

## CONTROL SYSTEM USING MULTI-PERIOD DISTURBANCE OBSERVER FOR NON-MINIMUM PHASE SYSTEMS

SIRIPONG SANGSARPAN<sup>1,2</sup>, DAISUKE KOYAMA<sup>2</sup>, NGHIA THI MAI<sup>3</sup>  
MD ABDUS SAMAD KAMAL<sup>2</sup>, IWANORI MURAKAMI<sup>2</sup> AND KOU YAMADA<sup>2</sup>

<sup>1</sup>Faculty of Engineering  
Thai-Nichi Institute of Technology  
1771/1 Suanluang, Bangkok 10250, Thailand  
siripong@tni.ac.th

<sup>2</sup>Graduate School of Science and Technology  
Gunma University  
1-5-1 Tenjincho, Kiryu 376-8515, Japan  
{ t202b004; maskamal; murakami; yamada }@gunma-u.ac.jp

<sup>3</sup>Faculty of Electronics Engineering 1  
Posts and Telecommunications Institute of Technology  
122 Hoang Quoc Viet Road, Cau Giay District, Hanoi 122300, Vietnam  
nghiamt@ptit.edu.vn

Received November 2025; revised March 2026

**ABSTRACT.** *In practical systems, control systems are often required to attenuate periodic output disturbances and make the output follow the non-periodic reference input. A control system to attenuate the periodic disturbance often has a high-order controller. A high-order controller increases computational and tuning burden and can complicate stability margins. In order to design the control system with low-order controller, a control system using a periodic disturbance observer is proposed. Periodic disturbance observer is an observer that estimates periodic disturbances from available measurements. However, no paper considers a design method of a control system using the multi-period disturbance observer to attenuate the periodic output disturbance for the non-minimum phase plants. This paper examines the control system using the multi-period disturbance observer to attenuate the periodic output disturbances, to make the transfer function from the disturbance to the output have finitely many poles and for the output to follow the non-periodic reference input.*

**Keywords:** Disturbance attenuation, Non-minimum phase systems, Periodic disturbance, Multi-period disturbance observer, Two-degree-of-freedom control system

**1. Introduction.** Disturbance attenuation is an important issue of control system design. From the internal model principle, the non-periodic disturbance, such as step disturbances or ramp disturbances, can be attenuated by making a controller have an integrator. Disturbances are not only non-periodic but also periodic. In some practical applications, periodic disturbances arise, for example, in optoelectronic precision positioning systems [1], active noise control [2], and robotic manipulators [3]. Achieving disturbance attenuation is a core requirement in precision control. By the internal model principle, effective rejection of step or ramp disturbances requires embedding the corresponding dynamics in the controller [4]. However, in practice, many systems are perturbed by periodic disturbances containing multiple harmonics, which severely degrade accuracy in precision

motion and power-conversion applications [5]. Such periodic disturbances often contain multiple harmonics, and effective attenuation is nontrivial in practical feedback designs.

It is well-known that attenuating the periodic disturbance is difficult. A classical approach to attenuating periodic disturbances is repetitive control. Repetitive control embeds an internal model of periodic signals and thus achieves strong attenuation of periodic components [6]. Repetitive control essentially places an infinite set of poles on the imaginary axis corresponding to the period of harmonics, which enables high-performance tracking for periodic reference and attenuation for periodic disturbances. To follow the output to a non-periodic reference input, Pipeleers et al. have proposed a robust higher-order repetitive control [7]. Nevertheless, repetitive control has some problems [8]:

- 1) its controller is often high-order;
- 2) the exact knowledge of the period-time of reference or disturbance signals is required [8, 9];
- 3) the perfect reduction at harmonic frequencies is counteracted by amplification of noise at intermediate frequencies [8, 10].

Recent studies have extended repetitive control in several directions, including applications to nonlinear/Lur'e-type systems [11] and designs using fractional time-delay elements in discrete-time settings [12]. Moreover, phase-compensated equivalent-input-disturbance approaches have been proposed to improve disturbance rejection within repetitive-control structures [13].

The other approach for attenuating the periodic disturbances is to use the disturbance observer. Disturbance-observer approaches are attractive due to their low structural complexity and direct disturbance estimation [3, 14, 15, 16]. Representative advances include higher-order/time-series disturbance observer [17, 18], the disturbance observer-aided repetitive designs for delay systems [19, 20], and broad surveys covering linear and nonlinear developments [15, 21]. For periodic disturbances in particular, adaptive/periodic disturbance observer schemes using time-delay elements have been validated on multi-axis systems [22]. Disturbance-observer-based control is closely related to extended  $H_\infty$ -type design viewpoints and has been discussed in comparison with  $H_\infty$  control [23, 24]. Using the extended  $H_\infty$  method, control systems can have disturbance attenuation or rejection. Kobayashi et al. clarify the relationship of tradeoffs between the practical tuning and robustness based on the phase-compensation analysis [25].

In parallel with these papers, research on the parameterization of all disturbance observers has emerged [5, 26, 27, 28, 29]. The parameterization of all disturbance observers is a problem to clarify all disturbance observers to estimate a disturbance. The parameterization for all disturbance observers allows for a unified design of disturbance observers. Early studies characterized the characteristics of all disturbance observers for a finite number of frequency components [26]. More recent papers have provided the parameterization of a disturbance observer using the period of periodic disturbances [27, 28, 29]. Phukapak et al. clarify the parameterization of all disturbance observers for periodic input or output disturbance [27, 28]. In [29], the parameterization of all disturbance observers for the periodic input and output disturbances is provided. The results of [27, 28, 29] have a remaining problem in that their method uses only one signal of the periodic time before the present time. Phukapak et al. provide a Multi-Period Disturbance Observer (MP-DOB) using multiple signals before the present time, and clarify the parameterization of all multi-period disturbance observers for the output periodic disturbances [5]. Compared with the single-period case, which uses only one delayed signal from the most recent period, the MP-DOB utilizes multiple delayed signals taken from several past periods with delays synchronized to the disturbance period. Because these delayed signals correspond to the

same repeating pattern of the disturbance, their periodic components add constructively when combined, which reinforces the periodic disturbance estimate while non-periodic components do not accumulate in the same manner. By combining several period-aligned delayed components, the MP-DOB can improve attenuation of periodic disturbances containing multiple harmonics, while preserving a low-order structure. This is the theoretical origin of the core advantage of the MP-DOB over the traditional single-period observer; see [5].

The parameterization of all disturbance observers is applicable to designing a control system to follow a non-periodic reference input and to attenuate periodic disturbance without using repetitive control. Yamada et al. consider a design method of control system based on the result of [27] for the periodic output disturbance [30]. From [30], the controller using the multi-period disturbance observer can be designed to make the transfer function from the disturbance to the output have finitely many poles. Thus, the control system using the multi-period disturbance observer has an advantage in making the controller more easily designed than using repetitive control. Related studies for non-minimum-phase plants include attenuation of periodic disturbances without using repetitive control [31] and low-sensitivity stabilization using double feedback control [32].

However, a design method of a control system using the multi-period disturbance observer for non-minimum phase plants has not been established in the existing literature. Since many practical plants are non-minimum phase, it is important to develop such a design method.

This paper proposes a design method for control system for non-minimum-phase plants using the multi-period disturbance observer for the periodic disturbance. This paper is organized as follows. In Section 2, the problems in this paper are explained. In Section 3, the condition that the transfer function from the disturbance to the output has a finite number of poles is shown. In Section 4, the necessary and sufficient condition that the control system is internally stable is clarified. In Sections 5 and 6, a design method for the feedback controllers and the controller for disturbance observer are examined. In Section 7, a design procedure is provided. In Section 8, numerical examples are illustrated to show the effectiveness of the proposed method. Section 9 gives some concluding remarks.

**2. Problem Formulation.** Consider the single-input and single-output linear time invariant plant denoted by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + d(t) \end{cases}, \quad (1)$$

where  $x(t) \in R^n$  is the state variable,  $u(t) \in R$  is the input,  $y(t) \in R$  is the output,  $d(t)$  is the disturbance and  $A \in R^{n \times n}$ ,  $B \in R^n$  and  $C \in R^{1 \times n}$ . Assume that  $(A, B)$  is stabilizable,  $(C, A)$  is detectable, and  $A$  has no eigenvalues on the imaginary axis. The plant in (1) is non-minimum phase, that is,

$$\det \begin{bmatrix} A - sI & B \\ C & 0 \end{bmatrix} = 0 \quad (2)$$

has roots in the closed right-half plane.  $u(t)$  and  $y(t)$  are assumed to be available but  $d(t)$  is unavailable.  $d(t) \in R$  in (1) is a periodic disturbance with period  $T > 0$  satisfying

$$d(t + T) = d(t). \quad (3)$$

The Laplace transformation of (1) gives

$$y(s) = G(s)u(s) + d(s), \quad (4)$$

where

$$G(s) = C(sI - A)^{-1}B, \quad (5)$$

$y(s) = \mathcal{L}[y(t)]$ ,  $u(s) = \mathcal{L}[u(t)]$  and  $d(s) = \mathcal{L}[d(t)]$ .

To make  $y(t)$  follow a non-periodic reference  $r(t)$  and attenuate the periodic disturbance  $d(t)$ , we propose a control system that adopts the multi-period disturbance observer for output disturbances [5]. The proposed control structure is shown in Figure 1. We also define  $d_1(s) \in R(s)$  and  $d_2(s) \in R(s)$  as exogenous (aperiodic) signals. They represent unknown inputs that are not generated by the nominal plant model and are not assumed to be periodic with period  $T$  (e.g., aperiodic load disturbances, unmodeled dynamics, and sensor noise/bias). These signals enter the summing junctions in Figure 1, perturb the measured output and/or the control input, and hence affect the closed-loop response.

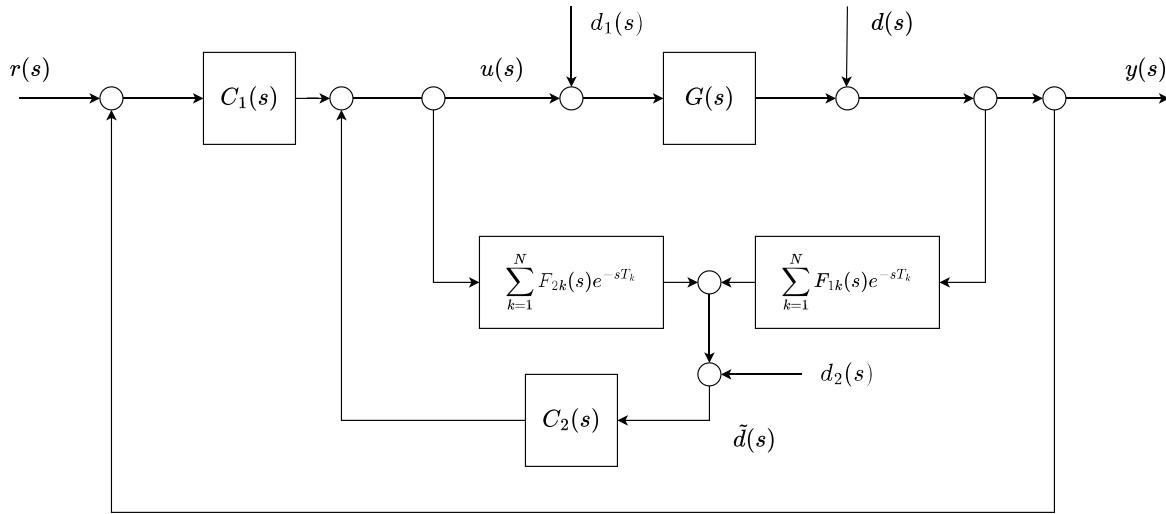


FIGURE 1. Control system using multi-period disturbance observer

Here,  $C_1(s) \in R(s)$  is the stabilizing controller for  $G(s)$  and  $C_2(s) \in R(s)$  is the controller that attenuates the periodic disturbance  $d(s)$ . Without loss of generality,  $C_2(s)$  is assumed to satisfy  $C_2(s) \neq 0$ .  $r(s) \in R(s)$  is the non-periodic reference input. For example, in motor speed control, the command  $r(t)$  is often non-periodic, whereas the periodic disturbance  $d(t)$  can represent rotation-synchronous effects such as torque ripple or encoder ripple.

The estimator to estimate the periodic disturbance  $d(s)$  is denoted by

$$\tilde{d}(s) = \sum_{k=1}^N F_{1k}(s)e^{-sT_k}y(s) + \sum_{k=1}^N F_{2k}(s)e^{-sT_k}u(s), \quad (6)$$

where

$$T_k = kT \quad (k = 1, 2, \dots, N). \quad (7)$$

The estimator  $\tilde{d}(s)$  in (6) for the periodic disturbance  $d(s) \in R(s)$  is called the multi-period disturbance observer when

$$\lim_{t \rightarrow \infty} e(t) = \lim_{t \rightarrow \infty} (d(t) - \tilde{d}(t)) = 0 \quad (8)$$

holds true. According to [5], the parameterization of all disturbance observer for the periodic disturbance  $d(s)$  is given by

$$\sum_{k=1}^N F_{1k}(s) = D(s)(1 + Q(s)), \quad (9)$$

and

$$\sum_{k=1}^N F_{2k}(s) = -N(s)(1 + Q(s)), \quad (10)$$

respectively, where  $N(s) \in RH_\infty$  and  $D(s) \in RH_\infty$  are coprime factors of  $G(s)$  on  $RH_\infty$  satisfying

$$G(s) = \frac{N(s)}{D(s)}. \quad (11)$$

$Q(s) \in RH_\infty$  is any function satisfying

$$D(s_l) + Q(s_l)D(s_l) = 1, \quad (12)$$

$$s_l = j\omega_l \quad (13)$$

and

$$\omega_l = \frac{2\pi l}{T} \quad (\forall l = 1, 2, \dots). \quad (14)$$

The problem in this paper is to propose a design method of  $C_1(s)$  and  $C_2(s)$  for the output  $y(t)$  to follow the non-periodic reference input  $r(t)$  without steady-state error, the periodic disturbance  $d(t)$  in (3) is attenuated, and the transfer function from  $d(s)$  to  $y(s)$  has finitely many poles.

**3. Condition for Finitely Many Poles.** We derive the necessary and sufficient condition that the transfer function from the disturbance  $d(s)$  to the output  $y(s)$  in Figure 1 has finitely many poles.

The transfer function from  $d(s)$  to  $y(s)$  is given by

$$\frac{y(s)}{d(s)} = \frac{1 + C_2(s) \sum_{k=1}^N F_{2k}(s)e^{-sT_k}}{1 + G(s)C_1(s) + \sum_{k=1}^N C_2(s) (F_{2k}(s) + G(s)F_{1k}(s)) e^{-sT_k}}. \quad (15)$$

From this equation, the condition that the transfer function from  $d(s)$  to  $y(s)$  in (15) has finitely many poles is summarized in the following theorem.

**Theorem 3.1.** *The transfer function in (15) has finitely many poles if and only if*

$$\sum_{k=1}^N (F_{2k}(s) + G(s)F_{1k}(s)) = 0 \quad (16)$$

*holds true.*

**Proof:** The necessity is shown. That is, if the transfer function from  $d(s)$  to  $y(s)$  in (15) has finitely many poles, then (16) holds true. The denominator of transfer function from  $d(s)$  to  $y(s)$  in (15) is given as

$$1 + G(s)C_1(s) + C_2(s) \left( \sum_{k=1}^N F_{2k}(s)e^{-sT_k} + G(s) \sum_{k=1}^N F_{1k}(s)e^{-sT_k} \right). \quad (17)$$

The proof of necessity is to show that if the transfer function in (15) has finitely many poles, then (16) holds. From (17) and the assumption that  $C_2(s) \neq 0$  and the transfer function in (15) has finitely many poles,

$$\sum_{k=1}^N F_{2k}(s)e^{-sT_k} + G(s) \sum_{k=1}^N F_{1k}(s)e^{-sT_k} = 0 \quad (18)$$

has to be satisfied. From (18), we have

$$\sum_{k=1}^N (F_{2k}(s) + G(s)F_{1k}(s)) e^{-sT_k} = 0. \quad (19)$$

This implies that

$$F_{2k}(s) + G(s)F_{1k}(s) = 0. \quad (20)$$

A solution of  $F_{2k}(s) \in RH_\infty$  and  $F_{1k}(s) \in RH_\infty$  are given as

$$F_{2k}(s) = -N(s) - N(s)Q_k(s) \quad (21)$$

and

$$F_{1k}(s) = D(s) + D(s)Q_k(s), \quad (22)$$

where  $Q_k(s) \in RH_\infty$  is any function. The sum of  $F_{2k}(s)$  in (21) and the sum of  $F_{1k}(s)$  in (22) are described as

$$\sum_{k=1}^N F_{2k}(s) = -N(s)[1 - \{(N-1) + NQ_k(s)\}] \quad (23)$$

and

$$\sum_{k=1}^N F_{1k}(s) = D(s)[1 - \{(N-1) + NQ_k(s)\}], \quad (24)$$

respectively. Since  $Q_k(s) \in RH_\infty$ ,  $\sum_{k=1}^N F_{2k}(s)$  in (23) and  $\sum_{k=1}^N F_{1k}(s)$  in (24) are the disturbance observer for the periodic disturbance. Using  $F_{2k}(s)$  in (21) and  $F_{1k}(s)$  in (22), we have

$$\sum_{k=1}^N (F_{2k}(s) + G(s)F_{1k}(s)) = \sum_{k=1}^N (-N(s) - N(s)Q_k(s) + N(s) + N(s)Q_k(s)) = 0. \quad (25)$$

Thus, the necessity has been shown.

The sufficiency is shown. That is, if (16) holds true, then the transfer function from  $d(s)$  to  $y(s)$  has finitely many poles. From the assumption that (16) holds true, the denominator of transfer function from  $d(s)$  to  $y(s)$  is reduced to  $1 + G(s)C_1(s)$ . Thus, it is clear that the transfer function from  $d(s)$  to  $y(s)$  has finitely many poles.

We have thus proved Theorem 3.1.  $\square$

**4. Stability Condition.** In this section, we clarify the stability condition of control system in Figure 1.

The stability condition is summarized in Theorem 4.1.

**Theorem 4.1.** *Assume that Theorem 3.1 holds true. The control system shown in Figure 1 is internally stable if and only if following expressions hold true.*

- 1) The controller  $C_1(s)$  makes  $G(s)$  internally stable.
- 2)  $C_2(s) \in RH_\infty$ .

**Proof:** The necessity is shown. That is, we show that if the control system is internally stable, then the expressions 1) and 2) hold true. From the assumption that (16) holds true, the transfer function from  $r(s)$ ,  $d(s)$ ,  $d_1(s)$  and  $d_2(s)$  to  $u(s)$  and  $y(s)$  is given as

$$\begin{bmatrix} y(s) \\ u(s) \end{bmatrix} = \begin{bmatrix} V_1(s) & V_2(s) & V_3(s) & V_4(s) \\ V_5(s) & V_6(s) & V_7(s) & V_8(s) \end{bmatrix} \begin{bmatrix} r(s) \\ d_1(s) \\ d_2(s) \\ d(s) \end{bmatrix}, \quad (26)$$

where  $V_i(s)$  ( $i = 1, 2, \dots, 8$ ) are written by

$$V_1(s) = \frac{G(s)C_1(s)}{1 + G(s)C_1(s)}, \quad (27)$$

$$V_2(s) = \frac{G(s) \left( 1 + C_2(s) \sum_{k=1}^N F_{2k}(s)e^{-sT_k} \right)}{1 + G(s)C_1(s)}, \quad (28)$$

$$V_3(s) = \frac{-G(s)C_2(s) \left( 1 + C_2(s) \sum_{k=1}^N F_{2k}(s)e^{-sT_k} \right)}{1 + G(s)C_1(s)}, \quad (29)$$

$$V_4(s) = \frac{1 + C_2(s) \sum_{k=1}^N F_{2k}(s)e^{-sT_k}}{1 + G(s)C_1(s)}, \quad (30)$$

$$V_5(s) = \frac{C_1(s)}{1 + G(s)C_1(s)}, \quad (31)$$

$$V_6(s) = \frac{G(s) \left( C_1(s) + C_2(s) \sum_{k=1}^N F_{2k}(s)e^{-sT_k} \right)}{1 + G(s)C_1(s)}, \quad (32)$$

$$V_7(s) = \frac{-C_2(s) \left( 1 + C_2(s) \sum_{k=1}^N F_{2k}(s)e^{-sT_k} \right)}{1 + G(s)C_1(s)} \quad (33)$$

and

$$V_8(s) = \frac{C_1(s) + C_2(s) \sum_{k=1}^N F_{2k}(s)e^{-sT_k}}{1 + G(s)C_1(s)}. \quad (34)$$

From the assumption that the control system in Figure 1 is internally stable, all  $V_i(s)$  ( $i = 1, 2, \dots, 8$ ) from (27) to (34) are stable. From the assumption that  $F_{1k}(s) \in RH_\infty$  ( $k = 1, \dots, N$ ) and  $F_{2k}(s) \in RH_\infty$  ( $k = 1, \dots, N$ ) and equations from (27) to (34), we have  $G(s)C_1(s)/(1 + G(s)C_1(s)) \in RH_\infty$ ,  $G(s)/(1 + G(s)C_1(s)) \in RH_\infty$ ,  $C_1(s)/(1 + G(s)C_1(s)) \in RH_\infty$  and  $1/(1 + G(s)C_1(s)) \in RH_\infty$ . This corresponds to the expression 1). In the same reason,  $C_2(s) \in RH_\infty$ . This corresponds to the expression 2).

Sufficiency is shown. That is, if expressions 1) and 2) hold true, then the control system in Figure 1 is internally stable. From the assumption that expressions 1) and 2) hold true, it is obvious that  $V_i(s)$  ( $i = 1, 2, \dots, 8$ ) from (27) to (34) are stable. The control system in Figure 1 is internally stable.

We have thus proved Theorem 4.1.  $\square$

**5. Design of  $C_1(s)$ .** In this section, we examine a design method of  $C_1(s)$  to make the output  $y(t)$  follow the non-periodic reference input  $r(t)$  without steady-state error.

The transfer function from  $r(s)$  to  $y(s)$  in Figure 1 is written by

$$\frac{y(s)}{r(s)} = \frac{G(s)C_1(s)}{1 + G(s)C_1(s)}. \quad (35)$$

From this equation and the internal model principle [4], in order for  $y(t)$  to follow  $r(t)$  without steady-state error,  $C_1(s)$  needs to be

$$C_1(s) = C_r(s)\bar{C}_1(s), \quad (36)$$

where  $C_r(s) \in R(s)$  is the bi-proper internal model of  $r(s)$  and  $\bar{C}_1(s) \in R(s)$  is a function that does not have the unstable poles of  $C_r(s)$  as its zeros.

Then the problem to design  $C_1(s)$  for  $y(t)$  to follow  $r(t)$  without steady state error is reduced to design  $\bar{C}_1(s)$  to make  $G(s)C_r(s)$  stable.  $\bar{C}(s)$  is obtained from the parameterization [33, 34] of all stabilizing controllers for  $G(s)C_r(s)$ .

**6. Design of  $C_2(s)$ .** In this section, we provide a design method of  $C_2(s)$ .

Consider the influence of  $d(s)$  to  $y(s)$ . The transfer function from  $d(s)$  to  $y(s)$  is written by

$$y(s) = \frac{1 + C_2(s) \sum_{k=1}^N F_{2k}(s) e^{-sT_k}}{1 + G(s)C_1(s)} d(s). \quad (37)$$

From (37) and  $e^{-s_l T_k} = 1$ , if

$$1 + C_2(s) \sum_{k=1}^N F_{2k}(s) \Big|_{s=s_l} = 0, \quad (38)$$

then  $d(t)$  in (3) is attenuated effectively.

A solution to (38) is written by

$$C_2(s) = -\frac{1}{\sum_{k=1}^N F_{2k}(s)}. \quad (39)$$

Since  $G(s)$  is of non-minimum phase and (10),  $F_{2k}(s)$  ( $k = 1, \dots, N$ ) is of non-minimum phase.  $C_2(s)$  in (39) is unstable.

From Theorem 4.1,  $C_2(s)$  needs to satisfy the expression 2) in Theorem 4.1. However, there exists  $C_2(s) \in RH_\infty$  satisfying

$$1 + C_2(s) \sum_{k=1}^N F_{2k}(s) \Big|_{s=s_l} \simeq 0 \quad (l = 1, \dots, l_{\max}), \quad (40)$$

where  $l_{\max}$  is the maximum frequency component to attenuate the disturbance  $d(t)$ . A solution of (40) is obtained by using  $C_2(s)$  that satisfies

$$C_2(s) \sum_{k=1}^N F_{2k}(s) = -N_i(s)\bar{Q}(s), \quad (41)$$

where  $\bar{Q}(s) \in RH_\infty$  is a function satisfying  $\bar{Q}(j\omega_l) \simeq 1$  in the wide frequency range, and makes  $C_2(s)$  in (41) proper and

$$\frac{\bar{Q}(s)}{N_o(s)(1 + Q(s))} \in RH_\infty. \quad (42)$$

Here,  $N_o(s) \in RH_\infty$  is the outer function of  $N(s)$  satisfying

$$N(s) = N_i(s)N_o(s) \quad (43)$$

and  $N_i(s) \in RH_\infty$  is an inner function satisfying  $N_i(0) = 1$ .

Using  $C_2(s)$  satisfying (41) and  $F_{2k}(s)$  satisfying (10), (40) can be written by the form in

$$1 - N_i(s)Q(s) \Big|_{s=s_l} \simeq 0. \quad (44)$$

The solution to (41) is given by

$$C_2(s) = \frac{-N_i(s)\bar{Q}(s)}{\sum_{k=1}^N F_{2k}(s)} = \frac{\bar{Q}(s)}{N_o(s)(1 + Q(s))}. \quad (45)$$

Since  $\bar{Q}(s)$  makes  $C_2(s)$  proper and (42),  $C_2(s) \in RH_\infty$  in (45). Note that from (45),  $\bar{Q}(s)$  has zeros of  $1 + Q(s)$  in the closed right half plane, since  $N_o(s)$  has no zeros in the closed right half plane.

**7. Design Procedure.** In this section, we provide a design procedure of the control system in Figure 1.

The design procedure of the control system in Figure 1 is summarized as follows.

- Step 1 Obtain coprime factors  $N(s) \in RH_\infty$  and  $D(s) \in RH_\infty$  of  $G(s)$  satisfying (11).
- Step 2 Obtain  $N_i(s)$  and  $N_o(s)$  satisfying (43).
- Step 3 Find the period  $T$  of  $d(t)$  in (3).
- Step 4 We settle  $C_r(s)$  in (36) from  $r(t)$ .
- Step 5 Settle  $N$  and  $T_k$  ( $k = 1, \dots, N$ ) satisfying (7), where  $T_k = kT$  is chosen to align the delay branches with the disturbance period  $T$ , and  $N$  is selected as a trade-off between attenuation performance and implementation complexity.
- Step 6  $F_{1k}(s)$  ( $k = 1, \dots, N$ ) and  $F_{2k}(s)$  ( $k = 1, \dots, N$ ) are designed as (9) and (10), respectively.  $Q(s) \in RH_\infty$  is any function to make  $1/(1 + Q(s)) \in RH_\infty$ .
- Step 7 Design  $\bar{Q}(s)$  in (41) as

$$\bar{Q}(s) = \frac{1}{(1 + \tau s)^\alpha}, \quad (46)$$

where  $\tau \in R$  is any real number and  $\alpha$  is any integer satisfying  $\bar{Q}(j\omega_k) \simeq 1$  and (42).

Step 8 Using  $\bar{Q}(s)$  in (46),  $C_2(s)$  is settled by (45).

Step 9 Using the internal model of  $r(s)$ ,  $C_1(s)$  in (36) is designed from the parameterization of all stabilizing controllers for  $G(s)C_r(s)$ .

**8. Numerical Examples.** In this section, numerical examples are illustrated to show the effectiveness of the proposed method.

Consider the problem to design the control system in Figure 1 for the non-minimum phase plant  $G(s)$  written by

$$G(s) = \frac{s - 100}{s^2 - 43s - 350}. \quad (47)$$

The objective is that the output  $y(t)$  follows the non-periodic reference  $r(t) = t$  and attenuates the periodic output disturbance  $d(t)$  given as

$$d(t) = \sum_{i=1}^3 \sin(it). \quad (48)$$

The period  $T$  of the disturbance  $d(t)$  in (48) is  $T = 2\pi$ .

Coprime factors  $N(s) \in RH_\infty$  and  $D(s) \in RH_\infty$  of  $G(s)$  in (11) are given as

$$N(s) = \frac{-2s + 200}{s^2 + 1007s + 7000} \quad (49)$$

and

$$D(s) = \frac{-2s + 100}{s + 1000}. \quad (50)$$

An inner function  $N_i(s) \in RH_\infty$  and an outer function  $N_o(s) \in RH_\infty$  in (43) are obtained as

$$N_i(s) = \frac{-2s + 200}{2s + 200} \quad (51)$$

and

$$N_o(s) = \frac{2s + 200}{s^2 + 1007s + 7000}. \quad (52)$$

To attenuate  $d(t)$  in (48), we design the control system in Figure 1 with  $N = 3$ .  $F_{1k}(s)$  ( $k = 1, 2, 3$ ) and  $F_{2k}(s)$  ( $k = 1, 2, 3$ ) are designed as

$$F_{1k}(s) = \frac{1}{N} \frac{-5s^2 - 750s + 50000}{s^2 + 1050s + 50000} \quad (k = 1, 2, 3) \quad (53)$$

and

$$F_{2k}(s) = \frac{1}{N} \frac{5s^2 + 500s - 100000}{s^3 + 1057s^2 + 57350s + 350000} \quad (k = 1, 2, 3). \quad (54)$$

$Q(s)$  is settled by

$$Q(s) = \frac{1.5s + 450}{s + 50}. \quad (55)$$

$\bar{Q}(s)$  is designed as

$$\bar{Q}(s) = \frac{1}{(0.0166s + 1)^3}. \quad (56)$$

Using  $\bar{Q}(s)$  in (56),  $C_2(s)$  is settled by (45) and written by

$$C_2(s) = \frac{2.16 \times 10^5 s^3 + 2.283 \times 10^8 s^2 + 1.239 \times 10^{10} s + 7.56 \times 10^{10}}{5s^5 + 2400s^4 + 42.4 \times 10^4 s^3 + 3.528 \times 10^7 s^2 + 1.404 \times 10^9 s + 2.16 \times 10^{10}} \quad (57)$$

Using the parameterization of all stabilizing controllers for  $G(s)C_r(s)$ ,  $C_1(s)$  is designed as

$$C_1(s) = \frac{-10^{11}(-3.092 \times 10^{11} s^3 + 70.26 s^2 + 1176 s + 6524)}{s^2 (s^3 + 3.154 \times 10^4 s^2 + 2.713 \times 10^8 s + 3.474 \times 10^{11})}. \quad (58)$$

The response of  $y(t)$  for  $d(t)$  in (48) is shown in Figure 2. Here the dotted line shows the periodic disturbance  $d(t)$  in (48), and the solid line shows that of  $y(t)$  for  $d(t)$ . Figure 2 implies that  $d(t)$  is attenuated effectively.

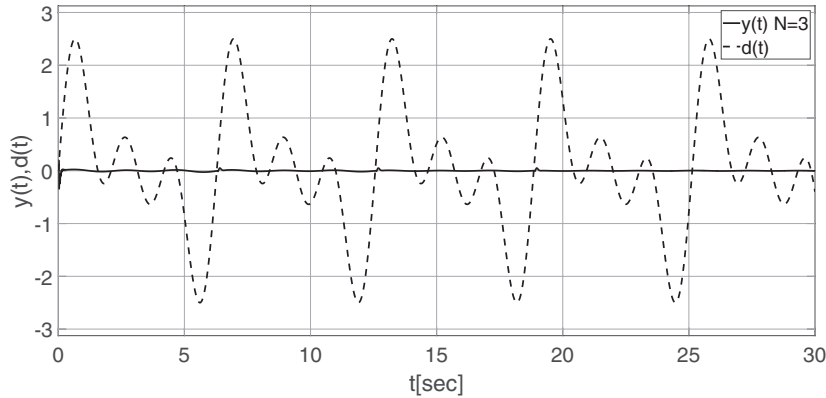


FIGURE 2. The response of  $y(t)$  for the disturbance  $d(t)$

The reference tracking performance of the control system is shown. Figure 3 shows that the error  $e(t)$  for  $r(t) = t$ . From Figure 3, the error  $e(t)$  converges to 0. This result confirms that the proposed method not only attenuates periodic disturbances but also maintains accurate ramp tracking despite the structural constraints.

To compare the effectiveness of the multi-period disturbance observer for the periodic output disturbance, two numerical examples are shown. One numerical example is for the control system in Figure 1 when  $N = 3$ . This is the control system using the disturbance

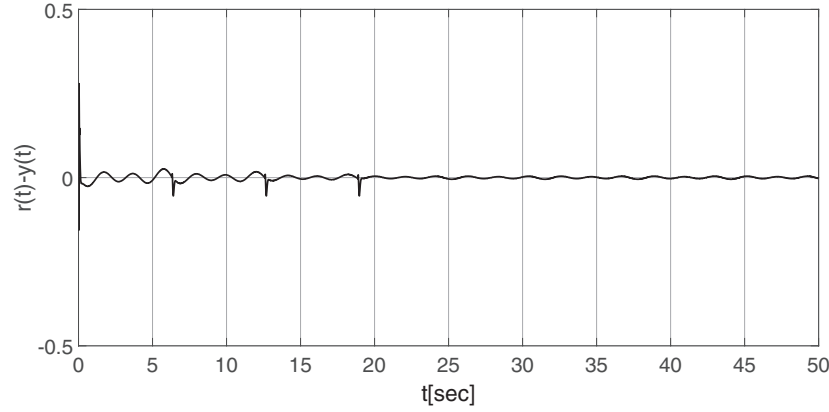


FIGURE 3. Response of error  $e(t) = r(t) - y(t)$  for the ramp reference input  $r(t) = t$  under the proposed controller

observer that uses three signals of the periodic time before the present time. The other numerical example is for the control system in Figure 1 when  $N = 1$ . This is the control system using the disturbance observer that uses only one signal of the periodic time before the present time. These two numerical examples use the same controllers  $C_1(s)$  in (58) and  $C_2(s)$  in (57).

To show the numerical example for the control system in Figure 1 when  $N = 1$ ,  $F_{11}(s)$  and  $F_{21}(s)$  when  $N = 1$  are designed as

$$F_{11}(s) = \frac{-5s^2 - 750s + 50000}{s^2 + 1050s + 50000}, \quad (59)$$

$$F_{21}(s) = \frac{5s^2 + 500s - 100000}{s^3 + 1057s^2 + 57350s + 350000}. \quad (60)$$

The gain plot of the transfer function from  $d(s)$  to  $y(s)$  of the control system in Figure 1 when  $N = 1$  and that of when  $N = 3$  is shown in Figure 4. Here the solid line shows the gain plot when  $N = 3$ , and the dashed line shows that when  $N = 1$ .

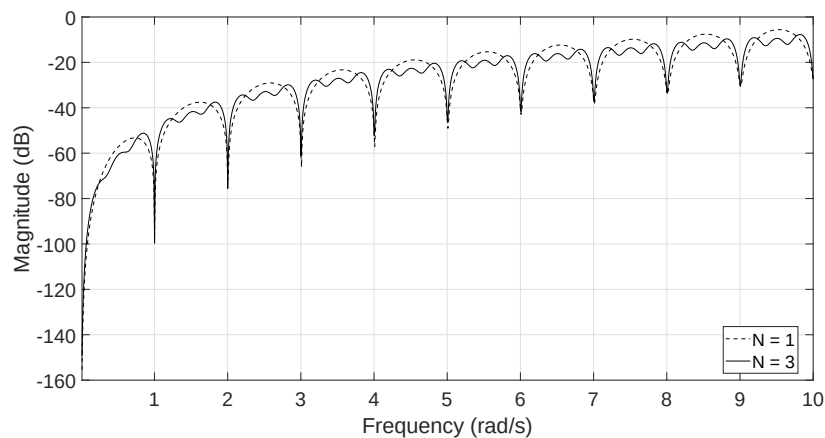


FIGURE 4. Gain plot of transfer function from  $d(s)$  to  $y(s)$  (solid:  $N = 3$ , dashed:  $N = 1$ )

The response of  $y(t)$  to the periodic disturbance  $d(t)$  in (48) is shown in Figure 5. The dotted line indicates the response for  $N = 1$ , whereas the solid line indicates that for  $N = 3$ . As observed in Figure 5, the dominant transient peak of  $y(t)$  for  $N = 1$  is approximately

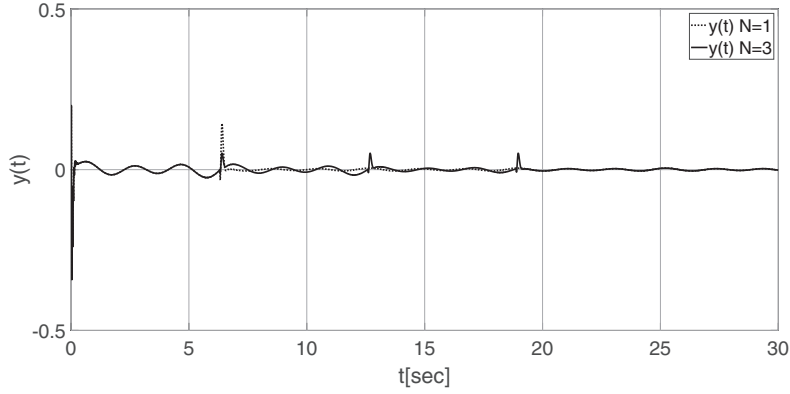


FIGURE 5. Response of  $y(t)$  for  $d(t)$  in (48) (solid:  $N = 3$ , dashed:  $N = 1$ )

three times larger than that for  $N = 3$ , indicating a significant reduction of the peak excursion by the proposed multi-period design. Regarding the decay of the oscillation envelope, the case  $N = 1$  exhibits a slightly faster settling in the early stage; however, the difference is small, and the responses for  $N = 1$  and  $N = 3$  become comparable after the transient. Therefore, the main advantage of increasing  $N$  to 3 in this example is the substantial peak reduction while maintaining a similar settling behavior, which is particularly beneficial when peak constraints (e.g., saturation limits or safety margins) are critical under periodic disturbances.

Furthermore, it is shown the performances of the proposed controller and the other four control methods for  $d(t)$  in (48) and the plant  $G(s)$  in (47) as follows:

- 1) **OL** – a control only using the feed forward without controller;
- 2) **DF** – feedback only using  $y(t)$ ;
- 3) **DF+ $C_1(s)$**  – one degree of freedom control system using  $C_1(s)$  in (58);
- 4) **DOB** – the control system in Figure 1 when  $N = 1$ ;
- 5) **Proposed DOB** – multi-period disturbance observer ( $N = 3$ ).

The performance is evaluated by the response of  $y(t)$  of  $r(t) = 0$  of the each control method with  $d(t)$  in (48) and that of the control system in Figure 1 when  $N = 3$  with  $d(t)$  in (48) on the time interval from 0 to 30 seconds. For indicating the performance, we use four indicators: the Integral Square Error (ISE), the Integral Absolute Error (IAE), the Root Mean Square (RMS) and the percentage improvement based on RMS. The ISE is given by

$$\text{ISE} := \int_0^{30} (y(t))^2 dt. \quad (61)$$

The IAE is given by

$$\text{IAE} := \int_0^{30} |y(t)| dt. \quad (62)$$

The RMS is given by

$$\text{RMS} := \sqrt{\frac{1}{30} \int_0^{30} y(t)^2 dt}. \quad (63)$$

The percentage improvement based on RMS is given by

$$\text{Improvement} = \frac{\text{RMS}_{\text{other}} - \text{RMS}_{\text{Proposed}}}{\text{RMS}_{\text{other}}} \times 100[\%], \quad (64)$$

where  $\text{RMS}_{\text{other}}$  is the RMS of OL, DF,  $\text{DF}+C_1(s)$  or DOB,  $\text{RMS}_{\text{proposed}}$  is the control system in Figure 1 when  $N = 3$ .

The performance is summarized in Table 1. From Table 1, the proposed multi-period disturbance observer with  $N = 3$  has the lowest RMS than the other three control methods. In the OL case, the RMS decreases from 1.28866 to 0.014876, corresponding to a 98.85% reduction. In the  $\text{DF}+C_1(s)$  case, the RMS is reduced by 19.32%. In the DOB case, the RMS is reduced by 0.97%.

TABLE 1. Steady-state tracking error metrics

| Control method                           | ISE              | IAE             | RMS             | Improvement (%) |
|------------------------------------------|------------------|-----------------|-----------------|-----------------|
| OL                                       | 42.6384          | 26.0275         | 1.28866         | 98.85           |
| DF                                       | $\infty$         | $\infty$        | $\infty$        | 100             |
| $\text{DF}+C_1(s)$                       | 0.00855712       | 0.358094        | 0.0184371       | 19.32           |
| DOB ( $N = 1$ )                          | 0.00609553       | 0.15773         | 0.0150214       | 0.97            |
| <b>Proposed DOB (<math>N = 3</math>)</b> | <b>0.0057742</b> | <b>0.203574</b> | <b>0.014876</b> | —               |

These results imply that the proposed control system using the multi-period disturbance observer when  $N = 3$  provides a good performance about the disturbance attenuation for the periodic disturbance.

Next, we evaluate robustness against disturbance-period uncertainty by a period-mismatch test, as shown in Figure 6. In this test, the MP-DOB is designed with the assumed period  $T = \pi$ , while the actual disturbance  $d(t)$  has period  $2\pi$ . The dotted line indicates the response for  $N = 1$ , whereas the solid line indicates that for  $N = 3$ . As seen in Figure 6 (and its zoomed view), a mismatch in the period leads to a residual periodic component that cannot be perfectly canceled; nevertheless, the proposed multi-period design ( $N = 3$ ) yields a smaller output oscillation than the single-delay design ( $N = 1$ ). In particular, the transient peak excursion is reduced for  $N = 3$ , and the subsequent steady oscillation amplitude remains consistently lower than that of  $N = 1$ . These observations demonstrate that increasing  $N$  improves attenuation performance even when the actual disturbance period deviates from the design period, thereby providing practical robustness to period perturbations.

Next, to further verify the universality of the proposed design for non-minimum phase systems with different pole distributions, we consider another non-minimum phase plant

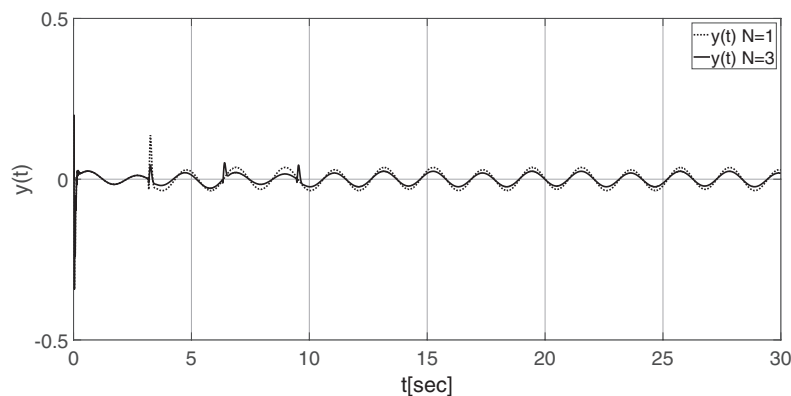


FIGURE 6. The response of  $y(t)$  for  $d(t)$  with period  $2\pi$  when  $T = \pi$ : Comparison between  $N = 1$  (dashed) and  $N = 3$  (solid)

that has *stable complex-conjugate poles*:

$$G_{\text{new}}(s) = \frac{s - 100}{s^2 + 20s + 350}. \quad (65)$$

Note that  $G_{\text{new}}(s)$  is *stable* (all poles are in the open left-half plane) while it is still *non-minimum phase* due to the right-half-plane zero at  $s = 100$ . Applying the same design procedure in Section 7 with the same fundamental period  $T = 2\pi$ , we compare the gain of the disturbance-to-output transfer function in (37) for  $N = 1$  and  $N = 3$ .

As shown in Figure 7, the delay-coordinated structure produces notch behavior at harmonic frequencies determined by the period  $T$ . Moreover, the three-delay configuration ( $N = 3$ ) yields a lower gain profile around these harmonic components compared with the single-delay case ( $N = 1$ ), which implies stronger attenuation of periodic disturbance components in the output.

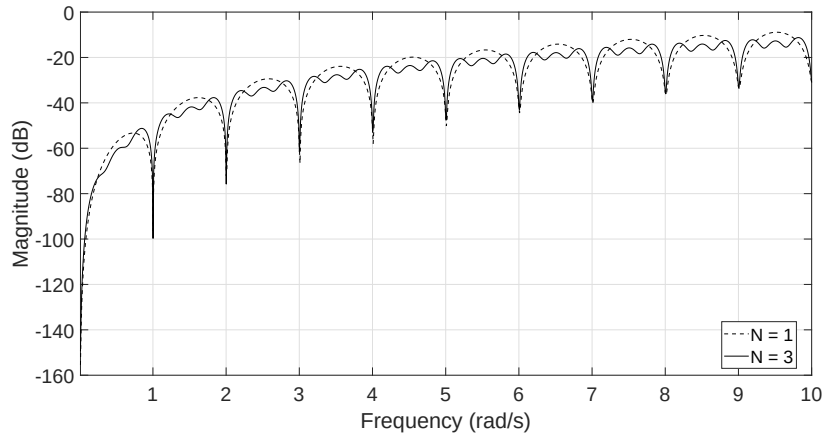


FIGURE 7. Gain plot of the disturbance-to-output transfer function for  $G_{\text{new}}(s)$  (solid:  $N = 3$ , dashed:  $N = 1$ )

The time-domain response in Figure 8 is consistent with the frequency-domain result in Figure 7: after the transient, the output oscillation under  $N = 3$  becomes smaller than that under  $N = 1$ , demonstrating that the proposed multi-period disturbance observer maintains its disturbance-attenuation advantage even for a different non-minimum phase plant with stable complex poles.

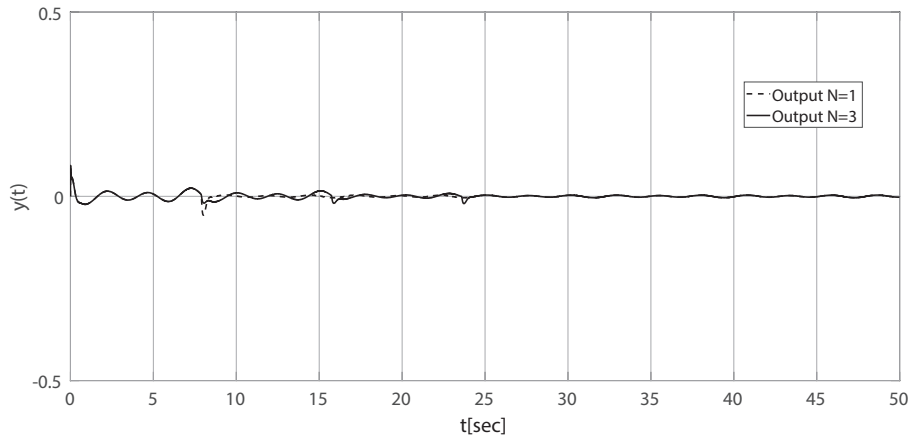


FIGURE 8. Output response comparison for  $G_{\text{new}}(s)$  (solid:  $N = 3$ , dashed:  $N = 1$ )

**9. Conclusion.** This paper proposed a design method for a control system using a multi-period disturbance observer to attenuate periodic output disturbances in non-minimum-phase plants. The proposed structure makes the disturbance-to-output transfer function have finitely many poles, and the internal stability condition of the overall control system was clarified. A simple design procedure was presented, and numerical examples demonstrated the effectiveness of the proposed method. The numerical examples also confirmed that the proposed control system remains effective even when the disturbance period does not coincide with the design period.

From an implementation viewpoint, the proposed control system is suitable for real-time deployment because it consists of low-order rational controllers/filters and a finite number of delay branches. In digital implementation, each delay element can be realized by a sample buffer corresponding to the required delay length, and the rational controllers/filters can be implemented by standard discrete-time realizations (difference equations or state-space forms). Therefore, the required memory and per-sample computations increase approximately linearly with the number of delay branches  $N$  and the filter orders, while avoiding the high-order internal model typically associated with repetitive control. These properties make the proposed structure compatible with common motion-control hardware (DSP/MCU/FPGA) and practical control platforms. When the disturbance period varies, the controller can be retuned by updating the period parameter and the corresponding delays, and the numerical example indicates that meaningful attenuation is still achieved even under period mismatch.

Although this paper focused on numerical verification, the proposed control system is applicable to non-minimum-phase systems subject to periodic disturbances, such as vibration control for 3D printers or washing machines during spin cycles, and precision motion stages in semiconductor manufacturing. Future work will investigate implementation issues on actual hardware platforms and experimental validation in practical applications.

## REFERENCES

- [1] K. Nie, W. Xue, C. Zhang and Y. Mao, Disturbance observer-based repetitive control with application to optoelectronic precision positioning system, *Journal of the Franklin Institute*, vol.358, no.16, pp.8443-8469, DOI: 10.1016/j.jfranklin.2021.08.042, 2021.
- [2] M. Bodson, J. Jensen and S. C. Douglas, Active noise control for periodic disturbance, *IEEE Transactions on Control Systems Technology*, vol.9, no.1, pp.200-205, 2001.
- [3] A. Mohammadi, M. Tavakoli, H. J. Marquez and F. Hashemzadeh, Nonlinear disturbance observer design for robotic manipulators, *Control Engineering Practice*, vol.21, no.3, pp.253-267, 2013.
- [4] B. A. Francis and W. M. Wonham, The internal model principle of control theory, *Automatica*, vol.12, no.5, pp.457-465, 1976.
- [5] S. Phukapak, T. H. Do, D. Koyama, C. Phukapak, N. T. Mai, K. Hashikura, M. A. S. Kamal, I. Murakami and K. Yamada, Proposal of multi-period disturbance observers for periodic output disturbances, *ICIC Express Letters*, vol.18, no.7, pp.711-720, 2024.
- [6] M. Nakano, T. Inoue, Y. Yamamoto and S. Hara, *Repetitive Control*, The Society of Instrument and Control Engineers, 1989 (in Japanese).
- [7] G. Pipeleers, B. Demeulenaere, J. De Schutter and J. Swevers, Robust high-order repetitive control: Optimal performance trade-offs, *Automatica*, vol.44, no.10, pp.2628-2634, 2008.
- [8] Z. Chen, K. Yamada and T. Sakanushi, A new design method of high-order modified repetitive control systems for reference inputs with uncertain period-time, *Mathematical Problems in Engineering*, vol.2013, Article ID 374328, 2013.
- [9] B. Mahawan and Z. H. Luo, Repetitive control of tracking systems with time-varying periodic references, *International Journal of Control*, vol.73, no.1, pp.1-10, 2000.
- [10] J. Freudenberg and D. Looze, Right half plane poles and zeros and design tradeoffs in feedback systems, *IEEE Transactions on Automatic Control*, vol.30, no.6, pp.555-565, 1985.

- [11] J. Reinders, M. Giaccagli, B. Hunnekens, D. Astolfi, T. Oomen and N. van de Wouw, Repetitive control for Lur'e-type systems: Application to mechanical ventilation, *IEEE Transactions on Control Systems Technology*, vol.31, no.4, pp.1819-1829, DOI: 10.1109/TCST.2023.3250966, 2023.
- [12] E. Kurniawan, H. Wang, J. A. Prakosa, P. Purwowibowo and E. B. Pratiwi, An improved repetitive controller with fractional time-delay for discrete-time linear systems: Synthesis and comparison study, *ISA Transactions*, vol.146, pp.511-527, DOI: 10.1016/j.isatra.2023.12.035, 2024.
- [13] M. Zhang, C. Lu, S. Tian, Y. Wang, M. Wu and M. Iwasaki, Improved disturbance rejection in repetitive-control systems: Phase-compensated equivalent-input-disturbance approach, *IEEE Transactions on Industrial Electronics*, vol.71, no.10, pp.12942-12951, DOI: 10.1109/TIE.2024.3355510, 2024.
- [14] S. Ding, W.-H. Chen, K. Mei and D. J. Murray-Smith, Disturbance observer design for nonlinear systems represented by input-output models, *IEEE Transactions on Industrial Electronics*, vol.67, no.2, pp.1222-1232, 2020.
- [15] W.-H. Chen, J. Yang, L. Guo and S. Li, Disturbance-observer-based control and related methods – An overview, *IEEE Transactions on Industrial Electronics*, vol.63, no.2, pp.1083-1095, 2016.
- [16] K. Ohnishi, M. Shibata and T. Murakami, Motion control for advanced mechatronics, *IEEE/ASME Transactions on Mechatronics*, vol.1, no.1, pp.56-67, 1996.
- [17] K.-S. Kim, K.-H. Rew and S. Kim, Disturbance observer for estimating higher order disturbances in time series expansion, *IEEE Transactions on Automatic Control*, vol.55, no.8, pp.1905-1911, 2010.
- [18] I. Peñarrocha-Alós, D. Tena and R. Sanchis, Cascade multi-resonant disturbance observer design. Application to a distillation column, *Journal of Process Control*, vol.132, DOI: 10.1016/j.jprocont.2023.103105, 2023.
- [19] J. Na, R. Grino, R. C. Castello, X. Ren and Q. Chen, Repetitive controller for time-delay systems based on disturbance observer, *IET Control Theory & Applications*, vol.4, no.11, pp.2391-2404, 2010.
- [20] L. Wang, C. T. Freeman, E. Rogers and P. C. Young, Disturbance observer-based repetitive control system with nonminimal state-space realization and experimental evaluation, *IEEE Transactions on Control Systems Technology*, vol.31, no.2, pp.961-968, DOI: 10.1109/TCST.2022.3202299, 2023.
- [21] E. Sariyildiz, R. Oboe and K. Ohnishi, Disturbance observer-based robust control and its applications: 35th anniversary overview, *IEEE Transactions on Industrial Electronics*, vol.67, no.3, pp.2042-2053, 2020.
- [22] H. Muramatsu and S. Katsura, An adaptive periodic disturbance observer for periodic disturbance suppression, *IEEE Transactions on Industrial Informatics*, vol.14, no.10, pp.4446-4456, 2018.
- [23] M. Zheng, S. Zhou and M. Tomizuka, A design methodology for disturbance observer with application to precision motion control: An  $H_\infty$ -based approach, *Proc. of American Control Conference*, pp.3524-3529, 2017.
- [24] T. Mita, M. Hirata, K. Murata and H. Zhang,  $H_\infty$  control versus disturbance-observer-based control, *IEEE Transactions on Industrial Electronics*, vol.45, no.3, pp.488-495, 1998.
- [25] H. Kobayashi, S. Katsura and K. Ohnishi, An analysis of parameter variations of disturbance observer for motion control, *IEEE Transactions on Industrial Electronics*, vol.54, no.6, pp.3413-3421, 2007.
- [26] K. Yamada, I. Murakami, Y. Ando, T. Hagiwara, Y. Imai and M. Kobayashi, The parameterization of all disturbance observers, *ICIC Express Letters*, vol.2, no.4, pp.421-426, 2008.
- [27] S. Phukapak, D. Koyama, K. Hashikura, M. A. S. Kamal and K. Yamada, The parameterization of all disturbance observers for periodic output disturbances, *International Journal of Innovative Computing, Information and Control*, vol.19, no.1, pp.163-180, 2023.
- [28] S. Phukapak, D. Koyama, K. Hashikura, M. A. S. Kamal, I. Murakami and K. Yamada, The parameterization of all disturbance observers for periodic input disturbances, *ECTI Transactions on Electrical Engineering, Electronics, and Communications*, vol.21, no.2, Article no.249809, 2023.
- [29] S. Phukapak, D. Koyama, K. Hashikura, M. A. S. Kamal and K. Yamada, The parameterization of all disturbance observers for periodic input and output disturbances, *International Journal of Innovative Computing, Information and Control*, vol.19, no.3, pp.637-654, 2023.
- [30] Y. Yamada, H. M. Tien, S. Phukapak, C. Phukapak, N. T. Mai, K. Hashikura, M. A. S. Kamal, I. Murakami and K. Yamada, Control system to attenuate periodic disturbance without using repetitive control, *International Journal of Innovative Computing, Information and Control*, vol.20, no.5, pp.1381-1397, 2024.
- [31] Y. Yamada, H. M. Tien, S. Phukapak, C. Phukapak, N. T. Mai, K. Hashikura, M. A. S. Kamal, I. Murakami and K. Yamada, Control system to attenuate periodic disturbance without using repetitive control for non-minimum phase plant, *International Journal of Innovative Computing, Information and Control*, vol.20, no.6, pp.1819-1835, 2024.

- [32] D. Koyama, X. Yu, J. Li, N. T. Mai, K. Hashikura, M. A. S. Kamal, I. Murakami and K. Yamada, Low sensitivity control for non minimum-phase systems using double feedback control, *International Journal of Innovative Computing, Information and Control*, vol.20, no.4, pp.997-1018, 2024.
- [33] D. C. Youla, H. A. Jabr and J. J. Bongiorno, Modern Wiener-Hopf design of optimal controllers: Part II, *IEEE Transactions on Automatic Control*, vol.AC-21, no.3, pp.319-338, 1976.
- [34] M. Vidyasagar, *Control System Synthesis – A Factorization Approach*, MIT Press, Cambridge, MA, 1985.

## Author Biography



**Siripong Sangsarpan** received the B.Eng. degree in Engineering from King Mongkut's Institute of Technology Ladkrabang (KMUTL), Thailand, in 2012, and the M.Eng. degree from Thai-Nichi Institute of Technology (TNI), Thailand, in 2017. He is currently a doctoral student in the Graduate School of Science and Technology at Gunma University, Japan, and a lecturer at Thai-Nichi Institute of Technology, Thailand. His research interests include disturbance observers, robust control, and adaptive systems.



**Daisuke Koyama** received the B.S. degree in Science from Gunma University, Japan, in 2017, and the M.S. degree in Science and Technology from Gunma University, Japan, in 2020. Mr. Koyama is currently a doctoral candidate in Mechanical Science and Technology at Gunma University, Japan. His research interests include robust control system design.



**Nghia Thi Mai** received the B.S., M.S., and Dr. Eng. degrees from Gunma University, Japan, in 2009, 2011, and 2014, respectively. She is currently a lecturer at the Faculty of Electronics Engineering 1, Posts and Telecommunications Institute of Technology (PTIT), Vietnam, and a visiting associate professor at Gunma University, Japan. Her research interests include Smith predictor, internal model control, and robotics.



**Md Abdus Samad Kamal** received the B.Sc. degree in Electrical and Electronic Engineering from Khulna University of Engineering and Technology (KUET), Bangladesh, in 1997; and the M.S. and Dr. Eng. degrees from Kyushu University, Japan, in 2003 and 2006, respectively. He is currently an associate professor at Graduate School of Science and Technology, Gunma University, Japan. His current research interests are reinforcement learning, intelligent transportation systems, and multiagent systems. He is a member of IEEE and SICE.



**Iwanori Murakami** received the B.E., M.E. and Dr. Eng. degrees from Gunma University, Japan, in 1992, 1994 and 1997, respectively. He is currently an associate professor at Gunma University, Japan. His research interests include robotics, applied electromagnetics, machines, and superconducting levitation applications.



**Kou Yamada** received his B.S. and M.S. degrees from Yamagata University, Japan, in 1987 and 1989, respectively, and a Dr. Eng. degree from Osaka University, Japan, in 1997. From 1991 to 2000, he was with the Department of Electrical and Information Engineering, Yamagata University, Japan, as a research associate. From 2000 to 2008, he was an associate professor in the Graduate School of Science and Technology, Gunma University, Japan. Since 2008, he has been a professor in the Graduate School of Science and Technology, Gunma University, Japan. His research interests include robust control, repetitive control, process control, and control theory for inverse systems and infinite-dimensional systems. Dr. Yamada received the 2005 Yokoyama Award in Science and Technology, the 2005 Electrical Engineering/Electronics, Computer, Telecommunication, and Information Technology International Conference (ECTI-CON2005) Best Paper Award, the Japanese Ergonomics Society Encouragement Award for an Academic Paper in 2007, the 2008 Electrical Engineering/Electronics, Computer, Telecommunication, Information Technology International Conference (ECTI-CON2008) Best Paper Award, and the 4th International Conference on Innovative Computing, Information and Control Best Paper Award in 2009, the 14th International Conference on Innovative Computing, Information and Control Best Paper Award in 2019, and Outstanding Achievement Award from Kanto Branch of Japanese Society for Engineering Education in 2022 and JSME (The Japan Society of Mechanical Engineers) Education Award in 2023. He is a member of IEEE and SICE, and a fellow of JSME.