

STATE OF CHARGE ESTIMATION OF LITHIUM BATTERIES USING A MULTI-INNOVATION ADAPTIVE UNSCENTED KALMAN FILTER WITH QR DECOMPOSITION

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ABSTRACT. *Accurate estimation of a lithium-ion battery's State of Charge (SOC) is critical for safety, reliability, and performance in battery systems. This paper proposes an SOC estimation method based on a second-order RC equivalent circuit model, whose parameters are identified online using a recursive least squares algorithm with forgetting factor. The open-circuit voltage versus SOC relationship is fitted via polynomial regression, enabling conversion between terminal voltage and SOC. To address nonlinearities, uncertainties, and numerical instability, an enhanced Unscented Kalman Filter (UKF) is developed incorporating three improvements: adaptive adjustment of system and measurement noise covariances, multi-innovation usage of historical residuals for gain updating, and QR decomposition for square-root filtering instead of conventional Cholesky decomposition. Simulation experiments under Hybrid Pulse Power Characterization (HPPC) and Urban Dynamometer Driving Schedule (UDDS) load profiles demonstrate that this QR-MIAUKF method achieves superior SOC estimation accuracy and robustness compared to the Extended Kalman Filter (EKF), with estimation errors maintained within a few thousandths of full scale. The results suggest the proposed algorithm is a promising approach for real-time battery management systems.*

Keywords: Lithium-ion battery, State of charge estimation, Recursive least squares, Unscented Kalman filter, Multi-innovation, Adaptive filtering, QR decomposition

1. Introduction. Lithium-ion batteries, with high energy density, long cycle life, and environmental friendliness, are widely applied in electric vehicles and energy storage systems. The Battery Management System (BMS) ensures safe and efficient battery operation, where the State of Charge (SOC) is a key parameter for energy management and charge/discharge control. Accurate SOC estimation is crucial for determining remaining battery energy and directly affects battery life, safety, and vehicle reliability [1].

SOC estimation methods are generally classified into two categories. The first includes traditional methods based on battery output parameters, such as the ampere-hour integral

method and the Open-Circuit Voltage (OCV) method. The ampere-hour integral method [2] is computationally simple but suffers from cumulative errors and sensitivity to external disturbances [3]. The OCV method offers higher accuracy but requires long rest periods to obtain the OCV value at a specific SOC, making it unsuitable for real-time applications [4].

The second category involves model-based approaches using intelligent or filtering algorithms, such as neural networks, Particle Filters (PF), Extended Kalman Filters (EKF), and Unscented Kalman Filters (UKF). Neural networks can achieve high accuracy by training on current and voltage data but require large datasets and high computational resources [5]. Therefore, the application of neural networks to online real-time SOC estimation remains challenging [6]. PF handles strong nonlinearities and non-Gaussian noise but suffers from particle degeneracy, resampling issues, and heavy computation [7]. EKF is efficient for online estimation but requires system linearization and Jacobian computation, leading to limited accuracy and stability [8, 9]. Studies have shown that the performance of EKF-based SOC estimation is highly sensitive to modeling errors, parameter variations, and noise characteristics under dynamic operating conditions, which may result in degraded robustness and estimation accuracy [10]. UKF avoids linearization via the Unscented Transformation (UT) and achieves higher accuracy [11]. Recent studies on adaptive UKF variants have demonstrated that properly optimized UKF algorithms can maintain strong stability and robustness across varying dynamic conditions while delivering highly accurate state estimates with lower SOC estimation errors compared to traditional filters [12], but modeling errors or uncertain noise may result in a non-semi-positive definite covariance matrix P , causing divergence [13, 14].

To overcome these issues, this paper proposes an improved SOC estimation algorithm based on a second-order RC equivalent circuit model. Model parameters are identified online using a Recursive Least Squares (RLS) algorithm with a forgetting factor, and the OCV-SOC relationship is obtained by polynomial fitting. At the filtering level, an adaptive strategy updates process and measurement noise covariance matrices online, while a multi-innovation technique enhances Kalman gain accuracy. QR decomposition is further employed to implement a square-root UKF for improved numerical stability. Simulations under Hybrid Pulse Power Characterization (HPPC) and Urban Dynamometer Driving Schedule (UDDS) conditions validate the proposed method, which outperforms conventional EKF and standard UKF in both accuracy and robustness of SOC estimation. The remainder of this paper is organized as follows. Chapter 2 presents the lithium battery modeling and parameter identification procedures. Chapter 3 details the proposed SOC estimation algorithm using a multi-innovation adaptive unscented Kalman filter with QR decomposition. Chapter 4 provides simulation results under various conditions to demonstrate the effectiveness of the proposed method. Finally, Chapter 5 concludes the paper and discusses potential directions for future work.

2. Modeling and Parameter Identification of Lithium-Ion Batteries.

2.1. Equivalent circuit model. In order to address the challenge of SOC estimation for lithium-ion batteries, an appropriate equivalent circuit model must first be established [15]. Equivalent circuit models, which consist of voltage sources, resistors, and capacitors, describe the external electrical characteristics of the battery through circuit network representations. Such models not only satisfy the real-time requirements for parameter identification but also enable efficient simulation of battery behavior. Typical models include the internal resistance (R_{int}) model and the Thevenin model. Among these, the second-order RC model extends the first-order RC model by incorporating an additional

RC network, thereby achieving improved accuracy while maintaining computational complexity at an acceptable level. Notably, under conditions of rapid current transients or prolonged charge/discharge cycles, the two RC networks capture distinct time constants of the battery response, thereby providing a more accurate representation of both the fast and slow dynamic behaviors. Considering the balance between modeling accuracy and parameter identifiability, this work employs the second-order RC model for battery characterization. The configuration of the proposed second-order RC model is illustrated in Figure 1.

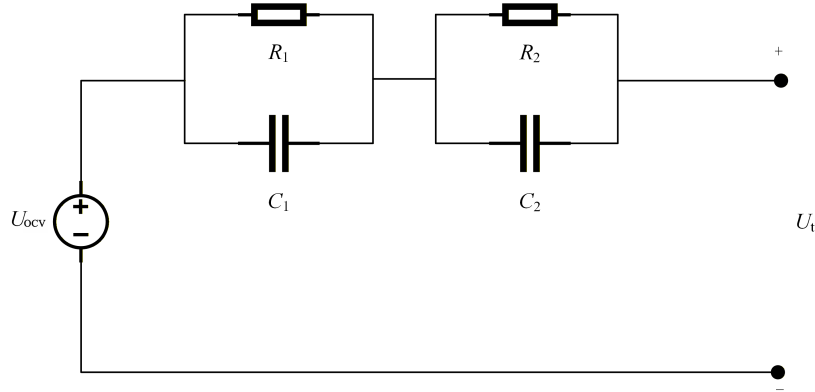


FIGURE 1. Second-order RC circuit model

In the equivalent circuit model, R represents the ohmic resistance of the battery, R_1 and R_2 denote the electrochemical polarization resistance and the concentration polarization resistance, respectively. Similarly, C_1 and C_2 correspond to the electrochemical and concentration polarization capacitances. I is the charge/discharge current, U_{OCV} is the open-circuit voltage, U_t is the terminal voltage, while U_1 and U_2 denote the electrochemical polarization voltage and the concentration polarization voltage, respectively. Based on the relationships defined in the equivalent circuit, the corresponding state-space equations and the terminal voltage expression can be formulated as follows:

$$\begin{cases} \dot{U}_1 = -\frac{U_1}{R_1 C_1} + \frac{I}{C_1}, \\ \dot{U}_2 = -\frac{U_2}{R_2 C_2} + \frac{I}{C_2} \end{cases} \quad (1)$$

$$U_t = U_{OCV} - U_1 - U_2 - IR \quad (2)$$

According to the ampere-hour integral method, when the initial SOC value is known, the current can be integrated to obtain the present SOC. Thus, the SOC expression of this model can be written as

$$SOC(t) = SOC(t_0) - \frac{\eta}{Q_n} \int_{t_0}^t I(\tau) d\tau \quad (3)$$

Here, η denotes the Coulombic efficiency, which is related to the operating temperature, cycle number, and discharge rate of the lithium-ion battery, and Q_n represents the nominal capacity. When applying the Kalman filter for SOC estimation, the battery system model needs to be discretized. By discretizing (1)-(3), we obtain

$$\begin{cases} \begin{bmatrix} U_{1,k+1} \\ U_{2,k+1} \\ SOC_{k+1} \end{bmatrix} = \begin{bmatrix} e^{-\frac{T_s}{\tau_1}} & 0 & 0 \\ 0 & e^{-\frac{T_s}{\tau_2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} U_{1,k} \\ U_{2,k} \\ SOC_k \end{bmatrix} + \begin{bmatrix} R_1 \left(1 - e^{-\frac{T_s}{C_1}}\right) \\ R_2 \left(1 - e^{-\frac{T_s}{C_2}}\right) \\ -\frac{\eta T_s}{Q_n} \end{bmatrix} I_k, \\ U_{t,k+1} = U_{OCV,k+1} - U_{1,k+1} - U_{2,k+1} - I_k R \end{cases} \quad (4)$$

Here, $\tau_1 = R_1 C_1$ and $\tau_2 = R_2 C_2$, T_s denotes the sampling period, which can be set to 1 s to ensure accurate SOC estimation while keeping the computational load manageable. The SOC estimation presented in this study is conducted on the basis of this model.

2.2. Identification of equivalent circuit model parameters. In system parameter identification, the RLS method is widely used due to its simplicity in programming, fast convergence, and low computational cost [16]. Compared with the ordinary least squares method, RLS updates the parameter estimates at the current time by directly incorporating the current experimental data and the parameter estimates from the previous time step. This reduces the computational load in online parameter identification, making it particularly suitable for real-time estimation of SOC in lithium-ion batteries.

During the parameter identification process using the RLS method, as the number of samples and the amount of data continue to increase, a phenomenon of “data saturation” may occur, which prevents the estimation algorithm from converging to the true parameter values. “Data saturation” refers to the situation where the continuous accumulation of historical data diminishes the contribution of new data, reducing or even eliminating its corrective effect [17, 18].

To decrease the influence of old data on the current estimate and highlight the correction effect of new data in the identification process, a forgetting factor λ is introduced. The recursive least squares formula with forgetting factor [19] is expressed as follows:

$$\begin{cases} e(k) = y(k+1) - \varphi^T(k+1) \hat{\theta}(k), \\ \hat{\theta}(k+1) = \hat{\theta}(k) + K(k+1) e(k), \\ K(k+1) = P(k) \varphi(k+1) [\lambda + \varphi^T(k+1) P(k) \varphi(k+1)]^{-1}, \\ P(k+1) = \lambda^{-1} [I - K(k+1) \varphi^T(k+1)] P(k) \end{cases} \quad (5)$$

In the equation, $\theta(k)$ denotes the coefficient matrix of the system’s estimated value at the previous time step; $y(k+1)$ is the actual system output; $\varphi(k+1)$ is the system measurement matrix; $e(k)$ represents the estimation error; K is the estimation gain matrix; P is the error covariance matrix; and I is the identity matrix. The forgetting factor λ ranges from 0 to 1; the smaller the value of λ , the stronger the algorithm’s tracking capability, but the larger the fluctuation. Typically, λ is chosen between 0.95 and 1. When $\lambda = 1$, the algorithm reduces to the standard RLS method. For lithium-ion batteries, the RC parameters typically vary slowly under normal operating conditions, so a large forgetting factor is appropriate. Therefore, λ is set to 0.999 to ensure smooth and accurate parameter estimation in this study.

By applying the Laplace transform to Equation (2), the corresponding transfer function can be obtained as follows:

$$G(s) = \frac{U_{OCV}(s) - U_t(s)}{I(s)} = R_0 + \frac{R_1}{\tau_1 s + 1} + \frac{R_2}{\tau_2 s + 1}$$

$$= \frac{R_0 s^2 + \frac{(R_0 \tau_1 + R_0 \tau_2 + R_1 \tau_2 + R_2 \tau_1)s}{\tau_1 \tau_2} + \frac{R_0 + R_1 + R_2}{\tau_1 \tau_2}}{s^2 + \frac{\tau_1 + \tau_2}{\tau_1 \tau_2} s + \frac{1}{\tau_1 \tau_2}} \quad (6)$$

In the above equation, $G(s)$ denotes the system transfer function, and s is the Laplace operator. By applying bilinear discretization and variable substitution, the equation can be converted into

$$G(z^{-1}) = \frac{\frac{b_4 T^2 + b_3 T + b_2 b_0}{T^2 + b_2 T + b_1} + \frac{-b_3 T - 2b_2 b_0}{T^2 + b_1 T + b_2} z^{-1} + \frac{b_2 b_0}{T^2 + b_1 T + b_2} z^{-2}}{1 + \frac{-b_1 T - 2b_2}{T^2 + b_1 T + b_2} z^{-1} + \frac{b_2}{T^2 + b_1 T + b_2} z^{-2}} \quad (7)$$

By performing a variable substitution, we obtain

$$G(z^{-1}) = \frac{a_3 + a_4 z^{-1} + a_5 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \quad (8)$$

Transform the discretized transfer function into a time-domain difference equation:

$$y(k) = U_{OCV}(k) - U_t(k) = a_1 y(k-1) + a_2 y(k-2) + a_3 I(k) + a_4 I(k-1) + a_5 I(k-2) \quad (9)$$

The state equations and parameters to be identified are

$$\begin{cases} \varphi(k) = \begin{bmatrix} y(k-1) & y(k-2) & I(k) & I(k-1) & I(k-2) \end{bmatrix}, \\ \theta^T = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \end{bmatrix} \end{cases} \quad (10)$$

Thus, Equation (11) can be written as

$$y(k) = \varphi^T(k) \theta \quad (11)$$

By setting the initial values θ_0 and $P(0)$, the identification yields $[a_1 \ a_2 \ a_3 \ a_4 \ a_5]$, and through variable substitution, the battery parameters to be identified can be obtained. The identification results are shown in Figure 2.

As shown in Figure 2, except for some spikes and negative values at the beginning and the end of the operation, the identified parameters remain relatively stable throughout the rest of the time, demonstrating satisfactory performance.

To verify the accuracy of the second-order RC equivalent circuit model and the parameter identification using the recursive least squares method with forgetting factor, the identified parameters were applied to the model for further experiments. The comparison between the actual measured voltage and the model output voltage is presented in Figure 3.

Figure 3 illustrates the error curve between the model output voltage and the actual measured voltage. It can be observed from the figure that most of the errors are within 0.1 V, which is sufficient to meet the accuracy requirements of the battery model.

3. SOC Estimation for Lithium-Ion Batteries.

3.1. Unscented Kalman filter. The UKF addresses nonlinear estimation problems through the use of the unscented transform. Its core idea is to approximate the mean and covariance of the state distribution by propagating a set of deterministically chosen sigma points with associated weights. Compared with the EKF, which requires linearization of nonlinear functions, the UKF avoids the cumbersome computation of Jacobian matrices while maintaining second-order accuracy in the propagation of the mean and covariance. As a result, the UKF not only improves the estimation accuracy for nonlinear systems but also preserves computational efficiency. The specific procedure of the unscented transform is given as follows:

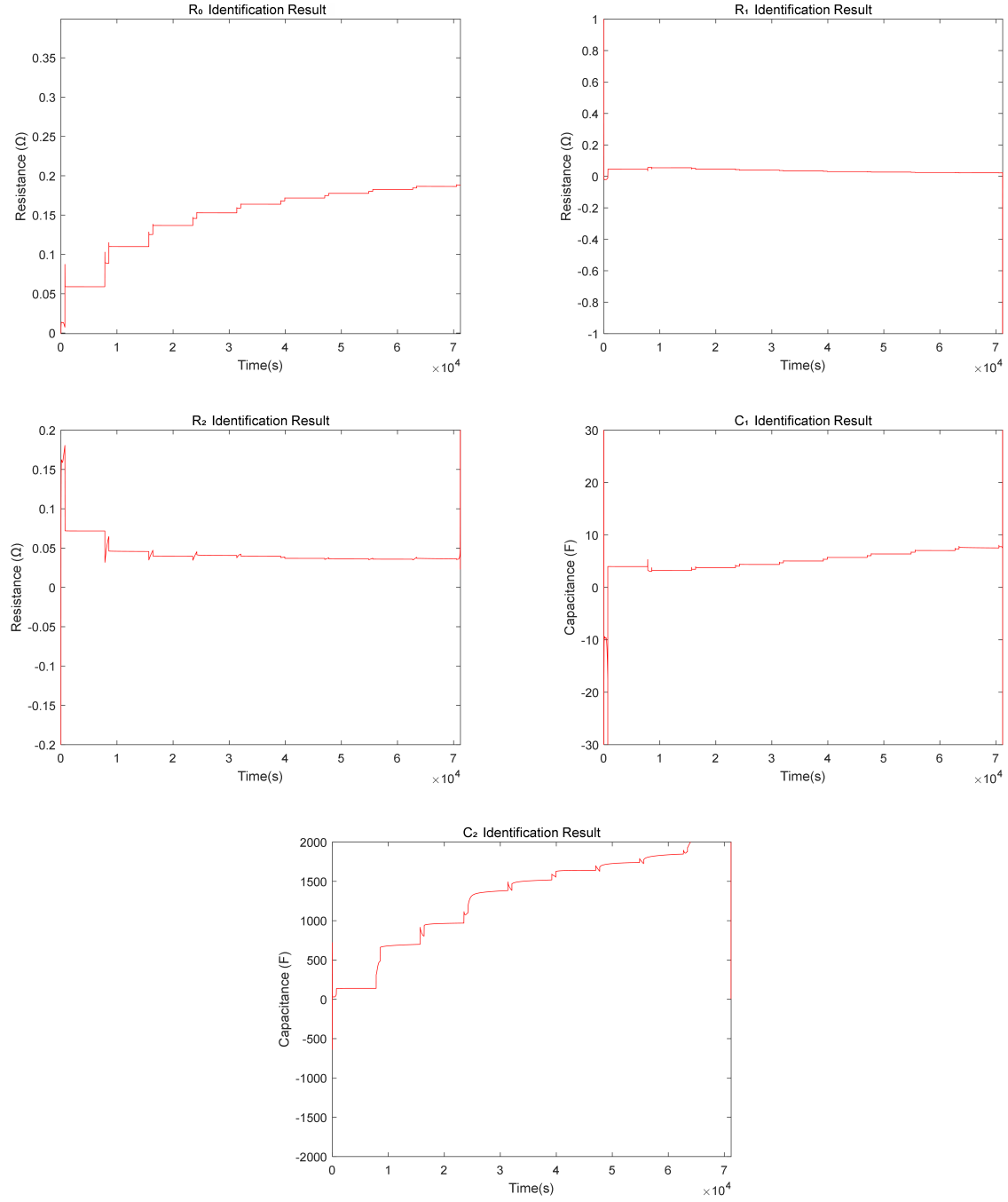


FIGURE 2. Identification results

(1) Calculation of Mean and Covariance Weights:

$$\left\{ \begin{array}{l} \lambda = \alpha^2(n+k) - n, \\ \omega_0^m = \frac{\lambda}{n+\lambda}, \\ \omega_i^m = \frac{1}{2(n+\lambda)}, \quad i = 1, 2, \dots, 2n, \\ \omega_0^c = \frac{\lambda}{n+\lambda} + (1 - \alpha^2 + \beta), \\ \omega_i^c = \frac{1}{2(n+\lambda)}, \quad i = 1, 2, \dots, 2n \end{array} \right. \quad (12)$$

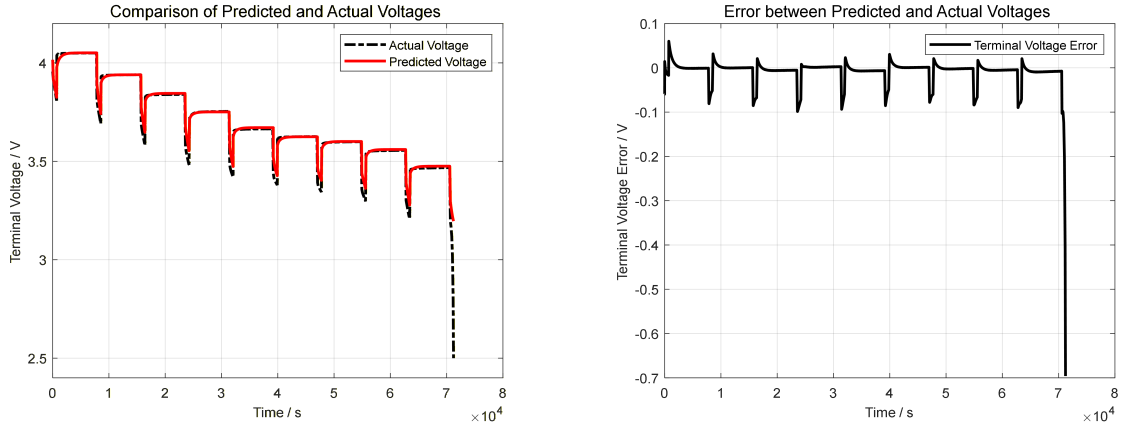


FIGURE 3. Comparison and error between actual voltage and model output voltage

In the above equations, ω_i^m and ω_i^c denote the weights for the mean and covariance, respectively. The parameter λ is the scaling factor, consistent with that defined in (12). The parameter α is the spread coefficient, which satisfies $\alpha \in (0, 1)$. The parameter β is introduced to incorporate the higher-order moments of the prior distribution into the covariance estimation of the unscented transform. For Gaussian-distributed process noise, setting $\beta = 2$ is theoretically optimal, as it enables accurate capture of the fourth-order moments and reduces covariance approximation errors under nonlinear transformations. This setting is widely adopted in the original UKF formulation and related literature. Therefore, β is set to 2 in this study. Finally, k is an auxiliary scaling parameter.

(2) Construction of Sigma Points:

$$\begin{cases} X_k^0 = \hat{x}_k, \\ X_k^i = \hat{x}_k + \left(\sqrt{(n + \lambda)P_k} \right)_i, & i = 1, 2, \dots, n, \\ X_k^{i+n} = \hat{x}_k - \left(\sqrt{(n + \lambda)P_k} \right)_i, & i = 1, 2, \dots, n \end{cases} \quad (13)$$

In the above equations, a symmetric sampling strategy is employed, generating a total of $2n + 1$ sigma points, where n denotes the dimension of the state vector. The term $\left(\sqrt{(n + \lambda)P_k} \right)_i$ represents the i -th column (or row) of the matrix $\sqrt{(n + \lambda)P_k}$.

(3) Sigma Point Propagation:

$$X_{k+1|k}^i = f(X_k^i, u_k), \quad i = 0, 1, 2, \dots, 2n \quad (14)$$

(4) Predicted State Mean and Covariance Update:

$$\begin{cases} \hat{x}_{k+1|k} = \sum_{i=0}^{2n} \omega_i^m X_{k+1|k}^i, \\ P_{xx,k+1|k} = \sum_{i=0}^{2n} \omega_i^c (X_{k+1|k}^i - \hat{x}_{k+1|k}) (X_{k+1|k}^i - \hat{x}_{k+1|k})^T + Q_k \end{cases} \quad (15)$$

Here, Q_k denotes the covariance matrix of the system noise ω_k .

(5) Calculation of Predicted Measurement Sigma Points:

$$\begin{cases} x_{k+1|k}^0 = \hat{x}_{k+1|k}, \\ x_{k+1|k}^i = \hat{x}_{k+1|k} + \left(\sqrt{(n + \lambda)P_{xx,k+1|k}} \right)_i, & i = 1, 2, \dots, n, \\ x_{k+1|k}^{i+n} = \hat{x}_{k+1|k} - \left(\sqrt{(n + \lambda)P_{xx,k+1|k}} \right)_i, & i = 1, 2, \dots, n \end{cases} \quad (16)$$

(6) Substituting Sigma Points into the Measurement Equation to Obtain System Observations:

$$Y_{k+1|k}^i = g(x_{k+1|k}^i, u_k), \quad i = 0, 1, 2, \dots, 2n \quad (17)$$

(7) Calculation of the Mean and Covariance Matrix of the System Observation Variables:

$$\begin{cases} \hat{y}_{k+1|k} = \sum_{i=0}^{2n} \omega_i^m Y_{k+1|k}^i, \\ P_{yy,k+1|k} = \sum_{i=0}^{2n} \omega_i^c (Y_{k+1|k}^i - \hat{y}_{k+1|k}) (Y_{k+1|k}^i - \hat{y}_{k+1|k})^T + R_k \end{cases} \quad (18)$$

(8) Joint Covariance between State Variables and Observation Variables:

$$P_{xy,k+1|k} = \sum_{i=0}^{2n} \omega_i^c (X_{k+1|k}^i - \hat{x}_{k+1|k}) (Y_{k+1|k}^i - \hat{y}_{k+1|k})^T \quad (19)$$

(9) Update Kalman Gain:

$$K_k = P_{xy,k+1|k} P_{yy,k+1|k}^{-1} \quad (20)$$

(10) Update State and Covariance:

$$e_{k+1} = y_k - \hat{y}_{k+1|k}, \quad (21)$$

$$\hat{x}_{k+1} = \hat{x}_{k+1|k} + K_{k+1} e_{k+1}, \quad (22)$$

$$P_{k+1} = P_{xx,k+1|k} - K_{k+1} P_{yy,k+1|k} K_{k+1}^T \quad (23)$$

The above describes the detailed computational procedure of the UKF algorithm.

3.2. Adaptive filtering algorithm. In each iteration of the algorithm, when errors arise due to changes in system dynamics or parameter fluctuations, it is necessary to correct the system noise covariance Q and measurement noise covariance R before updating the system state variables and covariances. This correction allows the noise characteristics to adaptively update with the estimation results, thereby reducing error accumulation and improving estimation accuracy.

When applying the UKF for estimation, the measurement noise covariance and system noise covariance are typically set as constants. As the algorithm iterates, this may cause error accumulation and even divergence. Therefore, in this study, an adaptive filtering mechanism is incorporated into the UKF to reduce the influence of system noise on the filtering results. The adaptive filtering algorithm enables online correction of the system noise, enhancing the accuracy of the UKF. The updating rules are as follows:

$$\begin{cases} H_{k+1} = \frac{1}{S} \sum_{i=k-s+2}^{k+1} e_i e_i^T, \\ Q_{k+1} = K_{k+1} H_{k+1} K_{k+1}^T, \\ R_{k+1} = H_{k+1} + \sum_{i=0}^{2n} \omega_i^c (Y_{k+1|k}^i - \hat{y}_{k+1|k}) (Y_{k+1|k}^i - \hat{y}_{k+1|k})^T \end{cases} \quad (24)$$

Here, S represents the window size, Q_{k+1} and R_{k+1} are the system noise covariance and measurement noise covariance, respectively, and H_{k+1} is the approximate innovation covariance. In this study, $S = 5$. The adaptive UKF algorithm is realized by incorporating the adaptive noise adjustment module of Equation (24) into Equation (23).

3.3. Multi-innovation algorithm. The multi-innovation theory reutilizes historical data, thereby improving the estimation accuracy. In the estimation algorithm, both the error between the measurements and the estimates (i.e., residuals) and the error between the measurements and the predictions (i.e., innovations) are used to update the system noise and covariance. The multi-innovation matrix is defined as in Equation (25):

$$E_{m,k+1} = \begin{bmatrix} e_{k+1} \\ e_k \\ \vdots \\ e_{k-m+2} \end{bmatrix} = \begin{bmatrix} y_{k+1} - \hat{y}_{k+1|k} \\ y_k - \hat{y}_k \\ \vdots \\ y_{k-m+2} - \hat{y}_{k-m+2} \end{bmatrix} \quad (25)$$

Here, m denotes the innovation length. Meanwhile, the Kalman filter gain K_k is extended to a gain matrix as in Equation (26):

$$K_{m,k+1} = [K_{k+1} \quad K_k \quad \cdots \quad K_{k-m+2}] \quad (26)$$

The weight vector is defined as in Equation (27):

$$\lambda_m = \left[0.5 \quad \frac{0.5}{m-1} \quad \frac{0.5}{m-1} \quad \cdots \quad \frac{0.5}{m-1} \right]^T \quad (27)$$

Based on the Adaptive Unscented Kalman Filter (AUKF) algorithm, Equation (25) is added after Equation (21), Equation (26) is added after Equation (20), and Equation (22) is replaced by Equation (28), thus completing the implementation of the Multi-Innovation Adaptive Unscented Kalman Filter (MIAUKF) algorithm.

$$\hat{x}_{k+1} = \hat{x}_{k+1|k} + K_{h,k+1} \lambda_m E_{m,k+1} \quad (28)$$

where m is the window size. Adjusting this value affects the utilization of historical data, thereby influencing the estimation accuracy.

3.4. QR decomposition. In the implementation of the UKF, the square root decomposition of the state covariance matrix is a critical step. Traditionally, the Cholesky decomposition is employed to obtain the square root of the covariance matrix. However, the Cholesky decomposition requires the covariance matrix to be strictly positive definite. In practical applications, due to inaccurate noise modeling, accumulation of sampling errors, and improper initialization, the covariance matrix may become non-strictly positive definite or even semi-definite, which can lead to decomposition failure and consequently affect the stability and convergence of the filter.

To address this problem, this study introduces QR decomposition to replace the Cholesky decomposition in the UKF. QR decomposition demonstrates better numerical stability and can maintain algorithmic feasibility even when the covariance matrix is close to singular.

During the filtering process, the weighted covariance matrix P needs to be decomposed. For a system with dimension L , the decomposition can be written as Equation (29):

$$P = QR \quad (29)$$

where Q is an orthogonal matrix satisfying $Q^T Q = I$, and R is an upper triangular matrix. Based on the QR decomposition results, the square root form of the covariance matrix can be expressed as

$$\Delta = \sqrt{L + \lambda} Q_P R_P \quad (30)$$

Equation (30) is then used to replace Equation (13) in the improved implementation of the UKF.

4. Simulation Results and Analysis. To verify the effectiveness of the proposed SVD-MIAUKF algorithm, a lithium-ion battery SOC estimation model was established in MATLAB. Both the EKF algorithm and the SVD-MIAUKF algorithm were employed for SOC estimation, and the estimation accuracy was compared. The HPPC profile was selected as the test condition. A 2 Ah 18650 battery was used for the experiments, with the ambient temperature fixed at 25°C. The short-term capacity fade and aging effects were neglected. Considering the discrepancy between the rated and actual capacity, the SOC variation was fixed at 10% for each charge/discharge cycle. The battery was first fully charged (SOC = 100%), followed by charge/discharge cycles at a current rate of 0.5C. The discharge capacity was set to 0.2 Ah, and each discharge cycle lasted for 720 s. After each discharge, the battery was left at rest until reaching equilibrium.

Figure 4 shows the SOC estimation results and error curve of the QR-MIAUKF algorithm under these conditions, while Figure 5 presents the corresponding results using the EKF algorithm. In both cases, the SOC estimated by the algorithms was compared with the reference values obtained using the ampere-hour integration method.

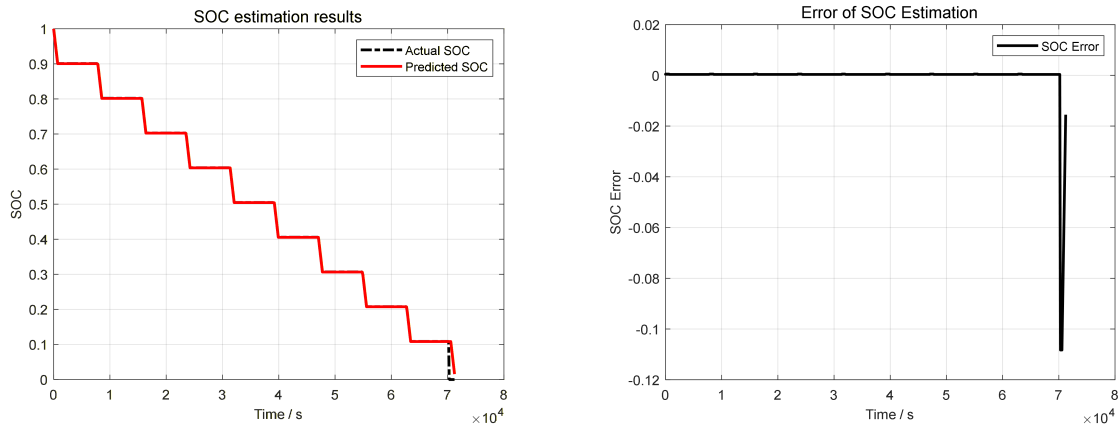


FIGURE 4. SOC estimation using the QR-MIAUKF algorithm

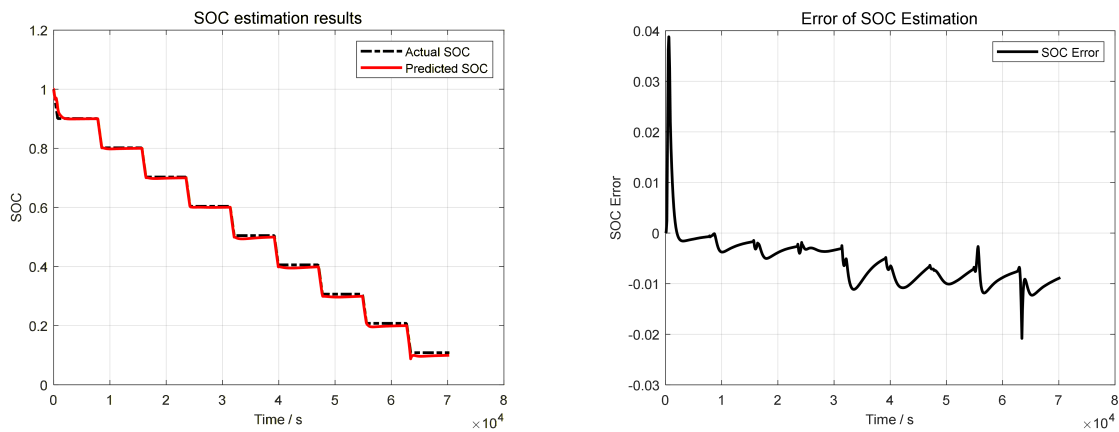


FIGURE 5. SOC estimation using the EKF algorithm

Figures 4 and 5 validate the effectiveness of the QR-MIAUKF and EKF algorithms. Under the same operating condition, except at the end of the discharge period, the QR-MIAUKF algorithm maintained the SOC estimation error within 2×10^{-3} , while the EKF algorithm kept the error around ± 0.02 . Both algorithms closely tracked the reference SOC

values. However, the comparison clearly shows that the QR-MIAUKF algorithm achieved smaller errors, higher accuracy, and better stability than the EKF, indicating superior tracking performance. Therefore, the QR-MIAUKF can be considered a more effective SOC estimation algorithm.

Figure 6 illustrates the SOC estimation results of the QR-MIAUKF algorithm under the UDDS. The estimation error between the algorithm and the reference values was controlled within 3×10^{-4} , which is acceptable. This simulation result demonstrates that the QR-MIAUKF algorithm can still achieve accurate SOC estimation for batteries under different operating conditions.

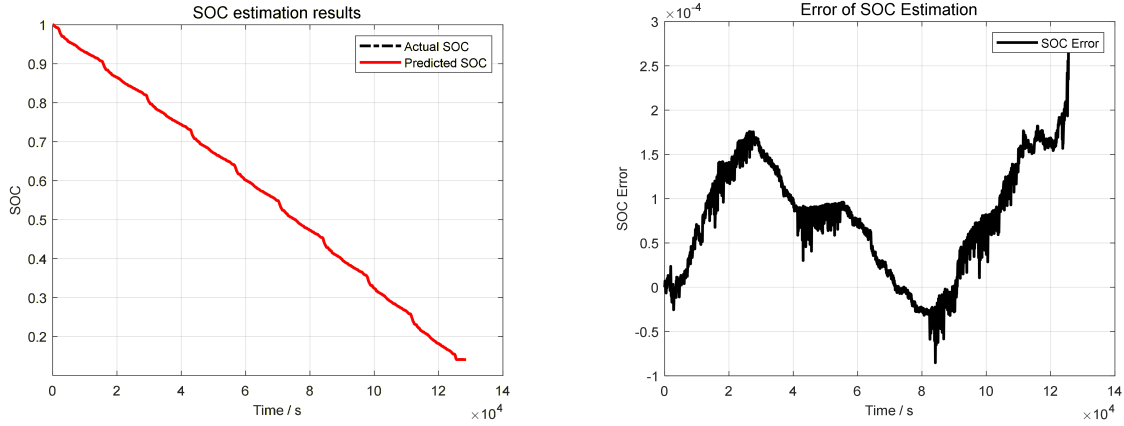


FIGURE 6. SOC estimation results of the QR-MIAUKF algorithm under UDDS condition

5. Conclusions. In this paper, a novel SOC estimation method for lithium-ion batteries has been proposed, which combines a second-order RC equivalent circuit model with an improved UKF. The model parameters were identified online using a recursive least squares algorithm with forgetting factor, and the open-circuit voltage versus SOC relationship was established via polynomial fitting. To enhance the reliability and accuracy of the estimation, three improvements were introduced into the UKF framework: adaptive adjustment of process and measurement noise covariances, multi-innovation utilization of historical residuals, and QR decomposition for square-root filtering.

Simulation results under HPPC and UDDS profiles have demonstrated that the proposed QR-MIAUKF algorithm achieves higher estimation accuracy and better reliability than the conventional EKF and standard UKF, with SOC errors maintained within a few thousandths of full scale. The findings indicate that the proposed method is promising for real-time applications in battery management systems, ensuring both reliability and computational efficiency.

Future research will focus on conducting more systematic experimental validations under different temperatures, aging conditions, and dynamic load profiles to further evaluate the robustness of the proposed algorithm in practical applications. In addition, the SOC estimation framework will be extended to the joint estimation of other critical battery states, such as state of health and internal resistance, in order to support more comprehensive battery management and further improve estimation accuracy.

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