

A DESIGN METHOD FOR CONTROL SYSTEM FOR NON-MINIMUM PHASE PLANT USING DISTURBANCE OBSERVER FOR PERIODIC DISTURBANCE

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ABSTRACT. *For attenuating periodic disturbances and making the output follow a non periodic reference input, repetitive control is often used. However, using repetitive control, a controller is high order. In order to design a controller that makes the output follow a non-periodic reference input without steady state error against periodic disturbances, this paper proposes a design method of a control system using disturbance observer for non-minimum-phase systems. Using the proposed design method, the control system can be constructed by using a low order controller. In this paper, the parameterization of all stabilizing controller is clarified. Furthermore, control characteristics and design procedure of the control system using disturbance observer for non-minimum-phase systems are clarified. Finally, we show the numerical example for the effectiveness of the proposed design method.*

Keywords: Parameterization, Disturbance observer, Non-periodic reference input, Periodic disturbance, Disturbance attenuation, Disturbance observer based control

1. Introduction. It is often an important problem to attenuate the periodic disturbances and the output follows a reference input without a steady-state error in practical control systems. To attenuate periodic disturbances, repetitive control has been proposed [1]. In repetitive control, a controller has the same infinite poles depending on the period of disturbance on the imaginary axis. From the same principle as the internal model one, the controller with infinite poles of periodic input on the imaginary axis can effectively attenuate periodic disturbances.

A reference input is not always periodic. Pipeleers et al. have proposed a robust higher-order repetitive control [7]. Using a method proposed in [7], a control system makes the output follow the non-periodic disturbance and attenuates the influence of the periodic disturbances. However, the method in [7] has a problem that the order of a controller has a large order. The above problem is caused by the controller that has the same infinite poles.

A disturbance observer is used to estimate the disturbances in the factory plant [13], and various papers are proposed [9, 10, 11, 12]. In addition, many papers proposed disturbance observer control systems [14, 15, 16, 17, 18], and the applications of disturbance observers have been proposed in many control systems such as a motion-control field [19, 20, 21]. A disturbance observer is used in the motion control field to cancel the disturbance or to make the closed-loop system robustly stable [22, 23, 24]. Typically, a disturbance observer includes a disturbance signal generator and an observer, and the disturbance that is normally considered as a step disturbance is estimated by the observer. Since the disturbance observer is easy to understand the structure, it is used in many cases [22, 23, 25].

Mita et al. pointed out that disturbance observers are not only alternative design of complete controllers [24]. Extended H_∞ control in [24] has therefore been proposed as an effective motion control method that cancels disturbances. From another point of view, Kobayashi et al. considered an observer design method for obtaining phase compensation based on disturbance observers [25].

Another important control problem is the parameterization problem which is the problem of finding all stabilizing controllers for the plant [26]. If the parameterization of all disturbance observers could be obtained, we could express previous studies of disturbance observers in a uniform manner. In addition, disturbance observers could be designed systematically. Yamada et al. examined the parameterization of all disturbance observers [27]. There exists another paper that motion control realization is based on the disturbance observer and the Kalman filter [19]. This paper realizes high robustness against disturbance, parameter variations, effective noise suppression and wide band force sensing by using disturbance observer and Kalman filter. However, there is a remaining problem that achieving both high responsiveness and noise-free force estimation in sensor-less motion control remains difficult due to limitations in disturbance observer bandwidth, noise in high-order estimations, and computational constraints for real-time implementation.

Recently, the parameterization of all disturbance observer for the periodic disturbance was clarified [8]. Using this parameterization in [8], we have a possibility to design a control system for the output to follow non-periodic reference input and to attenuate periodic disturbance effectively without using repetitive control. For minimum-phase plant, a design method for control system for the output to follow the non-periodic reference input and to attenuate the periodic disturbance for non-periodic reference input is examined by Yamada et al. [32]. In addition, Yamada et al. proposed a design method using a disturbance observer for periodic disturbance for non-minimum-phase plant without using repetitive control [33]. However, the method in [32, 33] remains a difficulty. Since the method in [32, 33] does not clarify the parameterization of all stabilizing controllers, we cannot design the controllers systematically. In order to overcome this problem, Duong et al. clarified the parameterization of all stabilizing controllers for minimum-phase plant [34]. However, the method in [34] cannot apply for non-minimum-phase plants. There exist a lot of non-minimum-phase plants: a pitch control of air planes [35], a control of inverted pendulum [36, 37], and so on. For non-minimum-phase plants, the results of [34] do not always make the control system stable because the unstable zeros in a plant often makes a closed-loop system unstable.

This paper clarifies the parameterization of all stabilizing controllers for non-minimum-phase plant using the disturbance observer for the periodic disturbances. In addition,

a design method is proposed for the control system using disturbance observer for the periodic reference input for non-minimum-phase plants such that the output follows the non-periodic reference input and the periodic disturbances are attenuated effectively.

This paper is organized as follows. In Section 2, the preliminary result and the problem formulation in this paper are explained. In Section 3, we examine the parameterization of all controllers to make functions in the controller for the disturbance observer stable. In Section 4, we clarify that the obtained controllers in Section 3 make the control system stable. In Section 5, we summarize the control characteristics of the control system. In Section 6, a design procedure of the control system is presented. In Section 7, we provide a numerical example to illustrate the effectiveness of the proposed method. Section 8 gives concluding remarks.

2. Preliminary Result and Problem Formulation. Consider the plant written by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + d(t) \end{cases}, \quad (1)$$

where $x(t) \in R^n$ is the state variable, $u(t) \in R$ is the control input, $y(t) \in R$ is the output, $d(t) \in R$ is the periodic output disturbance with period $T > 0$ satisfying

$$d(t + T) = d(t) \quad (\forall t \geq 0), \quad (2)$$

$A \in R^{n \times n}$, $B \in R^n$ and $C \in R^{1 \times n}$. It is assumed that (A, B) is stabilizable, (C, A) is detectable, A has no eigenvalue on the imaginary axis,

$$\det \begin{bmatrix} A - sI & B \\ C & 0 \end{bmatrix} = 0 \quad (3)$$

has roots in the closed right half plane, and $u(t)$ and $y(t)$ are available, but $d(t)$ is unavailable. The transfer function in (1) is denoted by

$$y(s) = G(s)u(s) + d(s), \quad (4)$$

where

$$G(s) = C(sI - A)^{-1}B \in R(s), \quad (5)$$

$y(s) = \mathcal{L}\{y(t)\}$, $u(s) = \mathcal{L}\{u(t)\}$ and $d(s) = \mathcal{L}\{d(t)\}$. Note that the assumption that (3) has roots in the closed right half plane implies that $G(s)$ has zeroes in the closed right half plane, that is, $G(s)$ in (5) is of non-minimum-phase.

For the non-minimum-phase systems, in order to attenuate the periodic disturbance and to follow the non-periodic reference input $r(s)$ without steady-state error, the control system using the disturbance observer shown in Figure 1 is proposed in [33]. Here, $C_1(s) \in$

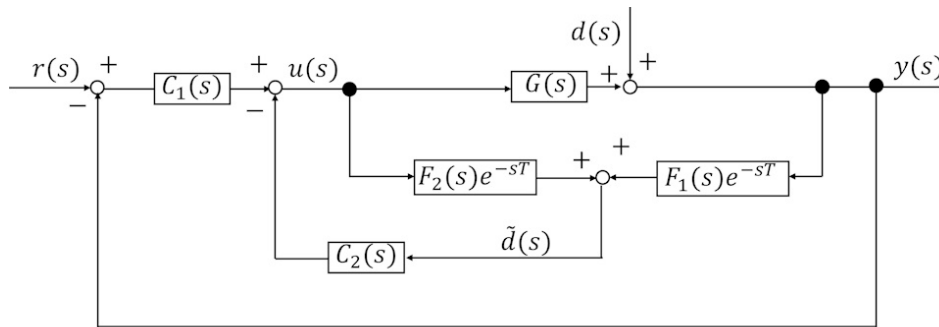


FIGURE 1. Structure of a disturbance observer system

$R(s)$ and $C_2(s)$ are controllers. $\tilde{d}(s)$ is disturbance observer for periodic disturbance [8] written by the form in

$$\tilde{d}(s) = F_1(s)e^{-sT}y(s) + F_2(s)e^{-sT}u(s) \quad (6)$$

and satisfies

$$\lim_{t \rightarrow \infty} (d(t) - \tilde{d}(t)) = 0 \quad (7)$$

for any initial state $x(0)$, control input $u(t)$ and periodic output disturbance $d(t)$. According to [8], the parameterization of all $F_1(s)$ and $F_2(s)$ is given by

$$F_1(s) = D(s) + Q(s)D(s) \in RH_\infty \quad (8)$$

and

$$F_2(s) = -N(s) - Q(s)N(s) \in RH_\infty, \quad (9)$$

where $D(s) \in RH_\infty$ and $N(s) \in RH_\infty$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \frac{N(s)}{D(s)} \quad (10)$$

and $Q(s) \in RH_\infty$ is any function satisfying

$$D(s_i) + Q(s_i)D(s_i) = 1 \quad \forall s_i \ (i = 0, 1, \dots), \quad (11)$$

$$s_i = j\omega_i, \quad (12)$$

$$\omega_i = \frac{2\pi i}{T} \ (i = 0, 1, \dots) \quad (13)$$

and j is the imaginary unit.

Using the disturbance observer for periodic disturbance, in general, the transfer function from the disturbance $d(s)$ to the output $y(s)$ has an infinite number of poles. In order to make the transfer function from $d(s)$ to $y(s)$ have a finite number of poles, $C_2(s) \in R(s)$ is designed as the form in

$$C_2(s) = \frac{C_n(s)}{1 + C_d(s)e^{-sT}}, \quad (14)$$

where $C_n(s) \in R(s)$ and $C_d(s) \in R(s)$. Using $C_2(s)$ in (14), the transfer function from $d(s)$ to $y(s)$ in Figure 1 is given by

$$\frac{y(s)}{d(s)} = \frac{1 + (C_d(s) + F_2(s)C_n(s))e^{-sT}}{1 + C_1(s)G(s) + \{(1 + C_1(s)G(s))C_d(s) + (F_1(s)G(s) + F_2(s))C_n(s)\}e^{-sT}}. \quad (15)$$

The condition that the transfer function from $d(s)$ to $y(s)$ has a finite number of poles is summarized in Lemma 2.1.

Lemma 2.1. *The transfer function from $d(s)$ to $y(s)$ in Figure 1 has a finite number of poles if and only if*

$$(1 + C_1(s)G_m(s))C_d(s) + (F_1(s)G_m(s) + F_2(s))C_n(s) = 0 \quad (16)$$

holds true [32].

The stability of the control system in Figure 1 satisfying Lemma 2.1 is summarized in the following lemma.

Lemma 2.2. *The control system in Figure 1 is stable if and only if the following expressions hold true [32].*

- 1) $C_1(s)$ stabilizes $G(s)$. That is, all transfer functions $C_1(s)G(s)/(1 + C_1(s)G(s))$, $G(s)/(1 + C_1(s)G(s))$, $C_1(s)/(1 + C_1(s)G(s))$ and $1/(1 + C_1(s)G(s))$ are stable.

- 2) $C_d(s) \in RH_\infty$ in (14).
 3) $C_n(s) \in RH_\infty$ in (14).

In order to attenuate periodic disturbance $d(s)$ effectively, from (15), $C_n(s)$ and $C_d(s)$ have to satisfy

$$1 + (C_d(s) + F_2(s)C_n(s))|_{s_i=j\frac{2\pi i}{T}} = 0 \quad (i = 0, 1, 2, \dots). \quad (17)$$

According to [33], from (17), $C_n(s)$ and $C_d(s)$ are designed to satisfy

$$C_d(s) + F_2(s)C_n(s) = -\bar{q}(s)N_i(s), \quad (18)$$

where $\bar{q}(s) \in RH_\infty$ is a strictly proper low-pass filter written by

$$\bar{q}(s) = \frac{1}{(1 + \tau_{\bar{q}}s)^{n_{\bar{q}}}}, \quad (19)$$

$n_{\bar{q}} > 0$ is positive integer to make $\bar{q}(s)/G(s)$ and $\bar{q}(s)/F_2(s)$ proper, and $\tau_{\bar{q}} > 0$ is small real positive number. $N_i(s) \in RH_\infty$ is an inner function of $N(s)$ satisfying $N_i(0) = 1$ and satisfies

$$N(s) = N_i(s)N_o(s) \quad (20)$$

and $N_o(s) \in RH_\infty$ is an outer function of $N(s)$. Then in the frequency range satisfying

$$\bar{q}(s_i)N_i(s_i) \simeq 1 \quad (i = 0, 1, \dots, k_{\max}), \quad (21)$$

for s_i ($i = 0, 1, \dots, k_{\max}$), (17) is satisfied, where $k_{\max}$ is maximum integer. From (9), $F_2(s)$ is factorized as

$$F_2(s) = N_i(s)\bar{F}_2(s), \quad (22)$$

where

$$\bar{F}_2(s) = -N_o(s) - Q(s)N_o(s). \quad (23)$$

This yields $\bar{F}_2(s) \in RH_\infty$. In order to make $C_d(s) \in RH_\infty$ satisfying (18), we settle

$$C_d(s) = N_i(s)\bar{C}_d(s), \quad (24)$$

where $\bar{C}_d(s) \in RH_\infty$. By substituting (22) and (24) to (18), we have

$$C_n(s) = -\frac{\bar{q}(s) + \bar{C}_d(s)}{\bar{F}_2(s)}, \quad (25)$$

and by substituting (20), (22), (24) and (25) to (16), we have

$$\bar{C}_d(s) = \frac{(F_1(s)N_o(s) + \bar{F}_2(s)D(s))\bar{q}(s)D_c(s)}{(\bar{F}_2(s)N_i(s)N_c(s) - F_1(s)D_c(s))N_o(s)}, \quad (26)$$

where $N_c(s) \in RH_\infty$ and $D_c(s) \in RH_\infty$ are coprime factors of $C_1(s)$ satisfying

$$C_1(s) = \frac{N_c(s)}{D_c(s)}. \quad (27)$$

Using the method proposed in [33], even if the controller $C_1(s)$ stabilizes the plant $G(s)$, $C_1(s)$ does not necessarily make $\bar{C}_d(s)$ in (26) stable.

The problem considered in this paper is to overcome this problem and clarify the parameterization of all $C_1(s)$ to make $\bar{C}_d(s)$ in (26) stable and the control system in Figure 1 stable.

3. Parameterization of All $C_1(s)$ to Make $\bar{C}_d(s)$ Stable. In this section, we clarify the parameterization of all $C_1(s)$ that makes $\bar{C}_d(s)$ in (26) stable.

The parameterization of all $C_1(s)$ that makes $\bar{C}_d(s)$ in (26) stable is summarized in the following theorem.

Theorem 3.1. $C_1(s)$ written in (27) makes $\bar{C}_d(s)$ in (26) stable if and only if $\bar{q}(s)/N_o(s) \in RH_\infty$, and $N_c(s)$ and $D_c(s)$ in (27) are written by

$$N_c(s) = \bar{X}(s) - F_1(s)\hat{Q}(s) \quad (28)$$

and

$$D_c(s) = \bar{Y}(s) - \bar{F}_2(s)N_i(s)\hat{Q}(s), \quad (29)$$

where $\bar{X}(s) \in RH_\infty$ and $\bar{Y}(s) \in RH_\infty$ satisfy

$$\bar{X}(s)\bar{F}_2(s)N_i(s) - \bar{Y}(s)F_1(s) = 1 \quad (30)$$

and $\hat{Q}(s) \in RH_\infty$ is any function.

Proof: The necessity is shown, that is, we show that if $\bar{C}_d(s)$ in (26) is stable, $N_c(s)$ and $D_c(s)$ in (27) are written by (28) and (29), respectively. Since $D_c(s) \in RH_\infty$ and $F_1(s) \in RH_\infty$ are biproper and $N_o(s) \in RH_\infty$, $\bar{F}_2(s) \in RH_\infty$, $D(s) \in RH_\infty$, $\bar{q}(s) \in RH_\infty$, $F_2(s) \in RH_\infty$, $N_i(s) \in RH_\infty$ and $N_c(s) \in RH_\infty$, $\bar{C}_d(s) \in RH_\infty$ is satisfied if $\bar{q}(s)/N_o(s) \in RH_\infty$ and

$$\bar{F}_2(s)N_i(s)N_c(s) - F_1(s)D_c(s) \in \mathcal{U}. \quad (31)$$

According to [29], the condition of (31) is equivalent to satisfy

$$\bar{F}_2(s)N_i(s)N_c(s) - F_1(s)D_c(s) = I. \quad (32)$$

Using the result in [29] and simple manipulations, all solutions of $N_c(s)$ and $D_c(s)$ to (32) are written by (28) and (29), respectively, where $\bar{X}(s) \in RH_\infty$ and $\bar{Y}(s) \in RH_\infty$ satisfy (30) and $\hat{Q}(s) \in RH_\infty$ is any function. Since $\bar{C}_d(s)$ in (26) is proper, $\bar{q}(s)/N_o(s) \in RH_\infty$ holds true. The necessity has been shown.

Next, the sufficiency is shown. That is, we show that if $N_c(s)$ and $D_c(s)$ are written by (28) and (29), respectively, and $\bar{q}(s)/N_o(s) \in RH_\infty$, then $\bar{C}_d(s)$ in (26) is stable. Substituting (28) and (29) to (26), we have

$$\bar{C}_d(s) = \frac{(F_1(s)N_o(s) + \bar{F}_2(s)D(s))\bar{q}(s)(\bar{Y}(s) - \bar{F}_2(s)N_i(s)\hat{Q}(s))}{(\bar{X}(s)\bar{F}_2(s)N_i(s) - \bar{Y}(s)F_1(s))N_o(s)}. \quad (33)$$

Since $\bar{X}(s)\bar{F}_2(s)N_i(s) - \bar{Y}(s)F_1(s) = I$ and $\bar{q}(s)/N_o(s) \in RH_\infty$, $\bar{C}_d(s) \in RH_\infty$ in (33) is satisfied. The sufficiency has been shown.

In this way, we have thus proved Theorem 3.1. $\square$

Using the result in Theorem 3.1, we can design $C_1(s)$ to make $\bar{C}_d(s)$ stable. From Lemma 2.2, $C_1(s)$ also needs to make $G(s)$ stable. In the next section, we will clarify that $C_1(s)$ designed in Theorem 3.1 makes $G(s)$ stable.

4. Stability of Control System. In this section, we will clarify that $C_1(s)$ satisfying conditions in Theorem 3.1 makes the control system in Figure 1 stable.

From the definition of internal stability, if all transfer functions $C_1(s)G(s)/(1+C_1(s)G(s))$, $G(s)/(1+C_1(s)G(s))$, $C_1(s)/(1+C_1(s)G(s))$ and $1/(1+C_1(s)G(s))$ are stable, then $C_1(s)$ makes $G(s)$ stable. From (27), (28), (29) and (30), all transfer functions $C_1(s)G(s)/(1+C_1(s)G(s))$, $G(s)/(1+C_1(s)G(s))$, $C_1(s)/(1+C_1(s)G(s))$ and $1/(1+C_1(s)G(s))$ are written by

$$\frac{C_1(s)G(s)}{1+C_1(s)G(s)} = -\left(\bar{X}(s) - F_1(s)\hat{Q}(s)\right)N_i(s)N_o(s), \quad (34)$$

$$\frac{G(s)}{1 + C_1(s)G(s)} = - \left(\bar{Y}(s) - \bar{F}_2(s)N_i(s)\hat{Q}(s) \right) N_i(s)N_o(s), \quad (35)$$

$$\frac{C_1(s)}{1 + C_1(s)G(s)} = - \left(\bar{X}(s) - F_1(s)\hat{Q}(s) \right) D(s) \quad (36)$$

and

$$\frac{1}{1 + C_1(s)G(s)} = - \left(\bar{Y}(s) - \bar{F}_2(s)N_i(s)\hat{Q}(s) \right) D(s). \quad (37)$$

Since all transfer functions $C_1(s)G(s)/(1 + C_1(s)G(s))$, $G(s)/(1 + C_1(s)G(s))$, $C_1(s)/(1 + C_1(s)G(s))$ and $1/(1 + C_1(s)G(s))$ are stable, $C_1(s)$ stabilizes $G(s)$.

Thus, we have found that $C_1(s)$ satisfying Theorem 3.1 makes the control system in Figure 1 stable.

5. Control Characteristics. In this section, we explain control characteristics of the control system in Figure 1.

First, the input output characteristic of the control system in Figure 1 is shown. The transfer function from the reference input $r(s)$ to the output $y(s)$ and that from the reference input $r(s)$ to the error $e_r(s) = r(s) - y(s)$ are written by

$$y(s) = \frac{C_1(s)G(s)}{1 + C_1(s)G(s)} r(s) \quad (38)$$

and

$$e_r(s) = r(s) - y(s) = \frac{1}{1 + C_1(s)G(s)} r(s), \quad (39)$$

respectively. From the internal model principle [30], in order to make the output $y(s)$ to follow the non-periodic reference input $r(s)$ without steady-state error, $C_1(s)$ should have an internal model of the reference input $r(s)$. $D_c(s)$ is written by the form

$$D_c(s) = \frac{\tilde{Q}(s)}{C_r(s)}, \quad (40)$$

where $C_r(s)$ is internal model of the reference input $r(s)$ and assumed to be of minimum phase and bi-proper, and $\tilde{Q}(s) \in RH_\infty$ is a function that makes $\tilde{Q}(s)/C_r(s)$ has no unstable pole-zero cancellation.

In order to design $D_c(s)$ in (40), $\hat{Q}(s)$ is designed such as satisfying

$$\hat{Q}(s) = \frac{1}{\bar{F}_2(s)N_i(s)} \left(\bar{Y}(s) - \frac{\tilde{Q}(s)}{C_r(s)} \right), \quad (41)$$

where $\tilde{Q}(s)$ is any function to make (41) stable. Using $\hat{Q}(s)$ in (41), we have

$$\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} se(s) = \lim_{s \rightarrow 0} s \frac{\tilde{Q}(s)}{C_r(s)} D(s) (1 + Q(s)) r(s) = 0. \quad (42)$$

This implies that the input output characteristics of the control system in Figure 1 are improved by using the controller $C_1(s)$.

Next, about the disturbance attenuation characteristic, from (17), the disturbance characteristic is specified using controllers $C_d(s)$ and $C_n(s)$ of $C_2(s)$ in (14). From (18), the periodic disturbance with period T is attenuated effectively.

In this way, the purpose of $C_1(s)$ is to specify the input/output characteristics, and that of $C_2(s)$ is to specify the disturbance attenuation characteristics. This implies that the control system in Figure 1 is a kind of two-degree-of-freedom control system.

6. Design Procedure. In this section, we show a design procedure of the control system in Figure 1.

A design procedure of the control system in Figure 1 is summarized as follows.

- Step 1) Obtain the coprime factors $N(s)$ and $D(s)$ satisfying (10).
 Step 2) Obtain the inner function $N_i(s)$ and the outer function $N_o(s)$ in (20) by factorizing $N(s)$.
 Step 3) Design $F_1(s) \in RH_\infty$ and $F_2(s) \in RH_\infty$, and then obtain $\bar{F}_2(s)$:
 (a) Settle $Q(s)$ in (8) and (9) to satisfy (11).
 (b) Design $F_1(s) \in RH_\infty$ and $F_2(s) \in RH_\infty$ using the method in [8].
 (c) Obtain $\bar{F}_2(s)$ by using (22).
 Step 4) Define $\bar{q}(s)$ by (19).
 Step 5) Obtain $\hat{Q}(s)$ to make $D_c(s)$ have the form in (40):
 (a) Design $\tilde{Q}(s)$ to make $\hat{Q}(s)$ in (41) proper.
 (b) Obtain $\hat{Q}(s)$ by (41).
 Step 6) Find $N_c(s)$ and $D_c(s)$, and then obtain $C_1(s)$:
 (a) Design $C_r(s)$ which is internal model of the reference input $r(s)$, and of minimum phase and bi-proper.
 (b) Find $\bar{X}(s)$ and $\bar{Y}(s)$ satisfying (30).
 (c) Obtain $N_c(s)$ and $D_c(s)$ by (28) and (29) with $\hat{Q}(s)$ which is obtained by previous step.
 (d) Obtain $C_1(s)$ by (27) with the obtained $N_c(s)$ and $D_c(s)$.
 Step 7) Design $C_d(s)$ and $C_n(s)$:
 (a) Obtain $\bar{C}_d(s)$ by (26).
 (b) Obtain $C_d(s)$ and $C_n(s)$ by (24) and (25), respectively.

7. Numerical Example. In this section, a numerical example is shown to illustrate the effectiveness of the proposed method.

Consider a problem to design the system in Figure 1 to attenuate periodic disturbance $d(s)$ with period $T = \pi$ [sec], and follow the reference input $r(t) = t$ for the non minimum-phase plant $G(s)$ given by

$$G(s) = \frac{s-3}{(s+3)(s-1)}. \quad (43)$$

Coprime factors $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ are given as

$$N(s) = \frac{s-30}{(s+10)(s+3)} \quad (44)$$

and

$$D(s) = \frac{s-1}{s+1}, \quad (45)$$

respectively. The inner function and outer function of $N(s)$ in (44) is given as

$$N_i(s) = \frac{-(s-30)}{s+30} \quad (46)$$

and

$$N_o(s) = \frac{-(s+30)}{(s+3)(s+10)}, \quad (47)$$

respectively.

$F_1(s)$ in (8) and $F_2(s)$ in (9) are designed based on the results of [8]. To design $F_1(s)$ in (8) and $F_2(s)$ in (9) using $N(s)$ in (44) and $D(s)$ in (45), $Q(s)$ is settled as

$$Q(s) = \frac{2s + 9}{s + 1}. \quad (48)$$

Using $Q(s)$ in (48), $F_1(s)$ in (8) and $F_2(s)$ in (9) are designed as

$$F_1(s) = \frac{-3s^2 - 7 * s + 10}{s^2 + 11 * s + 10} \quad (49)$$

and

$$F_2(s) = \frac{3 * s^2 - 80 * s - 300}{s^3 + 14 * s^2 + 43 * s + 30}. \quad (50)$$

From $F_2(s)$ in (50) and $N_i(s)$ in (46), $\bar{F}_2(s)$ is given by

$$\bar{F}_2(s) = \frac{-3(s + 30)(s + 3.333)}{(s + 10)(s + 3)(s + 1)}. \quad (51)$$

$\bar{q}(s)$ in (19) is settled by

$$\bar{q}(s) = \frac{1}{6.2832s + 1}. \quad (52)$$

$\bar{X}(s)$ and $\bar{Y}(s)$ are given by the form in

$$\bar{X}(s) = \frac{-22(s + 3)}{29(3s + 10)} \quad (53)$$

and

$$\bar{Y}(s) = \frac{29s + 370}{29(3s + 10)}, \quad (54)$$

respectively. Since $C_1(s)$ should have twice poles at $s = 0$ due to following $r(s) = \mathcal{L}[t] = 1/s^2$, $C_r(s)$ is designed as

$$C_r(s) = \frac{-0.44444s^2(s + 43.86)(s + 2.693)(s + 2.203)(s + 0.9569)}{(s + 10)^3(s + 3.333)(s + 3)(s + 1)}. \quad (55)$$

$\hat{Q}(s)$ that makes $\tilde{Q}(s)$ in (41) proper is setteled as

$$\hat{Q}(s) = \frac{116s + 111}{87(s + 10)}. \quad (56)$$

Using $\hat{Q}(s)$ in (56), $\tilde{Q}(s)$ in (41) is given as

$$\tilde{Q}(s) = \frac{-116s - 111}{87s + 870}. \quad (57)$$

A pair of $\bar{X}(s)$ and $\bar{Y}(s)$ satisfying (30) is given by

$$\bar{X}(s) = \frac{-0.25287(s + 3)}{s + 3.333} \quad (58)$$

and

$$\bar{Y}(s) = \frac{0.33333(s + 12.76)}{s + 3.333}. \quad (59)$$

Using $\bar{X}(s)$ in (53), $\bar{Y}(s)$ in (54) and $\hat{Q}(s)$ in (56), $N_c(s)$ and $D_c(s)$ are found as

$$N_c(s) = \frac{-4.2529(s^2 + 1.776s + 0.8876)(s^2 + 5.881s + 8.83)}{(s + 10)^2(s + 3.333)(s + 1)} \quad (60)$$

and

$$D_c(s) = \frac{0.33333s^2(s + 43.86)(s + 30)(s + 2.693)(s + 2.203)}{(s + 30)(s + 10)^2(s + 3.333)(s + 3)(s + 1)}, \quad (61)$$

respectively. From $N_c(s)$ in (60) and $D_c(s)$ in (61), we have

$$C_1(s) = \frac{-12.759(s+3)(s^2+1.776s+0.8876)(s^2+5.881s+8.83)}{s^2(s+43.86)(s+2.693)(s+2.203)}. \quad (62)$$

Using $F_1(s)$ in (49), $F_2(s)$ in (50), $C_1(s)$ in (62), $C_2(s)$ and period $T = \pi$ [sec], the control system in Figure 1 is constructed. The response of the error $e(t) = r(t) - y(t)$ for the ramp reference input $r(t) = t$ in Figure 1 is shown in Figure 2. Figure 2 shows that the output $y(t)$ follows the ramp reference input $r(t) = t$ without steady-state error. This is an advantage of system in Figure 1 using a disturbance observer over a repetitive control with steady-state error for a ramp reference input.

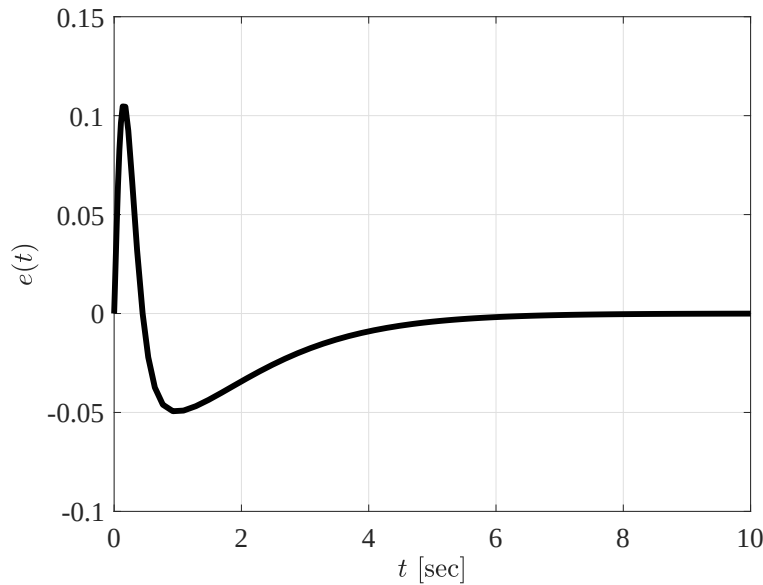


FIGURE 2. The response of the error $e(t) = r(t) - y(t)$ for $r(t) = t$

Next, the disturbance attenuation characteristic is shown. When the periodic disturbance $d(t)$ is given by

$$d(t) = \begin{cases} \frac{2}{\pi}t & 0 \leq t - kt < \frac{\pi}{2} \\ \frac{2}{\pi}(\pi - t) & \frac{\pi}{2} \leq t - kt < \pi \end{cases}, \quad (63)$$

where k is the maximum integer which will not exceed t/T , the disturbance $d(t)$ in (63) is a triangular wave with the period T . The response of the output $y(t)$ for the disturbance $d(t)$ in (63) is shown in Figure 4. The solid line in Figure 4 shows the output $y(t)$ for the disturbance $d(t)$ in (63), and the dotted line in Figure 4 shows the disturbance $d(t)$. From Figure 4, the periodic disturbance $d(t)$ in (63) is attenuated effectively.

To show the effectiveness of the control system in Figure 1, we compare the proposed control system in Figure 1 with a modified repetitive control in [39]. The modified repetitive controller is given as

$$C_{rep}(s) = C_1(s) + \bar{C}(s), \quad (64)$$

where $\bar{C}(s)$ is given as

$$C_{rep}(s) = \frac{\hat{q}(s)e^{-sT}}{1 - \hat{q}(s)e^{-sT}}, \quad (65)$$

with

$$\hat{q}(s) = \frac{1}{0.05s + 1}. \quad (66)$$

The responses of the output $y(t)$ for $d(t)$ in (63) from the control system in Figure 1 and those of a modified control system are shown in Figure 5. The solid line in Figure 5 shows the responses of the output $y(t)$ for $d(t)$ in (63) from the control system in Figure 1, and the dotted line shows the responses of the output $y(t)$ for $d(t)$ in (63) from the modified control system. From Figure 5, the distribution of response of $y(t)$ for $d(t)$ in (63) from the control system in Figure 1 is smaller than that of the modified repetitive control system. Furthermore, the maximum value of the error from the control system in Figure 1 is smaller than that of the modified repetitive controller.

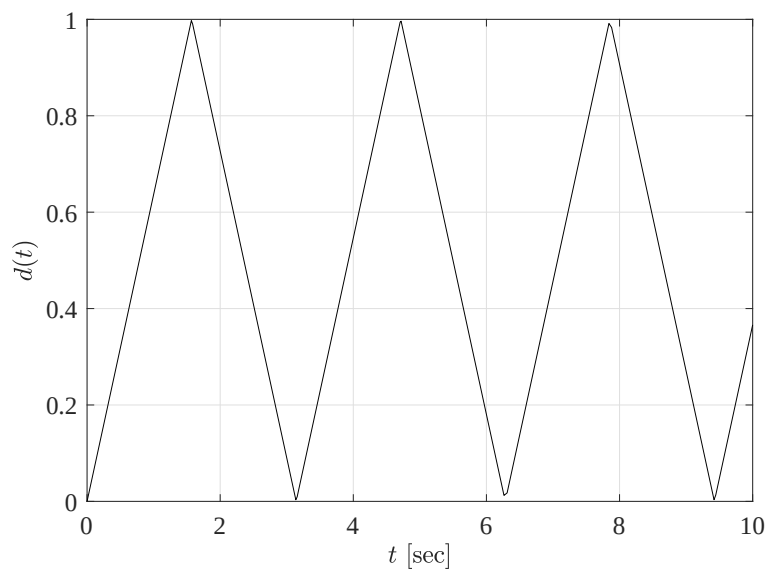


FIGURE 3. Triangular wave disturbance with period $T = \pi$ [sec]

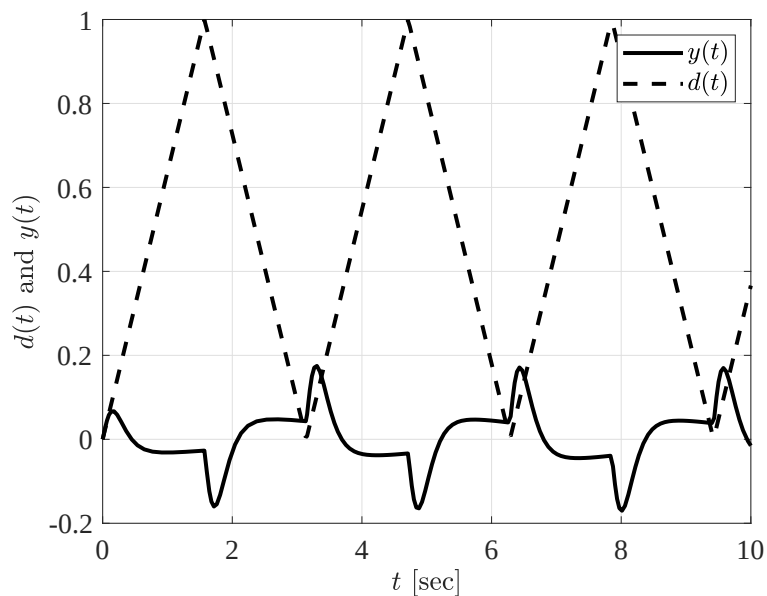


FIGURE 4. The response of the output $y(t)$ for the disturbance $d(t)$ in (63)

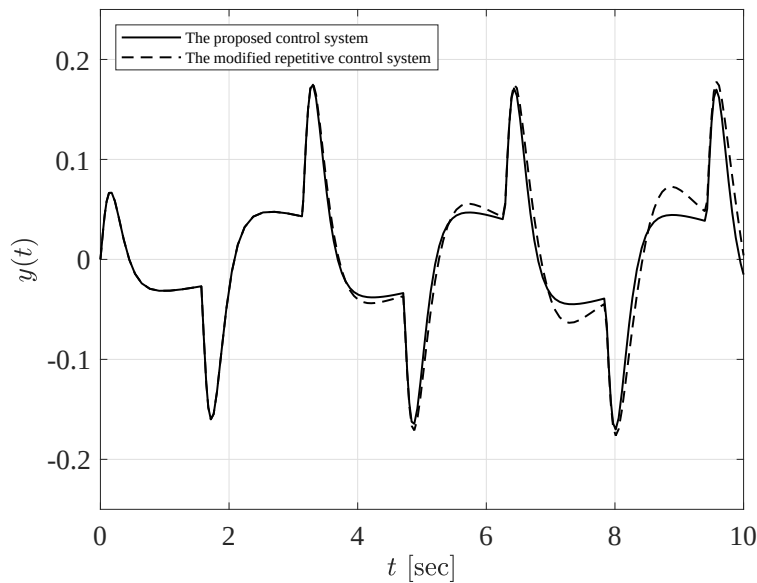


FIGURE 5. The responses of the output $y(t)$ for $d(t)$ in (63) from the control system in Figure 1 and those of a modified repetitive control system

In this way, using the proposed method, we can design the stable control system to attenuate the periodic disturbance and to follow the reference input without steady-state error.

8. Conclusion. In this paper, we have considered a design method for control system using disturbance observer for non-minimum phase plants such that the output follows a non-periodic reference input and the periodic disturbance is attenuated without using repetitive control for periodic disturbances. This paper has clarified the parameterization of all feedback controllers to stabilize a plant to ensure the transfer function from the disturbance to the output has a finite number of poles. In addition, it has been considered the control characteristics of the control system using a disturbance observer to follow the output to a non-periodic reference input, and to attenuate periodic disturbances. Considering the control characteristics of the control system, a design method for the control system using a disturbance observer has been proposed. Finally, in order to illustrate the effectiveness of the design method for the control system using a disturbance observer for periodic disturbance, a numerical example has been shown.

Using the proposed design method, we can construct a control system using a disturbance observer for non-minimum phase plants to follow the output to a non-periodic reference input and to attenuate periodic disturbances without using repetitive control for periodic disturbances. However, this proposed design method often amplifies the influence of disturbances with a period that is different from the period used for the controller design on the output. In addition, when the disturbance has not only periodic signals but also non-periodic signals, the proposed design method does not always attenuate non-periodic disturbances. Thus, a design method for a control system using a disturbance observer to overcome these problems should be considered. These design methods will be discussed in other articles.

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