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DECENTRALIZED CONTROLLER DESIGN FOR LARGE-SCALE UNCERTAIN LINEAR SYSTEMS WITH NO BLOCK DIAGONAL OUTPUT MATRIX

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Abstract. *This paper is devoted to the problem of a decentralized controller design for linear large-scale system (LSS) continuous time uncertain polytopic system, for the case when the system output variables are functions of all system states, i.e., the system output matrix is not in the form of a decentralized structure. The original decentralized controller design procedure on the subsystem level is introduced where the interaction (either the strong or the weak) between subsystems does not play any role. Such design procedure will take place in three steps. In the first step of decentralized controller design, the LSS model is transformed to a decentralized structure. In the second step, robust stability boundary of the complex transformed system is calculated. Based on the results obtained, the method of robust decentralized controller design is selected, which accepts the obtained above stability boundary. This procedure will allow more precise and better implementation of the third step of the robust decentralized controller design. The whole robust decentralized controller design process will be performed on the subsystem level, without taking account of any interactions between the subsystems. Finally, the effectiveness of the proposed method is documented using an example with three scenarios and three subsystems.*
Keywords: Decentralized controller design, Robust controller design, Uncertain polytopic system, Output feedback, PID controller

1. **Introduction.** Belonging to challenging topics of control theory and practice, robust and decentralized control reached its maturity during past decades. The essence of robust and decentralized control of large-scale system responds to the needs of control practice, where real-live applications have to cope with system uncertainties, and the control is expected to keep the required performance within the specified uncertainty domain. The notion of large-scale systems (LSS) indicates such characteristics as complex system, uncertainty, and information structure constraints. LSS is too complex to be effectively controlled in centralized way [4]. LSS are controlled by decentralized algorithms with information-constraints structure. Since 1970s, decentralized controller design procedures are being developed in the frequency and time domains. In the frequency domain, interesting results have been obtained in the area of the robust decentralized controller design. They are covered by the following papers: [5, 6, 7] and others. Original results are obtained in the following three main groups: independent design [5], sequential design [6], and the method of equivalent subsystem [7].

In the time domain the obtained results could be broken down into three groups as follows: 1) methods using the aggregation matrix approach for linear and nonlinear LSS

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[4]; 2) methods based on the vector Lyapunov function approach [15]; and 3) methods using the LMI-BMI approach, where great progress has been reached in the design of decentralized robust controllers. The survey of decentralized controller design procedures for continuous-time and discrete-time systems may be found in the survey [8] and in the book [11]. Various approaches have been developed for robust decentralized control stability analysis and robust control design. Excellent books and papers exist that cover the fundamentals of the robust control theory. The linear (bilinear) matrix inequalities (LMI (BMI)) play an important role in the area of procedures of design of time domain robust and decentralized controller design procedures [9]. They played a particular role in the development of the theory of robust and decentralized controllers design [12, 16, 17, 18] and others. All decentralized design methods have the following very important problem: the plant model, specially in the time domain, needs to be in the decentralized structure [4, 11]. The methods on frequency [5, 6] and time domain which take account of interactions between subsystems belong to the class of "highly conservative approach". Decentralized controller design procedures on the subsystem level for continuous-time system are described in [3, 7], and for discrete-time systems in [7].

In this article, we have proposed an original procedure that determines the mutual relationship of stability between a complex system and subsystems, that is the stability of a complex system in relation to the stability, parameters and quality of subsystems. The proposed method should define such quality of the subsystems which ensures the stability of the complex system. The proposed original approach allows to solve the design of the decentralized control regulators at the level of subsystems without considering sizes of interaction links. Proposed in this paper the method of designing robust decentralized controller will take place in three steps as follows. In the first step, the complex uncertain plant model will be transformed to the decentralized structure, while using an original approach. Obtained in this way, the decentralized structure of the complex system will be in the form of descriptor system. In the second step, the robust stability boundary of the complex descriptor system without controller will be calculated. Based on the results obtained in the second step, an appropriate method of controller design will be selected, to use to design a robust decentralized controller that accepts the calculated stability boundary. This procedure will allow more precise and better implementation of the method of the robust decentralized controller design that will be used in the third step. While the results obtained in the first and second steps of our design procedure meet the necessary and sufficient condition, the main value of conservatives will be found in the third steps of the robust decentralized controller design on the subsystems level.

This paper is organized as follows. Section 2 provides the preliminary results and problem formulation. Section 3, Main Results, gives the theoretical bases to transformation of the complex linear model to a decentralized structure and calculation of the robust stability boundary for the uncertain polytopic system. In Section 4 of this paper, the method of H_2 is used to design the decentralized controller. The above mentioned example shows the effectiveness of the proposed method. In the Conclusion, Section 5 presents the advantages of the proposed method.

Hereafter, the following notation conditions will be adopted. Given a symmetric matrix $P = P^T \in R^{n\times n}$, the inequality $P > 0$ ($P < 0$) denotes matrix positive (negative) definiteness, respectively. I_n, 0_n denote the identity and zero matrices respectively, of corresponding dimensions.

2. **Preliminaries and Problem Formulation.** The considered uncertain polytopic LSS continuous time system is in the form:

$$\begin{aligned}\dot{x}(t) &= \overline{A(\xi)}x(t) + \overline{B(\xi)}u(t),\\ y(t) &= \overline{C}x(t),\end{aligned} \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^m$, $y(t) \in R^l$ are the state, control input, and controlled output, respectively. System matrices:

$$\overline{A(\xi)}, \overline{B(\xi)} = \sum_{i=1}^{N} \left(\overline{A_i}, \overline{B_i}\right) \xi_i \tag{2}$$

$$\overline{A(\xi)} = \overline{Ad(\xi)} + \overline{Am(\xi)}$$

belong to a polytopic uncertainty domain with N-vertices, and uncertainty ξ_i, $i = 1, 2, \ldots,$ N are constant or time varying but unknown parameters belonging to the set $\xi \in \Omega_\xi$

$$\Omega_\xi = \left\{\xi_i \geq 0, i = 1, 2, \ldots, N, \sum_{i=1}^{N} \xi_i = 1, \sum_{i=1}^{N} \dot{\xi}_i = 0\right\} \tag{3}$$

We assume that matrix $\overline{B_i}$, $i = 1, 2, \ldots, N$ is in the form of a decentralized structure [4]:

$$\overline{B_i} = blockdiag\left[\overline{B_{i1}}, \ldots, \overline{B_{im}}\right] \in R^{n\times m} \tag{4}$$

Output matrix is given as follows:

$$\overline{C} = \begin{bmatrix} \overline{C_1} \\ \vdots \\ \overline{C_m} \end{bmatrix} \in R^{l\times n}$$

where the l_jth, $j = 1, 2, \ldots, m$ output matrix C_j is with dimension $R^{l_j\times n}$, system matrix

$$\overline{Ad(\xi)} = blockdiag\left\{\overline{Ad_1(\xi)}, \ldots, \overline{Ad_m(\xi)}\right\}$$

is diagonal part and interaction matrix $\overline{Am(\xi)}$ is diagonal off part of the matrix $\overline{A(\xi)}$. Polytopic representation is one of the most general ways to describe uncertainty of the physical parameters without any conservatism. In this presentation, the uncertain system belongs to a polytope which is the convex hull of the parameters of a set of models (vertices). Polytopic uncertainty can cover the well-known interval and linear parameter uncertainty as well as multi-model structures. Problem studied in this article is to design such robust decentralized PI (PID) controller for the jth, $j = 1, 2, \ldots, m$ subsystem which ensures the stability of the closed-loop subsystems as well as stability and performance of closed-loop complex system. PID control algorithm for the jth subsystem is given as follows:

$$u_j = kp_j y_j + kI_j \int_{t_o}^{t} y_j + kd_j \dot{y}_j \tag{5}$$

where kp_j, kI_j, kd_j are the jth PID controller parameters.

3. Main Results.

3.1. Transform system output matrix to blockdiagonal.

For another form for matrix $\overline{A}_i$, $i = 1, 2, \ldots, N$ one obtains

$$\overline{A}_i = \begin{bmatrix} \overline{A}_{i11} & \overline{A}_{i12} & \cdots & \overline{A}_{i1m} \\ \overline{A}_{i21} & \overline{A}_{i22} & \cdots & \overline{A}_{i2m} \\ \overline{A}_{im1} & \overline{A}_{im2} & \cdots & \overline{A}_{imm} \end{bmatrix}$$

Note that $\overline{A}_{ijj} = \overline{Ad_{ij}}$. For the ith vertex one obtains the following plant model:

$$\dot{x} = \left(\overline{Ad_i} + \overline{Am_i}\right) x + \overline{B}_i u, \quad y = \overline{C}x, \quad i = 1, 2, \ldots, N \tag{6}$$

Assume that each subsystem has l_j outputs, that is $y_j = \left[\overline{C}_{j1}; \dots; \overline{C}_{jn}\right] x$, $j = 1, 2, \dots, m$. In order to obtain decentralized-blockdiagonal structure to output matrix $\overline{C}$, let us define the following new state variable for global system as follows:

$$x_d' = \left[\begin{array}{cccccccc} x_1' & y_1' & x_2' & y_2' & \dots & x_m' & y_m' \end{array}\right]$$

where $x_j \in R^{n_j}$, $y_j \in R^{l_j}$ are the jth state and output subsystem variables. Instead of the large scale linear polytopic uncertain system, for the new state variable x_d the following uncertain polytopic large-scale descriptor system is obtained:

$$E\dot{x}_d = A_i x_d + B_i u_d \quad i = 1, 2, \dots, N \tag{7}$$

where

$$A_i = \left[\begin{array}{ccc} A_{i11} & \dots & A_{i1m} \\ \vdots & \ddots & \vdots \\ A_{im1} & \dots & A_{imm} \end{array}\right]$$

$$A_{ijj} = \left[\begin{array}{cc} \overline{A}_{ijj} & 0_{jj} \\ \overline{C}_{jj} & -I_j \end{array}\right]$$

$A_{ijj} \in R^{n_j \times n_j} = Ad_{ij}$, $n_j = \overline{n}_j + l_j$

$$A_{ijk} = \left[\begin{array}{cc} \overline{A}_{ijk} & 0_{jj} \\ \overline{C}_{jk} & 0 \end{array}\right]$$

Input matrix

$$B_i = blockdiag \left[\begin{array}{ccc} B_{i1} & \dots & B_{im} \end{array}\right]$$

$$B_{ij} = \left[\begin{array}{c} \overline{B}_{ij} \\ 0 \end{array}\right]$$

and output matrix in decentralized structure is

$$C = blockdiag \left[\begin{array}{cccc} O_{n_1} I_1; & O_{n_2} I_2; & \dots; & O_{nm} I_m \end{array}\right]$$

Descriptor matrix

$$E = blockdiag \left[\begin{array}{cc} I_1 & 0_1 \\ I_2 & 0_2 \\ \vdots & \vdots \\ I_m & 0_m \end{array}\right]$$

where $I_j \in R^{n_j \times n_j}$ is unite matrix and $0_j \in R^{l_j \times l_j}$ is zero matrix. $E \in R^{n \times n}$, $rank(E) \leq n$. Model (7) describes the uncertain polytopic descriptor system at the ith, $i = 1, 2, \dots, N$ vertex in the decentralized structure. Let us assume that the complex system (1) is centralized controllable, and observable and there are no unstable fixed modes [10]. Note that we do not change the matrix dimensions description.

In order to obtain the plant output feedback integral for robust PI controller design, it is necessary to expand the jth plant state. In the PID controller (5), let us put the following integral part of the output feedback:

$$\dot{z}_j = y_j = C_j x_j \tag{8}$$

Now, let us define the jth plant with the new state as

$$E_{nj}\dot{x}_{nj} = E_{nj} \left[\begin{array}{c} \dot{x}_j \\ \dot{z}_j \end{array}\right] = Adn_j(\xi) x_{nj} + Bn_j(\xi) u_j \tag{9}$$

$$y_{nj} = Cn_j x_{nj}$$

where

$$Adn_j(\xi) = \begin{bmatrix} Ad_j(\xi) & 0 \\ Cn_j & 0 \end{bmatrix}, \quad Bn_j(\xi) \begin{bmatrix} B_j(\xi) \\ 0 \end{bmatrix}, \quad Cn_j = \begin{bmatrix} C_j & 0 \\ 0 & I_z \end{bmatrix}$$

When substituting (8) to (5) the following PI control algorithm is obtained:

$$u_j = [kp_j C_j \quad kI_j] x_{nj} = F_j x_{nj} \tag{10}$$

and for PID controller one obtains

$$u_j = F_j x_{nj} + F_D E \dot{x}_{nj} \tag{11}$$

where $F_D = [k_d C_j \quad 0]$. See [13] for more details.

Lemma 3.1. *Assume that the uncertain system is described by (1) and gain's static output feedback controller matrix is given as K. Then, the necessary and sufficient conditions for the existence of a robust decentralized controller with gain K such that closed-loop system is asymptotically stable, are as follows:*

$$\lambda(E, A(\xi), B(\xi), C, K) \in C^- \tag{12}$$

where C^- is the left hand side of the complex plane.

3.2. **Decentralized controller design.** In order to obtain the robust stability conditions for the descriptor systems, let us introduce the following theorems and lemma [2].

Definition 3.1. *Descriptor system $E\dot{x} = Ax$ is regular if there is such z*

$$det(zE - A) \neq 0$$

existing.

Definition 3.2. *A regular descriptor system is asymptotically stable if $\sigma(E, A) \in C^-$.*

Theorem 3.1. [2] *Let (E, A) be regular and consider the following generalized Lyapunov function*

$$A^T PE + E^T PA + E^T QE = 0 \tag{13}$$

If matrices $P \geq 0$ and $Q > 0$ satisfying Lyapunov equality exist, then (E, A) is impulse free and stable.

Theorem 3.2. *If an $n \times n$ symmetric positive definite matrix P exists, such that the derivative of the function $V(Ex) = (Ex)^T PEx$ along the solution of the system given as $E\dot{x} = Ax$, i.e., $\dot{V}(Ex)$ is negative definite for all variate of Ex, then the equilibrium $x = 0$ of the system is stable.*

Theorem 3.3. *Complex descriptor system (7) is impulse-free and asymptotically stable if there exist matrices $N_1, N_2 \in R^{n\times n}$, positive definite matrix $P_i(P) \in R^{n\times n}$ such that for each $i = 1, 2, \ldots, N$ the following inequality holds for:*
a) complex polytopic descriptor system, for the ith vertex put $A_i = Ad_i + \alpha I + Am_i$

$$\begin{bmatrix} N_1 A_i + A_i^T N_1^T & * \\ P_i E - N_1^T + N_2 A_i & -N_2 - N_2^T \end{bmatrix} < 0, \quad i = 1, 2, \ldots, N \tag{14}$$

b) if number of polytopic vertices $N = 1$, put $N = 1$ to Equation (14) in a) or use the following inequality [2]

$$A = A_i, \; A^T PE + E^T PA < 0 \tag{15}$$

Proof: Inequality a) can be easily obtained from the inequality b).
Inequality (14) implies the following.

- If $\alpha \geq 0$ is obtained, then the uncertain descriptor complex system is asymptotically stable. On the subsystem level the designer should use any controller design method to design the decentralized controller. The subsystems decentralized controller should be designed with such parameters that the quality, (the degree of stability) of the closed loop subsystem with decentralized controller, is equal to or better than that of the corresponding open loop.
- Obtained $\alpha < 0$, the uncertain complex system is not stable. The value of α indicates by how much it is necessary to move the closed loop subsystems dominant eigenvalues to the left, to preserve the stability of the complex system. For unstable complex system, the authors recommended using the equivalent subsystem approach. Note that subsystems need to be controllable with designed decentralized controllers.

From decentralized review paper [8], complex systems are split into two large groups: a complex system with strong interaction and a complex system with weak one. In this paper the division of complex systems is realized according to whether the complex system is stable or not. The value of the obtained α plays an important role for robust decentralized controller design and stability of complex systems. Robust stability boundary condition of complex systems with respect to subsystems properties Ad_i, $i = 1, 2, \dots, N$ eigenvalues is given as follows:

$$S_c = \alpha + \max_i(real(eig(Ad_i))) \tag{16}$$

Note that the whole decentralized design procedure performs on the subsystem level. Let us introduce the notion of equivalent subsystems for unstable ($\alpha < 0$) LSS.

Definition 3.3. *Equivalent subsystem is an auxiliary subsystem that serves as a tool to design the decentralized controller for unstable complex descriptor systems. The designed decentralized controller guarantees stability of the closed-loop and the performance of both the closed-loop subsystems, as well as stability of the LSS, i.e., the stability and performance of the diagonal equivalent subsystem matrix and the stability of the complex system.*

Choose $\beta = |\alpha| + \delta$, where $\delta \geq 0$ is a small tuning parameter (for the first step $\delta = 0$). The equivalent subsystem is defined as

$$Ae_i = Ad_i + I\beta, \quad i = 1, 2, \dots, N \tag{17}$$

Lemma 3.2. *Large-scale uncertain descriptor system (7) should be stabilized by equivalent subsystems for the given parameter δ if all equivalent subsystems are stable and controllable by the chosen decentralized controller structure.*

Let the jth descriptor subsystems with PID controller be described by (9). In order to obtain closed-loop performance quality in the frame of H_2 norm the following cost function for the jth descriptor subsystem will be used in the form

$$J_c = \int_{t=0}^{\infty} J_d\left(\dot{x}_n, x_n, u\right) dt \tag{18}$$

where

$$J_d = x_{nj}^T Q x_{nj} + \left(E\dot{x}_{nj}\right)^T S\left(E\dot{x}_{nj}\right) + u_j^T R u_j$$

Note that for the next all theoretical results hold for the jth descriptor subsystem, the jth will be omitted. Let the Lyapunov function for descriptor systems in (9) be given as follows:

$$V(Ex)_j = (Ex_{nj})^T P_j(\xi) E x_{nj}, \quad j = 1, 2, \dots, m \tag{19}$$

where

$$P(\xi) = \sum_{i=1}^{N} P_i \xi_i$$

Time derivative of (19) gives

$$\frac{dV(Ex)}{dt} = \begin{bmatrix}(E\dot{x}_n)^T & x_n^T & u^T\end{bmatrix} \begin{bmatrix} 0 & P(\xi)E & 0 \\ E^T P(\xi) & E^T \dot{P}(\xi) E & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E\dot{x}_n \\ x_n \\ u \end{bmatrix} \tag{20}$$

To decrease the conservatives and obtain the convex robust stability conditions with respect to uncertainties, separate matrix $P(.)$ from the system matrices $A_j(\xi)$ and $B_j(\xi)$, the following new slack matrices N_i, $i = 1, 2 \in R^{n\times n}$, $N_i \in R^{m\times n}$, $i = 4, 5$ and $N_6 \in R^{m\times m}$ are introduced in the following way.

$$\overline{v}^T \begin{bmatrix} 2N_1^T \\ 2N_2^T \\ 2N_3^T \end{bmatrix} \begin{bmatrix} I & -A(\xi) & -B(\xi) \end{bmatrix} \overline{v} = 0 \tag{21a}$$

$$\overline{v}^T \begin{bmatrix} 2N_4^T \\ 2N_5^T \\ 2N_6^T \end{bmatrix} \begin{bmatrix} -F_D & -F & I \end{bmatrix} \overline{v} = 0 \tag{21b}$$

$$\overline{v}^T = \begin{bmatrix} (E\dot{x}_n)^T & x_n^T & u^T \end{bmatrix}$$

Summarizing (21), (18) and substituting the obtained results to the well-known Bellman-Lyapunov equation [21], the following procedure will be obtained, which will be used to design the robust decentralized controller for the descriptor system.

$$B_e = \max_{u\in\Omega_u} \left(\overline{v}^T W(\xi) \overline{v}\right) < 0 \tag{22}$$

Due to convex conditions, (22) can be split as follows:

$$W(\xi) = \sum_{i=1}^{N} W_i \xi_i < 0$$

and

$$W_i = \{w_{ikl}\}_{3\times 3}$$

Inequality (22) holds if and only if

$$W_i < 0 \tag{23}$$

for all $i = 1, 2, \ldots, N$, where

$$\begin{aligned}
w_{i11} &= N_1^T + N_1 - N_4^T F_D - F_D^T N_4 + S \\
w_{i12} &= E^T P_i - N_1^T A_i + N_2 - N_4^T F - F_D^T N_5 \\
w_{i13} &= -N_1^T B_i + N_3 + N_4^T - F_D^T N_6 \\
w_{i22} &= E^T P_i - N_2^T A_i - Ad_i^T N_2 - N_5^T F - F^T N_5 + Q \\
w_{i23} &= -N_2^T B_i - A_i^T N_3 + N_5^T - F_1^T N_6 \\
w_{i33} &= -N_3^T B_i - B_i^T N_3 + N_6^T + N_6 + R
\end{aligned}$$

Summary of the obtained main results of this chapter to robust decentralized controller design for descriptor subsystems which ensures robust properties of closed-loop subsystems, robust parameter dependent quadratic stability and in the frame of H_2 guaranteed cost is given in the next theorem.

Theorem 3.4. *Let the uncertain polytopic descriptor system be given by (7) and corresponding equivalent subsystem (17). The* j*th closed-loop uncertain descriptor subsystem and complex system is impulse free and asymptotically stable if for each* j*th,* $j = 1, 2, \dots, m$ *there are auxiliary matrices* $N_1, \dots, N_6$*, positive definite matrix* P_{ij}*, performance matrices* Q*,* S*,* R*, decentralized controller parameters* k_{pj}*,* k_{Ij}*,* k_{dj} *and positive scalar* δ *existing, such that Inequality (23) holds for* $i = 1, 2, \dots, N$.

4. **Example.** This example with three scenarios aims to show in detail the sequence of the decentralized controller design, for the case of the non decentralized output matrix structure and thus to explain and confirm the validity of the derived theory to control complex systems. Let us have a complex system 6×6 order with 3 subsystems where input subsystems matrices B_j, $j = 1, 2, 3$ are in the form of a decentralized structure and system output variables are functions of all complex system state variables, the matrices C_j being in the form of a not decentralized structure. The system model is in the form of (1) as follows:

$$\dot{x}(t) = \overline{A}x(t) + \overline{B}u(t), \quad y(t) = \overline{C}x(t)$$

where

$$\overline{A} = \begin{bmatrix} \overline{A}_{11} & \overline{A}_{12} & \overline{A}_{13} \\ \overline{A}_{21} & \overline{A}_{22} & \overline{A}_{23} \\ \overline{A}_{31} & \overline{A}_{32} & \overline{A}_{33} \end{bmatrix}$$

Note that $Ad_i = \overline{A}_{ii}$

$$B = blockdiagonal[B_1, B_2, B_3]$$

$$\overline{A}_{11} = \begin{bmatrix} -0.5 & 0.62 \\ 0.12 & -0.45 \end{bmatrix} \quad \overline{A}_{12} = \begin{bmatrix} 0.1 & 0.11 \\ 0.1 & 0.02 \end{bmatrix} \quad \overline{A}_{13} = \begin{bmatrix} 0.05 & -0.1 \\ 0.12 & 0.04 \end{bmatrix}$$

$$\overline{A}_{21} = \begin{bmatrix} 0.1 & 0.035 \\ 0.21 & 0.01 \end{bmatrix} \quad \overline{A}_{22} = \begin{bmatrix} -1 & 0.6 \\ 0.45 & -2 \end{bmatrix} \quad \overline{A}_{23} = \begin{bmatrix} 0.22 & 0.0031 \\ 0.11 & 0.07 \end{bmatrix}$$

$$\overline{A}_{31} = \begin{bmatrix} 0.021 & 0.022 \\ 0.1 & 0.012 \end{bmatrix} \quad \overline{A}_{32} = \begin{bmatrix} 0.12 & 0.17 \\ 0.022 & 0.05 \end{bmatrix} \quad \overline{A}_{33} = \begin{bmatrix} -0.32 & 0.41 \\ 0.25 & -0.75 \end{bmatrix}$$

$$\overline{B}_1 = \begin{bmatrix} 0.1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \overline{B}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0.2 \\ 0 \\ 0 \end{bmatrix} \quad \overline{B}_3^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0.1 & 1 \end{bmatrix}$$

are input matrices. Output matrices are

$$\begin{aligned} y_1 &= \overline{C}_1 x & \overline{C}_1 &= \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \end{bmatrix} \\ y_2 &= \overline{C}_2 x & \overline{C}_2 &= \begin{bmatrix} C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \end{bmatrix} \\ y_3 &= \overline{C}_3 x & \overline{C}_3 &= \begin{bmatrix} C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \end{bmatrix} \end{aligned}$$

where dates for output matrices are

$$\overline{C}_1 = \begin{bmatrix} 1 & 0 & 0 & 0.1 & 0.2 & 0.3 \end{bmatrix} \quad \overline{C}_2 = \begin{bmatrix} 0.15 & 0 & 1 & 0 & 0.05 & 0.1 \end{bmatrix}$$
$$\overline{C}_3 = \begin{bmatrix} 0.05 & 0.02 & 0.1 & 0 & 1 & 0.12 \end{bmatrix}$$

For the purposes of our example, let us define a new global system with the following new state variable as follows:

$$x_d^T = \begin{bmatrix} x_1^T & y_1 & x_2^T & y_2 & x_3^T & y_3 \end{bmatrix}$$

where x_j, y_j, $x_1 = [x_{11}, x_{12}]$ are the jth state vector of the jth subsystem and output subsystem variables. In this way, the output variable y_j becomes a state variable of the jth subsystem. Using the approach described in Section 3.1 we have obtained new subsystem matrices as follows:

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

Note that $Ad_i = \overline{A}_{ii}$ where the subsystems are with I part of controller and the transform data to the decentralized structure are as follows:

$$A_{11} = \begin{bmatrix} -0.5 & 0.62 & 0 & 0 \\ 0.12 & -0.45 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad B_{11} = \begin{bmatrix} 0.1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$A_{22} = \begin{bmatrix} -1 & 0.6 & 0 & 0 \\ 0.45 & -2 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad B_{22} = \begin{bmatrix} 1 \\ 0.2 \\ 0 \\ 0 \end{bmatrix}$$

$$A_{33} = \begin{bmatrix} -0.32 & 0.41 & 0 & 0 \\ 0.25 & -0.75 & 0 & 0 \\ 1 & 0.12 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad B_{33} = \begin{bmatrix} 0.1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

and output matrices for descriptor subsystems

$$C_1 = C_2 = C_3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For the new state variable we obtain the large scale linear descriptor system. With I part of controller we have large-scale descriptor system 12th order.

$$E\dot{x}_d = A_i x_d + B_i u_d, \quad i = 1, 2, \ldots, N \tag{24}$$

where the subsystem matrices are given above. First of all, let us calculate the stability conditions α of complex system by (14). For non uncertain system the obtained results are from (15). $\alpha = -0.0218$. The above result indicates that the complex system is unstable but it is very near to the stability boundary. Because the $\max(real(eig(A_d))) = -0.1494$, the stability boundary (16) is equal to $S_c = -0.1712$. To guarantee the stability of complex systems, it is sufficient that for all subsystems closed-loop eigenvalues meet $subsrealeig < S_c$. For the first case the tuning parameter is $\delta = 0.01$. Using H_2 performance for the case of $s = 0$, $q = 10^{-6}$, $r = 1$, $ro = 1000 < P$ the PID controllers have to be designed for descriptor subsystems, and the following decentralized controllers will be obtained:

$$R_1(s) = -11.5358 + \frac{-21.1753}{s} - 12.095s$$
$$R_2(s) = -7.7933 + \frac{-51.2208}{s} - 8.1714s$$
$$R_3(s) = -9.8212 + \frac{-37.6202}{s} - 10.2987s$$

Other approaches to designing the decentralized controller are regional pole placement, L_2 gain, predictive control [19] and others. With the above controllers, all original subsystems are stable, with the following eigenvalues

$$\begin{aligned} \mathit{Eigsubsystems} = \{&-27.65 \pm 3.00i, -1.55 - \mathit{inf}, -4.3328 \pm 5.6799i, -2.1275, -\mathit{inf}, \\ &-0.716 \pm 3.2397i, -1.7989, -\mathit{inf}\} \end{aligned}$$

Comparison of the stability boundary of the complex system S_c with the closed-loop subsystem eigenvalues, leads to the conclusion that complex system with the proposed is proven. Closed-loop eigenvalues for the complex system are as follows:

$$\begin{aligned} \mathit{Closed\text{-}loopeigen} = \{&-4.2552 \pm 5.669i, -0.2741 \pm 3.0266i, -0.7821 \pm 3.1314i, -1.5746, \\ &-2.083, -1.8537, -\mathit{inf}, -\mathit{inf}, -\mathit{inf}\} \end{aligned}$$

For the next case, to see the impact of δ to the dynamic properties of the closed-loop system. Let $\delta = 0.3$, other parameters remaining unchanged. Then, the following controllers are obtained:

$$\begin{aligned} R_1(s) &= -55.14432 - \frac{18.6217}{s} - 83.163s \\ R_2(s) &= -54.959 - \frac{-18.655}{s} - 82.886s \\ R_3(s) &= -55.3488 - \frac{18.6475}{s} - 83.4718s \end{aligned}$$

closed-loop eigenvalues being

$$\begin{aligned} \mathit{Closed\text{-}loopeigen} = \{&-55.7339, -9.7207, -2.965 \pm 5.1566i, -3.0728, -2.207, -0.43493, \\ &-0.2835 \pm 0.0365i, -\mathit{inf}, -\mathit{inf}, -\mathit{inf}\} \end{aligned}$$

For the third case, let us increase the value if interactions between subsystems by hundred percent and put $\delta = 0.08$. Stability condition changes as follows: $\alpha = -0.2864$ with $S_c = -0.4357$. Obtained PID controller parameters are

$$\begin{aligned} R_1(s) &= -8.366 - \frac{10.8594}{s} - 13.4726s \\ R_2(s) &= -4.5543 - \frac{-51.059}{s} - 7.3343s \\ R_3(s) &= -7.363 - \frac{24.8548}{s} - 11.8573s \end{aligned}$$

Eigenvalues of closed-loop subsystems are as follows:

$$\begin{aligned} \mathit{Eigclosedsubs} = \{&-0.3002 \pm 2.449i, -1.1862, -\mathit{inf}, -2.7139 \pm 6.5979i, -2.1266, -\mathit{inf}, \\ &-0.5046 \pm 2.744i, -1.6806, -\mathit{inf}\}. \end{aligned}$$

The following sufficient stability conditions guarantee the complex stability, that all closed-loop subsystems eigenvalues are less than S_c. In this case these conditions are not met. In order to check the stability of complex system, the corresponding eigenvalues will be calculated as follows:

$$\begin{aligned} \mathit{Eigclosedsubs} = \{&-2.6292 \pm 6.5808i, -0.2582 \pm 2.5602i, -0.619 \pm 2.5523i, -1.1922, \\ &-2.1050, -1.7138, -\mathit{inf}, -\mathit{inf}, -\mathit{inf}\} \end{aligned}$$

Obtained for the above three cases, the eigenvalues imply that closed-loop complex system with designed decentralized PID controllers is asymptotically stable. By simulation of the original closed-loop system for the three cases, the above assertion will be proven in the following three figures. The last picture documents that really complex system is relatively close to the stability limit.

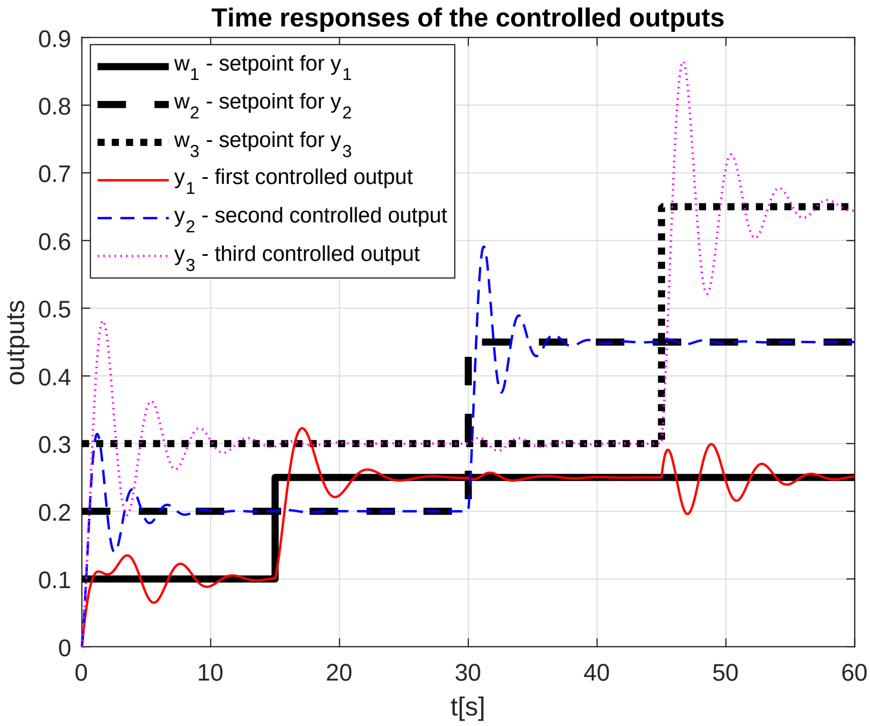

FIGURE 1. Time response of the controlled output – 1st case ($\delta = 0.01$)

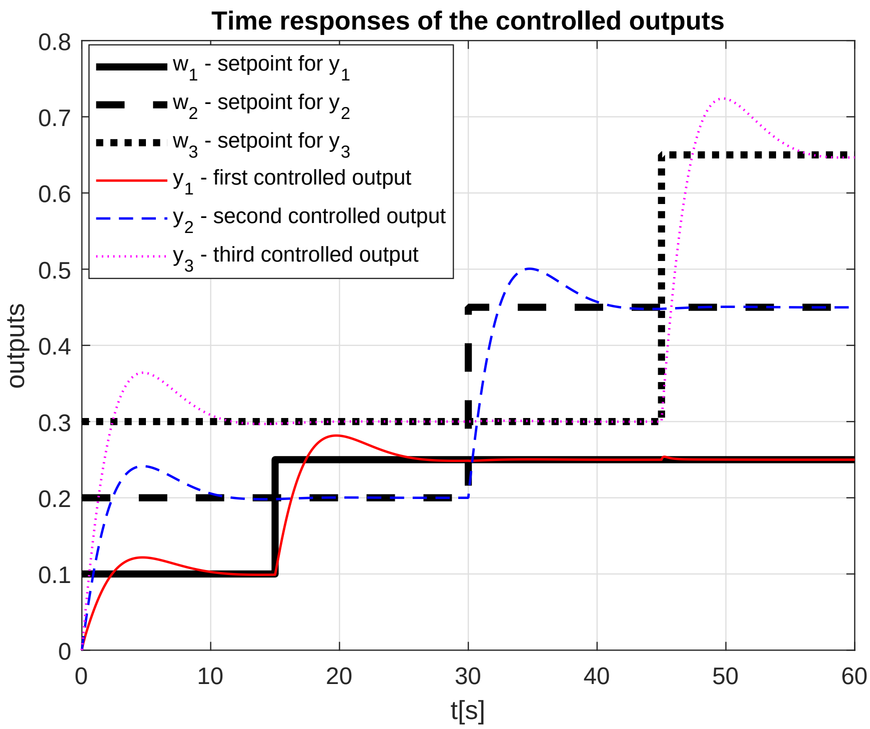

FIGURE 2. Time responses of the decentralized controlled system for the case of $\delta = 0.3$

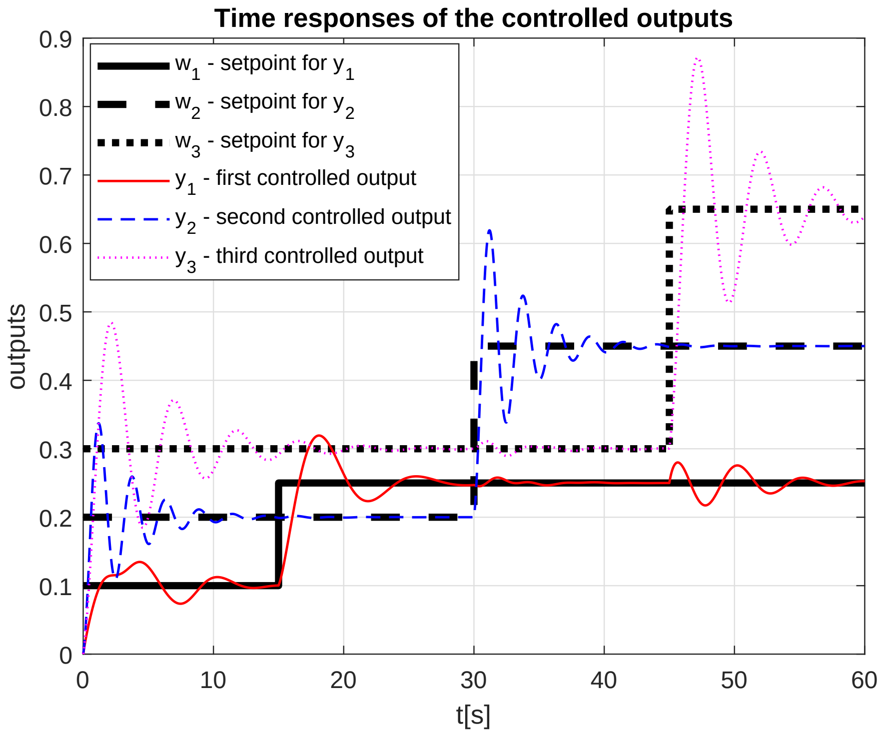

FIGURE 3. Time responses of the decentralized controlled system for the last case of $\delta = 0.08$. The interactions have been increased by hundred percent.

5. **Conclusion.** In this paper we solved the fundamental issue of decentralized control of linear complex systems. When designing decentralized control of dynamic systems, it is required that the input and output matrices of such complex systems should be in a decentralized structure. In practice, many complex plants do not meet this requirement. In this work, we proposed a suitable transformation of the plant model for the case that plant output matrix is not in a decentralized structure, so that the stated requirements are met after the complex system transformation. We verified the proposed object transformation method using the original decentralized control design method [3]. In this method, we do not follow the question of strong or weak interaction ties between subsystems. Rather, we are focused on the relationship between the subsystems quality and parameters to the stability and performance of the complex system. Achieved in this paper, our results could be summarized as follows.

1) We have proposed the new LSS transformation method so that the LSS is transformed to a decentralized structure, that is suitable for the decentralized controller design.
2) For the purposes of verification of the proposed LSS transformation to the decentralized structure we have used the original decentralized control design method which is performed on the subsystem level [3].
3) Derive conditions to solving stability of a complex descriptor system in relation to subsystem parameters and the interaction matrix.
4) Solving three cases of the example shows the high efficiency of the method proposed in this work.
5) Due to results of (16), all decentralized controller design process is performed on the subsystem level, without taking account of any interactions between the subsystems.

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