

ELASTIC CURVE FOR RECTANGULAR BEAMS WITH STRAIGHT HAUNCHES UNDER A CONCENTRATED LOAD APPLIED ANYWHERE ON THE BEAM

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ABSTRACT. *This work shows the EC (Elastic Curve) for rectangular beams with straight haunches that support a concentrated load located at any point of the beam and a moment at each end that considers the shear and bending deformations (Timoshenko Theory) to determine the MD (Maximum Deflections), which is the main contribution of this study. Some authors determine the EC of non-prismatic rectangular beams subjected to a concentrated load, assuming bending deformations using the Euler-Bernoulli Theory, which is the CM (Current Model). The methodology is developed by integration, since the integral of the moment is the slope and the integral of the slope is the EC. The EC diagrams are presented for simply supported beams, for perfectly fixed beams at their ends and for the same moments at the ends for PM (Proposed Model) and CM. Results indicate that the smallest MD difference is $PM = 1.0018CM$ and the largest difference is $PM = 1.0276CM$ for simply supported beams, the smallest MD difference is $PM = 1.0071CM$ and the largest is $PM = 1.2334CM$ for the same moments at the ends, and the smallest MD difference is $PM = 0.9896CM$ and the largest is $PM = 0.5443CM$ for beams perfectly fixed at their ends. Therefore, the model developed in this work is more suitable and safer than the CM for structural analysis, because all structures are subjected to bending moments and shear forces, producing shear and bending deformations.*

Keywords: Elastic curve, Rectangular beams, Straight haunches, Concentrated load, Shear and bending deformations, Euler-Bernoulli theory, Timoshenko theory

1. Introduction. Deflections in steel beams and reinforced concrete beams are important for their performance in service, as specifically required by current performance-based design codes. Generally, the deformations of a beam consist of shear and bending deformations.

Many researchers have investigated the EC to obtain the MD, but only consider the bending deformations.

More relevant studies on non-prismatic beams are the following. Yuksel [1] developed a study of symmetrical non-prismatic beams under temperature variations using the FEM

(Finite Element Method). Yuksel [2] estimated the non-prismatic beams with symmetrically shaped parabolic haunches and a uniform haunch length ratio of 0.5. Ponnada and Vipparthy [3] presented a method to determine the deflection in I-shaped prestressed beams. Luévanos-Rojas [4] proposed a model to determine the EC and MD for simply supported beams supporting a uniformly distributed load, considering shear and bending deformations for uniform sections. Ponnada and Thonangi [5] presented the deflections for simply supported non-prismatic prestressed concrete beams. Razavi et al. [6] described the load-deflection relationship to predict the behavior of a CFRP (Carbon Fiber Reinforced Polymer) reinforced concrete slab using RNN (Recurrent Neural Network). Akbas [7] evaluated the large deflections of several simply supported beams with edge cracks. Luévanos-Rojas et al. [8] estimated an analytical model to determine the EC in simply supported beams under a concentrated load considering the shear and bending deformations for uniform sections. Li and Chen [9] presented the deflections in slat beams with shear effect and the discrete effect of slats on the transverse deflection of the beam. Barquero-Cabrero et al. [10] described the EC to obtain the MD of non-prismatic rectangular beams with straight haunches at their ends under moments at their ends and a uniformly distributed load. Unsal et al. [11] experimentally studied the load-deflection behavior of eight two-span continuous concrete beams of reinforced with GFRP (Glass Fiber Reinforced Polymer) and steel bars. Carrera et al. [12] presented the large deflections for composite beams using the node-dependent kinematics concept by FEM. Lin et al. [13] studied the deformations and nonlinear behavior of large deflections in a Euler-Bernoulli beam using the Laplace-Adomian decomposition method. Hakim and Abramovich [14] investigated thin-walled plates under transverse loading due to in-plane stresses. Ramirez-Muñoz et al. [15] presented the EC to obtain the MD of non-prismatic rectangular beams with parabolic haunches at their ends under moments at their ends and a uniformly distributed load. Barraza-Saucedo and Luévanos-Rojas [16] studied the EC to obtain the MD of non-prismatic rectangular beams with parabolic haunches at their ends subjected to moments at their ends and a concentrated load.

Some works describe models for beams with “rectangular”, “T” and “I” cross-sections under uniformly distributed load and/or concentrated load with straight and parabolic haunches, considering shear and bending deformations to determine the fixed-end moments factors, carry-over factors and stiffness factors [17-26]. These factors are used to determine the moments at the ends of the beam and thus generate the EC.

Other researchers have presented design aids for rectangular cross-section beams under uniformly distributed load and/or concentrated load with straight and parabolic haunches considering shear and bending deformations to determine the fixed-end moments factors, carry-over factors and stiffness factors [27-30]. These design aids are used to determine the moments at the ends of the beam and thus generate the EC.

This work shows a model to obtain the EC and MD for rectangular beams with straight haunches under a concentrated load located at any point of the beam and a moment at each end considering shear and bending deformations (Timoshenko Theory). The methodology is developed in three parts: first, the load is located on the left haunches (0 to a); second, the load is located in the central section or uniform section (a to $L - c$); third, the load is located on the right haunches ($L - c$ to L). The CM to obtain the EC of rectangular beams under concentrated load takes account of only bending deformations (Euler-Bernoulli Theory). A comparison is developed between the PM and the CM for simply supported beams, for perfectly fixed beams at their ends and for the same moments at the ends with respect to the MD of the beam to observe the behavior of both models.

The paper is organized as follows. Section 2 presents the methodology to determine the EC and MD of a concentrated load located at any point of the beam, assuming the shear and bending deformations (Timoshenko Theory). Subsection 2.1 describes the equations when the load is located on the left haunches (0 to a). Subsection 2.2 shows the equations when the load is located in the central section or uniform section (a to $L - c$). Subsection 2.3 describes the equations when the load is located on the right haunches ($L - c$ to L). Section 3 shows the numerical studies applied to PM and CM. Section 4 indicates the results. Section 5 describes the conclusions to complete the paper.

2. Methodology. The development is carried out by integration, since the integral of the moment between EI is the slope and the integral of the slope gives the EC, and the MD is determined by differentiating the equation of the EC to obtain the position of the MD and then substituting it into the equation of the EC.

Figure 1 presents the Euler-Bernoulli theory and the Timoshenko theory. The first theory includes only bending deformations ($dy/dx = dy_b/dx$) and the second theory includes bending and shear deformations ($dy/dx = dy_s/dx + dy_b/dx$) [31,32].

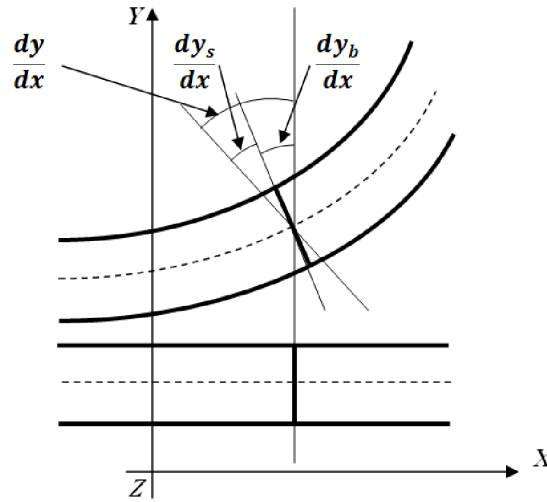


FIGURE 1. Deformation of a portion of a beam

Timoshenko [31] considered shear and bending deformations for short and long elements. The EC is described below [31,32]:

$$\frac{d^2y}{dx^2} = \frac{d^2y_s}{dx^2} + \frac{d^2y_b}{dx^2} \quad (1)$$

$$\frac{d^2y_s}{dx^2} = \frac{1}{GA_{sx}} \frac{dV_x}{dx}; \quad \frac{d^2y_b}{dx^2} = \frac{M_z}{EI_z} \quad (2)$$

where G = shear modulus, y = total deformation, y_s = shear deformation by shear force, y_b = bending deformation by moment, A_{sx} = shear area, E = modulus of elasticity, V_x = shear force measured on the “X” axis, M_z = moment about the “Z” axis, and I_z = moment of inertia about the “Z” axis.

Substituting Equation (2) into Equation (1) and integrating, the rotations at any point on the beam are obtained:

$$\frac{dy}{dx} = \frac{V_x}{GA_{sx}} + \int \frac{M_z}{EI_z} dx \quad (3)$$

The values of R_A and R_B in function of M_{AB} , M_{BA} , P , L and e (distance from support A to the location of load P) are obtained by the following equations:

$$R_A = \frac{M_{AB} - M_{BA}}{L} + \frac{P(L - e)}{L} \quad (4)$$

$$R_B = \frac{Pe}{L} - \frac{M_{AB} - M_{BA}}{L} \quad (5)$$

Figure 2 presents a beam “AB” under a moment at each end and a concentrated load at any point of the beam for a rectangular section with constant width “ b ”, height of the beam in the central part “ h ”, height of the straight haunch of the beam at support A “ u ”, height of the straight haunch of the beam at support B “ z ”, total length of the beam “ L ” and the variable height “ h_x ” that varies in a straight line in three different parts.

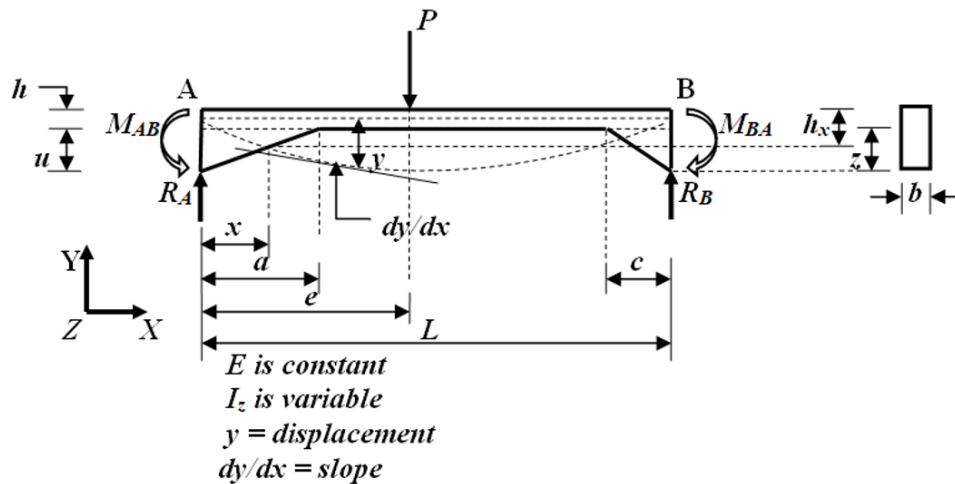


FIGURE 2. Beam under a moment at each end and a concentrated load

The shear forces and moments at any point on the beam are

$$V_x = R_A \rightarrow (0 \leq x \leq e) \quad (6)$$

$$V_x = R_A - P \rightarrow (e \leq x \leq L) \quad (7)$$

$$M_z = R_A x - M_{AB} \rightarrow (0 \leq x \leq e) \quad (8)$$

$$M_z = (R_A - P)x + Pe - M_{AB} \rightarrow (e \leq x \leq L) \quad (9)$$

Substituting Equations (6) and (8) into Equation (3), and Equations (7) and (9) into Equation (3) gives the following equations:

$$\frac{dy}{dx} = \frac{R_A}{GA_{sx}} + \frac{1}{E} \int \frac{(R_A x - M_{AB})}{I_z} dx \rightarrow (0 \leq x \leq e) \quad (10)$$

$$\frac{dy}{dx} = \frac{R_A - P}{GA_{sx}} + \frac{1}{E} \int \frac{[(R_A - P)x + Pe - M_{AB}]}{I_z} dx \rightarrow (e \leq x \leq L) \quad (11)$$

Table 1 shows the equations for the moment of inertia “ I_z ” about the “ Z ” axis, shear areas “ A_{sx} ”, and the heights “ h_x ” at a distance “ x ” in each range [20].

2.1. Load located in the range $0 \leq x \leq a$. Substitute the properties from Table 1 into Equations (10) and (11) to obtain the rotations “ dy/dx ” at any point on the beam, and then integrate these equations to find the “ y ” deflections anywhere along the beam. The simplified equations for each interval are shown below.

TABLE 1. Rectangular cross section properties

Concept	Equations		
Range	$0 \leq x \leq a$	$a \leq x \leq L - c$	$L - c \leq x \leq L$
h_x	$\frac{a(h+u)-ux}{a}$	h	$\frac{c(h+z)-z(L-x)}{c}$
A_{sx}	$\frac{5b[a(h+u)-ux]}{6a}$	$\frac{5bh}{6}$	$\frac{5b[c(h+z)-z(L-x)]}{6c}$
I_z	$\frac{b[a(h+u)-ux]^3}{12a^3}$	$\frac{bh^3}{12}$	$\frac{b[c(h+z)-z(L-x)]^3}{12c^3}$

For $0 \leq x \leq e$:

$$\frac{dy}{dx} = \frac{6R_A a}{5Gb[a(h+u)-ux]} - \frac{6a^3\{R_A[a(h+u)-2ux] + M_{AB}u\}}{Ebu^2[a(h+u)-ux]^2} + C_1 \quad (12)$$

$$y = -\frac{6R_A a \ln[a(h+u)-ux]}{5Gbu} + \frac{12R_A a^3 \ln[a(h+u)-ux]}{Ebu^3} + \frac{6a^3[R_A a(h+u) - M_{AB}u]}{Ebu^3[a(h+u)-ux]} + C_1 x + C_2 \quad (13)$$

For $e \leq x \leq a$:

$$\frac{dy}{dx} = \frac{6(R_A - P)a}{5Gb[a(h+u)-ux]} - \frac{6a^3\{(R_A - P)[a(h+u)-2ux] - (Pe - M_{AB})u\}}{Ebu^2[a(h+u)-ux]^2} + C_3 \quad (14)$$

$$y = -\frac{6(R_A - P)a \ln[a(h+u)-ux]}{5Gbu} + \frac{12(R_A - P)a^3 \ln[a(h+u)-ux]}{Ebu^3} + \frac{6a^3[(R_A - P)a(h+u) + (Pe - M_{AB})u]}{Ebu^3[a(h+u)-ux]} + C_3 x + C_4 \quad (15)$$

For $a \leq x \leq L - c$:

$$\frac{dy}{dx} = \frac{6(R_A - P)}{5Gbh} + \frac{6x[(R_A - P)x + 2(Pe - M_{AB})]}{Ebh^3} + C_5 \quad (16)$$

$$y = \frac{6(R_A - P)x}{5Gbh} + \frac{2x^2[(R_A - P)x + 3(Pe - M_{AB})]}{Ebh^3} + C_5 x + C_6 \quad (17)$$

For $L - c \leq x \leq L$:

$$\frac{dy}{dx} = \frac{6(R_A - P)c}{5Gb[c(h+z)-z(L-x)]} - \frac{6c^3\{(R_A - P)[c(h+z)-z(L-2x)] + (Pe - M_{AB})z\}}{Ebz^2[c(h+z)-z(L-x)]^2} + C_7 \quad (18)$$

$$y = \frac{6(R_A - P)c \ln[c(h+z)-z(L-x)]}{5Gbz} - \frac{12(R_A - P)c^3 \ln[c(h+z)-z(L-x)]}{Ebz^3} - \frac{6c^3\{(R_A - P)[c(h+z)-zL] - z(Pe - M_{AB})\}}{Ebz^3[c(h+z)-z(L-x)]} + C_7 x + C_8 \quad (19)$$

The eight known beam conditions are as follows: 1) $x = 0$ and $y = 0$ is substituted into Equation (13). 2) $x = e$ is substituted into Equations (12) and (14), dy/dx must be the same. 3) $x = e$ is substituted into Equations (13) and (15), y must be equal. 4) $x = a$ is substituted into Equations (14) and (16), dy/dx must be the same. 5) $x = a$ is substituted into Equations (15) and (17), y must be equal. 6) $x = L - c$ is substituted into Equations (16) and (18), dy/dx must be the same. 7) $x = L - c$ is substituted into Equations (17) and (19), y must be equal. 8) $x = L$ and $y = 0$ is substituted into Equation (19).

Now, the eight known conditions are substituted into corresponding equations to generate eight equations that are used to determine the integration constants.

The integration constants are presented below:

$$\begin{aligned}
C_1 = & -\frac{6a\{R_A \ln[a(h+u)] - P \ln[a(h+u) - ue] - (R_A - P) \ln(ah)\}}{5GLbu} \\
& -\frac{6(R_A - P)(L - a - c)}{5GLbh} - \frac{6(R_A - P)c\{\ln[c(h+z)] - \ln(ch)\}}{5GLbz} \\
& -\frac{6Pa(L - e)}{5GLb[a(h+u) - ue]} + \frac{6M_{AB}a^2u}{ELbh^3(h+u)} + \frac{12(R_A - P)c^3\{\ln[c(h+z)] - \ln(ch)\}}{ELbz^3} \\
& -\frac{6(Pe - M_{AB})[L^2(h+z) - zc^2]}{ELbh^3(h+z)} \\
& +\frac{12a^3\{R_A \ln[a(h+u)] - P \ln[a(h+u) - ue] - (R_A - P) \ln(ah)\}}{ELbu^3} \\
& -\frac{2(R_A - P)a^2[2a(3h^2 + u^2) - 3L(h^2 + u^2) + 3hu(L - a)]}{ELbh^3u^2} \\
& -\frac{2(R_A - P)\{z^2L[L^2(h+z) - 3zc^2] + c^3[z^2(2z - h) + 3h^2(2h + z)]\}}{ELbh^3z^2(h+z)} \\
& +\frac{6Pa^2[ah^3(L - 2e) - eu^3(a - e) + hue^2(h - u)]}{ELbu^2h^3[a(h+u) - ue]} - \frac{6a(Pe - M_{AB})(h - 2u)}{Ebh^3u} \quad (20)
\end{aligned}$$

$$C_2 = \frac{6R_Aa \ln[a(h+u)]}{5Gbu} - \frac{12R_Aa^3 \ln[a(h+u)]}{Ebu^3} - \frac{6a^2[R_Aa(h+u) - M_{AB}u]}{Ebu^3(h+u)} \quad (21)$$

$$\begin{aligned}
C_3 = & -\frac{6a\{R_A \ln[a(h+u)] - P \ln[a(h+u) - ue] - (R_A - P) \ln(ah)\}}{5GLbu} \\
& +\frac{6Pae}{5GLb[a(h+u) - ue]} - \frac{6(R_A - P)c\{\ln[c(h+z)] - \ln(ch)\}}{5GLbz} \\
& -\frac{6(R_A - P)(L - a - c)}{5GLbh} + \frac{12(R_A - P)c^3\{\ln[c(h+z)] - \ln(ch)\}}{ELbz^3} \\
& -\frac{6(Pe - M_{AB})[L^2(h+z) - zc^2]}{ELbh^3(h+z)} \\
& +\frac{12a^3\{R_A \ln[a(h+u)] - P \ln[a(h+u) - ue] - (R_A - P) \ln(ah)\}}{ELbu^3} \\
& -\frac{2(R_A - P)a^2[2a(3h^2 + u^2) - 3L(h^2 + u^2) + 3hu(L - a)]}{ELbh^3u^2} + \frac{6M_{AB}a^2u}{ELbh^3(h+u)} \\
& -\frac{6a(Pe - M_{AB})(h - 2u)}{Ebh^3u} - \frac{6Pa^2e[2ah^3 + u^3(a - e) - hue(h - u)]}{ELbu^2h^3[a(h+u) - ue]} \\
& -\frac{2(R_A - P)\{z^2L[L^2(h+z) - 3zc^2] + c^3[z^2(2z - h) + 3h^2(2h + z)]\}}{ELbh^3z^2(h+z)} \quad (22)
\end{aligned}$$

$$\begin{aligned}
C_4 = & \frac{6a\{R_A \ln[a(h+u)] - P \ln[a(h+u) - ue]\}}{5Gbu} - \frac{6Pae}{5Gb[a(h+u) - ue]} \\
& -\frac{6R_Aa^3}{Ebu^3} + \frac{6M_{AB}a^2}{Ebu^2(h+u)} + \frac{6Pa^4(h+u)}{Ebu^3[a(h+u) - ue]} \\
& -\frac{12a^3\{R_A \ln[a(h+u)] - P \ln[a(h+u) - ue]\}}{Ebu^3} \quad (23)
\end{aligned}$$

$$\begin{aligned}
C_5 = & -\frac{6a\{R_A \ln[a(h+u)] - P \ln[a(h+u) - ue] - (R_A - P) \ln(ah)\}}{5GLbu} \\
& -\frac{6(R_A - P)(L - a - c)}{5GLbh} - \frac{6(R_A - P)c\{\ln[c(h+z)] - \ln(ch)\}}{5GLbz} \\
& +\frac{6Pae}{5GLb[a(h+u) - ue]} + \frac{6M_{AB}a^2u}{ELbh^3(h+u)} + \frac{12(R_A - P)c^3\{\ln[c(h+z)] - \ln(ch)\}}{ELbz^3}
\end{aligned}$$

$$\begin{aligned}
& - \frac{2(R_A - P)a^3(6h^2 + 2u^2 - 3hu)}{ELbh^3u^2} \\
& + \frac{12a^3\{R_A \ln[a(h+u)] - P \ln[a(h+u) - ue] - (R_A - P) \ln(ah)\}}{ELbu^3} \\
& - \frac{6(Pe - M_{AB})[L^2(h+z) - zc^2]}{ELbh^3(h+z)} - \frac{6Pa^2e[2ah^3 + u^3(a-e) - hue(h-u)]}{ELbu^2h^3[a(h+u) - ue]} \\
& - \frac{2(R_A - P)\{z^2L[L^2(h+z) - 3zc^2] + c^3[z^2(2z-h) + 3h^2(2h+z)]\}}{ELbh^3z^2(h+z)} \quad (24)
\end{aligned}$$

$$\begin{aligned}
C_6 = & \frac{6a\{R_A \ln[a(h+u)] - P \ln[a(h+u) - ue] - (R_A - P) \ln(ah)\}}{5Gbu} \\
& - \frac{6Pae}{5Gb[a(h+u) - ue]} - \frac{6(R_A - P)a}{5Gbh} + \frac{6Pa^2e[2ah^3 + u^3(a-e) - hue(h-u)]}{Ebu^2h^3[a(h+u) - ue]} \\
& + \frac{2(R_A - P)a^3(6h^2 + 2u^2 - 3hu)}{Ebh^3u^2} \\
& - \frac{12a^3\{R_A \ln[a(h+u)] - P \ln[a(h+u) - ue] - (R_A - P) \ln(ah)\}}{Ebu^3} \\
& - \frac{6M_{AB}a^2u}{Ebh^3(h+u)} \quad (25)
\end{aligned}$$

$$\begin{aligned}
C_7 = & - \frac{6a\{R_A \ln[a(h+u)] - P \ln[a(h+u) - ue] - (R_A - P) \ln(ah)\}}{5GLbu} \\
& - \frac{6(R_A - P)(L - a - c)}{5GLbh} - \frac{6(R_A - P)c\{\ln[c(h+z)] - \ln(ch)\}}{5GLbz} \\
& + \frac{6Pae}{5GLb[a(h+u) - ue]} + \frac{12(R_A - P)c^3\{\ln[c(h+z)] - \ln(ch)\}}{ELbz^3} \\
& - \frac{2(R_A - P)a^3(6h^2 + 2u^2 - 3hu)}{ELbh^3u^2} \\
& + \frac{12a^3\{R_A \ln[a(h+u)] - P \ln[a(h+u) - ue] - (R_A - P) \ln(ah)\}}{ELbu^3} \\
& + \frac{6c^2(R_A - P)(L - 2c)}{ELbz^2(h+z)} - \frac{6Pa^2e[2ah^3 + u^3(a-e) - hue(h-u)]}{ELbu^2h^3[a(h+u) - ue]} \\
& + \frac{2(R_A - P)[2z^2(L-c)^3 + 3ch^2(L^2 - c^2) + zh(2L^3 - 3L^2c + c^3)]}{ELbh^3z(h+z)} \\
& + \frac{6(Pe - M_{AB})\{Lh[c(h-z) + zL] + z^2(L-c)^2\}}{ELbh^3z(h+z)} + \frac{6M_{AB}a^2u}{ELbh^3(h+u)} \quad (26)
\end{aligned}$$

$$\begin{aligned}
C_8 = & \frac{6a\{R_A \ln[a(h+u)] - P \ln[a(h+u) - ue] - (R_A - P) \ln(ah)\}}{5Gbu} \\
& - \frac{6(R_A - P)c \ln(ch)}{5Gbz} - \frac{6Pae}{5Gb[a(h+u) - ue]} + \frac{6(R_A - P)(L - a - c)}{5Gbh} \\
& + \frac{12(R_A - P)c^3 \ln(ch)}{Ebz^3} \\
& - \frac{12a^3\{R_A \ln[a(h+u)] - P \ln[a(h+u) - ue] - (R_A - P) \ln(ah)\}}{Ebu^3} \\
& + \frac{2(R_A - P)a^3(6h^2 + 2u^2 - 3hu)}{Ebh^3u^2} + \frac{6Pa^2e[2ah^3 + u^3(a-e) - hue(h-u)]}{Ebu^2h^3[a(h+u) - ue]} \\
& - \frac{2(R_A - P)\{2z^3(L-c)^3 + 3z^2ch(L-c)^2 + 3c^2h^2[2z(L-c) - ch]\}}{Ebh^3z^3}
\end{aligned}$$

$$- \frac{6(Pe - M_{AB}) [ch[Lz + c(h - z)] + z^2(L - c)^2]}{Ebh^3z^2} - \frac{6M_{AB}a^2u}{Ebh^3(h + u)} \quad (27)$$

2.2. Load located in the range $a \leq x \leq L - c$. Substituting the properties from Table 1 into Equations (10) and (11) to obtain the rotations “ dy/dx ” at any point on the beam, and then integrating these equations to find the “ y ” deflections anywhere along the beam. The simplified equations for each interval are shown below.

For $0 \leq x \leq a$:

$$\frac{dy}{dx} = \frac{6R_{Aa}}{5Gb[a(h + u) - ux]} - \frac{6a^3\{R_A[a(h + u) - 2ux] + M_{AB}u\}}{Ebu^2[a(h + u) - ux]^2} + C_9 \quad (28)$$

$$y = -\frac{6R_{Aa} \ln[a(h + u) - ux]}{5Gbu} + \frac{12R_{Aa}^3 \ln[a(h + u) - ux]}{Ebu^3} + \frac{6a^3[R_{Aa}(h + u) - M_{AB}u]}{Ebu^3[a(h + u) - ux]} + C_9x + C_{10} \quad (29)$$

For $a \leq x \leq e$:

$$\frac{dy}{dx} = \frac{6R_A}{5Gbh} + \frac{6x(R_Ax - 2M_{AB})}{Ebh^3} + C_{11} \quad (30)$$

$$y = \frac{6R_Ax}{5Gbh} + \frac{2x^2(R_Ax - 3M_{AB})}{Ebh^3} + C_{11}x + C_{12} \quad (31)$$

For $e \leq x \leq L - c$:

$$\frac{dy}{dx} = \frac{6(R_A - P)}{5Gbh} + \frac{6x[(R_A - P)x + 2(Pe - M_{AB})]}{Ebh^3} + C_{13} \quad (32)$$

$$y = \frac{6(R_A - P)x}{5Gbh} + \frac{2x^2[(R_A - P)x + 3(Pe - M_{AB})]}{Ebh^3} + C_{13}x + C_{14} \quad (33)$$

For $L - c \leq x \leq L$:

$$\frac{dy}{dx} = \frac{6(R_A - P)c}{5Gb[c(h + z) - z(L - x)]} - \frac{6c^3\{(R_A - P)[c(h + z) - z(L - 2x)] + (Pe - M_{AB})z\}}{Ebz^2[c(h + z) - z(L - x)]^2} + C_{15} \quad (34)$$

$$y = \frac{6(R_A - P)c \ln[c(h + z) - z(L - x)]}{5Gbz} - \frac{12(R_A - P)c^3 \ln[c(h + z) - z(L - x)]}{Ebz^3} - \frac{6c^3\{(R_A - P)[c(h + z) - zL] - z(Pe - M_{AB})\}}{Ebz^3[c(h + z) - z(L - x)]} + C_{15}x + C_{16} \quad (35)$$

The eight known beam conditions are as follows: 1) $x = 0$ and $y = 0$ is substituted into Equation (29). 2) $x = e$ is substituted into Equations (28) and (30), dy/dx must be the same. 3) $x = e$ is substituted into Equations (29) and (31), y must be equal. 4) $x = a$ is substituted into Equations (30) and (32), dy/dx must be the same. 5) $x = a$ is substituted into Equations (31) and (33), y must be equal. 6) $x = L - c$ is substituted into Equations (32) and (34), dy/dx must be the same. 7) $x = L - c$ is substituted into Equations (33) and (35), y must be equal. 8) $x = L$ and $y = 0$ is substituted into Equation (35).

Now, the eight known conditions are substituted into corresponding equations to generate eight equations that are used to determine the integration constants.

The integration constants are presented below:

$$C_9 = \frac{6R_{Aa}\{\ln(ah) - \ln[a(h + u)]\}}{5GLbu} + \frac{6(R_A - P)c\{\ln(ch) - \ln[c(h + z)]\}}{5GLbz} - \frac{6[R_A(L - a - c) - Pc]}{5GLbh} - \frac{12R_{Aa}^3\{\ln(ah) - \ln[a(h + u)]\}}{ELbu^3} + \frac{2Pe^2(3L - e)}{ELbh^3}$$

$$\begin{aligned}
 & + \frac{6M_{AB}a [L(h^2 - 2u^2 - hu) + au^2]}{ELbh^3u(h+u)} - \frac{6(Pe - M_{AB}) [L^2(h+z) - zc^2]}{ELbh^3(h+z)} \\
 & - \frac{2(R_A - P) \{z^2L [L^2(h+z) - 3zc^2] + c^3 [3h^2(2h+z) - z^2(h-2z)]\}}{ELbh^3z^2(h+z)} \\
 & + \frac{2R_Aa^2 [3L(h^2 + u^2 - hu) - a(6h^2 + 2u^2 - 3hu)]}{ELbu^2h^3} \\
 & - \frac{12(R_A - P)c^3 \{\ln(ch) - \ln[c(h+z)]\}}{ELbz^3}
 \end{aligned} \tag{36}$$

$$C_{10} = \frac{6R_Aa \ln[a(h+u)]}{5Gbu} - \frac{12R_Aa^3 \ln[a(h+u)]}{Ebu^3} - \frac{6a^2[R_Aa(h+u) - M_{AB}u]}{Ebu^3(h+u)} \tag{37}$$

$$\begin{aligned}
 C_{11} = & - \frac{6[R_A(L-a-c) - Pc]}{5GLbh} + \frac{6R_Aa \{\ln(ah) - \ln[a(h+u)]\}}{5GLbu} \\
 & + \frac{6(R_A - P)c \{\ln(ch) - \ln[c(h+z)]\}}{5GLbz} - \frac{12R_Aa^3 \{\ln(ah) - \ln[a(h+u)]\}}{ELbu^3} \\
 & - \frac{12(R_A - P)c^3 \{\ln(ch) - \ln[c(h+z)]\}}{ELbz^3} - \frac{2R_Aa^3 (6h^2 + 2u^2 - 3hu)}{ELbh^3u^2} \\
 & + \frac{2Pe^2(3L-e)}{ELbh^3} - \frac{6(Pe - M_{AB}) [L^2(h+z) - zc^2]}{ELbh^3(h+z)} + \frac{6M_{AB}a^2u}{ELbh^3(h+u)} \\
 & - \frac{2(R_A - P) \{z^2L [L^2(h+z) - 3zc^2] + c^3 [3h^2(2h+z) - z^2(h-2z)]\}}{ELbh^3z^2(h+z)}
 \end{aligned} \tag{38}$$

$$\begin{aligned}
 C_{12} = & - \frac{6R_Aa}{5Gbh} - \frac{6R_Aa \{\ln(ah) - \ln[a(h+u)]\}}{5Gbu} + \frac{12R_Aa^3 \{\ln(ah) - \ln[a(h+u)]\}}{Ebu^3} \\
 & + \frac{2R_Aa^3 (2u^2 + 6h^2 - 3hu)}{Ebh^3u^2} - \frac{6M_{AB}a^2u}{Ebh^3(h+u)}
 \end{aligned} \tag{39}$$

$$\begin{aligned}
 C_{13} = & - \frac{6[R_A(L-a-c) - P(L-c)]}{5GLbh} + \frac{6R_Aa \{\ln(ah) - \ln[a(h+u)]\}}{5GLbu} \\
 & + \frac{6(R_A - P)c \{\ln(ch) - \ln[c(h+z)]\}}{5GLbz} - \frac{12R_Aa^3 \{\ln(ah) - \ln[a(h+u)]\}}{ELbu^3} \\
 & - \frac{12(R_A - P)c^3 \{\ln(ch) - \ln[c(h+z)]\}}{ELbz^3} - \frac{2Pe^3}{ELbh^3} + \frac{6M_{AB}a^2u}{ELbh^3(h+u)} \\
 & - \frac{2(R_A - P) \{z^2L [L^2(h+z) - 3zc^2] + c^3 [3h^2(2h+z) - z^2(h-2z)]\}}{ELbh^3z^2(h+z)} \\
 & - \frac{2R_Aa^3 (6h^2 + 2u^2 - 3hu)}{ELbh^3u^2} - \frac{6(Pe - M_{AB}) [L^2(h+z) - zc^2]}{ELbh^3(h+z)}
 \end{aligned} \tag{40}$$

$$\begin{aligned}
 C_{14} = & - \frac{6R_Aa}{5Gbh} - \frac{6R_Aa \{\ln(ah) - \ln[a(h+u)]\}}{5Gbu} \\
 & + \frac{12R_Aa^3 \{\ln(ah) - \ln[a(h+u)]\}}{Ebu^3} + \frac{2R_Aa^3 (2u^2 + 6h^2 - 3hu)}{Ebh^3u^2} + \frac{2Pe^3}{Ebh^3} \\
 & - \frac{6M_{AB}a^2u}{Ebh^3(h+u)}
 \end{aligned} \tag{41}$$

$$\begin{aligned}
 C_{15} = & - \frac{6[R_A(L-a-c) - P(L-c)]}{5GLbh} + \frac{6R_Aa \{\ln(ah) - \ln[a(h+u)]\}}{5GLbu} - \frac{6(R_A - P)c^2}{Ebz^2(h+z)} \\
 & + \frac{6(R_A - P)c \{\ln(ch) - \ln[c(h+z)]\}}{5GLbz} - \frac{12R_Aa^3 \{\ln(ah) - \ln[a(h+u)]\}}{ELbu^3} \\
 & - \frac{12(R_A - P)c^3 \{\ln(ch) - \ln[c(h+z)]\}}{ELbz^3} - \frac{2R_Aa^3 (6h^2 + 2u^2 - 3hu)}{ELbh^3u^2} - \frac{2Pe^3}{ELbh^3}
 \end{aligned}$$

$$\begin{aligned}
& + \frac{2(R_A - P)(L - c) [6c^2h^2 + 3chz(L - c) + 2z^3(L - c)^3]}{ELbh^3z^2} + \frac{6M_{AB}a^2u}{ELbh^3(h + u)} \\
& + \frac{6(Pe - M_{AB}) \{Lh[ch + z(L - c)] + z^2(L - c)^2\}}{ELbh^3z(h + z)} \quad (42) \\
C_{16} = & \frac{6[R_A(L - a - c) - P(L - c)]}{5Gbh} - \frac{6R_Aa\{\ln(ah) - \ln[a(h + u)]\}}{5Gbu} \\
& - \frac{6(R_A - P)c \ln(ch)}{5Gbz} + \frac{2Pe^3}{Ebh^3} + \frac{12R_Aa^3\{\ln(ah) - \ln[a(h + u)]\}}{Ebu^3} \\
& + \frac{2R_Aa^3(6h^2 + 2u^2 - 3hu)}{Ebh^3u^2} - \frac{6M_{AB}a^2u}{Ebh^3(h + u)} + \frac{6(R_A - P)c^2[ch - z(L - c)]}{Ebh^3z^3} \\
& - \frac{6(Pe - M_{AB}) [ch[ch + z(L - c)] + z^2(L - c)^2]}{Ebh^3z^2} \\
& - \frac{2(R_A - P) \{3ch(L - c)[ch + z(L - c)] + 2z^2(L - c)^3\}}{Ebh^3z^2} \\
& + \frac{12(R_A - P)c^3 \ln(ch)}{Ebz^3} \quad (43)
\end{aligned}$$

2.3. Load located in the range $L - c \leq x \leq L$. Substituting the properties from Table 1 into Equations (10) and (11) to obtain the rotations “ dy/dx ” at any point on the beam, and then integrating these equations to find the “ y ” deflections anywhere along the beam. The simplified equations for each interval are shown below.

For $0 \leq x \leq a$:

$$\frac{dy}{dx} = \frac{6R_Aa}{5Gb[a(h + u) - ux]} - \frac{6a^3\{R_A[a(h + u) - 2ux] + M_{AB}u\}}{Ebu^2[a(h + u) - ux]^2} + C_{17} \quad (44)$$

$$\begin{aligned}
y = & -\frac{6R_Aa \ln[a(h + u) - ux]}{5Gbu} + \frac{12R_Aa^3 \ln[a(h + u) - ux]}{Ebu^3} \\
& + \frac{6a^3[R_Aa(h + u) - M_{AB}u]}{Ebu^3[a(h + u) - ux]} + C_{17}x + C_{18} \quad (45)
\end{aligned}$$

For $a \leq x \leq L - c$:

$$\frac{dy}{dx} = \frac{6R_A}{5Gbh} + \frac{6x(R_Ax - 2M_{AB})}{Ebh^3} + C_{19} \quad (46)$$

$$y = \frac{6R_Ax}{5Gbh} + \frac{2x^2(R_Ax - 3M_{AB})}{Ebh^3} + C_{19}x + C_{20} \quad (47)$$

For $L - c \leq x \leq e$:

$$\frac{dy}{dx} = \frac{6R_Ac}{5Gb[c(h + z) - z(L - x)]} - \frac{6c^3\{R_A[c(h + z) - z(L - 2x)] - M_{AB}z\}}{Ebz^2[c(h + z) - z(L - x)]^2} + C_{21} \quad (48)$$

$$\begin{aligned}
y = & \frac{6R_Ac \ln[c(h + z) - z(L - x)]}{5Gbz} - \frac{12R_Ac^3 \ln[c(h + z) - z(L - x)]}{Ebz^3} \\
& - \frac{6c^3\{R_A[c(h + z) - zL] + M_{AB}z\}}{Ebz^3[c(h + z) - z(L - x)]} + C_{21}x + C_{22} \quad (49)
\end{aligned}$$

For $e \leq x \leq L$:

$$\begin{aligned}
\frac{dy}{dx} = & \frac{6(R_A - P)c}{5Gb[c(h + z) - z(L - x)]} - \frac{6c^3\{(R_A - P)[c(h + z) - z(L - 2x)] + (Pe - M_{AB})z\}}{Ebz^2[c(h + z) - z(L - x)]^2} \\
& + C_{23} \quad (50) \\
y = & \frac{6(R_A - P)c \ln[c(h + z) - z(L - x)]}{5Gbz} - \frac{12(R_A - P)c^3 \ln[c(h + z) - z(L - x)]}{Ebz^3}
\end{aligned}$$

$$- \frac{6c^3\{(R_A - P)[c(h + z) - zL] - z(Pe - M_{AB})\}}{Ebz^3[c(h + z) - z(L - x)]} + C_{23}x + C_{24} \quad (51)$$

The eight known beam conditions are as follows: 1) $x = 0$ and $y = 0$ is substituted into Equation (45). 2) $x = e$ is substituted into Equations (44) and (46), dy/dx must be the same. 3) $x = e$ is substituted into Equations (45) and (47), y must be equal. 4) $x = a$ is substituted into Equations (46) and (48), dy/dx must be the same. 5) $x = a$ is substituted into Equations (47) and (49), y must be equal. 6) $x = L - c$ is substituted into Equations (48) and (50), dy/dx must be the same. 7) $x = L - c$ is substituted into Equations (49) and (51), y must be equal. 8) $x = L$ and $y = 0$ is substituted into Equation (51).

Now, the eight known conditions are substituted into corresponding equations to generate eight equations that are used to determine the integration constants.

The integration constants are presented below:

$$\begin{aligned} C_{17} = & -\frac{6R_A(L - a - c)}{5GLbh} - \frac{6Pc(L - e)}{5GLb[c(h + z) - z(L - e)]} \\ & + \frac{6R_Aa\{\ln(ah) - \ln[a(h + u)]\}}{5GLbu} + \frac{6R_Ac^2[c(h + z) - zL]}{ELbz^3(h + z)} \\ & - \frac{6(R_A - P)c\ln[c(h + z)]}{5GLbz} + \frac{6R_Ac\ln(ch)}{5GLbz} - \frac{6Pc\ln[c(h + z) - z(L - e)]}{5GLbz} \\ & - \frac{12R_Aa^3\{\ln(ah) - \ln[a(h + u)]\}}{ELbu^3} - \frac{12R_Ac^3\ln(ch)}{ELbz^3} + \frac{12(R_A - P)c^3\ln[c(h + z)]}{ELbz^3} \\ & + \frac{6M_{AB}[L^2(h + z) - zc^2]}{ELbh^3(h + z)} + \frac{12Pc^3\ln[c(h + z) - z(L - e)]}{ELbz^3} \\ & - \frac{6Pc^2(L - e)[z(L - e) - 2c(h + z)]}{ELbz^2(h + z)[c(h + z) - z(L - e)]} \\ & + \frac{2R_Aa^2[3L(h^2 + u^2 - hu) - a(2u^2 + 6h^2 - 3hu)]}{Ebu^2h^3} \\ & + \frac{6M_{ABa}[L(h^2 - hu - 2u^2) + au^2]}{ELbuh^3(h + u)} - \frac{6R_Ac^2[ch - z(L - 2c)]}{ELbz^3h} \\ & - \frac{2R_A(L - c)\{3c[z(L - c) + ch] + z(L - c)^2\}}{ELbzh^3} \end{aligned} \quad (52)$$

$$C_{18} = \frac{6R_Aa\ln[a(h + u)]}{5Gbu} - \frac{12R_Aa^3\ln[a(h + u)]}{Ebu^3} - \frac{6a^2[R_Aa(h + u) - M_{AB}u]}{Ebu^3(h + u)} \quad (53)$$

$$\begin{aligned} C_{19} = & -\frac{6R_A(L - a - c)}{5GLbh} - \frac{6Pc(L - e)}{5GLb[c(h + z) - z(L - e)]} + \frac{6R_Aa\{\ln(ah) - \ln[a(h + u)]\}}{5GLbu} \\ & + \frac{6R_Ac\ln(ch)}{5GLbz} - \frac{6Pc\ln[c(h + z) - z(L - e)]}{5GLbz} + \frac{6R_Ac^2[c(h + z) - zL]}{ELbz^3(h + z)} \\ & - \frac{6Pc^2(L - e)[z(L - e) - 2c(h + z)]}{ELbz^2(h + z)[c(h + z) - z(L - e)]} - \frac{6(R_A - P)c\ln[c(h + z)]}{5GLbz} \\ & + \frac{6M_{AB}[L^2(h + z) - zc^2]}{ELbh^3(h + z)} - \frac{2R_Aa^3(2u^2 + 6h^2 - 3hu)}{ELbh^3u^2} + \frac{6M_{AB}a^2u}{ELbh^3(h + u)} \\ & - \frac{12R_Aa^3\{\ln(ah) - \ln[a(h + u)]\}}{ELbu^3} - \frac{12R_Ac^3\ln(ch)}{ELbz^3} + \frac{12(R_A - P)c^3\ln[c(h + z)]}{ELbz^3} \\ & + \frac{12Pc^3\ln[c(h + z) - z(L - e)]}{ELbz^3} - \frac{6R_Ac^2[ch - z(L - 2c)]}{ELbz^3h} \\ & - \frac{2R_A(L - c)\{3c[z(L - c) + ch] + z(L - c)^2\}}{ELbzh^3} \end{aligned} \quad (54)$$

$$C_{20} = -\frac{6R_Aa}{5Gb h} - \frac{6R_Aa\{\ln(ah) - \ln[a(h+u)]\}}{5Gb u} + \frac{12R_Aa^3\{\ln(ah) - \ln[a(h+u)]\}}{Ebu^3} \\ + \frac{2R_Aa^3(2u^2 + 6h^2 - 3hu)}{Ebh^3u^2} - \frac{6M_{AB}a^2u}{Ebh^3(h+u)} \quad (55)$$

$$C_{21} = -\frac{6R_A(L-a-c)}{5GLbh} - \frac{6Pc(L-e)}{5GLb[c(h+z) - z(L-e)]} \\ + \frac{6R_Aa\{\ln(ah) - \ln[a(h+u)]\}}{5GLbu} + \frac{6R_Ac\ln(ch)}{5GLbz} + \frac{6R_Ac^2[c(h+z) - zL]}{ELbz^3(h+z)} \\ - \frac{6(R_A-P)c\ln[c(h+z)]}{5GLbz} - \frac{6Pc\ln[c(h+z) - z(L-e)]}{5GLbz} \\ - \frac{12R_Aa^3\{\ln(ah) - \ln[a(h+u)]\}}{ELbu^3} + \frac{12(R_A-P)c^3\ln[c(h+z)]}{ELbz^3} - \frac{12R_Ac^3\ln(ch)}{ELbz^3} \\ - \frac{2R_Aa^3(2u^2 + 6h^2 - 3hu)}{ELbh^3u^2} + \frac{12Pc^3\ln[c(h+z) - z(L-e)]}{ELbz^3} + \frac{6M_{AB}a^2u}{ELbh^3(h+u)} \\ - \frac{6Pc^2(L-e)[z(L-e) - 2c(h+z)]}{ELbz^2(h+z)[c(h+z) - z(L-e)]} - \frac{6M_{AB}\{Lh[ch + z(L-c)] + z^2(L-c)^2\}}{ELbh^3z(h+z)} \\ - \frac{2R_A\{6c^3h^3 - 3ch[ch + z(L-c)]^2 - 2z^3(L-c)^3\}}{ELbz^3h^3} \quad (56)$$

$$C_{22} = \frac{6R_A(L-a-c)}{5Gb h} - \frac{6R_Aa\{\ln(ah) - \ln[a(h+u)]\}}{5Gb u} - \frac{6R_Ac\ln(ch)}{5Gb z} + \frac{12R_Ac^3\ln(ch)}{Ebz^3} \\ - \frac{6M_{AB}a^2u}{Ebh^3(h+u)} + \frac{12R_Aa^3\{\ln(ah) - \ln[a(h+u)]\}}{Ebu^3} \\ + \frac{2R_A\{6c^3h^3 - 3ch[ch + z(L-c)]^2 - 2z^3(L-c)^3\}}{Ebz^3h^3} + \frac{2R_Aa^3(2u^2 + 6h^2 - 3hu)}{Ebh^3u^2} \\ + \frac{6M_{AB}\{z^2(L-c)^2 + ch[ch + z(L-c)]\}}{Ebh^3z^2} \quad (57)$$

$$C_{23} = -\frac{6R_A(L-a-c)}{5GLbh} + \frac{6Pce}{5GLb[c(h+z) - z(L-e)]} + \frac{6R_Ac\ln(ch)}{5GLbz} \\ - \frac{6(R_A-P)c\ln[c(h+z)]}{5GLbz} + \frac{6R_Aa\{\ln(ah) - \ln[a(h+u)]\}}{5GLbu} \\ - \frac{6R_Ac\{2c^2h^2 - [ch + z(L-c)]^2\}}{ELbz^3h^2} + \frac{4R_A(L-c)^3}{ELbh^3} - \frac{2R_Aa^3(2u^2 + 6h^2 - 3hu)}{ELbh^3u^2} \\ + \frac{12Pc^3\ln[c(h+z) - z(L-e)]}{ELbz^3} - \frac{6Pc^2[z(L-e)^2 - c(h+z)(L-2e)]}{ELbz^2(h+z)[c(h+z) - z(L-e)]} \\ + \frac{6R_Ac^2[c(h+z) - zL]}{ELbz^3(h+z)} - \frac{12R_Ac^3\ln(ch)}{ELbz^3} + \frac{6M_{AB}a^2u}{ELbh^3(h+u)} \\ + \frac{12(R_A-P)c^3\ln[c(h+z)]}{ELbz^3} - \frac{6Pc\ln[c(h+z) - z(L-e)]}{5GLbz} \\ - \frac{12R_Aa^3\{\ln(ah) - \ln[a(h+u)]\}}{ELbu^3} - \frac{6M_{AB}\{Lh[ch + z(L-c)] + z^2(L-c)^2\}}{ELbh^3z(h+z)} \quad (58)$$

$$C_{24} = \frac{6R_A(L-a-c)}{5Gb h} - \frac{6Pce}{5Gb[c(h+z) - z(L-e)]} - \frac{6R_Aa\{\ln(ah) - \ln[a(h+u)]\}}{5Gb u} \\ - \frac{6R_Ac\ln(ch)}{5Gb z} + \frac{6Pc\ln[c(h+z) - z(L-e)]}{5Gb z} \\ + \frac{6M_{AB}\{z^2(L-c)^2 + ch[ch + z(L-c)]\}}{Ebh^3z^2} + \frac{12R_Aa^3\{\ln(ah) - \ln[a(h+u)]\}}{Ebu^3}$$

$$\begin{aligned}
& - \frac{12Pc^3 \ln[c(h+z) - z(L-e)]}{Ebz^3} + \frac{12R_A c^3 \ln(ch)}{Ebz^3} - \frac{6Pc^3[c(h+z) - z(L-e)]}{Ebz^3[c(h+z) - z(L-e)]} \\
& + \frac{2R_A a^3(2u^2 + 6h^2 - 3hu)}{Ebh^3u^2} + \frac{2R_A \{6c^3h^3 - 3ch[ch + z(L-c)]^2 - 2z^3(L-c)^3\}}{Ebz^3h^3} \\
& - \frac{6M_{AB}a^2u}{Ebh^3(h+u)}
\end{aligned} \tag{59}$$

3. Numerical Studies. The values of the EC for simply supported rectangular beams with straight haunches that support a concentrated load located at any point on the beam are obtained by substituting into the corresponding equations: $M_{AB} = 0$; $R_{AB} = (L-e)P$; $G = 5E/12$; $a = 0.2L$; $u = h$; $c = 0.2L$; $z = h$; $e = 0.1L, 0.2L, 0.3L, 0.4L, 0.5L, 0.6L, 0.7L, 0.8L, 0.9L$; $h = 0.1L, 0.2L$ of the two models (PM and CM).

The moment M_{AB} and the reaction R_A at support “A” for beams perfectly fixed at their ends are obtained by substituting into the corresponding equations: $dy/dx = 0$; $x = 0, L$; $G = 5E/12$; $a = 0.2L$; $u = h$; $c = 0.2L$; $z = h$; $e = 0.1L, 0.2L, 0.3L, 0.4L, 0.5L, 0.6L, 0.7L, 0.8L, 0.9L$; $h = 0.1L, 0.05L$ of the two models (PM and CM), and these two equations are solved to obtain “ $M_{AB} = m_{AB}PL$ ” and reaction “ $R_A = r_{AB}P$ ” on support “A”. Now, the moment and the reaction on support “B” are obtained by static equilibrium “ $M_{BA} = m_{BA}PL$ ” and “ $R_B = r_{BA}P$ ” (see Table 2).

TABLE 2. Moment M_{AB} and reaction R_A at support “A”

e	Bending and shear deformations				Bending deformations			
	m_{AB}	r_{AB}	m_{BA}	r_{BA}	m_{AB}	r_{AB}	m_{BA}	r_{BA}
$h = 0.1L, a = 0.2L, c = 0.2L$								
0.1L	0.0967	0.9945	0.0022	0.0055	0.0939	0.9900	0.0039	0.0100
0.2L	0.1738	0.9577	0.0161	0.0423	0.1668	0.9470	0.0198	0.0530
0.3L	0.2030	0.8494	0.0536	0.1506	0.1973	0.8412	0.0561	0.1588
0.4L	0.1923	0.6880	0.1043	0.3120	0.1885	0.6836	0.1049	0.3164
0.5L	0.1549	0.5000	0.1549	0.5000	0.1533	0.5000	0.1533	0.5000
0.6L	0.1043	0.3120	0.1922	0.6880	0.1049	0.3164	0.1884	0.6836
0.7L	0.0536	0.1506	0.2030	0.8494	0.0561	0.1588	0.1973	0.8412
0.8L	0.0161	0.0423	0.1738	0.9577	0.0198	0.0530	0.1668	0.9470
0.9L	0.0022	0.0055	0.0967	0.9945	0.0039	0.0100	0.0939	0.9900
$h = 0.05L, a = 0.2L, c = 0.2L$								
0.1L	0.0946	0.9911	0.0035	0.0089	0.0939	0.9900	0.0039	0.0100
0.2L	0.1685	0.9496	0.0189	0.0504	0.1668	0.9470	0.0198	0.0530
0.3L	0.1987	0.8432	0.0555	0.1568	0.1973	0.8412	0.0561	0.1588
0.4L	0.1894	0.6846	0.1048	0.3154	0.1885	0.6836	0.1048	0.3164
0.5L	0.1537	0.5000	0.1537	0.5000	0.1533	0.5000	0.1533	0.5000
0.6L	0.1048	0.3154	0.1894	0.6846	0.1049	0.3164	0.1884	0.6836
0.7L	0.0555	0.1568	0.1987	0.8432	0.0561	0.1588	0.1973	0.8412
0.8L	0.0189	0.0504	0.1685	0.9496	0.0198	0.0530	0.1668	0.9470
0.9L	0.0035	0.0089	0.0946	0.9911	0.0039	0.0100	0.0939	0.9900

where: $M_{AB} = m_{AB}PL$; $R_A = r_{AB}P$.

The values of the EC for beams perfectly fixed at their ends with straight haunches that support a concentrated load located at any point of the beam are obtained by substituting into the corresponding equations: M_{AB} and R_A provided by Table 2; $G = 5E/12$; $a = 0.2L$; $u = h$; $c = 0.2L$; $z = h$; $e = 0.1L, 0.2L, 0.3L, 0.4L, 0.5L, 0.6L, 0.7L, 0.8L, 0.9L$; $h = 0.1L, 0.05L$ of the two models (PM and CM).

The values of the EC for the same moment at the ends for both cases with straight haunches under a concentrated load located at any point of the beam are obtained by substituting into the corresponding equations: M_{AB} and R_A provided by Table 2 (shear and bending deformations are considered); $G = 5E/12$; $a = 0.2L$; $u = h$; $c = 0.2L$; $z = h$; $e = 0.1L, 0.2L, 0.3L, 0.4L, 0.5L, 0.6L, 0.7L, 0.8L, 0.9L$; $h = 0.1L, 0.05L$ of the two models (PM and CM).

Figures 3, 4 and 5 represent the EC of the three studies. The factor to obtain the maximum deflection is “ β ” to determine “ $y_{\max} = \beta P/bE$ ” and the factor to determine its position is “ α ” to obtain “ $x = \alpha L$ ”.

Figure 3 represents the EC for simply supported rectangular beams with straight haunches that support a concentrated load located at any point of the beam for the two models (PM and CM).

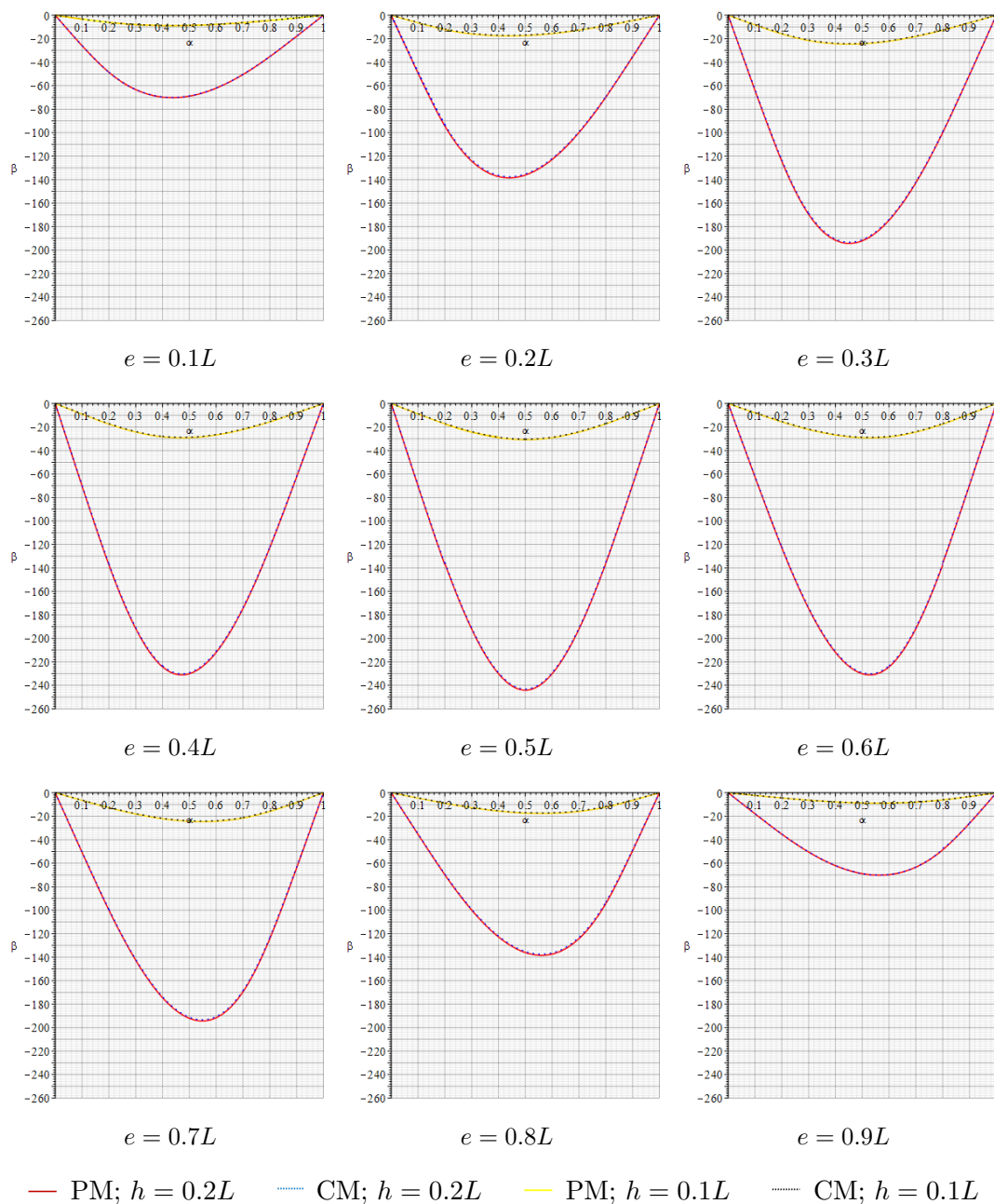


FIGURE 3. (color online) Elastic curve for simply supported beams at their ends

Figure 4 represents the EC for perfectly fixed beams at their ends with straight haunches that support a concentrated load located at any point of the beam for the two models (PM and CM).

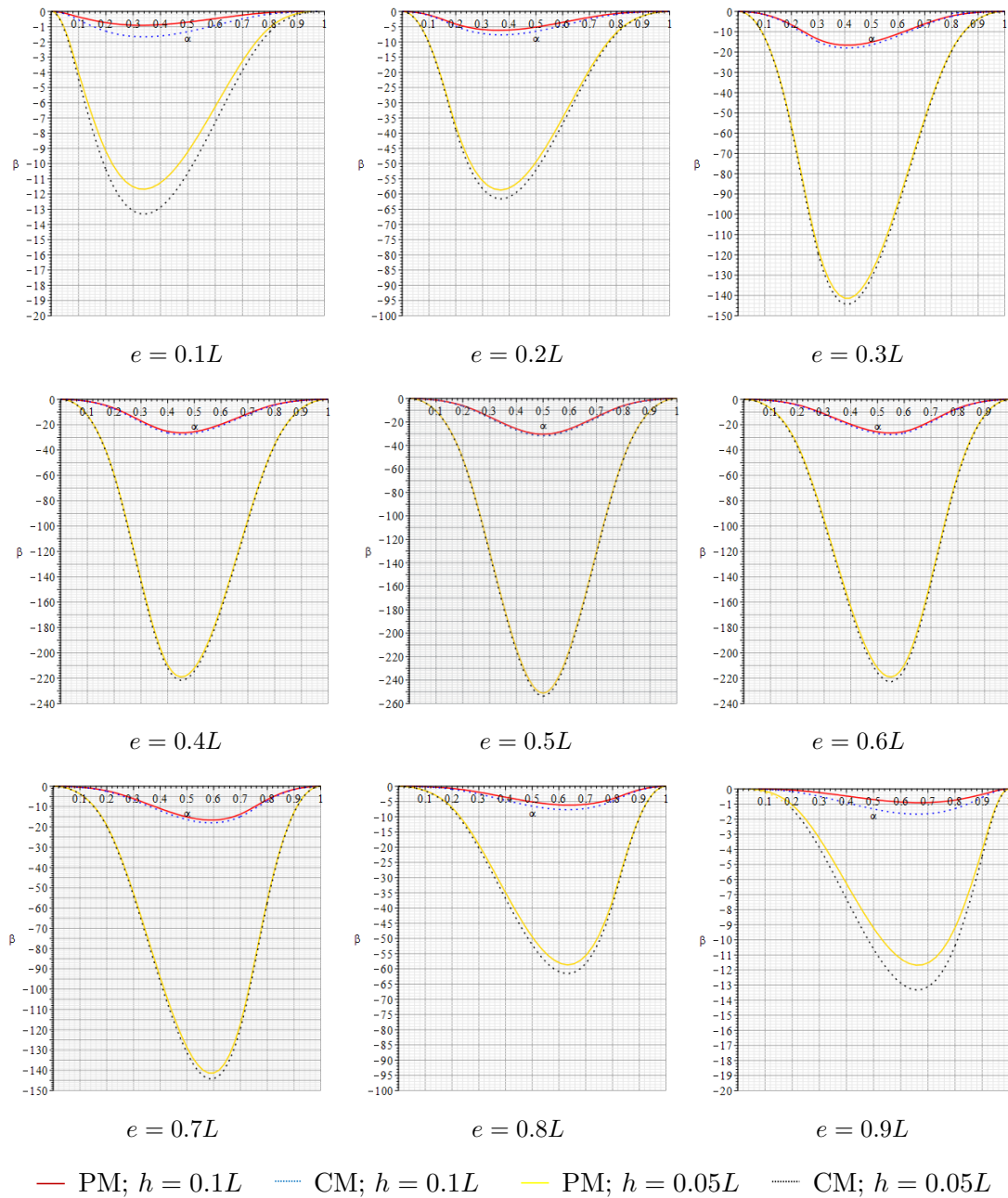


FIGURE 4. (color online) Elastic curve for perfectly fixed beams at their ends

Figure 5 represents the EC for the same moments provided by Table 2 (shear and bending deformations are considered) at the ends with straight haunches that support a concentrated load located at any point of the beam for the two models (PM and CM).

4. Results. The EC deflections for simply supported beams are larger in the PM than in the CM for $h = 0.1L$ and $h = 0.2L$ (see Figure 3).

The EC deflections for perfectly fixed beams at their ends are larger in the CM than in the PM for $h = 0.1L$ and $h = 0.05L$ (see Figure 4).

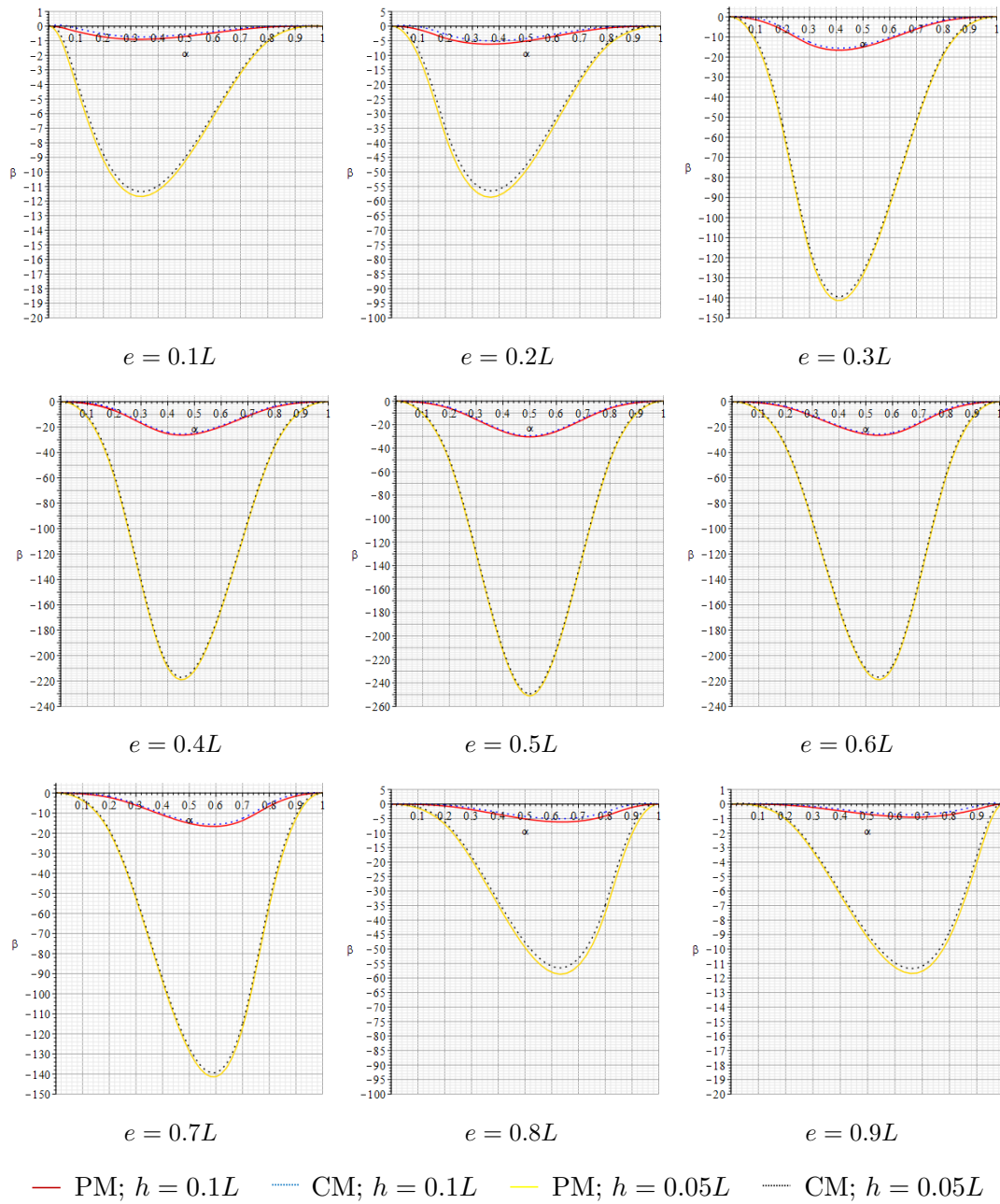


FIGURE 5. (color online) Elastic curve for the same moment at the ends for both cases

The EC deflections for the same moment at the ends for both cases are larger in the PM than in the CM for $h = 0.1L$ and $h = 0.05L$ (see Figure 5).

Tables 3, 4 and 5 present the factors to obtain the MD “ $y_{\max} = \beta P/bE$ ” and their position “ $x = \alpha L$ ” of the PM compared against the CM for the three cases mentioned above in Figures 3, 4 and 5. Table 3 presents the MD and their position for simply supported beams for $h = 0.1L$ and $h = 0.2L$. Table 4 shows the MD and their position for perfectly fixed beams at their ends for $h = 0.1L$ and $h = 0.05L$. Table 5 presents the MD and their position for the same moments at the ends for both cases for $h = 0.1L$ and $h = 0.05L$.

TABLE 3. Beam simply supported at its ends

e	Factors for the maximum deflections					
	α			β		
	PM	CM	PM/CM	PM	CM	PM/CM
$h = 0.1L$						
$0.1L$	0.4369	0.4368	1.0002	-70.2333	-70.1075	1.0018
$0.2L$	0.4395	0.4403	0.9982	-138.5277	-137.5802	1.0069
$0.3L$	0.4516	0.4520	0.9991	-194.3985	-193.4807	1.0047
$0.4L$	0.4718	0.4719	0.9998	-231.1470	-230.2533	1.0039
$0.5L$	0.5000	0.5000	1.0000	-244.1548	-243.2711	1.0036
$0.6L$	0.5282	0.5281	1.0002	-231.1470	-230.2533	1.0039
$0.7L$	0.5484	0.5480	1.0007	-194.3985	-193.4807	1.0047
$0.8L$	0.5605	0.5597	1.0014	-138.5277	-137.5802	1.0069
$0.9L$	0.5631	0.5632	0.9998	-70.2333	-70.1075	1.0018
$h = 0.2L$						
$0.1L$	0.4373	0.4368	1.0011	-8.8263	-8.7634	1.0072
$0.2L$	0.4371	0.4403	0.9927	-17.6719	-17.1975	1.0276
$0.3L$	0.4505	0.4520	0.9967	-24.6442	-24.1851	1.0190
$0.4L$	0.4714	0.4719	0.9989	-29.2286	-28.7817	1.0155
$0.5L$	0.5000	0.5000	1.0000	-30.8508	-30.4089	1.0145
$0.6L$	0.5286	0.5281	1.0009	-29.2286	-28.7817	1.0155
$0.7L$	0.5495	0.5480	1.0027	-24.6442	-24.1851	1.0190
$0.8L$	0.5629	0.5597	1.0057	-17.6719	-17.1975	1.0276
$0.9L$	0.5627	0.5632	0.9991	-8.8263	-8.7634	1.0072

where: x (location of the maximum deflection) = αL ; $y_{\max}$ (maximum deflection) = $\beta P/bE$.

The MD of simply supported beams are greater in the PM than in the CM for $h = 0.1L$ and $h = 0.2L$, being greater at $h = 0.1L$, and the greatest difference between both models occurs at $h = 0.2L$ for $e = 0.2L$ and $e = 0.8L$ of PM = 1.0276CM (see Table 3).

The MD of beams perfectly fixed at their ends are greater in the CM than in the PM for $h = 0.1L$ and $h = 0.05L$, being greater at $h = 0.05L$, and the greatest difference between both models occurs at $h = 0.1L$ for $e = 0.1L$ and $e = 0.9L$ of PM = 0.5443CM (see Table 4).

The MD of beams with the same moment at the ends for both cases are greater in the PM than in the CM for $h = 0.1L$ and $h = 0.05L$, being greater at $h = 0.05L$, and the greatest difference between both models occurs at $h = 0.1L$ for $e = 0.1L$ and $e = 0.9L$ of PM = 1.2334CM (see Table 5).

This model works for any relationship of a , c , e , h , u and z .

5. Conclusions. In this paper, the EC of a rectangular beam with straight haunches that support a concentrated load located at any point of the beam and a moment at each end has been investigated, considering the deformations due to shear and bending (Timoshenko theory) for the general case.

The main conclusions are the following.

- 1) The CM for the beams fixed at both ends for the moments and reactions at support "A" is not conditioned by the " h/L " ratios (see Table 2).
- 2) The greatest difference in maximum displacements for simply supported beams occurs at " $h = 0.2L$ ", for the beams perfectly fixed at their ends and for the same moment at the ends it occurs at " $h = 0.1L$ ", that is, at higher ratios of " h/L " there is the greatest difference between both models.

TABLE 4. Beam perfectly fixed at its ends

e	Factors for the maximum deflections					
	α			β		
	PM	CM	PM/CM	PM	CM	PM/CM
$h = 0.1L$						
$0.1L$	0.3287	0.3388	0.9702	-0.9067	-1.6657	0.5443
$0.2L$	0.3607	0.3683	0.9794	-6.1700	-7.6992	0.8014
$0.3L$	0.4093	0.4094	0.9998	-16.6220	-18.0457	0.9211
$0.4L$	0.4519	0.4517	1.0004	-26.3570	-27.7122	0.9511
$0.5L$	0.5000	0.5000	1.0000	-30.3928	-31.7171	0.9582
$0.6L$	0.5481	0.5483	0.9996	-26.3570	-27.7122	0.9511
$0.7L$	0.5907	0.5906	1.0002	-16.6220	-18.0457	0.9211
$0.8L$	0.6393	0.6317	1.0120	-6.1700	-7.6992	0.8014
$0.9L$	0.6713	0.6612	1.0153	-0.9067	-1.6657	0.5443
$h = 0.05L$						
$0.1L$	0.3363	0.3388	0.9926	-11.6918	-13.3260	0.8774
$0.2L$	0.3667	0.3683	0.9957	-58.6784	-61.5934	0.9527
$0.3L$	0.4093	0.4094	0.9998	-141.5282	-144.3654	0.9803
$0.4L$	0.4517	0.4517	1.0000	-219.0017	-221.6978	0.9878
$0.5L$	0.5000	0.5000	1.0000	-251.0880	-253.7365	0.9896
$0.6L$	0.5483	0.5483	1.0000	-219.0017	-221.6978	0.9878
$0.7L$	0.5907	0.5906	1.0002	-141.5282	-144.3654	0.9803
$0.8L$	0.6333	0.6317	1.0025	-58.6784	-61.5934	0.9527
$0.9L$	0.6637	0.6612	1.0038	-11.6918	-13.3260	0.8774

TABLE 5. Same moments at the ends for both cases

e	Factors for the maximum deflections					
	α			β		
	PM	CM	PM/CM	PM	CM	PM/CM
$h = 0.1L$						
$0.1L$	0.3287	0.3427	0.9591	-0.9067	-0.7351	1.2334
$0.2L$	0.3607	0.3733	0.9662	-6.1700	-5.0709	1.2167
$0.3L$	0.4093	0.4122	0.9930	-16.6220	-15.6280	1.0636
$0.4L$	0.4519	0.4527	0.9982	-26.3570	-25.4416	1.0360
$0.5L$	0.5000	0.5000	1.0000	-30.3928	-29.5091	1.0299
$0.6L$	0.5481	0.5473	1.0015	-26.3570	-25.4416	1.0360
$0.7L$	0.5907	0.5878	1.0049	-16.6220	-15.6280	1.0636
$0.8L$	0.6393	0.6267	1.0201	-6.1700	-5.0709	1.2167
$0.9L$	0.6713	0.6573	1.0213	-0.9067	-0.7351	1.2334
$h = 0.05L$						
$0.1L$	0.3363	0.3383	0.9941	-11.6918	-11.3536	1.0298
$0.2L$	0.3667	0.3693	0.9930	-58.6784	-56.4913	1.0387
$0.3L$	0.4093	0.4100	0.9983	-141.5282	-139.5416	1.0142
$0.4L$	0.4517	0.4519	0.9996	-219.0017	-217.1713	1.0084
$0.5L$	0.5000	0.5000	1.0000	-251.0880	-249.3205	1.0071
$0.6L$	0.5483	0.5481	1.0004	-219.0017	-217.1713	1.0084
$0.7L$	0.5907	0.5900	1.0012	-141.5282	-139.5416	1.0142
$0.8L$	0.6333	0.6307	1.0041	-58.6784	-56.4913	1.0387
$0.9L$	0.6637	0.6617	1.0030	-11.6918	-11.3536	1.0298

3) The PM is greater in all cases for the simply supported beams with respect to the EC and maximum deflections.

4) The PM is smaller in all cases for the beams perfectly fixed at their ends with respect to the EC and maximum deflections.

5) The PM is greater in all cases for beams with the same moment at the ends with respect to the EC and maximum deflections.

Then, the MD of the PM show a greater difference with respect to the CM. Therefore, MD acting on the beams by the PM in this work should be compared against the MD allowed by building codes, as in some situations they may not meet the standards set by building codes or in other situations they may be conservative.

Therefore, PM is safer and more appropriate than CM for structural analysis, because bending moments and shear forces are present in any type of structure and bending and shear deformations appear.

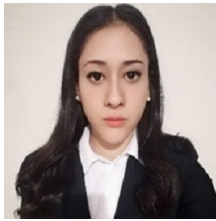
The suggestions for future research may be 1) when the applied load is different from a uniformly distributed load and/or concentrated load; 2) when the cross-section of the beam is different from a rectangular cross-section; 3) when the beam is constructed with other type of haunches such as elliptical or circular.

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Author Biography



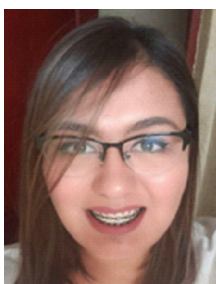
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