

SUBJECTIVE-OBJECTIVE WEIGHTING FOR AI PRODUCT SELECTION IN PULMONARY NODULE MANAGEMENT: A PLITHOGENIC TYPE-2 NEUTROSOPHIC PROBABILISTIC LINGUISTIC APPROACH

PEIWEN WANG¹ AND ZHI-PING FAN^{1,2,*}

¹School of Business Administration
Northeastern University

No. 195, Chuangxin Road, Hunnan New District, Shenyang 110169, P. R. China
wangpeiwen@mail.neu.edu.cn

²National Frontiers Science Center for Industrial Intelligence and Systems Optimization
Northeastern University

No. 3-11, Wenhua Road, Heping District, Shenyang 110819, P. R. China

*Corresponding author: zpfan@mail.neu.edu.cn

Received July 2025; revised October 2025

ABSTRACT. *Early detection and intervention of pulmonary nodules are crucial for enhancing lung cancer survival rates. AI has emerged as a transformative tool in healthcare, showcasing considerable potential in managing pulmonary nodules. However, the inherent complexity and uncertainty of medical decision-making present significant challenges in selecting optimal AI-assisted technology for pulmonary nodules diagnosis and treatment (PNDT). This research aims to propose an advanced model for AI product selection in PNDT by integrating type-2 neutrosophic numbers (T2NNs) and probabilistic linguistic term sets (PLTSs). This framework addresses the challenge through a combination of subjective and objective weighting methods, specifically the T2NN-fuzzy weighted zero inconsistency (FWZIC) and T2NN-logarithmic percentage change-driven objective weighting (LOPCOW). Furthermore, the T2NN Schweizer-Sklar weighted arithmetic mean and plithogenic T2NN (PT2NN) aggregation operators are employed to determine decision-maker (DM) weights and effectively synthesize heterogeneous data. AI product ranking is performed using a plithogenic T2NN-PLTS (PT2NNPLTS) extension of the measurement of alternative and ranking according to the compromise solution (MARCOS) approach, incorporating enhanced score and utility functions under the integrated weighting scheme. Comprehensive sensitivity and comparative analyses affirm the model's superiority in consistency and robustness. These advancements contribute to uncertainty-based theory and offer practical insights for implementing AI systems in complex clinical settings.*

Keywords: Pulmonary nodules diagnosis and treatment, T2NN, Plithogenic set, PLTS, Subjective and objective weighting, MARCOS method

1. Introduction. Lung cancer remains a leading cause of cancer-related mortality globally, a reality driven primarily by late-stage diagnosis limiting therapeutic options. Each year, about 2.48 million new lung cancer cases and 1.82 million deaths are reported, representing 12.4% of new diagnoses and 18.7% of cancer deaths globally [1]. The critical importance of early detection is underscored by the potential to increase five-year relative survival rates from under 30% to over 70% with timely intervention [2,3]. As most lung cancers originate as nodules, their prompt and accurate characterization is paramount and has become a significant focus of clinical research.

The rapid advancement of AI, particularly breakthroughs in algorithmic architectures such as deep learning and convolutional neural networks, has introduced powerful new tools for PNDT. These systems demonstrate significant potential in identifying suspicious lesions on computed tomography (CT) scans, predicting malignancy risk, and optimizing patient management strategies [4,5]. The proliferation of these AI tools, however, presents a new critical challenge: selecting the optimal system for a given clinical environment. This selection is inherently a complex multi-criteria group decision-making (MCDM) problem, as tools vary across conflicting criteria such as diagnostic accuracy, computational efficiency, and clinical interpretability. An effective decision framework is therefore essential for the rational adoption of these technologies in medical practice.

Addressing this MCDM problem requires a framework that can navigate the profound uncertainty inherent in clinical judgment. This uncertainty is twofold. First, there is quantitative ambiguity, where clinicians must grapple with imprecise probabilities and degrees of belief. Advanced theories like type-2 neutrosophic numbers (T2NNs) are well-suited to modeling this form of imprecision, ambiguity, and inconsistency by capturing the degrees of truth, indeterminacy, and falsity [6,7]. To enhance subjective evaluation accuracy, Smarandache [8] proposed the plithogenic set, an advancement of neutrosophic theory that incorporates contradiction and appurtenance. However, these theories that focus solely on quantitative uncertainty fall short of capturing linguistic imprecision. For instance, physicians may hesitate between linguistic terms (LTs) like “Lung-RADS Category 4b” and “4c” when evaluating nodules, ambiguity that exceeds the expressive capacity of T2NN alone. To address qualitative uncertainty, Pang et al. [9] introduced the probabilistic linguistic term set (PLTS), which enables DMs to express preferences using multiple possible LTs with associated probabilities. Wang et al. [10] extended this set to plithogenic PLTS, enhancing its ability to model complex linguistic preferences through the plithogenic principle. Given that clinical evaluations involve both numerical estimation and semantic interpretation, a hybrid approach that integrates the strengths of T2NN and plithogenic PLTS is necessary to model the full spectrum of diagnostic uncertainty.

1.1. Aims of the study. This study is the first to address auxiliary AI product selection for PNDT from an MCDM perspective. The main objective is to propose a practical and powerful framework for identifying the most suitable AI product. The framework is developed by integrating FWZIC, LOPCOW, and MARCOS methods under the T2NN-PLTS environment, incorporating plithogenic and T2NN-based aggregation operators. It encompasses expert weight determination, subjective-objective criteria weighting, and alternative ranking under uncertainty, enabling the analysis of complex decision processes. By leveraging the aggregation of T2NN and PLTS information, the model offers high versatility and adaptability for decision support, while minimizing the extent of human intervention. Furthermore, the model is validated through a real-world case study in a hospital’s critical care department, providing practical guidelines for selecting AI products in PNDT. Sensitivity and comparative analyses confirm the model’s robustness, applicability, and superiority.

The primary contribution of this study lies in the development of a synergistic and integrated decision-making framework. Its novelty and necessity are demonstrated through the following aspects. 1) The framework introduces a novel integration of T2NN and PLTS within a plithogenic environment, that is PT2NNPLTS. This fusion is not redundant but essential for addressing the heterogeneous nature of information in medical AI evaluation. T2NNs are specifically employed to manage high-order uncertainty in quantitative data, while PLTSs are indispensable for capturing the nuanced, qualitative judgments of clinical

experts, including their inherent hesitation. This synergistic approach prevents the information loss associated with single-method models. 2) The integration of FWZIC and LOPCOW methods is methodologically necessary to produce balanced criteria weights. Relying solely on subjective weights risks introducing expert bias, while purely objective weights would ignore invaluable clinical experience. Their combination is complementary, ensuring that the final weights reflect both the intrinsic data structure and the strategic priorities of the experts, thus enhancing the credibility and fairness of the evaluation. 3) The integrated model forms a logical pipeline where each component is indispensable. It systematically progresses from information modeling and weight determination to a stable final ranking using the MARCOS method, providing a transparent and defensible decision pathway. 4) Sensitivity and comparative analyses validate the model's robustness and effectiveness. These include i) testing with varying T2NN Schweizer-Sklar weighted arithmetic mean (T2NNSSWAM) coefficient values, ii) examining the impact of different types of criteria weights, and iii) benchmarking the model against representative MCDM methods – AROMAN, CPT-VIKOR, and EDAS using Spearman's correlation coefficient for consistency verification.

1.2. Organization of the study. This paper is organized into seven sections. Section 2 reviews the relevant literature. Section 3 introduces the theoretical foundations of T2NNs, PLTSs, and related aggregation operators. The proposed integrated framework is detailed in Section 4. Section 5 presents a case study to demonstrate the model's application and validates its robustness through sensitivity and comparative analyses. Section 6 provides a discussion of the findings and highlights both theoretical and managerial implications. Finally, Section 7 concludes the paper, outlining limitations and future research avenues.

2. Literature Review.

2.1. Applications of AI technologies in pulmonary nodule management. Advancements in deep learning, image processing, and large-scale data analytics have significantly enhanced the role of AI in the screening, diagnosis, treatment planning, and prognostic prediction of pulmonary nodules [11,12]. Among these, malignancy risk assessment has emerged as a promising area of development. For instance, Shi et al. [13] proposed a semi-supervised deep transfer learning framework based on a pre-trained classification network, which demonstrated robust diagnostic performance across 17,773 clinical cases and offered a scalable tool for managing suspected nodules. Similarly, Ardila et al. [14] developed a three-dimensional end-to-end deep learning model using LDCT volumes to estimate malignancy probabilities. The model achieved an area under the curve (AUC) slightly surpassing that of experienced doctors. To accurately differentiate malignant nodules by integrating multimodal information, Wang et al. [15] combined mobile CT scanning with a C-Lung-RADS-based system on a screening dataset, effectively stratifying high-risk nodules and optimizing medical resource allocation. Additionally, AI-powered products are increasingly entering the pulmonary nodule detection market [16]. These systems assist in reducing physicians' workload by automating lesion identification and reporting, improving workflow efficiency and ensuring clinically significant findings are systematically documented [17].

2.2. Applications of MCDM in healthcare. Healthcare decision-making is an inherently complex, multi-dimensional problem influenced by technological, social, economic, and environmental factors [18]. MCDM, which aims to provide transparent and accurate medical solutions, is a challenging task due to numerous conflicting criteria, the potential for equally preferred alternatives, and the involvement of multiple stakeholders

with diverse backgrounds and interests. To address vague and difficult-to-quantify factors in lung cancer screening decisions, Liao et al. [19] utilized hesitant fuzzy modeling with double-normalization and Delphi consensus, and later extended evidential reasoning method with LTs for diagnostic support while reducing clinician burden [20]. In broader healthcare decision contexts, MCDM has been applied to evaluating criteria such as medical data privacy and patient satisfaction [21,22]. These approaches enable DMs to assess complex scenarios by aligning solutions with the specific characteristics and preferences of alternatives. Recently, MCDM techniques have gained attention in evaluating AI applications in healthcare, enabling clinicians to make data-driven diagnostic and therapeutic decisions [23,24]. For instance, Alsalem et al. [25] proposed a q-ROF2TL-based framework incorporating FWZIC and CODAS to assess trustworthy healthcare AI under fuzzy conditions. Despite these advancements, research specifically targeting the selection of auxiliary AI products for PNDT remains limited.

2.3. Applications of T2NN sets. T2NNs have demonstrated superior capability in managing fuzziness and imprecise information, making them highly suitable for applications with incomplete or hesitant data. Consequently, they have received growing attention across diverse domains, such as renewable fuels, sustainable fish farming and green supplier selection [26–28]. In determining criteria weights, a critical component of MCDM modeling, T2NNs have been applied to a variety of subjective, objective, and integrated weighting techniques. Examples of subjective weighting methods include the BWM [29] AHP [30], and FWZIC [31]. Objective weighting methods have leveraged approaches such as CRITIC [28], the T2NN-Entropy approach [11], ITARA [32], and MEREC [33]. Hybrid approaches integrating both subjective and objective elements have also been proposed. For example, Simić et al. [32] developed an integrated weighting optimization model by combining the T2NN score function with the CEBOM, effectively capturing both data-driven objectivity and expert-derived subjectivity. Similarly, Bošković et al. [34] combined the CIMAS method with CRITIC to construct a comprehensive evaluation of criteria significance.

While various ranking methods like TOPSIS [35], MABAC [36], and WASPAS [37] are used with T2NN models, the MARCOS method is particularly advantageous due to its conceptual simplicity, computational efficiency, and flexibility in handling both qualitative expert evaluations and quantitative data, thus facilitating the generation of comprehensive and objective analytical results. Building on prior developments, this study incorporates T2NN, plithogenic PLTSs, and two aggregation operators: T2NN weighted averaging (T2NNWA) and T2NNSSWAM. Specifically, the model combines PT2NNPLTS with a hybrid FWZIC-LOPCOW weighting mechanism, utilizing MARCOS to rank the final AI-assisted solutions.

3. Preliminaries.

Definition 3.1. [35] *Let X be the limited universe of discourse, with a generic element in X denoted by x . Consider G as a set of triangular neutrosophic sets and $G \in [0, 1]$. A T2NN set $\tilde{N}$ in X can be given as follows:*

$$\tilde{N} = \left\{ \left\langle x, \left(\tilde{T}_{\tilde{N}}(x), \tilde{I}_{\tilde{N}}(x), \tilde{F}_{\tilde{N}}(x) \right) \mid x \in X \right\rangle \right\}, \quad (1)$$

where $\tilde{N}$ is characterized by a truth membership $\tilde{T}_{\tilde{N}}(x) = \left(T_{T_{\tilde{N}}}(x), T_{I_{\tilde{N}}}(x), T_{F_{\tilde{N}}}(x) \right)$, a indeterminacy membership $\tilde{I}_{\tilde{N}}(x) = \left(I_{T_{\tilde{N}}}(x), I_{I_{\tilde{N}}}(x), I_{F_{\tilde{N}}}(x) \right)$, and a falsity membership

$\tilde{F}_{\tilde{N}}(x) = (F_{T_{\tilde{N}}}(x), F_{I_{\tilde{N}}}(x), F_{F_{\tilde{N}}}(x))$. For simplicity and generality, let $\tilde{N} = \left\{ \left\langle (T_{T_{\tilde{N}}}(x), T_{I_{\tilde{N}}}(x), T_{F_{\tilde{N}}}(x)), (I_{T_{\tilde{N}}}(x), I_{I_{\tilde{N}}}(x), I_{F_{\tilde{N}}}(x)), (F_{T_{\tilde{N}}}(x), F_{I_{\tilde{N}}}(x), F_{F_{\tilde{N}}}(x)) \right\rangle \right\}$ be the T2NN.

Definition 3.2. [38] The T2NNSSWAM aggregation operator is defined as follows:

$$T2NNSSWAM \langle \tilde{N}_1, \tilde{N}_2, \dots, \tilde{N}_n \rangle = \varpi_1 \tilde{N}_1 \oplus \varpi_2 \tilde{N}_2 \oplus \dots \oplus \varpi_l \tilde{N}_l = \bigoplus_{\rho=1}^l \varpi_{\rho} \tilde{N}_{\rho}$$

$$= \left\langle \left(\begin{array}{l} 1 - \left(\left(\left(\sum_{\rho=1}^l \left(1 - \left(\sum_{\rho=1}^l \left((1 - (\varpi_{\rho} (1 - T_{T_{\tilde{N}}}(x))^{\psi} - (\varpi_{\rho} - 1)) \right)^{1/\psi} \right) \right)^{\psi} \right) - 1 \right)^{1/\psi} \right), \\ 1 - \left(\left(\left(\sum_{\rho=1}^l \left(1 - \left(\sum_{\rho=1}^l \left((1 - (\varpi_{\rho} (1 - T_{I_{\tilde{N}}}(x))^{\psi} - (\varpi_{\rho} - 1)) \right)^{1/\psi} \right) \right)^{\psi} \right) - 1 \right)^{1/\psi} \right), \\ 1 - \left(\left(\left(\sum_{\rho=1}^l \left(1 - \left(\sum_{\rho=1}^l \left((1 - (\varpi_{\rho} (1 - T_{F_{\tilde{N}}}(x))^{\psi} - (\varpi_{\rho} - 1)) \right)^{1/\psi} \right) \right)^{\psi} \right) - 1 \right)^{1/\psi} \right) \end{array} \right) \right. \right. \\ \left. \left. \left(\left(\left(\sum_{\rho=1}^l \left(\sum_{\rho=1}^l \left((\varpi_{\rho} (I_{T_{\tilde{N}}}(x))^{\psi} - (\varpi_{\rho} - 1))^{1/\psi} \right) \right)^{\psi} \right) - 1 \right)^{1/\psi} \right), \right. \right. \\ \left. \left(\left(\left(\sum_{\rho=1}^l \left(\sum_{\rho=1}^l \left((\varpi_{\rho} (I_{I_{\tilde{N}}}(x))^{\psi} - (\varpi_{\rho} - 1))^{1/\psi} \right) \right)^{\psi} \right) - 1 \right)^{1/\psi} \right), \right. \right. \\ \left. \left(\left(\left(\sum_{\rho=1}^l \left(\sum_{\rho=1}^l \left((\varpi_{\rho} (I_{F_{\tilde{N}}}(x))^{\psi} - (\varpi_{\rho} - 1))^{1/\psi} \right) \right)^{\psi} \right) - 1 \right)^{1/\psi} \right) \right. \right. \\ \left. \left(\left(\left(\sum_{\rho=1}^l \left(\sum_{\rho=1}^l \left((\varpi_{\rho} (F_{T_{\tilde{N}}}(x))^{\psi} - (\varpi_{\rho} - 1))^{1/\psi} \right) \right)^{\psi} \right) - 1 \right)^{1/\psi} \right), \right. \right. \\ \left. \left(\left(\left(\sum_{\rho=1}^l \left(\sum_{\rho=1}^l \left((\varpi_{\rho} (F_{I_{\tilde{N}}}(x))^{\psi} - (\varpi_{\rho} - 1))^{1/\psi} \right) \right)^{\psi} \right) - 1 \right)^{1/\psi} \right), \right. \right. \\ \left. \left(\left(\left(\sum_{\rho=1}^l \left(\sum_{\rho=1}^l \left((\varpi_{\rho} (F_{F_{\tilde{N}}}(x))^{\psi} - (\varpi_{\rho} - 1))^{1/\psi} \right) \right)^{\psi} \right) - 1 \right)^{1/\psi} \right) \right. \right. \end{array} \right) \right\rangle, \quad (2)$$

where $\varpi_{\rho} \in [0, 1]$ is the weight vector of $\tilde{N}_l$, and $\sum_{\rho=1}^l \varpi_{\rho} = 1$.

Definition 3.3. [35] The T2NNWA is defined as the following function:

$$T2NNWA \langle \tilde{N}_1, \tilde{N}_2, \dots, \tilde{N}_n \rangle = w_1 \tilde{N}_1 \oplus w_2 \tilde{N}_2 \oplus \dots \oplus w_n \tilde{N}_n = \bigoplus_{i=1}^n w_i \tilde{N}_i$$

$$= \left\langle \left(\begin{array}{l} \left(1 - \prod_{i=1}^n \left(1 - T_{T_{\tilde{N}}}(x) \right)^{w_i} \right), 1 - \prod_{i=1}^n \left(1 - T_{I_{\tilde{N}}}(x) \right)^{w_i}, 1 - \prod_{i=1}^n \left(1 - T_{F_{\tilde{N}}}(x) \right)^{w_i} \right), \\ \left(\prod_{i=1}^n \left(I_{T_{\tilde{N}}}(x) \right)^{w_i}, \prod_{i=1}^n \left(I_{I_{\tilde{N}}}(x) \right)^{w_i}, \prod_{i=1}^n \left(I_{F_{\tilde{N}}}(x) \right)^{w_i} \right), \\ \left(\prod_{i=1}^n \left(F_{T_{\tilde{N}}}(x) \right)^{w_i}, \prod_{i=1}^n \left(F_{I_{\tilde{N}}}(x) \right)^{w_i}, \prod_{i=1}^n \left(F_{F_{\tilde{N}}}(x) \right)^{w_i} \right) \end{array} \right) \right\rangle, \quad (3)$$

where $w_i \in [0, 1]$ is the weight vector of $\tilde{N}_i$, and $\sum_{i=1}^n w_i = 1$.

Definition 3.4. [35] The score function of T2NN $\tilde{N}$ is represented by $S(\tilde{N})$ and defined as

$$S(\tilde{\mathbb{N}}) = \frac{1}{12} \left\langle \begin{array}{l} 8 + \left(T_{T_{\tilde{\mathbb{N}}}}(x) + 2 \left(T_{I_{\tilde{\mathbb{N}}}}(x) \right) + T_{F_{\tilde{\mathbb{N}}}}(x) \right) - \left(I_{T_{\tilde{\mathbb{N}}}}(x) + 2 \left(I_{I_{\tilde{\mathbb{N}}}}(x) \right) + I_{F_{\tilde{\mathbb{N}}}}(x) \right) \\ - \left(F_{T_{\tilde{\mathbb{N}}}}(x) + 2 \left(F_{I_{\tilde{\mathbb{N}}}}(x) \right) + F_{F_{\tilde{\mathbb{N}}}}(x) \right) \end{array} \right\rangle. \quad (4)$$

Definition 3.5. [39] Suppose that $M = \{M_\alpha | \alpha = -\tau, \dots, -1, 0, 1, \dots, \tau\}$ be a finite and ordered discrete LT set, where τ is a positive integer and M_α reveals the possible value of an LV. A PLTS was proposed to connect qualitative information with probabilistic information:

$$\tilde{L}'(p) = \left\{ \tilde{L}'^{(k)}(p^{(k)}) \left| \tilde{L}'^{(k)} \in M, p^{(k)} \geq 0, k \in [1, \#\tilde{L}'(p)], \sum_{k=1}^{\#\tilde{L}'(p)} p^{(k)} \leq 1 \right. \right\}, \quad (5)$$

where $\tilde{L}'^{(k)}(p^{(k)})$ denotes the LT and its associated probability $p^{(k)}$, and $\#\tilde{L}'(p)$ represents the number of different LTs contained in $\tilde{L}'(p)$.

Definition 3.6. [9] Assuming that $\tilde{L}'(p)$ is a PLTS on M , the mean M_α and variance $\delta(\tilde{L}'(p))$ of $\tilde{L}'(p)$ are represented as follows:

$$M_\alpha = \sum_{k=1}^{\#\tilde{L}'(p)} \tilde{r}'^{(k)} p^{(k)} / \sum_{k=1}^{\#\tilde{L}'(p)} p^{(k)}, \quad (6)$$

$$\delta(\tilde{L}'(p)) = \sum_{k=1}^{\#\tilde{L}'(p)} (p^{(k)} (\tilde{r}'^{(k)} - M_\alpha))^2 / \sum_{k=1}^{\#\tilde{L}'(p)} p^{(k)}, \quad (7)$$

where $\tilde{r}'^{(k)}$ is the subscript of the LT $\tilde{L}'^{(k)}$.

Definition 3.7. [9] The score function of $\tilde{L}'(p)$ in $M = \{M_\alpha | \alpha = -\tau, \dots, -1, 0, 1, \dots, \tau\}$ can be calculated with following equation.

$$SF(\tilde{L}'(p)) = M_\alpha - \frac{\delta(\tilde{L}'(p))}{\delta(M)} = M_\alpha - \frac{12 \sum_{k=1}^{\#\tilde{L}'(p)} (p^{(k)} (\tilde{r}'^{(k)} - M_\alpha))^2}{\tau(\tau+2) \sum_{k=1}^{\#\tilde{L}'(p)} p^{(k)}}. \quad (8)$$

Different PLTSs can be compared by the value of the score function. For example, let $\tilde{L}'_1(p) = \{M_{-2} = 0.4, M_1 = 0.6\}$ and $\tilde{L}'_2(p) = \{M_{-2} = 0.5, M_1 = 0.2, M_3 = 0.3\}$ be two PLTSs in $M = \{M_k | k = -3, -2, -1, 0, 1, 2, 3\}$. According to Equations (6)-(8), the score functions of $\tilde{L}'_1(p)$ and $\tilde{L}'_2(p)$ are $SF(\tilde{L}'_1(p)) = -0.2006$ and $SF(\tilde{L}'_2(p)) = -0.3588$, respectively, so $\tilde{L}'_2(p) > \tilde{L}'_1(p)$.

Definition 3.8. [9] For PLTSs $\tilde{L}'_1(p)$ and $\tilde{L}'_2(p)$, the Hamming distance between them is

$$d(\tilde{L}'_1(p), \tilde{L}'_2(p)) = \left(\sum_{i=1}^{\#\tilde{L}'_1(p)} \sum_{j=1}^{\#\tilde{L}'_2(p)} \left((\tilde{p}_1^{(i)} \tilde{p}_2^{(j)}) | g(\tilde{L}'_1^{(i)}) - g(\tilde{L}'_2^{(j)}) \right) \right). \quad (9)$$

The Hamming distance between the normalized probabilistic linguistic decision matrix $\tilde{B}_1$ and $\tilde{B}_2$ is created with Equation (10).

$$d(\tilde{B}_1, \tilde{B}_2) = \frac{1}{m} \sum_{q=1}^m d(\tilde{L}'_q^1(p), \tilde{L}'_q^2(p)). \quad (10)$$

Definition 3.9. The degree of contradiction $c(v, D)$ and appurtenance $d(x, v)$ are the two main characteristics in plithogenic set, where $c(v, D)$ is calculated between criterion with reference to the main criterion v_D . Plithogenic aggregation is obtained by the linear combination of t_{norm} and t_{conorm} , symbolized by $\wedge_{\tilde{N}}$ and $\vee_{\tilde{N}}$, respectively. For T2NNs $\tilde{N}_1 = \{ \langle (T_{T_1}, T_{I_1}, T_{F_1}), (I_{T_1}, I_{I_1}, I_{F_1}), (F_{T_1}, F_{I_1}, F_{F_1}) \rangle \}$ and $\tilde{N}_2 = \{ \langle (T_{T_2}, T_{I_2}, T_{F_2}), (I_{T_2}, I_{I_2}, I_{F_2}), (F_{T_2}, F_{I_2}, F_{F_2}) \rangle \}$, the plithogenic intersection operation can be represented as

$$\begin{aligned} & \{ \langle (T_{T_1}, T_{I_1}, T_{F_1}), (I_{T_1}, I_{I_1}, I_{F_1}), (F_{T_1}, F_{I_1}, F_{F_1}) \rangle \} \\ & \wedge \{ \langle (T_{T_2}, T_{I_2}, T_{F_2}), (I_{T_2}, I_{I_2}, I_{F_2}), (F_{T_2}, F_{I_2}, F_{F_2}) \rangle \} \\ &= \left\{ \left\langle \begin{pmatrix} T_{T_1} \wedge_{\tilde{N}} T_{T_2}, \frac{1}{2}(T_{I_1} \wedge_{\tilde{N}} T_{I_2} + T_{I_1} \vee_{\tilde{N}} T_{I_2}), T_{F_1} \vee_{\tilde{N}} T_{F_2} \\ I_{T_1} \wedge_{\tilde{N}} I_{T_2}, \frac{1}{2}(I_{I_1} \wedge_{\tilde{N}} I_{I_2} + I_{I_1} \vee_{\tilde{N}} I_{I_2}), I_{F_1} \vee_{\tilde{N}} I_{F_2} \\ F_{T_1} \wedge_{\tilde{N}} F_{T_2}, \frac{1}{2}(F_{I_1} \wedge_{\tilde{N}} F_{I_2} + F_{I_1} \vee_{\tilde{N}} F_{I_2}), F_{F_1} \vee_{\tilde{N}} F_{F_2} \end{pmatrix} \right\rangle \right\}. \end{aligned} \quad (11)$$

The plithogenic union operation is

$$\begin{aligned} & \{ \langle (T_{T_1}, T_{I_1}, T_{F_1}), (I_{T_1}, I_{I_1}, I_{F_1}), (F_{T_1}, F_{I_1}, F_{F_1}) \rangle \} \\ & \wedge \{ \langle (T_{T_2}, T_{I_2}, T_{F_2}), (I_{T_2}, I_{I_2}, I_{F_2}), (F_{T_2}, F_{I_2}, F_{F_2}) \rangle \} \\ &= \left\{ \left\langle \begin{pmatrix} T_{T_1} \vee_{\tilde{N}} T_{T_2}, \frac{1}{2}(T_{I_1} \wedge_{\tilde{N}} T_{I_2} + T_{I_1} \vee_{\tilde{N}} T_{I_2}), T_{F_1} \wedge_{\tilde{N}} T_{F_2} \\ I_{T_1} \vee_{\tilde{N}} I_{T_2}, \frac{1}{2}(I_{I_1} \wedge_{\tilde{N}} I_{I_2} + I_{I_1} \vee_{\tilde{N}} I_{I_2}), I_{F_1} \wedge_{\tilde{N}} I_{F_2} \\ F_{T_1} \vee_{\tilde{N}} F_{T_2}, \frac{1}{2}(F_{I_1} \wedge_{\tilde{N}} F_{I_2} + F_{I_1} \vee_{\tilde{N}} F_{I_2}), F_{F_1} \wedge_{\tilde{N}} F_{F_2} \end{pmatrix} \right\rangle \right\}, \end{aligned} \quad (12)$$

where $T_{T_1} \wedge_{\tilde{N}} T_{T_2} = [1 - c(v_D, v_1)] \cdot t_{norm}(v_D, v_1) + c(v_D, v_1) \cdot t_{conorm}(v_D, v_1)$, $T_{T_1} \vee_{\tilde{N}} T_{T_2} = [1 - c(v_D, v_1)] \cdot t_{conorm}(v_D, v_1) + c(v_D, v_1) \cdot t_{norm}(v_D, v_1)$, $t_{norm} = T_{T_1} \wedge_{\tilde{N}} T_{T_2} = T_{T_1} \cdot T_{T_2}$, $t_{conorm} = T_{T_1} \vee_{\tilde{N}} T_{T_2} = T_{T_1} + T_{T_2} - T_{T_1} \cdot T_{T_2}$.

Definition 3.10. Suppose that

$$\tilde{\mathfrak{R}}_{ij} = \left\{ \left\langle \begin{pmatrix} T_{T_{\tilde{L}_{ij}}}(x), T_{I_{\tilde{L}_{ij}}}(x), T_{F_{\tilde{L}_{ij}}}(x) \\ I_{T_{\tilde{L}_{ij}}}(x), I_{I_{\tilde{L}_{ij}}}(x), I_{F_{\tilde{L}_{ij}}}(x) \\ F_{T_{\tilde{L}_{ij}}}(x), F_{I_{\tilde{L}_{ij}}}(x), F_{F_{\tilde{L}_{ij}}}(x) \end{pmatrix} \right\rangle, \begin{pmatrix} M_{\alpha}(p^1), M_{\beta}(p^2), M_{\gamma}(p^3) \end{pmatrix} \right\}$$

be a PT2NNPLTS set, where $T(x), I(x), F(x) \in [0, 1]$, $\alpha, \beta, \gamma \in [-\tau, \tau]$ and $\tau \in R$. Compared with plithogenic set and T2NN, this set adds consideration to the hesitancy and probability characteristics in probabilistic linguistic information when expressing decision evaluation, and is more suitable for fuzzy and uncertain decision environments. Compared with PLTS, the PT2NN set adds consideration of the truth, indeterminacy, and falsity memberships, appurtenance degree, and contradiction degree of the decision object.

Definition 3.11. Let $PT2NN_{agg}$ be a set that aggregates PT2NN information, and then the $PT2NNPLTS_{agg}$, which further integrates the PLTS information, is defined as

$$\begin{aligned} PT2NNPLTS_{agg} &= PT2NN_{agg} \otimes \left(2 + SF(M_{\alpha'}(p^1)) - SF(M_{\beta'}(p^2)) \right. \\ &\quad \left. - SF(M_{\gamma'}(p^3)) \right). \end{aligned} \quad (13)$$

Remark 3.1. Definition 3.11 presents a straightforward approach that integrates PLTS information and allows for the effective adjustment of PT2NN information through a nonlinear weighting function, simplified as $\Delta = 2 + \alpha' - \beta' - \gamma'$. This implies that if the PLTS component is highly positive, the credibility of the T2NN is enhanced; conversely, if it is less positive, the credibility is diminished.

Moreover, we further present the general form of Equation (13) to prevent information distortion: $PT2NNPLTS_{agg} = PT2NN_{agg} \otimes \Delta$, where $\Delta = \min(\max(2 + \alpha' - \beta' - \gamma', \nabla_{\min}), \nabla_{\max})$ and parameters $\nabla_{\min}$ and $\nabla_{\max}$ control the upper and lower limits of the correction range. The recommended setting is $\nabla_{\min} = 0.5$, $\nabla_{\max} = 2.5$. This mechanism reflects the weighted adjustment ability of linguistic judgment on fuzzy decision, which is conducive to capturing the dynamic tendencies in multi-source decision-making processes.

4. FWZIC-LOPCOW-MARCOS Method for PT2NNPLTS. This section explains the proposed integrated decision-making approach and the principles and main process steps of the PT2NNPLTS-FWZIC-LOPCOW model for expert and criteria weighting, followed by the MARCOS method for the problem of selecting auxiliary AI diagnostic and treatment products.

Notably, with the help of the T2NNSSWAM operator, we consider the criteria weights from both objective and subjective perspectives. A brief description of the model building process is provided and a detailed flowchart is presented to enhance the intuitiveness of the demonstration (Figure 1).

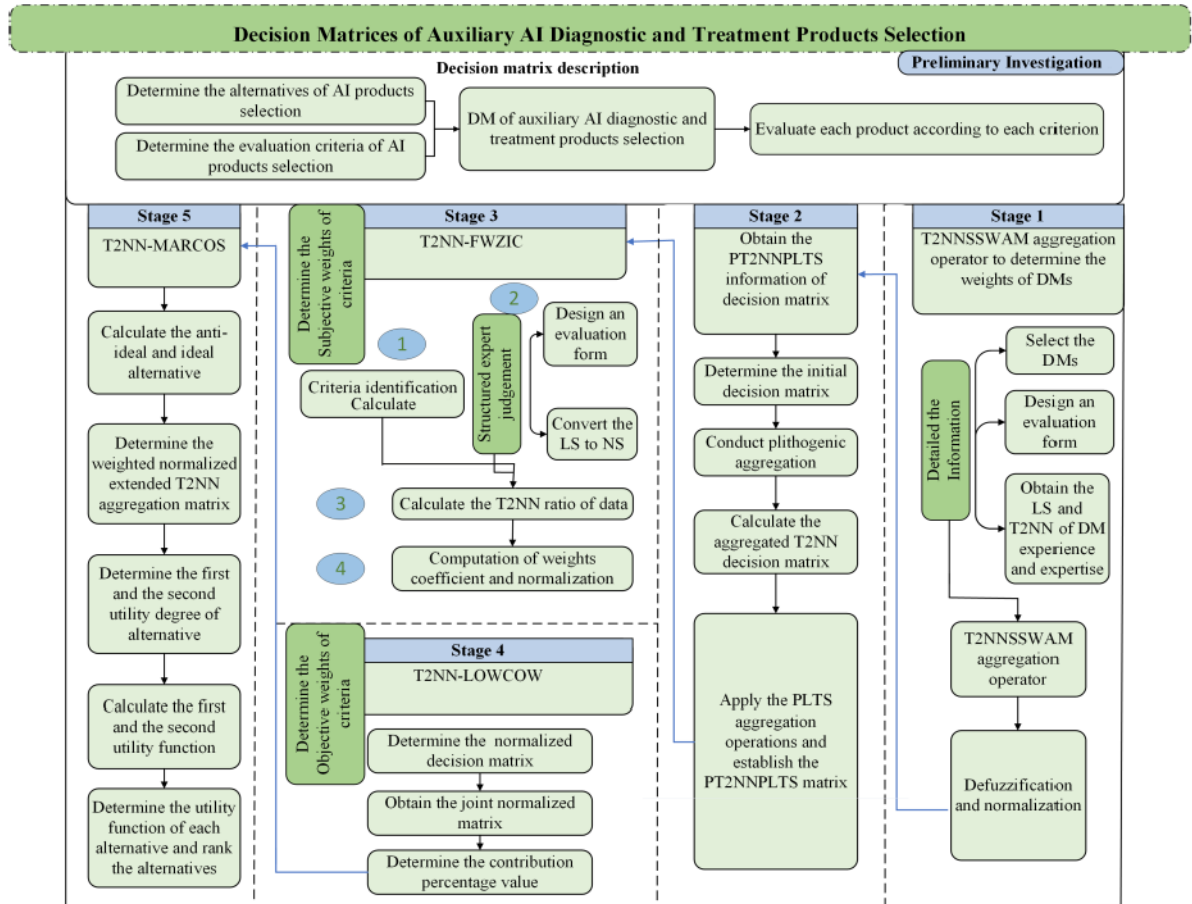


FIGURE 1. The flowchart of the proposed framework

For PT2NNPLTS MCDM problem, $A = \{A_1, A_2, \dots, A_m\}$ ($m \geq 2$) is a set of alternatives and $C = \{c_1, c_2, \dots, c_n\}$ ($n \geq 2$) denotes the criteria set. Let $w^* = (w_1^*, w_2^*, \dots, w_n^*)$ be the weight of criteria, where $\forall w_j^* \in [0, 1]$, $\sum_{j=1}^n w_j^* = 1$, and the subjective and objective weights are represented by $\tilde{w}_j$ and w_j , respectively. Let $D = \{DM_1, DM_2, \dots, DM_k\}$ ($k \geq 2$) be set of DMs and their weights are represented as $\lambda = \{\lambda_1, \lambda_2, \dots, \lambda_k\}$. Each alternative is evaluated by DMs according to criteria. The LVs for evaluation are shown in Table 1. As a result, we can obtain an initial T2NN-PLTS matrix $\mathfrak{R} = [\tilde{\mathfrak{R}}_{ij}]_{m \times n}$.

TABLE 1. The T2NN-PLTS evaluation scale

Influence linguistic scale	Abbreviation	Neutrosophic probabilistic linguistic scale
Very Weakly Influence	VWI	$\{ \langle (0.15, 0.20, 0.10), (0.70, 0.85, 0.85), (0.60, 0.80, 0.70) \rangle, M_1(0.48), M_{-2}(0.23), M_{-3}(0.29) \}$
Partially Influence	PI	$\{ \langle (0.35, 0.30, 0.25), (0.60, 0.70, 0.80), (0.50, 0.75, 0.65) \rangle, M_2(0.46), M_{-2}(0.11), M_{-1}(0.43) \}$
Equal Influence	EI	$\{ \langle (0.45, 0.35, 0.35), (0.50, 0.65, 0.55), (0.40, 0.65, 0.55) \rangle, M_2(0.71), M_3(0.18), M_{-3}(0.11) \}$
Strong Influence	SI	$\{ \langle (0.60, 0.55, 0.55), (0.25, 0.40, 0.45), (0.25, 0.45, 0.35) \rangle, M_4(0.57), M_2(0.27), M_{-2}(0.16) \}$
Very Strongly Influence	VSI	$\{ \langle (0.70, 0.65, 0.65), (0.20, 0.30, 0.40), (0.20, 0.25, 0.30) \rangle, M_3(0.73), M_4(0.09), M_1(0.18) \}$
Absolutely Influence	AI	$\{ \langle (0.95, 0.90, 0.95), (0.10, 0.10, 0.05), (0.05, 0.05, 0.10) \rangle, M_4(0.82), M_2(0.11), M_3(0.07) \}$

Stage 1. Aggregation of expert reputation ratings based on T2NNSSWAM operator and determination of DM's weight

Step 1-1: Given the varying levels of experience and expertise among DMs, differentiated weights are assigned. Experience is quantified based on years of professional practice, and evaluations are conducted using LSs. These LSs are then converted to T2NNs and aggregated using the T2NNSSWAM operator. The aggregated T2NNs are defuzzified using a score function and normalized to derive the final DM weight.

Step 1-2: The expertise and experience levels are aggregated using the T2NNSSWAM $(\tilde{Q}_1, \tilde{Q}_2, \dots, \tilde{Q}_l) = \oplus_{\rho=1}^l \varpi_{\rho} \tilde{Q}_{\rho}$ aggregation operator presented in Equation (2). Here, ϖ_1 and ϖ_2 represent the expertise and experience parameters of the DMs, respectively. $\varpi_1, \varpi_2 \in [0, 1]$ and $\varpi_1 + \varpi_2 = 1$. We assume $\varpi_1 = 0.4$ and $\varpi_2 = 0.6$. Parameter ξ is set to be $-\pi$ in this paper.

Step 1-3: The score function $S(\tilde{Q}_e)$ is obtained to transform the T2NN into a crisp value.

Step 1-4: A normalization process is needed in this step to minimize the effects of subjective weighting. For this, the score functions are normalized by Equation (14).

$$\lambda_e = \frac{S(\tilde{Q}_e)}{\sum_{l=1}^k S(\tilde{Q}_l)}. \quad (14)$$

TABLE 2. The LSs and T2NNs of expert experience and expertise

Experience (years)	Expertise	T2NNs
< 5	Very-poor (VP)	$\langle (0.15, 0.20, 0.10), (0.70, 0.85, 0.85), (0.60, 0.80, 0.70) \rangle$
[5, 10)	Poor (P)	$\langle (0.35, 0.30, 0.25), (0.60, 0.70, 0.80), (0.50, 0.75, 0.65) \rangle$
[10, 15)	Medium (M)	$\langle (0.45, 0.35, 0.35), (0.50, 0.65, 0.55), (0.40, 0.65, 0.55) \rangle$
[15, 20)	Medium-good (MG)	$\langle (0.60, 0.55, 0.55), (0.25, 0.40, 0.45), (0.25, 0.45, 0.35) \rangle$
[20, 25)	Good (G)	$\langle (0.70, 0.65, 0.65), (0.20, 0.30, 0.40), (0.20, 0.25, 0.30) \rangle$
≥ 25	Very-good (VG)	$\langle (0.95, 0.90, 0.95), (0.10, 0.10, 0.05), (0.05, 0.05, 0.10) \rangle$

Stage 2. Determine the aggregated matrix that contains PT2NNPLTS information

Step 2-1: Determine the initial T2NN-PLTS decision matrix $\tilde{\mathfrak{R}}^p = [\tilde{\mathfrak{R}}_{ij}^p]_{m \times n}$, where

$$\tilde{\mathfrak{R}}_{ij}^p = \left\langle \left(T_{T_{\tilde{L}_{ij}}^p}(x), T_{I_{\tilde{L}_{ij}}^p}(x), T_{F_{\tilde{L}_{ij}}^p}(x) \right), \left(I_{T_{\tilde{L}_{ij}}^p}(x), I_{I_{\tilde{L}_{ij}}^p}(x), I_{F_{\tilde{L}_{ij}}^p}(x) \right), \left(F_{T_{\tilde{L}_{ij}}^p}(x), F_{I_{\tilde{L}_{ij}}^p}(x), F_{F_{\tilde{L}_{ij}}^p}(x) \right) \right\rangle, \left. \begin{matrix} \\ \\ M_{\alpha}(p^1), M_{\beta}(p^2), M_{\gamma}(p^3) \end{matrix} \right\} \text{ is a}$$

basic form of T2NN-PLTS to express the evaluation opinion according to the LS.

Step 2-2: Perform plithogenic aggregation on the matrix $\tilde{\mathfrak{R}}^p$. Define the contradiction degree c ($c = (\frac{n-1}{n})$) for each criterion in accordance with Equations (11) and (12).

Step 2-3: Apply T2NNWA, presented in Equation (3), and calculate the aggregated T2NN decision matrix. The weight λ_e of each DM used in this step is calculated by Step 1-4.

Step 2-4: Obtain the integrated PT2NN decision matrix $\tilde{\mathfrak{R}}$ using Equation (12).

Step 2-5: For the PLTS component of $\tilde{\mathfrak{R}}^p = [\tilde{\mathfrak{R}}_{ij}^p]_{m \times n}$, the single evaluation of each DM is integrated through the PLTS score function with Equations (6)-(10). A T2NN aggregation matrix $\tilde{H} = [\tilde{H}_{ij}]_{m \times n}$ that contains PT2NNPLTS information is established based on Equation (13).

Stage 3. Determination of the subjective weights of criteria – the T2NN-FWZIC method

We utilize a competitive T2NN-FWZIC approach to subjectively weight criteria. FWZIC effectively captures the accumulated knowledge and perspectives of DMs, while preserving their true cognitive preferences to the greatest extent when addressing fuzzy problems. It is particularly useful in addressing inconsistencies arising from the high subjectivity involved in determining the significance of criteria through pairwise comparisons [40].

Step 3-1: The criteria for evaluating and selecting are defined and discussed in detail, which includes the definition of the criteria, numerical representation of measurement (whether it is a cost type or a benefit one). An evaluation form was created to collect the consensus of all DMs on each criterion of AI product, and then converted LS to its

equivalent numerical scale (NS), and finally transformed into T2NN scale. Next, an expert decision matrix (EDM) construction process is developed based on the selected criteria and the list of alternatives.

Step 3-2: Calculate the T2NN ratio of the data using Equation (15).

$$\frac{\text{importance} \left(\tilde{E}_k / C_j \right)}{\sum_{j=1}^n \text{importance} \left(E_k / C_{kj} \right)}, \quad (15)$$

where $\left(\tilde{E}_k / C_j \right)$ represents the k th DM's evaluation of the j th criterion in T2NN, and $\sum_{j=1}^n \text{importance} \left(E_k / C_{kj} \right)$ is the summation of the evaluation computed by the score function, presented in Equation (4). Equation (3) was conducted using the scalar multiplication operation of T2NN.

Step 3-3: Determine the weights of the criteria in the form of T2NN, the experts' evaluations of each criterion are aggregated according to the T2NNWA operator, and the weights λ_e of each DM are used to obtain the weights $(\tilde{w}'_1, \tilde{w}'_2, \dots, \tilde{w}'_n)$.

Step 3-4: Compute the final subjective weight $\tilde{w}_j$ of each criterion. The score function is used again to obtain the crisp value of $(\tilde{w}'_1, \tilde{w}'_2, \dots, \tilde{w}'_n)$. A normalization process is then implemented on these values to obtain the final weights $(\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_n)$, satisfying $\sum_{j=1}^n \tilde{w}_j = 1$.

Stage 4. Determination of the objective weights of criteria – the T2NN-LOPCOW method

Step 4-1: The T2NN aggregation matrix $\tilde{H}_{ij}$ can be represented as $\tilde{H}_{ij} = \left\langle \left(T_{\tilde{H}_{ij}}(x), T_{I_{\tilde{H}_{ij}}}(x), T_{F_{\tilde{H}_{ij}}}(x) \right), \left(I_{\tilde{H}_{ij}}(x), I_{I_{\tilde{H}_{ij}}}(x), I_{F_{\tilde{H}_{ij}}}(x) \right), \left(F_{\tilde{H}_{ij}}(x), F_{I_{\tilde{H}_{ij}}}(x), F_{F_{\tilde{H}_{ij}}}(x) \right) \right\rangle$. The normalized decision matrices are calculated with the following equations.

(i) Determine the max-min normalized decision evaluation matrix $F^{(1)} = \left[F_{ij}^{(1)} \right]_{m \times n}$.

$$F_{ij}^{(1)} = \begin{cases} \frac{\max_{1 \leq t \leq m} S \left(\tilde{H}_{tj} \right) - S \left(\tilde{H}_{ij} \right)}{\max_{1 \leq t \leq m} S \left(\tilde{H}_{tj} \right) - \min_{1 \leq t \leq m} S \left(\tilde{H}_{tj} \right)} & \text{for cost criteria,} \\ \frac{S \left(\tilde{H}_{ij} \right) - \min_{1 \leq t \leq m} S \left(\tilde{H}_{tj} \right)}{\max_{1 \leq t \leq m} S \left(\tilde{H}_{tj} \right) - \min_{1 \leq t \leq m} S \left(\tilde{H}_{tj} \right)} & \text{for benefit criteria.} \end{cases} \quad (16)$$

(ii) The sum normalized decision evaluation matrix $F^{(2)} = \left[F_{ij}^{(2)} \right]_{m \times n}$.

$$F_{ij}^{(2)} = \begin{cases} \frac{\frac{1}{S \left(\tilde{H}_{ij} \right)}}{\sum_{t=1}^m \frac{1}{S \left(\tilde{H}_{tj} \right)}} & \text{for cost criteria,} \\ \frac{S \left(\tilde{H}_{ij} \right)}{\sum_{t=1}^m S \left(\tilde{H}_{tj} \right)} & \text{for benefit criteria,} \end{cases} \quad (17)$$

where $i \in [1, m]$; $j \in [1, n]$, and this step utilizes two normalization techniques to enhance the flexibility and persuasiveness of decision-making in the T2NN environment.

Step 4-2: Obtain the joint normalized decision evaluation matrix $F = [F_{ij}]_{m \times n}$.

$$F_{ij} = \mu F_{ij}^{(1)} + (1 - \mu) F_{ij}^{(2)}. \quad (18)$$

DMs determine the parameter μ to make a trade-off between the two types of normalization techniques $F_{ij}^{(1)}$ and $F_{ij}^{(2)}$. DMs prefer the sum technique $F_{ij}^{(2)}$ when the value of μ belongs to $[0, 0.5)$, while the max-min technique holds an advantage in the DM's perspective when $\mu \in (0.5, 1]$. Considering that DMs have no obvious preference for the two matrices, the value of μ is set to 0.5 in this paper.

Step 4-3: The contribution percentage U_j of each criterion is calculated by Equation (19).

$$U_j = \left| \ln \left(\frac{RMS_j}{\sigma_j} \right) \times 100\% \right|, \quad (19)$$

where $RMS_j = \sqrt{\frac{1}{m} \sum_{i=1}^m (F_{ij}^2)}$ represents the root mean square value of the joint normalized evaluations F_{ij} and $\sigma_j = \sqrt{\frac{1}{m-1} \sum_{i=1}^m (F_{ij} - \frac{1}{m} \sum_{t=1}^m F_{tj})^2}$ is the standard deviation of F_{ij} .

Step 4-4: Determine the objective weights of criteria.

$$w_j = \frac{U_j}{\sum_{j=1}^n U_j}. \quad (20)$$

Stage 5. Determination of the ranking of each alternative with the T2NN-MARCOS method

Step 5-1: Determine the extended T2NN aggregation matrix $V = [\tilde{V}_{lj}]_{(m+2) \times n}$. This extension is implemented by defining the ideal and anti-ideal alternatives using the best and worst T2NN evaluations, respectively, and inserting them into the extended decision matrix.

(i) The anti-ideal alternative $A_0 = \{\tilde{V}_{01}, \dots, \tilde{V}_{0j}, \dots, \tilde{V}_{0n}\}$ is calculated with Equation (21).

$$\tilde{V}_{0j} = \begin{cases} \max_{1 \leq i \leq m} \tilde{H}_{ij} & \text{for cost criteria,} \\ \min_{1 \leq i \leq m} \tilde{H}_{ij} & \text{for benefit criteria,} \end{cases} \quad (21)$$

where $\tilde{V}_{0j}$ ($j \in [1, n]$) is the collection of criterion evaluations that represents anti-ideal values.

(ii) The ideal alternative $A_{m+1} = \{\tilde{V}_{m+11}, \dots, \tilde{V}_{m+1j}, \dots, \tilde{V}_{m+1n}\}$ is calculated with Equation (22).

$$\tilde{V}_{m+1j} = \begin{cases} \min_{1 \leq i \leq m} \tilde{H}_{ij} & \text{for cost criteria,} \\ \max_{1 \leq i \leq m} \tilde{H}_{ij} & \text{for benefit criteria,} \end{cases} \quad (22)$$

where $\tilde{V}_{m+1j}$ ($j \in [1, n]$) is the collection of evaluations that represents each criterion's ideal assessments. The comparisons are based on the T2NN score function and accuracy function.

Step 5-2: The calculation of the normalized extended T2NN aggregation matrix $Z = [\tilde{Z}_{lj}]_{(m+2) \times n}$ also requires the use of the T2NN score function.

$$\tilde{Z}_{lj} = \begin{cases} \left| \frac{\text{score}(\tilde{V}_{lj})}{\text{score}(\tilde{V}_{m+1j})} \right| & \text{for benefit criteria} \\ \left| \frac{\text{score}(\tilde{V}_{m+1j})}{\text{score}(\tilde{V}_{lj})} \right| & \text{for cost criteria} \end{cases} \quad l \in [0, m+1]. \quad (23)$$

Step 5-3: Obtain the weighted normalized extended aggregation matrix $O = [\tilde{O}_{lj}]_{(m+2) \times n}$.

$$\tilde{O}_{lj} = w_j^* \tilde{Z}_{lj}, \quad (24)$$

where $w_j^* = \eta \tilde{w}_j + (1 - \eta)w_j$ represents the comprehensive weight vector that integrates both the subjective weight $\tilde{w}_j$ and objective weight w_j . Given that the DMs expressed no clear preference between the two normalizations, we set $\eta = 0.5$.

Step 5-4: The utility degree is a crucial factor in comparing the pros and cons of alternatives. We determine two types of utility degrees, namely the first and the second utility degrees.

(i) The first utility degree

$$U_l^- = \frac{\sum_{j=1}^n O_{lj}}{\sum_{j=1}^n O_{0j}}, \quad (25)$$

where U_l^- denotes the utility degree of alternative to the anti-ideal alternative.

(ii) The second utility degree

$$U_l^+ = \frac{\sum_{j=1}^n O_{lj}}{\sum_{j=1}^n O_{m+1j}}, \quad (26)$$

where U_l^+ represents the utility degree of alternative to the ideal alternative.

Step 5-5: The functions corresponding to utility degrees are obtained by Equation (27) and Equation (28).

(i) The first utility function

$$f(U^-) = \frac{U_0^+}{U_0^+ + U_0^-}, \quad (27)$$

where $f(U^-)$ denotes the utility function of the anti-ideal alternative.

(ii) The second utility function

$$f(U^+) = \frac{U_{m+1}^-}{U_{m+1}^- + U_{m+1}^+}, \quad (28)$$

where $f(U^+)$ represents the utility function of the ideal alternative.

Step 5-6: Determine the utility function of each alternative and rank them in descending order.

$$f(U_i) = \frac{(U_i^+ + U_i^-) [f(U^-) \cdot f(U^+)]}{f(U^+) + f(U^+) - f(U^+) \cdot f(U^+)} \quad i \in [1, m]. \quad (29)$$

The optimal alternative has the highest utility function, while the worst has the lowest.

5. Case Study. To illustrate the effectiveness and feasibility of the proposed framework for auxiliary AI product selection in PNDT, a case study was conducted. The timely diagnosis of pulmonary nodules is critical, as delays can increase malignancy risk. Uncertainty in treatment decisions and disease severity often cause psychological distress

to patients and their families, limit treatment choices and reduce quality of life. Since AI-assisted systems support early detection of potentially malignant nodules, enabling timely interventions and improved prognosis, AI integration is essential for effective nodule management.

To enhance the quality of PNDT, the department of critical care medicine launched a project to integrate physicians' clinical expertise with digital healthcare advancement. This study aimed to identify the optimal AI-assisted medical product for PNDT through an MCDM-based evaluation. After a thorough market survey, systems were assessed based on technical sophistication, clinical application performance, and reputation. Three candidate products developed by leading tech firms were shortlisted. A panel of four experts evaluated these alternatives against six criteria using the proposed hybrid framework. Following expert input, the hybrid PT2NNPLTS-FWZIC-LOPCOW-MARCOS model was implemented. The evaluation process was designed to identify the most suitable AI solution for clinical adoption.

5.1. Criteria and alternatives of the case study.

5.1.1. *Definition of decision-making alternatives.* Based on market research and practical applications, three alternatives were identified and investigated in line with the principles of medical applicability, technological maturity, and regulatory compliance, as expressed below: 1) Aidoc (A_1): An AI platform that automatically analyzes chest CT images to identify and mark pulmonary nodules, generating reports to optimize workflow and improve diagnostic efficiency; 2) Zebra Medical Vision (A_2): An AI solution that detects pulmonary nodules in chest CT scans and performs feature analysis to assist with malignancy risk assessment; 3) Quantib (A_3): An AI product that automatically detects, measures, and tracks the volume, shape, and growth rate of pulmonary nodules to help assess their malignancy potential.

5.1.2. *Definition of decision-making criteria.* Drawing on expert interviews and existing literature, six criteria were identified for PNDT, as illustrated in Figure 2. (C_1): Patient data privacy and security; Assesses the product's capacity to safeguard sensitive patient information through robust encryption and access controls, ensuring compliance with regulations such as HIPAA and GDPR. (C_2): Diagnostic algorithm bias; Evaluates the risk of demographic bias in the algorithm. The criterion requires that the model be trained on diverse and representative datasets to ensure diagnostic accuracy across different patient populations. (C_3): Risk of AI technology dependence; Considers the potential for over-reliance on AI to degrade clinicians' diagnostic skills, emphasizing the continued need for expert judgment, particularly in atypical cases or system failures. (C_4): System complexity or vulnerabilities; Assesses the system's operational usability for clinicians and its technical stability. A user-friendly interface and rigorously tested, reliable performance is critical to prevent diagnostic errors. (C_5): Misunderstanding and misuse; Measures the risk of misinterpretation of AI-generated recommendations. It underscores the necessity of proper training for medical staff to use the system as a decision-support tool, not a definitive judgment. (C_6): Cost issues of diagnostic equipment; Evaluates the financial viability of adopting the AI solution. The criterion weighs the initial investment against long-term returns, such as improved diagnostic accuracy, operational efficiency, and patient outcomes.

5.2. **Experimental results.** Four physicians with extensive clinical experience and digital skills were invited to participate in the decision-making process (Table 3). All hold doctorates in medicine, and their respective research focuses are related to PNDT.

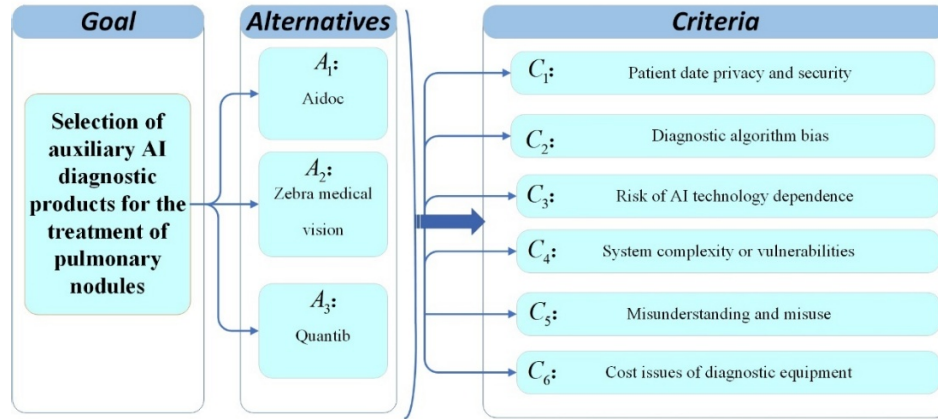


FIGURE 2. A practical framework for the AI treatment product selection problem

TABLE 3. The detailed information of experts

Expert	Professional position	Academic degree	Experience (years)	Expertise
DM_1	Chief Physician	Doctoral	20	VG
DM_2	Chief Physician	Doctoral	18	G
DM_3	Chief Physician	Doctoral	17	VG
DM_4	Associate Chief Physician	Doctoral	10	G

TABLE 4. The aggregated decision matrix using T2NNSSWAM and the weights of DMs

Expert	$\tilde{T}_{\theta_f}(\varphi)$			$\tilde{I}_{\theta_f}(\varphi)$			$\tilde{F}_{\theta_f}(\varphi)$			$S(\tilde{Q}_e)$	λ_p
	$T_{T_{\theta_f}}(\varphi)$	$T_{I_{\theta_f}}(\varphi)$	$T_{F_{\theta_f}}(\varphi)$	$I_{T_{\theta_f}}(\varphi)$	$I_{I_{\theta_f}}(\varphi)$	$I_{F_{\theta_f}}(\varphi)$	$F_{T_{\theta_f}}(\varphi)$	$F_{I_{\theta_f}}(\varphi)$	$F_{F_{\theta_f}}(\varphi)$		
DM_1	0.811	0.758	0.772	0.115	0.117	0.059	0.059	0.059	0.117	0.867	0.290
DM_2	0.652	0.601	0.601	0.215	0.327	0.417	0.215	0.285	0.316	0.672	0.225
DM_3	0.729	0.674	0.682	0.116	0.117	0.059	0.059	0.059	0.117	0.838	0.280
DM_4	0.544	0.440	0.440	0.233	0.347	0.439	0.230	0.291	0.342	0.612	0.205

Step 1-1: The experience and expertise levels of the DMs are presented in Table 3. With the help of Table 2 and Table 3, the T2NN matrix of DM's reputation can be obtained.

Step 1-2: The experience and expertise values of DMs were aggregated with the T2NNSSWAM aggregation operator. The aggregated T2NNs matrix is shown in Table 4.

Step 1-3: The score function $S(\tilde{Q}_e)$ of the aggregated matrix is calculated and the value of each expert is shown in the penultimate column of Table 3.

Step 1-4: The $S(\tilde{Q}_e)$ values are normalized using Equation (14) to obtain the weight of each DM. The results of the DMs' weights are as follows: $\lambda_1 = 0.290$, $\lambda_2 = 0.225$, $\lambda_3 = 0.280$, $\lambda_4 = 0.205$.

Step 2-1: The linguistics evaluations of DMs are presented in Table A.1. Then LSs are transferred into T2NN-PLTS evaluation scales.

Step 2-2: The $PT2NN_{agg}$ matrix is calculated by Equations (11) and (12), as shown in Table 5.

TABLE 5. The aggregated PT2NN evaluation matrix

CD	0	1/6	...	5/6
	C_1	C_2	...	C_6
A_1	$\langle(0.279, 0.209, 0.221), (0.163, 0.225, 0.238), (0.544, 0.706, 0.713)\rangle$	$\langle(0.282, 0.128, 0.128), (0.288, 0.413, 0.450), (0.574, 0.918, 0.870)\rangle$	...	$\langle(0.746, 0.665, 0.665), (0.469, 0.619, 0.538), (0.143, 0.335, 0.240)\rangle$
A_2	$\langle(0.442, 0.342, 0.381), (0.138, 0.175, 0.181), (0.422, 0.492, 0.603)\rangle$	$\langle(0.307, 0.156, 0.156), (0.256, 0.381, 0.438), (0.534, 0.885, 0.828)\rangle$	...	$\langle(0.803, 0.749, 0.749), (0.369, 0.513, 0.494), (0.114, 0.223, 0.187)\rangle$
A_3	$\langle(0.206, 0.151, 0.151), (0.206, 0.313, 0.406), (0.616, 0.768, 0.777)\rangle$	$\langle(0.367, 0.221, 0.221), (0.213, 0.325, 0.413), (0.467, 0.783, 0.756)\rangle$	...	$\langle(0.683, 0.629, 0.592), (0.544, 0.656, 0.725), (0.188, 0.408, 0.299)\rangle$

Step 2-3, Step 2-4: The aggregated T2NN decision matrix is obtained with the help of T2NNWA operator. The PT2NN matrix $\tilde{\mathcal{R}}$ is then calculated.

Step 2-5: The PLTS matrix $\tilde{\mathcal{R}}^p$ is computed to aggregate the probabilistic information, presented in Table 6. The PT2NNPLTS matrix $\tilde{H} = [\tilde{H}_{ij}]_{m \times n}$ is shown in Table A.2.

TABLE 6. The aggregated PLTS evaluation matrix

	A_1	A_2	A_3
C_1	(0.090, 0.058, 0.173)	(0.096, 0.049, 0.041)	(0.049, 0.090, 0.131)
C_2	(0.200, 0.040, 0.231)	(0.223, 0.036, 0.190)	(0.079, 0.051, 0.190)
C_3	(0.223, 0.036, 0.190)	(0.071, 0.023, 0.219)	(0.312, 0.026, 0.016)
C_4	(0.149, 0.060, 0.220)	(0.079, 0.043, 0.017)	(0.049, 0.090, 0.131)
C_5	(0.090, 0.058, 0.173)	(0.096, 0.049, 0.041)	(0.096, 0.049, 0.041)
C_6	(0.200, 0.027, 0.016)	(0.223, 0.036, 0.190)	(0.219, 0.318, 0.038)

Step 3-1: The set of the benefit criteria is $C^+ = \{C_1\}$, and the set of the cost criteria is $C^- = \{C_2, C_3, C_4, C_5, C_6\}$. Table 7 presents the evaluation of 6 criteria using LS and NS.

TABLE 7. All experts EDM

Criterion	DM_1		DM_2		DM_3		DM_4	
	LS	NS	LS	NS	LS	NS	LS	NS
C_1	PI	2	EI	3	EI	3	SI	4
C_2	EI	3	VSI	5	SI	4	AI	6
C_3	AI	6	EI	3	AI	6	AI	6
C_4	SI	4	AI	6	VSI	5	SI	4
C_5	EI	3	PI	2	EI	3	EI	3
C_6	AI	6	VSI	5	SI	4	VSI	5

Step 3-2 – Step 3-4: The EDM fuzzification process was completed upon completion of the LS conversion to T2NN process. The calculation of the T2NN ratio of the data consists of the differential preference of DM, T2NNWA operator, scalar multiplication operation and score function. The normalization was conducted to obtain the subjective weight, as shown in Table 8.

Step 4-1: The two normalized decision matrices $F^{(1)}$ and $F^{(2)}$ are calculated with Equations (16) and (17), provided in the first and second parts of Table A.3.

TABLE 8. The weights of criteria using LOPCOW-FWZIC

Criterion	Subjective weights			Objective weights	Integrated weights
	Fuzzy	Crisp	Final		
C_1	$\langle(0.152, 0.121, 0.116), (0.813, 0.874, 0.868), (0.776, 0.884, 0.843)\rangle$	0.141	0.085	0.169	0.127
C_2	$\langle(0.291, 0.242, 0.268), (0.684, 0.743, 0.729), (0.649, 0.717, 0.726)\rangle$	0.278	0.168	0.186	0.177
C_3	$\langle(0.482, 0.396, 0.477), (0.594, 0.604, 0.518), (0.507, 0.523, 0.598)\rangle$	0.440	0.266	0.180	0.223
C_4	$\langle(0.334, 0.284, 0.315), (0.636, 0.697, 0.692), (0.608, 0.667, 0.685)\rangle$	0.322	0.195	0.185	0.190
C_5	$\langle(0.141, 0.106, 0.102), (0.838, 0.894, 0.871), (0.791, 0.898, 0.859)\rangle$	0.126	0.076	0.103	0.089
C_6	$\langle(0.362, 0.307, 0.345), (0.621, 0.674, 0.665), (0.587, 0.630, 0.668)\rangle$	0.348	0.210	0.177	0.193

Step 4-2 – Step 4-4: Two types of normalized decision matrices are integrated to obtain the matrix $F = [F_{ij}]_{m \times n}$. The U_j of each criterion is calculated with Equation (19). We have $U_1 = 55.57\%$, $U_2 = 61.10\%$, $U_3 = 58.88\%$, $U_4 = 60.63\%$, $U_5 = 33.73\%$, and $U_6 = 57.96\%$. Equation (20) is employed to determine the objective weight of the criteria. Since DMs have no special preference for the ratio of subjective and objective weights, we integrate them evenly, and the comprehensive weights obtained are shown in the last column of Table 8, that is $w_1^* = 0.127$, $w_2^* = 0.177$, $w_3^* = 0.223$, $w_4^* = 0.190$, $w_5^* = 0.089$, $w_6^* = 0.193$.

Step 5-1: A_0 and A_4 are calculated with Equations (21) and (22). The matrix V is obtained by introducing the A_0 and A_4 into the set of alternatives.

Step 5-2, Step 5-3: All of the criteria are converted into the maximization-type criteria to calculate the normalized extended T2NN aggregation matrix Z by using Equation (23). Then, the weighted matrix O is obtained with the help of w_j^* . This matrix is shown in Table A.4.

Step 5-4, Step 5-5: U_l^- and U_l^+ are calculated by Equation (25) and Equation (26), respectively, presented in Table 9. Then, Equations (27) and (28) are employed to obtain the utility function of A_0 and A_4 . The value of $f(U^-)$ is 0.461, and the value of $f(U^+)$ is 0.539.

Step 5-6: The utility function $f(U_i)$ for each alternative is obtained by utilizing Equation (29). The results are shown in Table 9, and $f(U_1) = 0.684$, $f(U_2) = 0.638$, $f(U_3) = 0.675$. Since “ A_1 : Aidoc” has the highest utility function, the complete ranking order is $A_1 \succ A_3 \succ A_2$. Thus, according to the results of the proposed model, the alternative “ A_1 : Aidoc” can be regarded as the most ideal auxiliary AI product to achieve the decision objective for PNDT.

TABLE 9. Utility degrees, utility functions and ranks of the alternatives

Alternative	Utility degree		Utility function	Rank
	First	Second		
A_0 : Anti-ideal	1.000	0.856	—	—
A_1 : Aidoc	1.114	0.953	0.684	1
A_2 : Zebra medical vision	1.040	0.890	0.638	3
A_3 : Quantib	1.100	0.941	0.675	2
A_4 : Ideal	1.169	1.000	—	—

5.3. Sensitivity analyses. Considering that sensitivity analysis is a well-established method for checking the robustness of models, omitting it would compromise the completeness of the decision process. Accordingly, a scenario-based sensitivity analysis with three components was conducted. SAS-1 examined different variations in criteria weights, SAS-2 explored the trade-off coefficient μ in the LOPCOW method, while SAS-3 explored the effect of changes in the parameter $\bar{u}$ within the T2NNSSWAM operator.

For SAS-1, scenarios S1-S6 were generated by squaring the weight of a selected criterion, adding it to the weight of the subsequent criterion, and evenly subtracting the resulting increase from the other criteria. For instance, in S1, the recalculated weight of C_1 becomes $0.127 * 0.127 + 0.177 = 0.193$. This process was repeated across all criteria to generate six weight distributions, as shown in Table 10. Scenarios S7-S12 were based on a different principle.

TABLE 10. The twelve scenarios

Criterion	Weights	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12
C_1	0.127	0.193	0.112	0.124	0.140	0.105	0.133	0.127	0.193	0.112	0.124	0.140	0.105
C_2	0.177	0.164	0.254	0.174	0.190	0.155	0.183	0.177	0.164	0.254	0.174	0.190	0.155
C_3	0.223	0.209	0.207	0.239	0.236	0.200	0.228	0.223	0.209	0.207	0.239	0.236	0.200
C_4	0.190	0.177	0.174	0.187	0.126	0.167	0.196	0.190	0.177	0.174	0.187	0.126	0.167
C_5	0.089	0.076	0.074	0.086	0.102	0.201	0.095	0.089	0.076	0.074	0.086	0.102	0.201
C_6	0.193	0.180	0.178	0.190	0.206	0.171	0.165	0.193	0.180	0.178	0.190	0.206	0.171

Here, the weight of the most significant criterion (determined by T2NN-FWZIC-LOPCOW) was reduced, while the weights of the remaining criteria were proportionally increased by

$$w'_j = w_j^* \frac{(1 - w'_l)}{(1 - w_j^*)}, \quad j \in [1, n], \quad j \neq l \quad (30)$$

where w'_j represents the simulated weight value of C_j , w_j^* denotes the original calculated weight of T2NN-FWZIC-LOPCOW method, w'_l and w_j^* are the weight of the most important criterion for simulation and original, respectively. As seen from Table 8, the most important criterion for auxiliary AI product selection is “ C_3 : risk of AI technology dependence” with the value of $w_3^* = 0.223$. Using Equation (30), six additional scenarios were generated by reducing the weight of C_3 by 10%, with corresponding adjustments made to the remaining five criteria. The resulting weight configurations from scenarios S7-S12 are presented in Table 10. To further investigate the impact of these scenarios on the utility functions of alternatives, the results are visualized in Figure 3. As shown in Figure 3, the rankings of the AI alternatives exhibit no changes across all 12 scenarios, indicating the strong robustness of the model under varying weight distributions. For SAS-2, we set the step size of μ to 0.1 and analyzed 11 test scenarios, namely

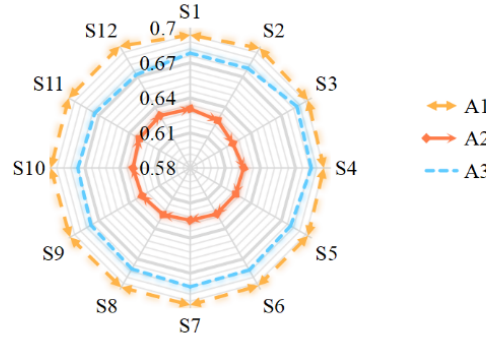
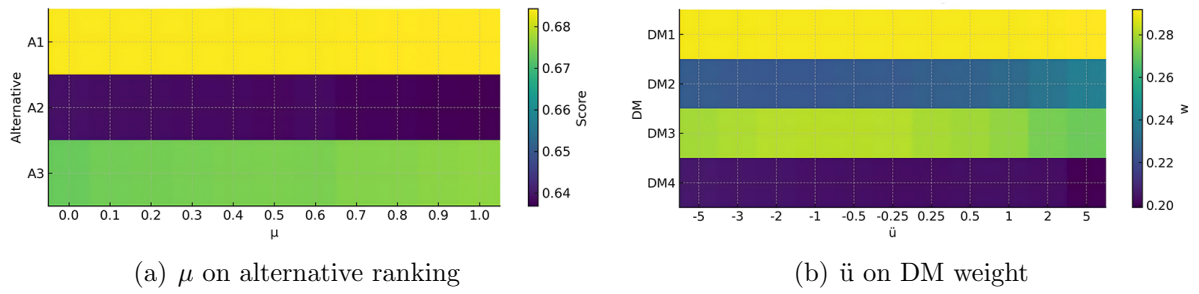


FIGURE 3. Sensitivity analysis of different alternatives to changes in criteria weights



(a) μ on alternative ranking

(b) $\ddot{u}$ on DM weight

FIGURE 4. Heatmap of the impact of coefficient μ and parameter $\ddot{u}$

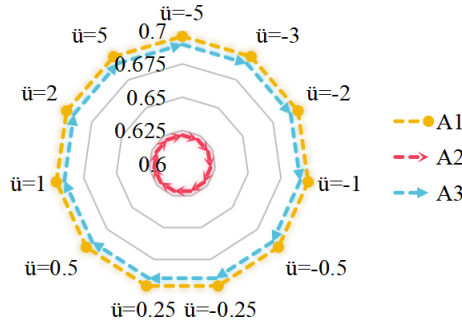


FIGURE 5. Influence of parameter $\ddot{u}$ on the utility function values

$\mu \in \{0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$. As shown in Figure 4(a), the ranking of the alternatives is highly stable and does not fluctuate with the change of coefficients.

SAS-3 examined the impact of DMs' experience and expertise by adjusting the " $\ddot{u}$ " parameter within the T2NNSSWAM aggregation operator. The heatmap of the impact of the change in coefficient parameter $\ddot{u}$ on the DM's weight is shown in Figure 4(b), where the weight ranking of DMs remains absolutely stable. As seen from Figure 5, eleven different values ($-5, -3, -2, -1, -0.5, -0.25, 0.25, 0.5, 1, 2$, and 5) were assigned to the parameter $\ddot{u}$. For each setting, the complete algorithmic procedure was executed to recalculate the utility function of each alternative. The results indicate that variations in the parameter μ had no impact on the ranking of the best alternative. The ranking order is A_1 (Aidoc) $>$ A_3 (Quantib) $>$ A_2 (Zebra Medical Vision) across all SAS-3, confirming the stability of the proposed model. Therefore, in both SASs, the proposed model demonstrated high robustness and reliability in AI product selection for PNDT.

TABLE 11. Comparative analysis ranking

Method	Performance value	The ranking value	Determination of criteria importance	Expression of decision information
The proposed model	$f(U_1) = 0.684$, $f(U_2) = 0.638$, $f(U_3) = 0.675$	$A_1 > A_3 > A_2$	FWZIC-LOPCOW	T2NN-PLTS
T2NNYWGM-Entropy-AROMAN	$\mathbb{R}_1 = 0.575$, $\mathbb{R}_2 = 0.330$, $\mathbb{R}_3 = 0.512$	$A_1 > A_3 > A_2$	Entropy	T2NN
T2NN-CDMT-CPT-VIKOR	$Q_1 = 0.060$, $Q_2 = 1$, $Q_3 = 0.140$	$A_1 > A_3 > A_2$	Entropy-CPT	T2NN
T2NN-MEREC-EDAS	$B_1 = 0.862$, $B_2 = 0.500$, $B_3 = 0.784$	$A_1 > A_3 > A_2$	MEREC	T2NN

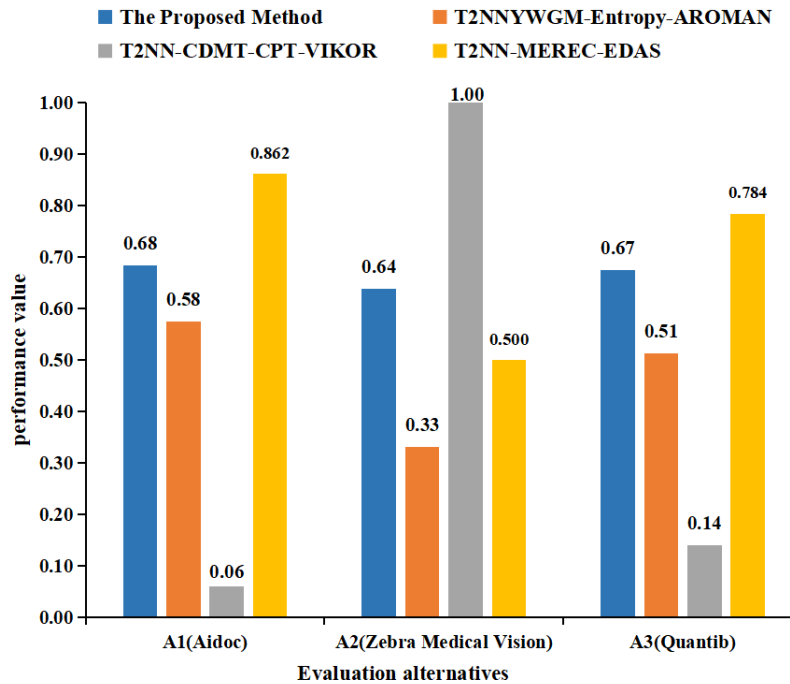


FIGURE 6. Comparison of alternative performance values

5.4. Comparative analyses. This section compares the decision outcomes of the developed model with several established T2NN-based methods on the same dataset. The rankings are shown in Table 11, and the corresponding performance values are visualized in Figure 6. As observed, T2NNYWGM-Entropy-AROMAN [38], T2NN-MEREC-EDAS [41], and T2NN-CDMT-CPT-VIKOR [42,43] yielded identical ranking orders to the proposed five-stage model, confirming its consistency. However, the proposed model exhibits superior performance compared to the existing methods, with detailed comparisons discussed below.

(i) T2NNYWGM-Entropy-AROMAN method for T2NN

This pipeline couples (a) T2NNYWGM, which builds on Yager's t-norm/t-conorm philosophy to aggregate uncertain linguistic information, with (b) objective Shannon-entropy weighting of criteria, and (c) AROMAN for final ranking. In contrast, the T2NN-FWZIC-LOPCOW method presented in this study integrates both subjective expert preferences and objective evaluation data, offering a more comprehensive weighting scheme. Moreover, T2NNYWGM-Entropy-AROMAN fails to incorporate probabilistic uncertainty in expert judgments, nor does it integrate such uncertainty with T2NNs, limiting its applicability in complex digital medical decision-making problems.

(ii) CDMT-CPT-VIKOR method for T2NN

This hybrid combines a cosine distance measure to quantify the similarity and divergence under T2NN information. CPT injects behavioral realism into the scoring stage, while VIKOR yields a solution closest to the ideal under conflicting criteria. This approach assumes equal importance among DMs, overlooking variations in experience and expertise. In real-world PNDT decision contexts, this assumption may distort results. The proposed model overcomes this limitation through the T2NNSSWAM operator, which adjusts weights based on quantified experience and expertise via a score function.

(iii) MEREC-EDAS method for T2NN

This framework combines a removal-effect-based weighting method with a distance-based ranking technique. The MEREC method is conceptually similar to our objective weighting component, whereas the EDAS ranking method scores alternatives based on their positive and negative distances from the average solution rather than from ideal points. However, this method shares similar limitations and further lacks flexibility, as it relies solely on the T2NNWA operator for information aggregation. Additionally, the use of zero as a reference point may neglect the attitudinal preferences of DMs, resulting in potential information loss. In contrast, the FWZIC-LOPCOW-MARCOS offers enhanced robustness and flexibility by incorporating diverse aggregation operators and better accommodating uncertainty.

According to the analysis of Table 11 and Figure 6, the proposed PT2NNPLTS-FWZIC-LOPCOW-MARCOS model produces highly consistent and reliable decision results. To obtain the ranking similarity of the proposed model with T2NNYWGM-Entropy-AROMAN, T2NN-CDMT-CPT-VIKOR, and T2NN-MEREC-EDAS, Spearman's correlation coefficient (SCC) [44] was applied as follows:

$$SCC = 1 - \frac{6 \sum_i^n l_i^2}{n(n^2 - 1)}, \quad (31)$$

where $l_i = s_i - t_i$, s_i and t_i are two ranking orders of A_i determined by different approaches for comparison. The SCC values of all methods are presented in Table 12.

Generally, an SCC value above 0.8 indicates high consistency across methods. All SCC values associated with the proposed model exceed this threshold, yielding a 100% match

TABLE 12. The SCC results

Methods	Proposed approach	Entropy-AROMAN	CDMT-CPT- VIKOR	MEREC- EDAS
The proposed approach	1	1	1	1
Entropy-AROMAN	/	1	1	1
CDMT-CPT- VIKOR	/	/	1	1
MEREC-TOPSIS	/	/	/	1

in ranking outcomes. These findings confirm the model's comprehensiveness and effectiveness in practical applications, providing high feasibility for researchers and stakeholders to make informed decisions under uncertainty.

6. Results and Implications.

6.1. Results. The application of the proposed framework yielded two primary findings. First, the criteria evaluation identified "Risk of AI technology dependence" as the most critical factor, followed by "System complexity or vulnerability" and "Cost issues". This result underscores a cautious and pragmatic stance from clinical experts, who prioritize the preservation of independent medical judgment and patient trust over a blind adoption of automated systems. It reflects the understanding that while AI is a powerful tool, its integration must not compromise the foundational role of clinical expertise, especially given the current limitations of AI in detecting atypical lesions. The second major finding is the identification of "Aidoc" as the most advantageous AI solution. Its superiority stems from its seamless integration with existing PACS and clinical workflows, high scalability, and advanced detection capabilities for complex cases, such as irregularly shaped or low-density nodules. These features make it a robust and reliable tool for high-volume clinical environments, directly addressing the practical needs of critical care physicians.

6.2. The managerial implications of the paper. The adoption of AI products is not merely an optimization tool for hospital's short-term operations; it also serves as a strategic enabler for achieving long-term goals. This study provides practical guidance for clinicians, AI developers, and policymakers engaged in PNDT and AI product deployment.

1) To our knowledge, this is the first study to investigate AI-assisted product selection for PNDT using a structured MCDM approach. It promotes more rational, evidence-based decision-making for hospitals seeking to adopt intelligent diagnostic technologies.

2) Project managers can leverage the insights from the model to significantly enhance PNDT service quality, improve decision efficiency, and bolster the hospital's overall competitiveness. Appropriate AI-assisted solutions enable clinical departments to efficiently process complex medical datasets and generate actionable insights. This, in turn, not only improves diagnostic accuracy but also enhances patient trust and satisfaction, streamlines workflows, and reinforces the hospital's reputation and market presence.

3) The introduction of AI products marks a crucial step towards the digital transformation of hospitals. By accelerating data digitization and enabling system automation, such tools facilitate efficient, hospital-wide information management. This not only enhances intelligent operations but also lays a solid foundation for sustainable development in modern healthcare.

6.3. The theoretical contributions of the paper. This paper makes several theoretical contributions to the MCDM literature. The primary contribution is the development of a novel integrated plithogenic T2NN-PLTS framework that, for the first time, addresses the AI product selection problem in PNDT. It has unparalleled advantages in capturing and processing complex uncertainties, contradictions and probabilistic characteristics. Methodologically, this study advances the field in three key ways.

1) The proposed PT2NNPLTS-FWZIC-LOPCOW-MARCOS model offers a powerful decision-support tool that provides many valuable advantages over state-of-the-art MCDM approaches for modeling the heterogeneous uncertainties inherent in clinical evaluations.

2) It proposes a comprehensive weighting approach that synthesizes subjective and objective insights, leading to more accurate and defensible criteria weights in complex environment.

3) The high consistency and reliability of the framework are verified through rigorous sensitivity and comparative analyses, demonstrating that the model maintains stable rankings even under fluctuating inputs. This work enriches the diversity of research in the T2NN-PLTS environment and offers a robust, adaptable model for other complex MCDM problems.

7. Conclusions. AI-assisted technologies are playing an increasing role in PNDT; however, their applicability in multidisciplinary clinical settings and their capacity for continuous improvement and to handle uncertainty in evolving healthcare environments remain challenging. This study proposes a PT2NNPLTS-FWZIC-LOPCOW-MARCOS decision support model to handle incomplete and uncertain information in selecting the optimal AI solution. The primary contribution lies in its ability to handle complex, heterogeneous information in high-stakes clinical environments. Application of the model in a real-world case study identified “Aidoc” (A_1) as the most suitable AI solution, with “Risk of AI technology dependence” (C_3) emerging as the most critical evaluation criterion. Furthermore, extensive sensitivity and comparative analyses confirmed the framework’s high robustness and the stability of its ranking results. The theoretical and managerial implications of this research are threefold: First, the novel integration of T2NN with plithogenic PLTS information; Second, the comprehensive weighting approach that synthesizes subjective and objective insights for better accuracy in complex uncertainty; Third, the flexible application of multiple aggregation operators to improve adaptability in real-world healthcare.

Despite these contributions, several avenues for future research remain to improve decision quality and theoretical development. First, the findings are based on a single case study with a limited dataset (three alternatives and six criteria), which may not fully capture all the challenges and potential opportunities involved in AI adoption for PNDT. Future work should involve multi-site or longitudinal studies with larger datasets and a more diverse group of stakeholders (e.g., patients, multidisciplinary clinicians, and regulators) to enhance generalizability. Second, while the model’s applicability was demonstrated, its theoretical properties of the combination of T2NN and PLTS, such as computational scalability and the trade-offs between complexity and performance, warrant deeper investigation. Finally, the proposed framework’s adaptability makes it a promising candidate for extension to other complex medical decision problems, including medical resource allocation and scheduling, AI-driven chronic disease management, and personalized health interventions.

Acknowledgment. This work was supported by the Fundamental Research Funds for the Central Universities (Grant No. N2406015).

REFERENCES

- [1] F. Bray, M. Laversanne, H. Sung, J. Ferlay, R. L. Siegel, I. Soerjomataram and A. Jemal, Global cancer statistics 2022: GLOBOCAN estimates of incidence and mortality worldwide for 36 cancers in 185 countries, *CA: A Cancer Journal for Clinicians*, vol.74, no.3, pp.229-263, 2024.
- [2] National Lung Screening Trial Research Team, Reduced lung-cancer mortality with low-dose computed tomographic screening, *The New England Journal of Medicine*, vol.365, no.5, pp.395-409, 2011.
- [3] H. Zeng, R. Zheng, K. Sun, M. Zhou, S. Wang, L. Li and J. He, Cancer survival statistics in China 2019-2021: A multicenter, population-based study, *Journal of the National Cancer Center*, vol.4, no.3, pp.203-213, 2024.

- [4] R. Li, C. Xiao, Y. Huang, H. Hassan and B. Huang, Deep learning applications in computed tomography images for pulmonary nodule detection and diagnosis: A review, *Diagnostics*, vol.12, no.2, 298, 2022.
- [5] C. de Margerie-Mellon and G. Chassagnon, Artificial intelligence: A critical review of applications for lung nodule and lung cancer, *Diagnostic and Interventional Imaging*, vol.104, no.1, pp.11-17, 2023.
- [6] M. Abdel-Basset, M. Saleh, A. Gamal and F. Smarandache, An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number, *Applied Soft Computing*, vol.77, pp.438-452, 2019.
- [7] I. Gokasar, V. Simic, M. Deveci and T. Senapati, Alternative prioritization of freeway incident management using autonomous vehicles in mixed traffic using a type-2 neutrosophic number based decision support system, *Engineering Applications of Artificial Intelligence*, vol.123, 106183, 2023.
- [8] F. Smarandache, Plithogenic set, an extension of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets-revisited, *Neutrosophic Sets and Systems*, vol.21, pp.153-166, 2018.
- [9] Q. Pang, H. Wang and Z. Xu, Probabilistic linguistic term sets in multi-attribute group decision making, *Information Sciences*, vol.369, pp.128-143, 2016.
- [10] P. Wang, Y. Lin, M. Fu and Z. Wang, VIKOR method for plithogenic probabilistic linguistic MAGDM and application to sustainable supply chain financial risk evaluation, *International Journal of Fuzzy Systems*, vol.25, no.2, pp.780-793, 2023.
- [11] W. Hendrix, N. Hendrix, E. T. Scholten, M. Mourits, J. Trap-de Jong, S. Schalekamp, M. Korst, M. van Leuken, B. van Ginneken, M. Prokop, M. Rutten and C. Jacobs, Deep learning for the detection of benign and malignant pulmonary nodules in non-screening chest CT scans, *Communications Medicine*, vol.3, no.1, 156, 2023.
- [12] A. Schreuder, E. T. Scholten, B. van Ginneken and C. Jacobs, Artificial intelligence for detection and characterization of pulmonary nodules in lung cancer CT screening: Ready for practice?, *Translational Lung Cancer Research*, vol.10, no.5, pp.2378-2388, 2021.
- [13] F. Shi, B. Chen, Q. Cao, Y. Wei, Q. Zhou, R. Zhang and D. Shen, Semi-supervised deep transfer learning for benign-malignant diagnosis of pulmonary nodules in chest CT images, *IEEE Transactions on Medical Imaging*, vol.41, no.4, pp.771-781, 2021.
- [14] D. Ardila, A. P. Kiraly, S. Bharadwaj, B. Choi, J. J. Reicher, L. Peng and S. Shetty, End-to-end lung cancer screening with three-dimensional deep learning on low-dose chest computed tomography, *Nature Medicine*, vol.25, no.6, pp.954-961, 2019.
- [15] C. Wang, J. Shao, Y. He, J. Wu, X. Liu, L. Yang, Y. Wei, X. S. Zhou, Y. Zhan, F. Shi, D. Shen and W. Li, Data-driven risk stratification and precision management of pulmonary nodules detected on chest computed tomography, *Nature Medicine*, vol.30, pp.3184-3195, 2024.
- [16] K. G. van Leeuwen, S. Schalekamp, M. J. Rutten, M. Huisman, C. M. Schaefer-Prokop and M. de Rooij, Project AIR working group: Comparison of commercial AI software performance for radiograph lung nodule detection and bone age prediction, *Radiology*, vol.310, no.1, e230981, 2024.
- [17] J. A. Liu, I. Y. Yang and E. B. Tsai, Artificial intelligence (AI) for lung nodules, from the AJR special series on AI applications, *American Journal of Roentgenology*, vol.219, no.5, pp.703-712, 2022.
- [18] S. Chakraborty, R. D. Raut, T. M. Rofin and S. Chakraborty, A comprehensive and systematic review of multi-criteria decision-making methods and applications in healthcare, *Healthcare Analytics*, vol.4, 100232, 2023.
- [19] H. Liao, Y. Long, M. Tang, D. Streimikiene and B. Lev, Early lung cancer screening using double normalization-based multi-aggregation (DNMA) and Delphi methods with hesitant fuzzy information, *Computers & Industrial Engineering*, vol.136, pp.453-463, 2019.
- [20] H. Liao, R. Fang, J. B. Yang and D. L. Xu, A linguistic belief-based evidential reasoning approach and its application in aiding lung cancer diagnosis, *Knowledge-Based Systems*, vol.253, 109559, 2022.
- [21] A. H. Seh, J. F. Al-Amri, A. F. Subahi, A. Agrawal, R. Kumar and R. A. Khan, Machine learning based framework for maintaining privacy of healthcare data, *Intell. Autom. Soft Computing*, vol.29, no.3, pp.697-712, 2021.
- [22] F. Pakdil, P. Tokta and G. F. Can, Six sigma project prioritization and selection: A multi-criteria decision making approach in healthcare industry, *International Journal of Lean Six Sigma*, vol.12, no.3, pp.553-578, 2020.
- [23] M. Dastjerdi, A. Keramati and N. Keramati, A novel framework for investigating organizational adoption of AI-integrated CRM systems in the healthcare sector; using a hybrid fuzzy decision-making approach, *Telematics and Informatics Reports*, vol.11, 100078, 2023.

- [24] S. A. Suha and T. F. Sanam, Exploring dominant factors for ensuring the sustainability of utilizing artificial intelligence in healthcare decision making: An emerging country context, *International Journal of Information Management Data Insights*, vol.3, no.1, 100170, 2023.
- [25] M. A. Alsalem, A. H. Alamoodi, O. S. Albahri, A. S. Albahri, L. Martínez, R. Yera, A. M. Duham and I. M. Sharaf, Evaluation of trustworthy artificial intelligent healthcare applications using multi-criteria decision-making approach, *Expert Systems with Applications*, vol.246, 123066, 2024.
- [26] A. E. Torkayesh, S. Hendiani, G. Walther and S. Venghaus, Fueling the future: Overcoming the barriers to market development of renewable fuels in Germany using a novel analytical approach, *European Journal of Operational Research*, vol.316, no.3, pp.1012-1033, 2024.
- [27] K. Kara, G. C. Yalçın, V. Simic, M. Erbay and D. Pamucar, A type-2 neutrosophic entropy-based group decision analytics model for sustainable aquaculture engineering, *Engineering Applications of Artificial Intelligence*, vol.134, 108615, 2024.
- [28] İ. Önden, M. Deveci, K. Kara, G. C. Yalçın, A. Önden, M. Eker and M. Hasseb, Supplier selection of companies providing micro mobility service under type-2 neutrosophic number based decision making model, *Expert Systems with Applications*, vol.245, 123033, 2024.
- [29] E. B. Tirkolaee, A. E. Torkayesh, M. Tavana, A. Goli, V. Simic and W. Ding, An integrated decision support framework for resilient vaccine supply chain network design, *Engineering Applications of Artificial Intelligence*, vol.126, 106945, 2023.
- [30] M. D. Alshehri, An integrated AHP MCDM based type-2 neutrosophic model for assessing the effect of security in fog-based IoT framework, *International Journal of Neutrosophic Science*, vol.20, no.2, 2023.
- [31] I. M. Sharaf, A. H. Alamoodi, O. S. Albahri, M. Deveci, M. Talal, A. S. Albahri and W. Pedrycz, Architecture selection for 5G-radio access network using type-2 neutrosophic numbers based decision making model, *Expert Systems with Applications*, vol.237, 121420, 2024.
- [32] V. Simić, B. Milovanović, S. Pantelić, D. Pamučar and E. B. Tirkolaee, Sustainable route selection of petroleum transportation using a type-2 neutrosophic number based ITARA-EDAS model, *Information Sciences*, vol.622, pp.732-754, 2023.
- [33] V. Simić, I. Gokasar, M. Deveci and L. Švadlenka, Mitigating climate change effects of urban transportation using a type-2 neutrosophic MEREC-MARCOS model, *IEEE Transactions on Engineering Management*, vol.71, pp.3233-3249, 2022.
- [34] S. Bošković, S. Jovčić, V. Simic, L. Švadlenka, M. Dobrodolac and N. Bacanin, A new criteria importance assessment (CIMAS) method in multi-criteria group decision-making: Criteria evaluation for supplier selection, *Facta Universitatis, Series: Mechanical Engineering*, vol.23, no.2, pp.335-349, 2025.
- [35] M. Abdel-Basset, M. Saleh, A. Gamal and F. Smarandache, An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number, *Applied Soft Computing*, vol.77, pp.438-452, 2019.
- [36] M. Deveci, N. Erdogan, U. Cali, J. Stekli and S. Zhong, Type-2 neutrosophic number based multi-attributive border approximation area comparison (MABAC) approach for offshore wind farm site selection in USA, *Engineering Applications of Artificial Intelligence*, vol.103, 104311, 2021.
- [37] Ö. F. Görçün, D. Pamucar, R. Krishankumar and H. Küçükönder, The selection of appropriate Ro-Ro Vessel in the second-hand market using the WASPAS' Bonferroni approach in type 2 neutrosophic fuzzy environment, *Engineering Applications of Artificial Intelligence*, vol.117, 105531, 2023.
- [38] K. Kara, G. C. Yalçın, V. Simic, M. Polat and D. Pamucar, An integrated neutrosophic Schweizer-Sklar-based model for evaluating economic activities in organized industrial zones, *Engineering Applications of Artificial Intelligence*, vol.130, 10772, 2024.
- [39] Z. S. Chen, K. S. Chin, Y. L. Li and Y. Yang, Proportional hesitant fuzzy linguistic term set for multiple criteria group decision making, *Information Sciences*, vol.357, pp.61-87, 2016.
- [40] N. Basil, M. Alqaysi, M. Deveci, A. Albahri, O. Albahri and A. Alamoodi, Evaluation of autonomous underwater vehicle motion trajectory optimization algorithms, *Knowledge-Based Systems*, vol.276, 110722, 2023.
- [41] V. Simic, S. Dabic-Miletic, E. B. Tirkolaee, Ž. Stević, M. Deveci and T. Senapati, Neutrosophic CEBOM-MACONT model for sustainable management of end-of-life tires, *Applied Soft Computing*, vol.143, 110399, 2023.
- [42] Z. Wang, Q. Cai and G. Wei, Modified TODIM method based on cumulative prospect theory with Type-2 neutrosophic number for green supplier selection, *Engineering Applications of Artificial Intelligence*, vol.126, 106843, 2023.

- [43] P. Wang, C. Y. Huang and Z. P. Fan, Prioritization of sustainable development plannings for supply chain finance using a novel triangular neutrosophic-based TOPSIS-CPT-VIKOR model, *International Journal of General Systems*, vol.54, pp.240-270, 2024.
- [44] P. Schober, C. Boer and L. A. Schwarte, Correlation coefficients: Appropriate use and interpretation, *Anesthesia & Analgesia*, vol.126, no.5, pp.1763-1768, 2018.

Online Supplementary Materials.

Appendix A. Calculation Results. The calculation results obtained from this study are shown in Tables A.1-A.4.

TABLE A.1. Evaluation of DMs on the criteria-based alternatives

Alternative	Expert	C_1	C_2	C_3	C_4	C_5	C_6
A_1	DM_1	VSI	EI	EI	VSI	SI	EI
	DM_2	VSI	EI	SI	SI	VSI	SI
	DM_3	SI	SI	SI	VSI	AI	EI
	DM_4	AI	VSI	VSI	EI	VSI	EI
A_2	DM_1	AI	EI	SI	VSI	VSI	SI
	DM_2	VSI	SI	VSI	AI	AI	VSI
	DM_3	VSI	SI	SI	VSI	AI	SI
	DM_4	AI	VSI	AI	VSI	VSI	EI
A_3	DM_1	SI	SI	EI	VSI	VSI	EI
	DM_2	VSI	SI	EI	VSI	AI	SI
	DM_3	VSI	VSI	SI	VSI	AI	PI
	DM_4	VSI	VSI	SI	SI	VSI	PI

TABLE A.2. The aggregated PT2NNPLTS evaluation matrix

	A_1	A_2	A_3
C_1	$\langle(0.779, 0.716, 0.747), (0.028, 0.054, 0.060), (0.081, 0.139, 0.166)\rangle$	$\langle(0.934, 0.879, 0.920), (0.020, 0.031, 0.026), (0.043, 0.056, 0.105)\rangle$	$\langle(0.678, 0.622, 0.622), (0.040, 0.093, 0.154), (0.120, 0.208, 0.223)\rangle$
C_2	$\langle(0.657, 0.530, 0.530), (0.095, 0.193, 0.211), (0.168, 0.427, 0.359)\rangle$	$\langle(0.713, 0.598, 0.598), (0.075, 0.165, 0.203), (0.146, 0.392, 0.320)\rangle$	$\langle(0.717, 0.634, 0.634), (0.044, 0.104, 0.161), (0.096, 0.244, 0.226)\rangle$
C_3	$\langle(0.804, 0.749, 0.749), (0.075, 0.165, 0.203), (0.088, 0.212, 0.170)\rangle$	$\langle(0.845, 0.808, 0.828), (0.030, 0.061, 0.062), (0.034, 0.072, 0.076)\rangle$	$\langle(0.833, 0.747, 0.747), (0.127, 0.270, 0.269), (0.145, 0.388, 0.265)\rangle$
C_4	$\langle(0.782, 0.728, 0.728), (0.084, 0.171, 0.197), (0.071, 0.162, 0.144)\rangle$	$\langle(0.962, 0.933, 0.947), (0.032, 0.065, 0.090), (0.024, 0.035, 0.059)\rangle$	$\langle(0.813, 0.780, 0.780), (0.043, 0.102, 0.159), (0.043, 0.090, 0.092)\rangle$
C_5	$\langle(0.902, 0.883, 0.893), (0.030, 0.058, 0.068), (0.014, 0.022, 0.035)\rangle$	$\langle(0.989, 0.976, 0.986), (0.023, 0.039, 0.038), (0.008, 0.011, 0.023)\rangle$	$\langle(0.989, 0.976, 0.986), (0.023, 0.039, 0.038), (0.008, 0.011, 0.023)\rangle$
C_6	$\langle(0.938, 0.862, 0.862), (0.216, 0.389, 0.305), (0.055, 0.216, 0.129)\rangle$	$\langle(0.920, 0.884, 0.884), (0.101, 0.212, 0.225), (0.030, 0.095, 0.069)\rangle$	$\langle(0.767, 0.717, 0.687), (0.237, 0.369, 0.426), (0.070, 0.244, 0.150)\rangle$

TABLE A.3. The max-min and sum normalized decision matrices

Criterion	Max-min normalized decision matrix			Sum normalized decision matrix		
	A_1	A_2	A_3	A_1	A_2	A_3
C_1	0.452	1.000	0.568	0.331	0.355	0.334
C_2	1.000	0.613	0.000	0.364	0.336	0.301
C_3	0.000	0.000	1.000	0.304	0.309	0.364
C_4	0.452	1.000	0.568	0.331	0.355	0.334
C_5	1.000	0.613	0.000	0.364	0.336	0.301
C_6	0.000	0.000	1.000	0.304	0.309	0.364

TABLE A.4. The weighted normalized extended T2NN aggregation matrix

Alternative	Criterion					
	C_1	C_2	C_3	C_4	C_5	C_6
A_0 : Anti-ideal	0.107	0.154	0.184	0.164	0.086	0.161
A_1 : Aidoc	0.116	0.177	0.204	0.190	0.089	0.177
A_2 : Zebra medical vision	0.127	0.167	0.184	0.164	0.086	0.161
A_3 : Quantib	0.107	0.154	0.223	0.178	0.086	0.193
A_4 : Ideal	0.127	0.177	0.223	0.190	0.089	0.193

Author Biography



Peiwen Wang received the Ph.D. degree in Management Science and Engineering from Dalian Maritime University, China, in 2023. He is a postdoctoral fellow at the Management Science and Engineering Mobile Station of Northeastern University, China. His current research interests include multi criteria decision-making and hypernetwork.



Zhi-Ping Fan received the Ph.D. degree in Control Theory and Applications from Northeastern University, China, in 1996. Dr. Fan is currently a full-time professor at the School of Business Administration, Northeastern University, China. His current research interests include operations management, e-commerce and information management, and decision-making. He has published over 200 papers in well-known SCI-indexed journals.