^{n} \left|\varpi^2_{\mathcal{G}(e_i)}(x) - \varpi^2_{\mathcal{H}(e_i)}(x)\right|}{\sum_{i=1}^{n} 1 + \left(\varpi^2_{\mathcal{G}(e_i)}(x) \cdot \varpi^2_{\mathcal{H}(e_i)}(x)\right)}}. \end{cases} \tag{8}$$

From Equations (5) and (8), we get

$$\varphi\left(\widetilde{\mathscr{F}}, \widetilde{\mathscr{H}}\right) \leq \varphi\left(\widetilde{\mathscr{G}}, \widetilde{\mathscr{H}}\right) \text{ and } \varphi(\mathcal{F}, \mathcal{H}) \leq \varphi(\mathcal{G}, \mathcal{H}). \tag{9}$$

By Equations (1) and (2), clearly $\left\langle \left[\alpha_i^L, \alpha_i^U\right], \alpha\right\rangle \leq \left\langle \left[\beta_i^L, \beta_i^U\right], \beta\right\rangle \leq \left\langle \left[\gamma_i^L, \gamma_i^U\right], \gamma\right\rangle$, where $\left\langle \left[\alpha_i^L, \alpha_i^U\right], \alpha\right\rangle = \left\langle \left[\frac{\eta^{2L}_{p(e_i)}(x)}{\eta^{2L}_{p(e_i)}(x) + \xi^{2L}_{p(e_i)}(x)}, \frac{\eta^{2U}_{p(e_i)}(x)}{\eta^{2U}_{p(e_i)}(x) + \xi^{2U}_{p(e_i)}(x)}\right], \frac{\pi^2_{p(e_i)}(x)}{\pi^2_{p(e_i)}(x) + \varpi^2_{p(e_i)}(x)}\right\rangle$ and $\left\langle \left[\beta_i^L, \beta_i^U\right], \beta\right\rangle = \left\langle \left[\frac{\eta^{2L}_{q(e_i)}(x)}{\eta^{2L}_{q(e_i)}(x) + \xi^{2L}_{q(e_i)}(x)}, \frac{\eta^{2U}_{q(e_i)}(x)}{\eta^{2U}_{q(e_i)}(x) + \xi^{2U}_{q(e_i)}(x)}\right], \frac{\pi^2_{q(e_i)}(x)}{\pi^2_{q(e_i)}(x) + \varpi^2_{q(e_i)}(x)}\right\rangle$ and $\left\langle \left[\gamma_i^L, \gamma_i^U\right], \gamma\right\rangle = \left\langle \left[\frac{\eta^{2L}_{r(e_i)}(x)}{\eta^{2L}_{r(e_i)}(x) + \xi^{2L}_{r(e_i)}(x)}, \frac{\eta^{2U}_{r(e_i)}(x)}{\eta^{2U}_{r(e_i)}(x) + \xi^{2U}_{r(e_i)}(x)}\right], \frac{\pi^2_{r(e_i)}(x)}{\pi^2_{r(e_i)}(x) + \varpi^2_{r(e_i)}(x)}\right\rangle$. Now, $\left\langle \left[\alpha_i^L, \alpha_i^U\right], \alpha\right\rangle - \left\langle \left[\gamma_i^L, \gamma_i^U\right],\right.$

$\gamma\rangle \leq \langle [\beta_i^L, \beta_i^U], \beta\rangle - \langle [\gamma_i^L, \gamma_i^U], \gamma\rangle$. $\langle [\alpha_i^L, \alpha_i^U], \alpha\rangle, \langle [\beta_i^L, \beta_i^U], \beta\rangle, \langle [\gamma_i^L, \gamma_i^U], \gamma\rangle$ are numerical values. Hence, $|\langle [\beta_i^L, \beta_i^U], \beta\rangle - \langle [\gamma_i^L, \gamma_i^U], \gamma\rangle| \leq |\langle [\alpha_i^L, \alpha_i^U], \alpha\rangle - \langle [\gamma_i^L, \gamma_i^U], \gamma\rangle|$ and

$$-\left|\langle [\alpha_i^L, \alpha_i^U], \alpha\rangle - \langle [\gamma_i^L, \gamma_i^U], \gamma\rangle\right| \leq -\left|\langle [\beta_i^L, \beta_i^U], \beta\rangle - \langle [\gamma_i^L, \gamma_i^U], \gamma\rangle\right| \tag{10}$$

and

$$\left|\langle [\alpha_i^L, \alpha_i^U], \alpha\rangle + \langle [\gamma_i^L, \gamma_i^U], \gamma\rangle\right| \leq \left|\langle [\beta_i^L, \beta_i^U], \beta\rangle + \langle [\gamma_i^L, \gamma_i^U], \gamma\rangle\right|. \tag{11}$$

Equation (10) is divided by Equation (11), and we get

$$\frac{-\left|\langle [\alpha_i^L, \alpha_i^U], \alpha\rangle - \langle [\gamma_i^L, \gamma_i^U], \gamma\rangle\right|}{\left|\langle [\alpha_i^L, \alpha_i^U], \alpha\rangle + \langle [\gamma_i^L, \gamma_i^U], \gamma\rangle\right|} \leq \frac{-\left|\langle [\beta_i^L, \beta_i^U], \beta\rangle - \langle [\gamma_i^L, \gamma_i^U], \gamma\rangle\right|}{\left|\langle [\beta_i^L, \beta_i^U], \beta\rangle + \langle [\gamma_i^L, \gamma_i^U], \gamma\rangle\right|}$$

and

$$1 - \frac{\left|\langle [\alpha_i^L, \alpha_i^U], \alpha\rangle - \langle [\gamma_i^L, \gamma_i^U], \gamma\rangle\right|}{\left|\langle [\alpha_i^L, \alpha_i^U], \alpha\rangle + \langle [\gamma_i^L, \gamma_i^U], \gamma\rangle\right|} \leq 1 - \frac{\left|\langle [\beta_i^L, \beta_i^U], \beta\rangle - \langle [\gamma_i^L, \gamma_i^U], \gamma\rangle\right|}{\left|\langle [\beta_i^L, \beta_i^U], \beta\rangle + \langle [\gamma_i^L, \gamma_i^U], \gamma\rangle\right|}.$$

Hence,

$$\psi(\widetilde{p}, \widetilde{r}) \leq \psi(\widetilde{q}, \widetilde{r}) \text{ and } \psi(p, r) \leq \psi(q, r). \tag{12}$$

From Equations (9) and (12), we get

$$\varphi\left(\widetilde{\mathscr{F}}, \widetilde{\mathscr{H}}\right) \cdot \psi(\widetilde{p}, \widetilde{r}) \leq \varphi\left(\widetilde{\mathscr{G}}, \widetilde{\mathscr{H}}\right) \cdot \psi(\widetilde{q}, \widetilde{r}) \text{ and } \varphi(\mathcal{F}, \mathcal{H}) \cdot \psi(p, r) \leq \varphi(\mathcal{G}, \mathcal{H}) \cdot \psi(q, r).$$

Hence, $\mathrm{Sim}\left(\left\langle \widetilde{\mathscr{F}}_p, \mathcal{F}_p\right\rangle, \left\langle \widetilde{\mathscr{H}}_r, \mathcal{H}_r\right\rangle\right) \leq \mathrm{Sim}\left(\left\langle \widetilde{\mathscr{G}}_q, \mathcal{G}_q\right\rangle, \left\langle \widetilde{\mathscr{H}}_r, \mathcal{H}_r\right\rangle\right)$.

4. **An Application of PPCFSSs in Decision-Making.** We encounter issues with decision-making in our daily lives related to politics, management, the economy, education, and the use of technology. The majority of individuals have personal computers in their homes, and they can use them for work, school, or home purposes. These computers have a lot of memory and storage capacity. In order to allow users to view more colors, computers themselves feature glass monitors similar to those used in televisions. Additionally, it has a greater resolution rate, which improves visibility. A personal computer may be equipped with some amazing functions. Printers, larger speakers, desktop scanner beds, and best of all a larger hard drive may all be added. Today's laptops are small, portable computers that are easy to transport, making life easier to take on business trips, vacations, and anywhere else make people want to take it. A laptop simply denotes the ability to place the computer on a flat surface, such as a desk or person's lap. The plastic screen on laptop computers themselves lowers the resolution rate. This explains why it is so difficult for certain individuals to perceive stuff on computers. The computer screen will always emit changing colours depending on where users are seated in front of it, making it more difficult to see the display. Out of a large number of options, we want to choose the best one based on professional evaluations against the criteria.

Any of the following factors may lead to the need to purchase a laptop:

1) He was having issues with his old laptop.
2) He needed a new laptop so he could check his emails from home.
3) He wished to give his wife a brand-new laptop.
4) He need a new laptop in order to launch his own company.

Before making a product or service purchase, a customer passes through a number of stages.

4.1. **Algorithm for PPCFSS model.** The algorithm for the selection of the best choice is given as follows.

1) Enter the values for PPCFSS $\left\langle \widetilde{\mathscr{F}_p}, \widetilde{\mathcal{F}}_p \right\rangle$ in tabular form.
2) Enter the selection of parameters $A \subseteq E$.
3) Determine the values of T and S.
4) Calculate the value $\varphi = \frac{T+S}{2}$.
5) Determine the value $\psi = 1 - \frac{\sum |\alpha_i - \beta_i|}{\sum |\alpha_i + \beta_i|}$ and $1 \leq i \leq 5$.
6) Take the product of φ and ψ to calculate the similarity measure.
7) Calculate the greatest similarity as $\mathrm{Max}\{similarity^i\}$ and $1 \leq i \leq 5$.
8) The best solution to the problem must be chosen as the final option.

4.2. **Survey study.** A consumer must choose the laptop he wants to purchase from the five types (alternatives) listed, which are L_1, L_2, L_3, L_4 and L_5. The score of the laptop evaluated by the experts is represented by $E = \{e_1$: Battery life, e_2: Storage capacity, e_3: Version of operating system, e_4: Over all cost, e_5: Speed of the processor$\}$.

Table 1. PPCFSS for the ideal laptop score

$\widetilde{\mathscr{L}_p}(e)$	e_1	e_2	e_3	e_4	e_5
$\left(\widetilde{\mathscr{L}}(e), \mathcal{L}(e)\right)$	$([0.8\ \ 0.95], 0.9)$, $([0.1\ \ 0.25], 0.15)$	$([0.85\ \ 0.9], 0.8)$, $([0.2\ \ 0.35], 0.2)$	$([0.9\ \ 0.95], 0.9)$, $([0.15\ \ 0.3], 0.15)$	$([0.85\ \ 0.9], 0.85)$, $([0.2\ \ 0.5], 0.2)$	$([0.8\ \ 0.85], 0.8)$, $([0.25\ \ 0.6], 0.3)$
$(\widetilde{p}(e), p(e))$	$([1\ \ 1], 1)$, $([0\ \ 0], 0)$	$([1\ \ 1], 1)$, $([0\ \ 0], 0)$	$([1\ \ 1], 1)$, $([0\ \ 0], 0)$	$([1\ \ 1], 1)$, $([0\ \ 0], 0)$	$([1\ \ 1], 1)$, $([0\ \ 0], 0)$

Table 2. PPCFSS for the first laptop score

$\widetilde{\mathscr{A}_{p_1}}(e)$	e_1	e_2	e_3	e_4	e_5
$\left(\widetilde{\mathscr{A}}(e), \mathcal{A}(e)\right)$	$([0.6\ \ 0.75], 0.75)$, $([0.5\ \ 0.55], 0.45)$	$([0.5\ \ 0.6], 0.55)$, $([0.45\ \ 0.55], 0.35)$	$([0.6\ \ 0.7], 0.65)$, $([0.5\ \ 0.6], 0.4)$	$([0.5\ \ 0.55], 0.55)$, $([0.4\ \ 0.5], 0.55)$	$([0.55\ \ 0.6], 0.45)$, $([0.4\ \ 0.65], 0.6)$
$(\widetilde{p}_1(e), p_1(e))$	$([0.3\ \ 0.45], 0.35)$, $([0.6\ \ 0.7], 0.5)$	$([0.4\ \ 0.6], 0.45)$, $([0.5\ \ 0.65], 0.4)$	$([0.5\ \ 0.7], 0.55)$, $([0.55\ \ 0.65], 0.5)$	$([0.4\ \ 0.45], 0.45)$, $([0.5\ \ 0.65], 0.7)$	$([0.4\ \ 0.5], 0.35)$, $([0.5\ \ 0.7], 0.8)$

Table 3. PPCFSS for the second laptop score

$\widetilde{\mathscr{B}_{p_2}}(e)$	e_1	e_2	e_3	e_4	e_5
$\left(\widetilde{\mathscr{B}}(e), \mathcal{B}(e)\right)$	$([0.6\ 0.65], 0.75)$, $([0.5\ 0.6], 0.4)$	$([0.65\ 0.75], 0.7)$, $([0.45\ 0.5], 0.5)$	$([0.7\ 0.8], 0.8)$, $([0.4\ 0.45], 0.6)$	$([0.6\ 0.65], 0.6)$, $([0.4\ 0.6], 0.4)$	$([0.5\ 0.7], 0.5)$, $([0.5\ 0.7], 0.5)$
$(\widetilde{p}_2(e), p_2(e))$	$([0.5\ 0.6], 0.5)$, $([0.7\ 0.75], 0.7)$	$([0.4\ 0.5], 0.4)$, $([0.6\ 0.65], 0.6)$	$([0.5\ 0.65], 0.5)$, $([0.6\ 0.75], 0.7)$	$([0.55\ 0.6], 0.5)$, $([0.5\ 0.7], 0.4)$	$([0.4\ 0.6], 0.4)$, $([0.65\ 0.75], 0.6)$

Table 4. PPCFSS for the third laptop score

$\widetilde{\mathscr{C}_{p_3}}(e)$	e_1	e_2	e_3	e_4	e_5
$\left(\widetilde{\mathscr{C}}(e), \mathcal{C}(e)\right)$	$([0.5\ \ 0.75], 0.7)$, $([0.55\ \ 0.6], 0.5)$	$([0.6\ \ 0.7], 0.6)$, $([0.6\ \ 0.65], 0.4)$	$([0.7\ \ 0.75], 0.65)$, $([0.45\ \ 0.6], 0.5)$	$([0.6\ \ 0.7], 0.55)$, $([0.5\ \ 0.7], 0.6)$	$([0.5\ \ 0.6], 0.5)$, $([0.6\ \ 0.75], 0.55)$
$(\widetilde{p}_3(e), p_3(e))$	$([0.4\ \ 0.6], 0.5)$, $([0.7\ \ 0.75], 0.5)$	$([0.25\ \ 0.4], 0.3)$, $([0.6\ \ 0.75], 0.4)$	$([0.45\ \ 0.5], 0.4)$, $([0.5\ \ 0.8], 0.6)$	$([0.5\ \ 0.6], 0.3)$, $([0.6\ \ 0.75], 0.7)$	$([0.3\ \ 0.4], 0.2)$, $([0.8\ \ 0.9], 0.65)$

TABLE 5. PPCFSS for the fourth laptop score

$\widetilde{\mathscr{D}}_{p_4}(e)$	e_1	e_2	e_3	e_4	e_5
$\left(\widetilde{\mathscr{D}}(e), \mathcal{D}(e)\right)$	$([0.75\ 0.8], 0.8)$, $([0.3\ 0.4], 0.25)$	$([0.7\ 0.75], 0.6)$, $([0.35\ 0.45], 0.35)$	$([0.75\ 0.8], 0.5)$, $([0.4\ 0.55], 0.4)$	$([0.6\ 0.7], 0.7)$, $([0.3\ 0.5], 0.45)$	$([0.5\ 0.65], 0.6)$, $([0.65\ 0.7], 0.55)$
$(\widetilde{p}_4(e), p_4(e))$	$([0.55\ 0.65], 0.4)$, $([0.45\ 0.6], 0.3)$	$([0.45\ 0.6], 0.5)$, $([0.5\ 0.55], 0.4)$	$([0.5\ 0.65], 0.3)$, $([0.6\ 0.65], 0.45)$	$([0.3\ 0.55], 0.6)$, $([0.35\ 0.55], 0.5)$	$([0.4\ 0.5], 0.5)$, $([0.7\ 0.75], 0.6)$

TABLE 6. PPCFSS for the fifth laptop score

$\widetilde{\mathscr{E}}_{p_5}(e)$	e_1	e_2	e_3	e_4	e_5
$\left(\widetilde{\mathscr{E}}(e), \mathcal{E}(e)\right)$	$([0.6\ 0.65], 0.7)$, $([0.3\ 0.4], 0.3)$	$([0.5\ 0.7], 0.5)$, $([0.35\ 0.4], 0.4)$	$([0.5\ 0.6], 0.7)$, $([0.3\ 0.5], 0.5)$	$([0.4\ 0.6], 0.6)$, $([0.4\ 0.55], 0.45)$	$([0.5\ 0.6], 0.5)$, $([0.6\ 0.75], 0.6)$
$(\widetilde{p}_5(e), p_5(e))$	$([0.4\ 0.55], 0.3)$, $([0.45\ 0.75], 0.4)$	$([0.3\ 0.5], 0.4)$, $([0.55\ 0.75], 0.6)$	$([0.4\ 0.5], 0.4)$, $([0.7\ 0.75], 0.7)$	$([0.25\ 0.3], 0.2)$, $([0.5\ 0.6], 0.5)$	$([0.3\ 0.35], 0.45)$, $([0.7\ 0.75], 0.65)$

To get the laptop score that is most similar to the ideal laptop score, we should compare the PPCFSSs in Table 2 through Table 6 with the one in Table 1. A formula for determining the similarity among laptop 1 to laptop 5 is provided in the table below.

	T	S
$\left\langle\left(\widetilde{\mathscr{L}, \mathscr{A}}\right), (\mathcal{L}, \mathcal{A})\right\rangle$	$\langle[0.83763181\ \ 0.85047941], 0.87047951\rangle$	$\langle[0.9119204\ \ 0.92633589], 0.90311816\rangle$
$\left\langle\left(\widetilde{\mathscr{L}, \mathscr{B}}\right), (\mathcal{L}, \mathcal{B})\right\rangle$	$\langle[0.89716962\ \ 0.90303363], 0.93000612\rangle$	$\langle[0.9119864\ \ 0.92430481], 0.89940736\rangle$
$\left\langle\left(\widetilde{\mathscr{L}, \mathscr{C}}\right), (\mathcal{L}, \mathcal{C})\right\rangle$	$\langle[0.87293315\ \ 0.90179578], 0.87820032\rangle$	$\langle[0.8618599\ \ 0.87110347], 0.88379586\rangle$
$\left\langle\left(\widetilde{\mathscr{L}, \mathscr{D}}\right), (\mathcal{L}, \mathcal{D})\right\rangle$	$\langle[0.92741362\ \ 0.93162693], 0.90089308\rangle$	$\langle[0.92688208\ \ 0.94968628], 0.93495378\rangle$
$\left\langle\left(\widetilde{\mathscr{L}, \mathscr{E}}\right), (\mathcal{L}, \mathcal{E})\right\rangle$	$\langle[0.78058587\ \ 0.83564405], 0.88111222\rangle$	$\langle[0.93351178\ \ 0.9470031], 0.91232082\rangle$

φ
$\langle[0.87477611\ \ 0.88840765], 0.88679884\rangle$
$\langle[0.90457801\ \ 0.91366922], 0.91470674\rangle$
$\langle[0.86739653\ \ 0.88644963], 0.88099809\rangle$
$\langle[0.92714785\ \ 0.94065661], 0.91792343\rangle$
$\langle[0.85704882\ \ 0.89132357], 0.89671652\rangle$

	ψ	*Similarity*
$\left\langle\left(\widetilde{\mathscr{L}, \mathscr{A}}\right), (\mathcal{L}, \mathcal{A})\right\rangle$	$\langle[0.53441639\ \ 0.56142158], 0.54821359\rangle$	$\langle[0.46749469\ \ 0.49877122], 0.48615517\rangle$
$\left\langle\left(\widetilde{\mathscr{L}, \mathscr{B}}\right), (\mathcal{L}, \mathcal{B})\right\rangle$	$\langle[0.54599504\ \ 0.57238679], 0.55089609\rangle$	$\langle[0.49389511\ \ 0.5229722], 0.50390837\rangle$
$\left\langle\left(\widetilde{\mathscr{L}, \mathscr{C}}\right), (\mathcal{L}, \mathcal{C})\right\rangle$	$\langle[0.43130375\ \ 0.44908062], 0.43977968\rangle$	$\langle[0.37411137\ \ 0.39808735], 0.38744506\rangle$
$\left\langle\left(\widetilde{\mathscr{L}, \mathscr{D}}\right), (\mathcal{L}, \mathcal{D})\right\rangle$	$\langle[0.59669666\ \ 0.64700762], 0.67680208\rangle$	$\langle[0.55322602\ \ 0.60861199], 0.62125249\rangle$
$\left\langle\left(\widetilde{\mathscr{L}, \mathscr{E}}\right), (\mathcal{L}, \mathcal{E})\right\rangle$	$\langle[0.40561350\ \ 0.42368948], 0.43156376\rangle$	$\langle[0.34763057\ \ 0.37764442], 0.38699035\rangle$

According to the aforementioned findings, the laptops may be compared using the order of $L_4 \geq L_2 \geq L_1 \geq L_3 \geq L_5$. As a result, we discover that the fourth laptop score, which has the greatest similarity measure value, that score is most similar to the ideal laptop score. This score is expressed as $\langle[\mathbf{0.55322602\ \ 0.60861199}], \mathbf{0.62125249}\rangle$.

5. **Pythagorean Cubic Fuzzy Soft Sets (PCFSSs).**

Algorithm for PCFSS model. The following is the best choice selection algorithm:

1) Enter the PCFSS $\left\langle \widetilde{\mathscr{F}}, \widetilde{\mathcal{F}} \right\rangle$ in tabular form.
2) Enter the selection of criteria $A \subseteq E$.
3) Determine the values of T and S.
4) Calculate $similarity = \frac{T+S}{2}$.
5) Calculate the greatest similarity using the formula $\mathrm{Max}\{similarity^i\}$ and $1 \leq i \leq 5$.
6) The best solution to the problem must be chosen as the final option.

We apply the PCFSS approach to evaluating the aforementioned survey research in order to investigate the effects of the possibility parameter. Using the following formula, we can determine how similar the aforementioned laptop 1 to laptop 5. We have

	T	S
$\left\langle \left(\widetilde{\mathscr{L}, \mathscr{A}}\right), (\mathcal{L}, \mathcal{A}) \right\rangle$	$\langle[0.83763181\ \ 0.85047941], 0.87047951\rangle$	$\langle[0.9119204\ \ 0.92633589], 0.90311816\rangle$
$\left\langle \left(\widetilde{\mathscr{L}, \mathscr{B}}\right), (\mathcal{L}, \mathcal{B}) \right\rangle$	$\langle[0.89716962\ \ 0.90303363], 0.93000612\rangle$	$\langle[0.9119864\ \ 0.92430481], 0.89940736\rangle$
$\left\langle \left(\widetilde{\mathscr{L}, \mathscr{C}}\right), (\mathcal{L}, \mathcal{C}) \right\rangle$	$\langle[0.87293315\ \ 0.90179578], 0.87820032\rangle$	$\langle[0.8618599\ \ 0.87110347], 0.88379586\rangle$
$\left\langle \left(\widetilde{\mathscr{L}, \mathscr{D}}\right), (\mathcal{L}, \mathcal{D}) \right\rangle$	$\langle[0.92741362\ \ 0.93162693], 0.90089308\rangle$	$\langle[0.92688208\ \ 0.94968628], 0.93495378\rangle$
$\left\langle \left(\widetilde{\mathscr{L}, \mathscr{E}}\right), (\mathcal{L}, \mathcal{E}) \right\rangle$	$\langle[0.78058587\ \ 0.83564405], 0.88111222\rangle$	$\langle[0.93351178\ \ 0.9470031], 0.91232082\rangle$

$Similarity$
$\langle[0.87477611\ \ 0.88840765], 0.88679884\rangle$
$\langle[0.90457801\ \ 0.91366922], 0.91470674\rangle$
$\langle[0.86739653\ \ 0.88644963], 0.88099809\rangle$
$\langle[0.92714785\ \ 0.94065661], 0.91792343\rangle$
$\langle[0.85704882\ \ 0.89132357], 0.89671652\rangle$

6. **Comparison of PPCFSSs and PCFSSs.** According to the aforementioned findings, the parameter significantly affects how the similarity measure of PPCFSSs is calculated. According to the similarity measure, it can be seen that the first, second, third, and fifth laptop scores are relatively far from the ideal laptop score. The fourth laptop score should be considered as a prospective laptop if the laptop score selects the threshold $[0.5, 0.65]$. However, without the generalization parameter, we are unable to determine which laptop score is the best when utilizing the PCFSS technique. As a result, the possibility parameter significantly affects the similarity of the fourth laptop score. The PPCFSS approach is consequently more logical and scientific because the PCFSS technique eliminates the generalization parameter.

7. **Discussion and Conclusion.** The primary objective of this study is to offer a possibility Pythagorean cubic fuzzy soft set to resolve the decision-related problems that include fuzzy soft sets and interval-valued soft sets. We discussed an algorithm for decision making and its use with this soft model. Additionally, in decision-making issues, we contrasted possibility Pythagorean cubic fuzzy soft sets with Pythagorean cubic fuzzy soft sets. Finally, we draw the conclusion that the PPCFSS strategy is more rational and scientific than the PCFSS approach, which does not include a generalization parameter in the decision-making process. We should thus think about the possibility Pythagorean spherical soft set theory in the future.

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