

COMPARATIVE ANALYSIS OF THE MINIMUM SOIL CONTACT AREAS BETWEEN RECTANGULAR AND ELLIPTICAL ISOLATED FOOTINGS WITH PARTIALLY AND FULLY COMPRESSED SURFACES

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Received February 2026; revised June 2026

ABSTRACT. *This paper shows a comparison of the minimum soil contact areas between RIF (rectangular isolated footings) and EIF (elliptical isolated footings) whose area is fully compressed considering the CM (current model) and/or the area is partially compressed assuming the NM (new model). Methodology is developed using the equations described in other works presented by the same author for RIF and EIF. Several numerical studies are presented for RIF and EIF subjected to biaxial and uniaxial bending. For biaxial bending, two types of studies are described: 1) the load P remains constant and the moments M_x and M_y vary; 2) the moments M_x and M_y remain constant and the load P varies. For uniaxial bending, when P is on the Y -axis, two types of studies are described: 1) the load P remains constant, the moment M_x varies and $M_y = 0$; 2) the moment M_x remains constant, $M_y = 0$ and the load P varies. For uniaxial bending, when P is on the X -axis, two types of studies are described: 1) the load P remains constant, the moment M_y varies and $M_x = 0$; 2) the moment M_y remains constant, $M_x = 0$ and the load P varies. The main results for biaxial bending indicate the CM shows a saving of up to 30.19% using the EIF, and for the NM it can reach up to 79.80% using the EIF. The main results for uniaxial bending indicate the CM shows a saving of up to 4.58% using the RIF, and for the NM it can reach up to 12.95% using the EIF. Therefore, it is recommended to use EIF for biaxial flexion, and for uniaxial flexion a prior study should be carried out to choose the EIF or RIF.*

Keywords: Minimum soil contact areas, Rectangular isolated footings, Elliptical isolated footings, Fully compressed area, Partially compressed area

1. Introduction. The foundation is the structural element (substructure) that transfers the loads from the superstructure to the ground. Footings are used in various types of construction in structural engineering, such as buildings and bridges.

The main objective of geotechnical and structural engineering is to obtain the smallest area and minimum cost of the footing to support the loads imposed by the superstructure.

The soil pressure below a foundation depends on the type of soil, the relative stiffness of the soil and the footing, and the depth of the foundation at the level of contact between the foundation and the soil [1].

The contact pressure depends on the position of the concentrated load “ P ” and the moments “ M_x ” and “ M_y ”. The base of the footing may be fully compressed, or a fraction of it may be without compression and without tension (zero pressure). The area that works under compression is called the active compression zone, and the zero pressure condition is defined by the NA (neutral axis) [2].

The most important contributions to the study of isolated footings have been developed by several researchers, such as square [3-6], rectangular [7-9], circular [10-14] and elliptical [15,16] isolated footings subjected to biaxial bending. These articles assume that the entire contact area works under compression.

The most outstanding research on combined footings presented by various researchers, such as rectangular [17-19], trapezoidal [20-22], L-shaped [23-26], strap [27-31], T-shaped [32-36] and elliptical [37] combined footings subjected to biaxial bending in each column. These articles assume that the entire contact area works under compression.

There are other studies to determine the dimensions and the contact area with the soil where it is partially compressed: for rectangular [38], circular [39,40] and elliptical [41] isolated footings subjected to biaxial bending, for rectangular [42], trapezoidal [43], T-shaped [44] and L-shaped [45,46] combined footings subjected to biaxial bending in each column, for rectangular foundation slabs that support an entire building [47], for annular strip footings or hollow circular footings [48], for strip footings [49].

The comparative studies that have been developed between different types of footings are as follows. Luévanos-Rojas [50] developed a study between isolated square, rectangular and circular footings, resulting in the finding that circular footings are more economical. Luévanos-Rojas et al. [51] conducted a study between trapezoidal and rectangular combined footings for design (assuming linear soil pressure), resulting in the finding that trapezoidal combined footings are more economical than rectangular combined footings. Pandey and Rai [52] analyzed an isolated footing with and without a strap beam by comparing the bending moment, bending shear, punching shear and settlement. Garay-Gallegos et al. [53] developed a study for T-shaped combined footings to compare for design of the current model (assuming uniform soil pressure) with the new model (assuming linear soil pressure), the new model being more economical than the current model. Raol and Suthar [54] presented an analysis of raft foundation, raft with drop and raft foundation with piles. Luévanos-Rojas et al. [55] presented a study between circular and rectangular isolated footings to determine the minimum area assuming that the area is totally and partially compressed, resulting in RIF under uniaxial bending having a smaller area than circular isolated footings in CM and NM; NM presents a smaller area than CM for RIF and circular isolated footings under biaxial bending in all cases; NM for circular isolated footings shows a smaller area than RIF, when the moments are equals and the ratio of axial load divided by the moment is 0.75 or greater, when the smallest moment divided by the larger moment is between 0.50 and 0.75, and the axial load is greater than the larger moment.

The articles that present the minimum contact area with the ground of the topic discussed here are for RIF [38] and EIF (This article shows the minimum area for biaxial bending, but not for uniaxial bending) [41] that work partially in compression.

This article describes a comparative study of minimum areas in contact with the ground for rectangular and elliptical isolated footings working partially and fully in compression. The comparisons are the followings: 1) The CM and the NM for RIF; 2) The CM and the NM for EIF; 3) The CM for RIF and EIF; 4) The NM for RIF and EIF. For RIF subjected

to biaxial bending, CM is Case I (area is totally compressed) and NM is the least of Cases II, III, IV and V (area is partially compressed). For RIF subjected to uniaxial bending, CM are Cases Y-I and X-I (area is totally compressed) and NM are Cases Y-II and X-II (area is partially compressed). For EIF subjected to biaxial bending, CM is Case I (area is totally compressed) and NM is Case II (area is partially compressed). For EIF subjected to uniaxial bending, CM are Cases Y-I and X-I (area is totally compressed) and NM are Cases Y-II and X-II (area is partially compressed). These numerical studies are performed using computer software to obtain the solution. These comparisons are made to observe which footing presents the minimum contact area with the ground, both in biaxial and uniaxial bending (main contribution of this work).

The paper is organized as follows. Section 2 indicates the methodology to determine the minimum area of an RIF and EIF using the CM (fully compressed area) and the NM (partially compressed area). Subsection 2.1 indicates the equations of an RIF for the five cases. Subsection 2.2 indicates the equations of an EIF for the two cases. Section 3 describes the numerical studies to obtain the dimensions and minimum area of RIF and EIF. Section 4 indicates the results. Section 5 indicates the conclusions to complete the article.

2. Methodology. The rigid footing deforms in a flat manner, that is, the distribution of soil pressure under the footing is considered linear.

The equation for biaxial and uniaxial bending is not valid when the resultant force P is located outside the central nucleus, a well-known and delimited area, indicating a non-compressed zone (see Figure 1).

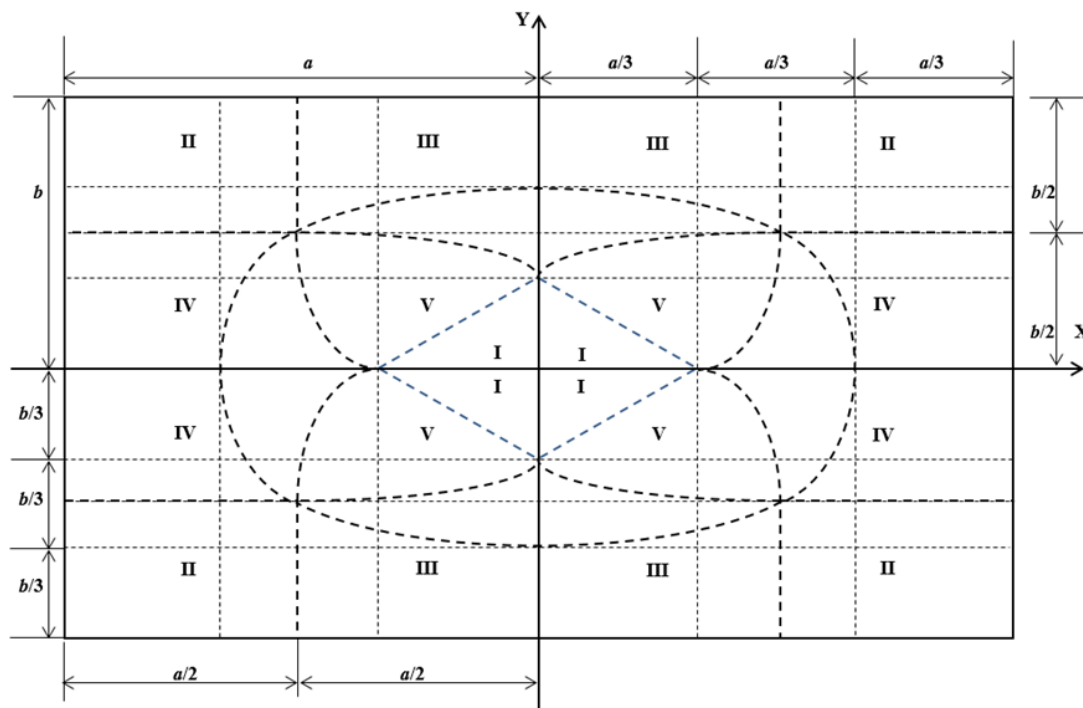


FIGURE 1. Eccentricities according to the location of P for RIF

2.1. Rectangular isolated footings. Figure 1 represents the complete eccentricity diagram resulting from the entire base of the RIF. The resulting eccentricities are measured from the center of the footing, i.e., e_x (distance in the X direction from the center of the RIF in m) = M_y/P , e_y (distance in the Y direction from the center of the RIF in m) =

M_x/P , where M_x = Moment about the X-axis in kN-m, M_y = Moment about the Y-axis in kN-m, P = Vertical force in kN. If P is located in zone I (central nucleus), the footing area works entirely under compression. If P is located in zones II, III, IV, and V, the footing area works partially under compression.

2.1.1. *Biaxial bending.* Figure 2 represents the five possible cases for an RIF subjected to biaxial bending.

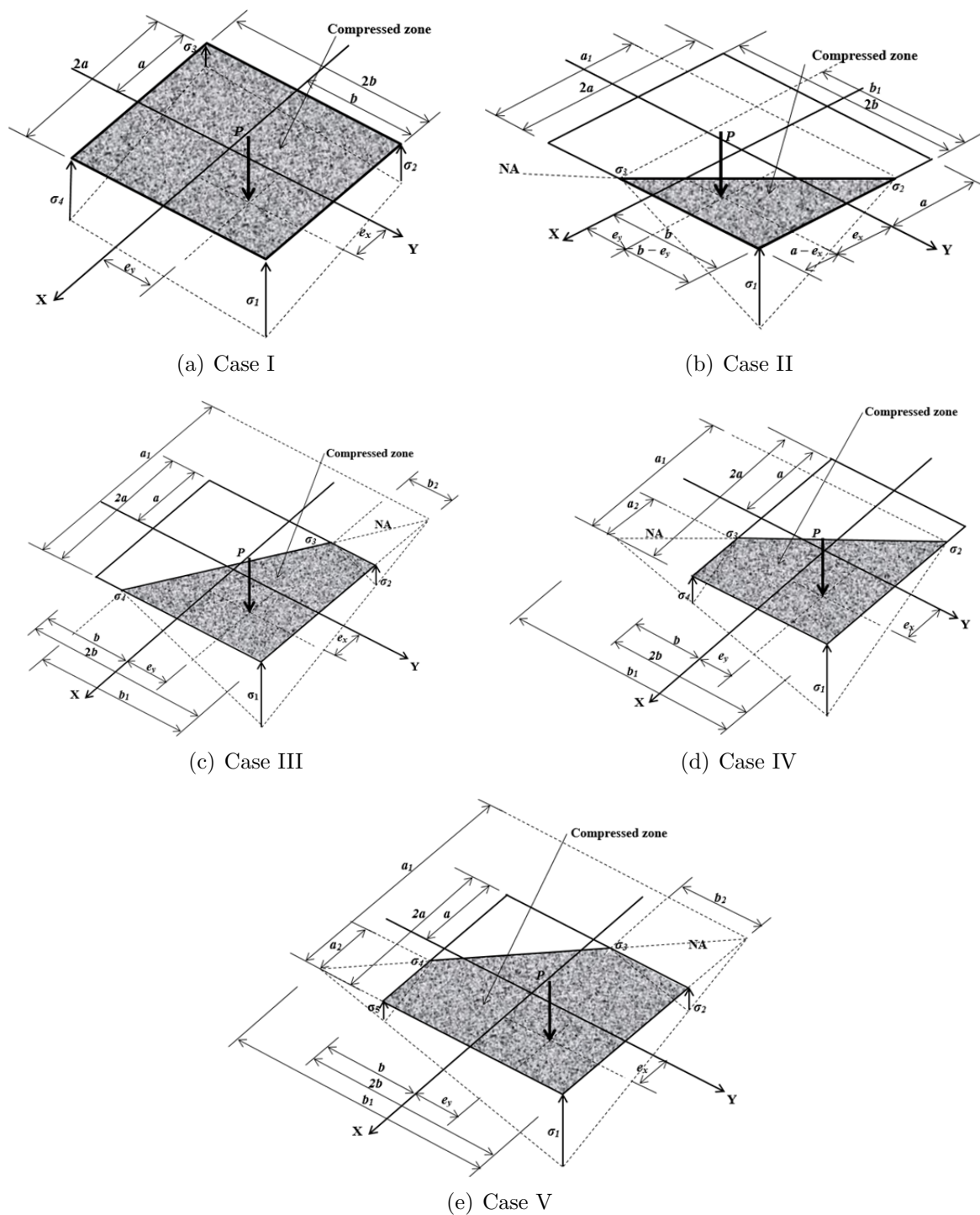


FIGURE 2. RIF subjected to biaxial bending

Case I assumes that the total area of the RIF works in compression. The pressure generated by the soil at any point of the footing “ σ_z ” is obtained using the biaxial bending equation [38]:

$$\sigma_z = \frac{P}{A} + \frac{M_x y}{I_x} + \frac{M_y x}{I_y} \quad (1)$$

where A = Area or contact surface of the footing with the soil in m, I_x = Moment of inertia about the X-axis in m⁴, I_y = Moment of inertia about the Y-axis in m⁴.

Cases II, III, IV and V assume that the total area of the RIF works partially in compression, that is, part of the area of the RIF has zero pressure.

The pressure below the soil at any part of the RIF is represented as follows [38]:

$$\sigma_z = \frac{\sigma_{\max} [b_1(x - a) + a_1(y - b) + a_1 b_1]}{a_1 b_1} \quad (2)$$

where $\sigma_{\max} = \sigma_1$ = Available allowable bearing capacity of the soil, a_1 = Distance in the X direction from $\sigma_{\max}$ to where it crosses the NA (neutral axis) in m, b_1 = Distance in the Y direction from $\sigma_{\max}$ to where it crosses the NA.

Figure 2(a) indicates when P is inside the central nucleus (Case I).

Substitute $A = 4ab$, $I_x = 4ab^3/3$ and $I_y = 4ba^3/3$ to determine the soil pressure under the RIF at the four corners subjected to biaxial bending [38]:

$$\sigma_1 = \frac{P}{4ab} + \frac{3M_x}{4ab^2} + \frac{3M_y}{4a^2b} \quad (3)$$

$$\sigma_2 = \frac{P}{4ab} + \frac{3M_x}{4ab^2} - \frac{3M_y}{4a^2b} \quad (4)$$

$$\sigma_3 = \frac{P}{4ab} - \frac{3M_x}{4ab^2} - \frac{3M_y}{4a^2b} \quad (5)$$

$$\sigma_4 = \frac{P}{4ab} - \frac{3M_x}{4ab^2} + \frac{3M_y}{4a^2b} \quad (6)$$

Figures 2(b), 2(c), 2(d) and 2(e) indicate when P is outside the central nucleus (Cases II, III, IV and V).

The general equations to determine P_R , M_{xR} and M_{yR} for each case are obtained [38]:

Case II

$$P_R = \frac{\sigma_{\max} a_1 b_1}{6} \quad (7)$$

$$M_{xR} = \frac{\sigma_{\max} a_1 b_1 (4b - b_1)}{24} \quad (8)$$

$$M_{yR} = \frac{\sigma_{\max} a_1 b_1 (4a - a_1)}{24} \quad (9)$$

Case III

$$P_R = \frac{\sigma_{\max} b_1 [a_1^3 - (a_1 - 2a)^3]}{6a_1^2} \quad (10)$$

$$M_{xR} = \frac{\sigma_{\max} b_1 \{ (2a - a_1)^3 [a_1(4b - b_1) + 2ab_1] + a_1^4(4b - b_1) \}}{24a_1^3} \quad (11)$$

$$M_{yR} = \frac{2\sigma_{\max} b_1 a^3 (a_1 - a)}{3a_1^2} \quad (12)$$

Case IV

$$P_R = \frac{\sigma_{\max} a_1 [b_1^3 - (b_1 - 2b)^3]}{6b_1^2} \quad (13)$$

$$M_{xR} = \frac{2\sigma_{\max}a_1b^3(b_1 - b)}{3b_1^2} \quad (14)$$

$$M_{yR} = \frac{\sigma_{\max}a_1 \{(2b - b_1)^3 [b_1(4a - a_1) + 2ba_1] + b_1^4(4a - a_1)\}}{24b_1^3} \quad (15)$$

Case V

$$P_R = \frac{\sigma_{\max}a_1 [a_1^3b_1^3 + b_1^3(2a - a_1)^3 + a_1^3(2b - b_1)^3]}{6a_1^2b_1^2} \quad (16)$$

$$M_{xR} = \frac{\sigma_{\max} \{b_1^3(2a - a_1)^3 [a_1(4b - b_1) + 2ab_1] - 16a_1^4b^3(b - b_1)\}}{24a_1^3b_1^2} \quad (17)$$

$$M_{yR} = \frac{\sigma_{\max} \{a_1^3(2b - b_1)^3 [b_1(4a - a_1) + 2ba_1] - 16a^3b_1^4(a - a_1)\}}{24a_1^2b_1^3} \quad (18)$$

where P_R , M_{xR} , M_{yR} represent the load and moments that the footing must support; P , M_x , M_y represent the load and moments acting on the footing.

2.1.2. *Uniaxial bending.* Figure 3 represents the four possible cases for an RIF subjected to uniaxial bending, when the axial load P is inside the central nucleus (Cases X-I and Y-I), and when the axial load P is outside the central nucleus (Cases X-II and Y-II).

Figures 3(a) and 3(c) indicate that P is inside the central nucleus (Case Y-I: when P is on the Y-axis, and Case X-I: when P is on the X-axis).

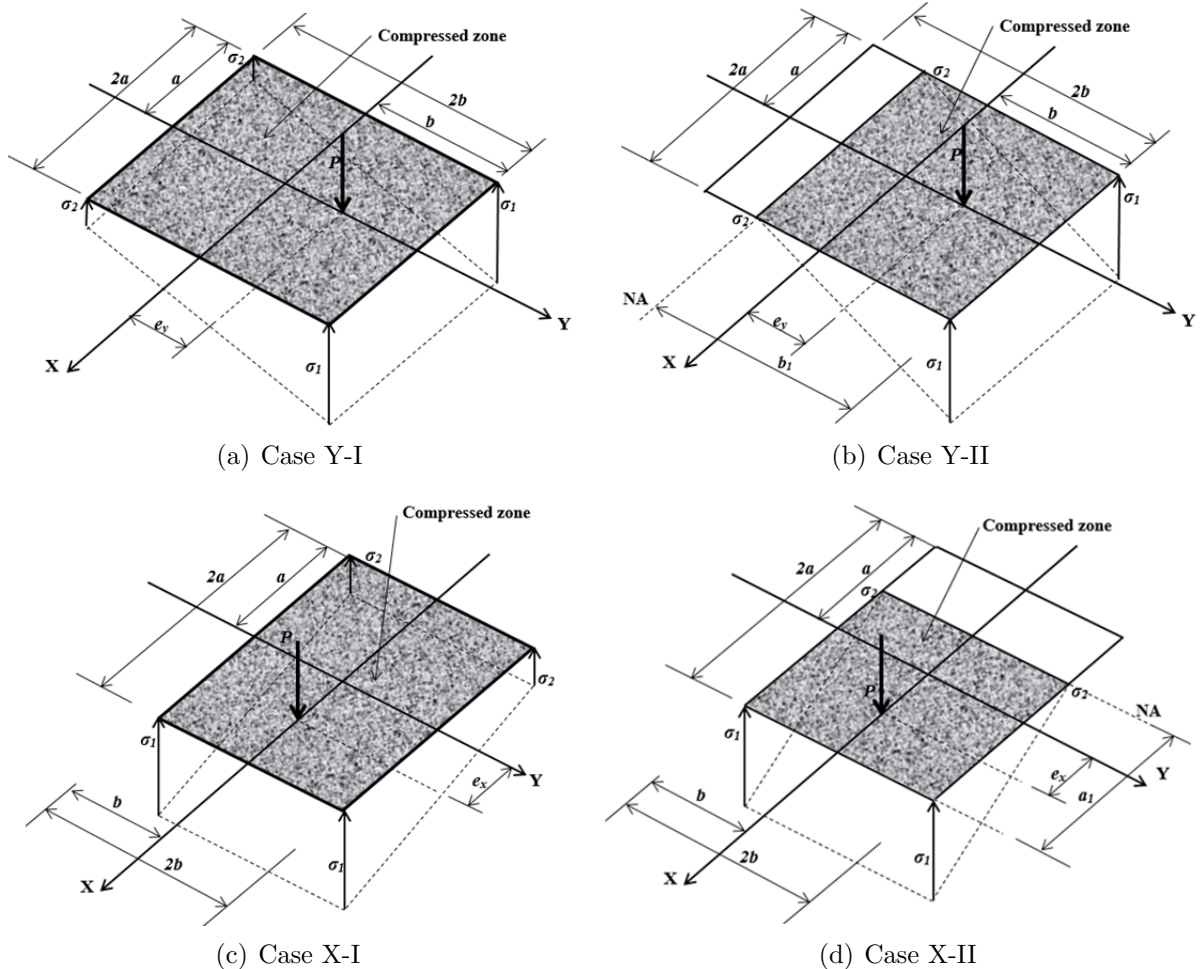


FIGURE 3. RIF subjected to uniaxial bending

Substitute $A = 4ab$, $I_x = 4ab^3/3$, $I_y = 4ba^3/3$ and $M_y = 0$ into Equation (1) to determine the soil pressure under the RIF at the corners subjected to uniaxial bending (Case Y-I) [38]:

$$\sigma_1 = \frac{P}{4ab} + \frac{3M_x}{4ab^2} \quad (19)$$

$$\sigma_2 = \frac{P}{4ab} - \frac{3M_x}{4ab^2} \quad (20)$$

Substitute $A = 4ab$, $I_x = 4ab^3/3$, $I_y = 4ba^3/3$ and $M_x = 0$ into Equation (1) to determine the soil pressure under the RIF at the corners subjected to uniaxial bending (Case X-I) [38]:

$$\sigma_1 = \frac{P}{4ab} + \frac{3M_y}{4a^2b} \quad (21)$$

$$\sigma_2 = \frac{P}{4ab} - \frac{3M_y}{4a^2b} \quad (22)$$

Figures 3(b) and 3(d) indicate that P is outside the central nucleus (Case Y-II: when P is on the Y-axis, and Case X-II: when P is on the X-axis).

The pressure below the soil at any part of the RIF for Case Y-II is represented as follows [38]:

$$\sigma_z = \frac{\sigma_{\max}(b_1 - b + y)}{b_1} \quad (23)$$

The general equations for Case Y-II to determine P_R and M_{xR} are obtained [38]:

$$P_R = \sigma_{\max}ab_1 \quad (24)$$

$$M_{xR} = \frac{\sigma_{\max}ab_1(3b - b_1)}{3} \quad (25)$$

The pressure below the soil at any part of the RIF for Case X-II is represented as follows [38]:

$$\sigma_z = \frac{\sigma_{\max}(a_1 - a + x)}{a_1} \quad (26)$$

The general equations for Case X-II to determine P_R and M_{yR} are obtained [38]:

$$P_R = \sigma_{\max}a_1b \quad (27)$$

$$M_{yR} = \frac{\sigma_{\max}a_1b(3a - a_1)}{3} \quad (28)$$

2.1.3. Minimum area for RIF. The minimum area " $A_{\min\text{RIF}}$ " (objective function) for all cases is

$$A_{\min\text{RIF}} = 4ab \quad (29)$$

Table 1 indicates the constraints for biaxial bending in each case (a , b , a_1 and b_1 are limited to meet the conditions of each case).

Table 2 indicates the constraints for uniaxial bending in each case (a and b are limited to avoid excessively disproportionate footings, i.e., footings that are too long in relation to their width).

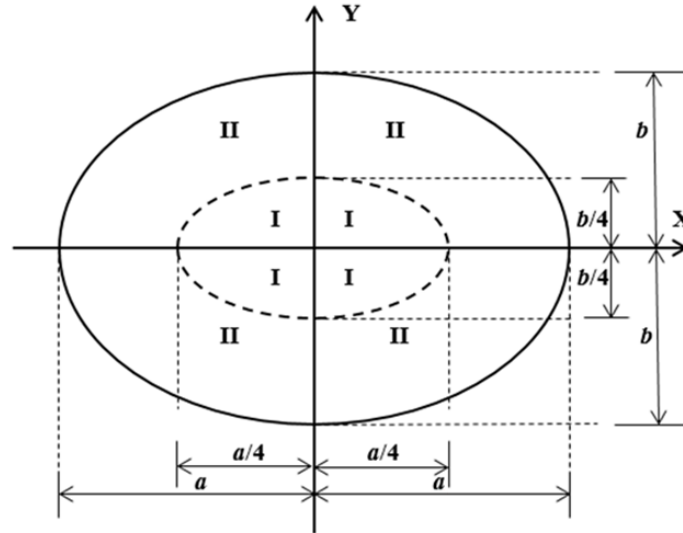
2.2. Elliptical isolated footings. Figure 4 represents the complete eccentricity diagram resulting from the entire base of the EIF. If P is in zone I, the footing area works entirely under compression. If P is in zone II, the footing area works partially under compression.

TABLE 1. Biaxial bending for RIF

Case	Constraints
I	Equations (3) to (6), $0 \leq \sigma_1, \sigma_2, \sigma_3, \sigma_4 \leq \sigma_{\max}$
II	Equations (7) to (9), $2a \geq a_1$ and $2b \geq b_1$
III	Equations (10) to (12), $(a_1 - 2a)b_1/a_1 \leq 2b$, $2a \leq a_1$ and $2b \geq b_1$
IV	Equations (13) to (15), $(b_1 - 2b)a_1/b_1 \leq 2a$, $2a \geq a_1$ and $2b \leq b_1$
V	Equations (16) to (18), $(a_1 - 2a)b_1/a_1 \leq 2b$, $(b_1 - 2b)a_1/b_1 \leq 2a$, $2a \leq a_1$ and $2b \leq b_1$

TABLE 2. Uniaxial bending for RIF

Case	Constraints
Y-I	Equations (19) and (20), $2a \geq \alpha$, $2b \geq \alpha$, $0 \leq \sigma_1$ and $\sigma_2 \leq \sigma_{\max}$
X-I	Equations (21) and (22), $2a \geq \alpha$, $2b \geq \alpha$, $0 \leq \sigma_1$ and $\sigma_2 \leq \sigma_{\max}$
Y-II	Equations (24) and (25), $2a \geq \alpha$, $2b \geq \alpha$, $2b \geq b_1$
X-II	Equations (27) and (28), $2a \geq \alpha$, $2b \geq \alpha$, $2a \geq a_1$

FIGURE 4. Eccentricities according to the location of P for EIF

2.2.1. *Biaxial bending.* Figure 5 represents the two possible cases for an EIF subjected to biaxial bending.

Case I assumes that the total area of the EIF works in compression. The pressure generated by the soil at any point of the footing “ σ_z ” is obtained using the biaxial bending equation [41].

Substituting $A = \pi ab$, $I_x = \pi ab^3/4$, $I_y = \pi a^3b/4$ and the corresponding coordinates $(x_{\max}, y_{\max})$ for the maximum pressure and $(x_{\min}, y_{\min})$ for the minimum pressure into Equation (1) is determined as follows:

$$\sigma_{\max 1} = \frac{P}{\pi ab} + \frac{4M_x y_{\max}}{\pi ab^3} + \frac{4M_y x_{\max}}{\pi a^3 b} \quad (30)$$

$$\sigma_{\min 2} = \frac{P}{\pi ab} + \frac{4M_x y_{\min}}{\pi ab^3} + \frac{4M_y x_{\min}}{\pi a^3 b} \quad (31)$$

where $\sigma_{\max 1}$ is the maximum pressure acting on EIF, and $\sigma_{\min 2}$ is the minimum pressure acting on EIF.

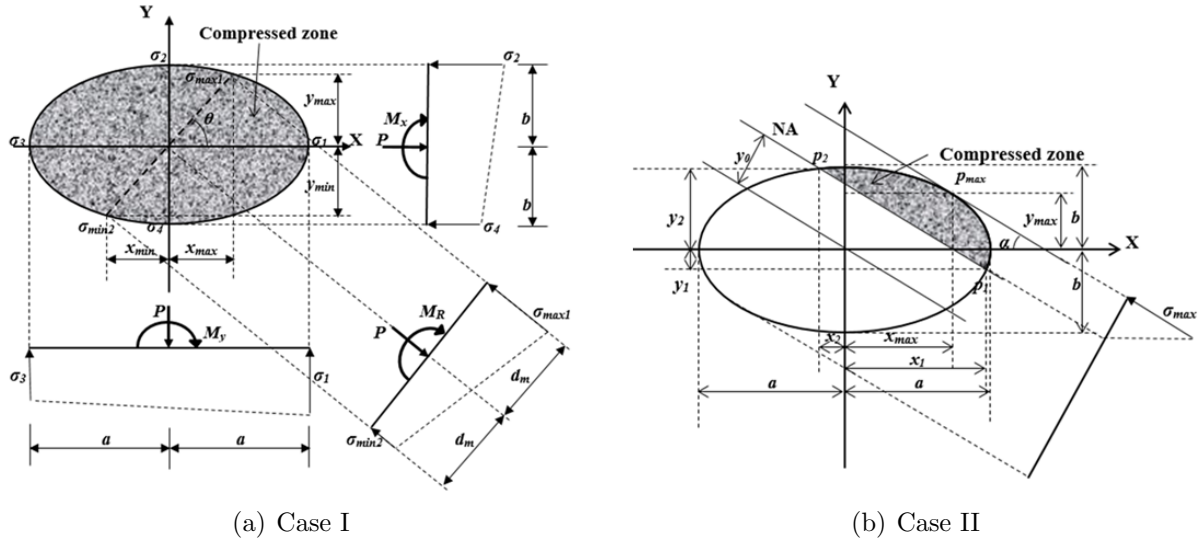


FIGURE 5. EIF subjected to biaxial bending. $p_{\max}$ represents the point of maximum pressure and $\sigma_{\max}$ represents the maximum allowable soil pressure.

The coordinates corresponding to $(x_{\max}, y_{\max})$ and the coordinates corresponding to $(x_{\min}, y_{\min})$ are [41]

$$x_{\max} = \frac{abM_y}{\sqrt{a^2M_x^2 + b^2M_y^2}}; \quad y_{\max} = \frac{abM_x}{\sqrt{a^2M_x^2 + b^2M_y^2}} \quad (32)$$

$$x_{\min} = -\frac{abM_y}{\sqrt{a^2M_x^2 + b^2M_y^2}}; \quad y_{\min} = -\frac{abM_x}{\sqrt{a^2M_x^2 + b^2M_y^2}} \quad (33)$$

Case II assumes that the total area of the EIF works partially in compression, that is, part of the area of the EIF has zero pressure.

The pressure below the soil at any part of the EIF is represented as follows [41]:

$$\sigma_z = \frac{\sigma_{\max} [a^2M_x y + b^2M_y x - y_0 \sqrt{a^4M_x^2 + b^4M_y^2}]}{ab\sqrt{a^2M_x^2 + b^2M_y^2} - y_0 \sqrt{a^4M_x^2 + b^4M_y^2}} \quad (34)$$

The general equations to determine P_R , M_{xR} and M_{yR} for each case are obtained [41]:
Case II

$$P_R = \int_{x_2}^{x_1} \int_{-\frac{b\sqrt{a^2-x^2}}{a}}^{\frac{b\sqrt{a^2-x^2}}{a}} \sigma_z dy dx + \int_{x_1}^a \int_{-\frac{b\sqrt{a^2-x^2}}{a}}^{\frac{b\sqrt{a^2-x^2}}{a}} \sigma_z dy dx \quad (35)$$

$$M_{xR} = \int_{x_2}^{x_1} \int_{-\frac{b\sqrt{a^2-x^2}}{a}}^{\frac{b\sqrt{a^2-x^2}}{a}} \sigma_z y dy dx + \int_{x_1}^a \int_{-\frac{b\sqrt{a^2-x^2}}{a}}^{\frac{b\sqrt{a^2-x^2}}{a}} \sigma_z y dy dx \quad (36)$$

$$M_{yR} = \int_{x_2}^{x_1} \int_{-\frac{b\sqrt{a^2-x^2}}{a}}^{\frac{b\sqrt{a^2-x^2}}{a}} \sigma_z x dy dx + \int_{x_1}^a \int_{-\frac{b\sqrt{a^2-x^2}}{a}}^{\frac{b\sqrt{a^2-x^2}}{a}} \sigma_z x dy dx \quad (37)$$

where x_1 is the X-coordinate at point p_1 and x_2 is the X-coordinate at point p_2 , and they are expressed as follows [41]:

$$x_1 = \frac{M_y b y_0 \sqrt{a^4M_x^2 + b^4M_y^2} + M_x a \sqrt{a^2b^2(a^2M_x^2 + b^2M_y^2) - y_0^2(a^4M_x^2 + b^4M_y^2)}}{b(a^2M_x^2 + b^2M_y^2)} \quad (38)$$

$$x_2 = \frac{M_y b y_0 \sqrt{a^4 M_x^2 + b^4 M_y^2} - M_x a \sqrt{a^2 b^2 (a^2 M_x^2 + b^2 M_y^2) - y_0^2 (a^4 M_x^2 + b^4 M_y^2)}}{b (a^2 M_x^2 + b^2 M_y^2)} \quad (39)$$

2.2.2. *Uniaxial bending.* Figure 6 represents the four possible cases for an EIF subjected to uniaxial bending. When the axial load is inside the central nucleus (Cases X-I and Y-I), and when the axial load is outside the central nucleus (Cases X-II and Y-II).

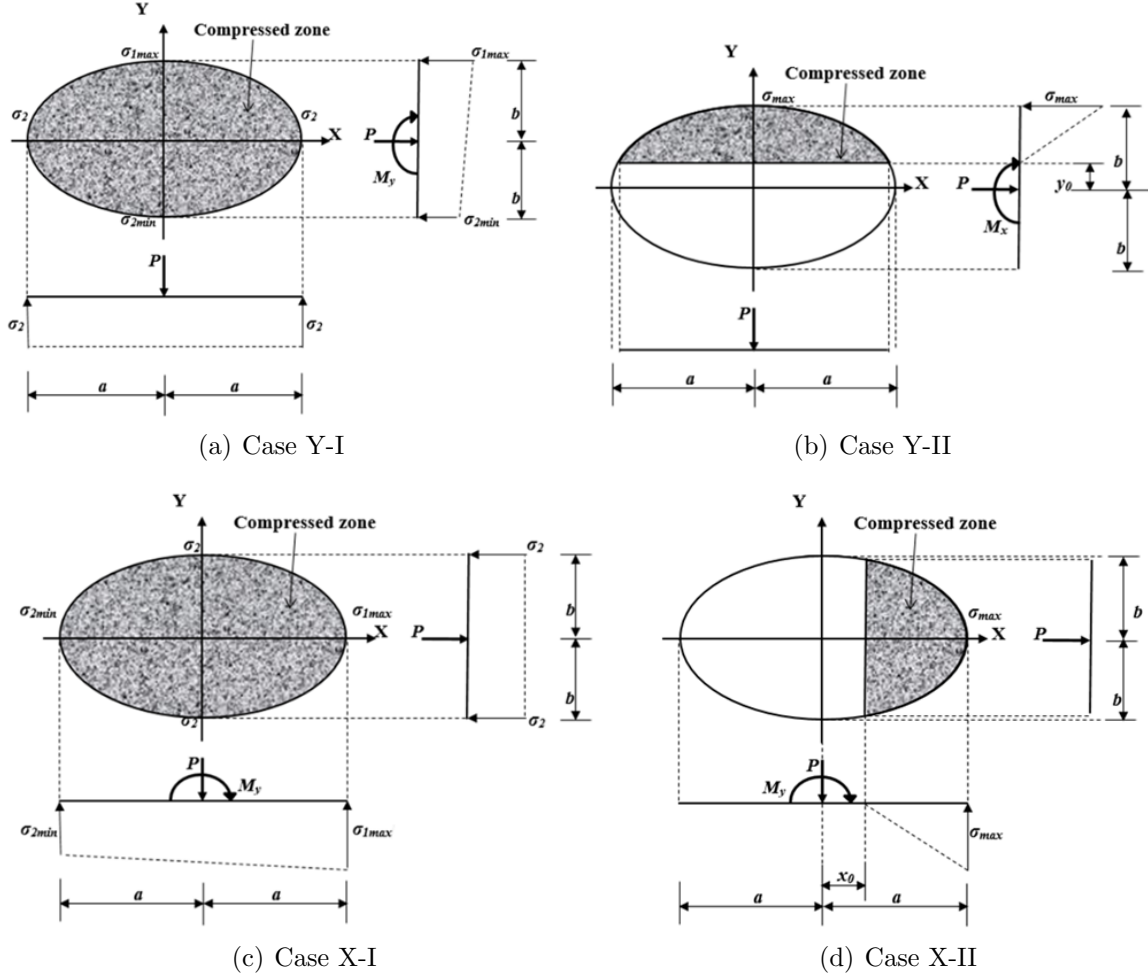


FIGURE 6. EIF subjected to uniaxial bending

Figures 6(a) and 6(c) indicate that P is inside the central nucleus (Case Y-I: when P is on the Y-axis, and Case X-I: when P is on the X-axis).

Substitute $A = \pi ab$, $I_x = \pi ab^3/4$, $I_y = \pi ba^3/4$ and $M_y = 0$ to determine the soil pressure under the EIF at the corners subjected to uniaxial bending (Case Y-I):

$$\sigma_1 = \frac{P}{\pi ab} + \frac{4M_x}{\pi ba^2} \quad (40)$$

$$\sigma_2 = \frac{P}{\pi ab} - \frac{4M_x}{\pi ba^2} \quad (41)$$

Substitute $A = \pi ab$, $I_x = \pi ab^3/4$, $I_y = \pi ba^3/4$ and $M_x = 0$ to determine the soil pressure under the EIF at the corners subjected to uniaxial bending (Case X-I):

$$\sigma_1 = \frac{P}{\pi ab} + \frac{4M_y}{\pi a^2 b} \quad (42)$$

$$\sigma_2 = \frac{P}{\pi ab} - \frac{4M_y}{\pi a^2 b} \quad (43)$$

Figures 6(b) and 6(d) indicate that P is outside the central nucleus (Case Y-II: when P is on the Y-axis, and Case X-II: when P is on the X-axis).

The pressure below the soil at any part of the EIF for Case Y-II is represented as follows:

$$\sigma_z = \frac{\sigma_{\max}(y - y_0)}{b - y_0} \quad (44)$$

The general equations for Case Y-II to determine P_R and M_{xR} are obtained:

$$P_R = \frac{\sigma_{\max} a \left\{ 2(2b^2 + y_0^2) \sqrt{b^2 - y_0^2} - 3b^2 y_0 [\pi - 2 \arcsin(\frac{y_0}{b})] \right\}}{6b(b - y_0)} \quad (45)$$

$$M_{xR} = \frac{\sigma_{\max} a \left\{ 3b^4 [\pi - 2 \arcsin(\frac{y_0}{b})] - 2y_0 \sqrt{b^2 - y_0^2} (5b^2 - 2y_0^2) \right\}}{24b(b - y_0)} \quad (46)$$

The pressure below the soil at any part of the EIF for Case X-II is represented as follows:

$$\sigma_z = \frac{\sigma_{\max}(x - x_0)}{a - x_0} \quad (47)$$

The general equations for Case X-II to determine P_R and M_{yR} are obtained:

$$P_R = \frac{\sigma_{\max} b \left\{ 2(2a^2 + x_0^2) \sqrt{a^2 - x_0^2} - 3a^2 x_0 [\pi - 2 \arcsin(\frac{x_0}{a})] \right\}}{6a(a - x_0)} \quad (48)$$

$$M_{yR} = \frac{\sigma_{\max} b \left\{ 3a^4 [\pi - 2 \arcsin(\frac{x_0}{a})] - 2x_0 \sqrt{a^2 - x_0^2} (5a^2 - 2x_0^2) \right\}}{24a(a - x_0)} \quad (49)$$

The equations for RIF have already been verified by Vela-Moreno et al. [38], and the equations for EIF have already been verified by Luévanos-Rojas et al. [41].

2.2.3. Minimum area for EIF. The minimum area “ $A_{\min \text{EIF}}$ ” (objective function) for all cases is

$$A_{\min \text{EIF}} = \pi ab \quad (50)$$

Table 3 indicates the constraints for biaxial bending in each case.

TABLE 3. Biaxial bending for EIF

Case	Constraints
I	Equations (30) and (31), $0 \leq \sigma_{\max 1}$ and $\sigma_{\min 2} \leq \sigma_{\max}$
II	Equations (35) to (37), $ y_0 \leq ab\sqrt{a^2 M_x^2 + b^2 M_y^2} / \sqrt{a^4 M_x^2 + b^4 M_y^2}$, $P_R \geq P$, $M_{xR} \geq M_x$, $M_{yR} \geq M_y$

Table 4 indicates the constraints for uniaxial bending in each case (a and b are limited to avoid excessively disproportionate footings, i.e., footings that are too long in relation to their width).

3. Numerical Studies. Tables 5 to 11 present the minimum areas and dimensions of RIF and EIF subjected to biaxial bending for Case I (fully compressed area) of the two

TABLE 4. Uniaxial bending for EIF

Case	Constraints
Y-I	Equations (40) and (41), $2a \geq \alpha$, $2b \geq \alpha$, $0 \leq \sigma_1, \sigma_2 \leq \sigma_{\max}$
X-I	Equations (42) and (43), $2a \geq \alpha$, $2b \geq \alpha$, $0 \leq \sigma_1, \sigma_2 \leq \sigma_{\max}$
Y-II	Equations (45) and (46), $2a \geq \alpha$, $2b \geq \alpha$, $b \geq y_0 \geq -b$
X-II	Equations (48) and (49), $2a \geq \alpha$, $2b \geq \alpha$, $a \geq x_0 \geq -a$

footings, and the smallest of Cases II, III, IV and V for RIF and Case II for EIF (partially compressed area).

The input data for each study is $\sigma_{\max} = 250$ kN/m², $P = 600$ kN, $M_x = 100, 300, 500, 700, 900, 1100$ kN-m; $M_y = 100$ kN-m (Table 5), 300 kN-m (Table 6), 500 kN-m (Table 7), 700 kN-m (Table 8), 900 kN-m (Table 9), 1100 kN-m (Table 10). Table 11 represents the results for $P = 200, 400, 600, 800, 1000, 1200, 1400, 1600, 1800, 2000$ kN; $M_x = M_y = 700$ kN-m and $\sigma_{\max} = 250$ kN/m².

Tables 12 to 15 present the minimum areas and dimensions of RIF and EIF subjected to uniaxial bending for Cases Y-I and X-I (fully compressed area), and for Cases Y-II and X-II (partially compressed area) of the two footings. Tables 12 and 13 indicate the Cases Y-I and Y-II. Table 12 represents the results for $P = 600$ kN, $M_x = 250, 500, 750, 1000, 1250, 1500$ kN-m, $M_y = 0$ kN-m and $\sigma_{\max} = 250$ kN/m². Table 13 indicates the results for $P = 250, 500, 750, 1000, 1250, 1500$ kN, $M_x = 500$ kN-m, $M_y = 0$ kN-m and $\sigma_{\max} = 250$ kN/m². Tables 14 and 15 indicate the Cases X-I and X-II. Table 14 indicates the results for $P = 600$ kN, $M_x = 0$ kN-m, $M_y = 250, 500, 750, 1000, 1250, 1500$ kN-m and $\sigma_{\max} = 250$ kN/m². Table 15 indicates the results for $P = 250, 500, 750, 1000, 1250, 1500$ kN, $M_x = 0$ kN-m, $M_y = 500$ kN-m and $\sigma_{\max} = 250$ kN/m².

4. Results. Table 5 for RIF indicates that when M_x increases for CM: a tends to decrease to $M_x = 300$ kN-m and then remains constant; b and $A_{\min\text{RIF}}$ tend to increase, and for NM, a and a_1 tend to decrease; b , b_1 and $A_{\min\text{RIF}}$ tend to increase. Table 5 for EIF indicates that when M_x increases for CM: a tends to decrease to $M_x = 300$ kN-m and then remains constant; b and $A_{\min\text{EIF}}$ tend to increase, and for NM, a and y_0 (absolute value) tend to decrease; b and $A_{\min\text{EIF}}$ tend to increase. Here, when it reaches the maximum value in Case I for RIF and EIF of $\sigma_{\max} = 250$ kN/m², it reaches the minimum value of $A_{\min}$ for both studies.

TABLE 5. Study 1 ($P = 600$ kN and $M_y = 100$ kN-m)

M_x (kN-m)	Case	$A_{\min\text{RIF}}$ (m ²)	a (m)	b (m)	a_1 (m)	b_1 (m)	σ_1 (kN/m ²)	σ_3 (kN/m ²)	Case	$A_{\min\text{EIF}}$ (m ²)	a (m)	b (m)	y_0 (m)	$\sigma_{\max 1}$ (kN/m ²)	$\sigma_{\min 2}$ (kN/m ²)	$A_{\min\text{RIF}} / A_{\min\text{EIF}}$
100	I	4.63	1.08	1.08	—	—	250	9.15	I	4.33	1.17	1.17	—	250	27.27	1.07
	V	4.80	1.10	1.10	4.38	4.38	250	0	II	4.92	1.25	1.25	-1.23	250	0	0.98
300	I	12.00	1.00	3.00	—	—	100	0	I	8.38	0.94	2.83	—	143.23	0	1.43
	V	5.90	0.70	2.10	2.35	7.05	250	0	II	5.42	0.76	2.27	-0.80	250	0	1.09
500	I	20.00	1.00	5.00	—	—	60	0	I	13.96	0.94	4.71	—	85.94	0	1.43
	V	6.82	0.58	2.92	1.77	8.84	250	0	II	6.22	0.63	3.15	-0.51	250	0	1.10
700	I	28.00	1.00	7.00	—	—	42.86	0	I	19.55	0.94	6.60	—	61.39	0	1.43
	V	7.60	0.52	3.65	1.47	10.30	250	0	II	6.92	0.56	3.93	-0.37	250	0	1.10
900	I	36.00	1.00	9.00	—	—	33.33	0	I	25.13	0.94	8.49	—	47.75	0	1.43
	V	8.30	0.48	4.32	1.29	11.57	250	0	II	7.53	0.52	4.64	-0.34	250	0	1.10
1100	I	44.00	1.00	11.00	—	—	27.27	0	I	30.72	0.94	10.37	—	39.07	0	1.43
	V	8.95	0.45	4.96	1.16	12.72	250	0	II	8.05	0.48	5.31	-0.32	250	0	1.11

Table 6 for RIF indicates that when M_x increases for CM: a remains constant; b and $A_{\min\text{RIF}}$ tend to increase, and for NM, a and a_1 tend to decrease; b , b_1 and $A_{\min\text{RIF}}$ tend to increase. Table 6 for EIF indicates that when M_x increases for CM: a remains constant; b and $A_{\min\text{EIF}}$ tend to increase, and for NM, a tends to decrease; y_0 (absolute value) tends to decrease to $M_x = 300$ kN-m, then increases to $M_x = 500$ kN-m and subsequently decreases; b and $A_{\min\text{EIF}}$ tend to increase.

TABLE 6. Study 2 ($P = 600$ kN and $M_y = 300$ kN-m)

M_x (kN-m)	Case	$A_{\min\text{RIF}}$ (m ²)	a (m)	b (m)	a_1 (m)	b_1 (m)	σ_1 (kN/m ²)	σ_3 (kN/m ²)	Case	$A_{\min\text{EIF}}$ (m ²)	a (m)	b (m)	y_0 (m)	$\sigma_{\max 1}$ (kN/m ²)	$\sigma_{\min 2}$ (kN/m ²)	$A_{\min\text{RIF}} / A_{\min\text{EIF}}$
100	I	12.00	3.00	1.00	—	—	100	0	I	8.38	2.83	0.94	—	143.24	0	1.43
	V	5.90	2.20	0.70	7.05	2.35	250	0	II	5.42	2.27	0.76	-0.80	250	0	1.09
300	I	36.00	3.00	3.00	—	—	33.33	0	I	25.13	2.83	2.83	—	47.75	0	1.43
	V	8.30	1.44	1.44	3.86	3.86	250	0	II	7.53	1.55	1.55	-0.73	250	0	1.10
500	I	60.00	3.00	5.00	—	—	20	0	I	41.89	2.83	4.71	—	28.65	0	1.43
	V	10.13	1.23	2.05	2.95	4.92	250	0	II	8.92	1.31	2.18	-0.75	250	0	1.14
700	I	84.00	3.00	7.00	—	—	14.29	0	I	58.64	2.83	6.60	—	20.46	0	1.43
	V	11.72	1.12	2.61	2.49	5.80	250	0	II	9.98	1.17	2.72	-0.71	250	0	1.17
900	I	108.00	3.00	9.00	—	—	11.11	0	I	75.40	2.83	8.49	—	15.92	0	1.43
	V	13.17	1.05	3.14	2.19	6.57	250	0	II	10.86	1.07	3.22	-0.68	250	0	1.21
1100	I	132.00	3.00	11.00	—	—	9.09	0	I	92.15	2.83	10.37	—	13.02	0	1.43
	II	14.53	0.99	3.65	1.98	7.27	250	0	II	11.61	1.00	3.68	-0.65	250	0	1.25

Table 7 for RIF indicates the same behavior as Table 6 for CM and NM. Table 7 for EIF shows the same behavior as Table 6 for CM and NM, except that y_0 (absolute value) tends to increase to $M_x = 700$ kN-m and then decreases.

TABLE 7. Study 3 ($P = 600$ kN and $M_y = 500$ kN-m)

M_x (kN-m)	Case	$A_{\min\text{RIF}}$ (m ²)	a (m)	b (m)	a_1 (m)	b_1 (m)	σ_1 (kN/m ²)	σ_3 (kN/m ²)	Case	$A_{\min\text{EIF}}$ (m ²)	a (m)	b (m)	y_0 (m)	$\sigma_{\max 1}$ (kN/m ²)	$\sigma_{\min 2}$ (kN/m ²)	$A_{\min\text{RIF}} / A_{\min\text{EIF}}$
100	I	20.00	5.00	1.00	—	—	60	0	I	13.96	4.71	0.94	—	85.94	0	1.43
	V	6.82	2.92	0.58	8.84	1.77	250	0	II	6.22	3.15	0.63	-0.51	250	0	1.10
300	I	60.00	5.00	3.00	—	—	20	0	I	41.89	4.71	2.83	—	28.65	0	1.43
	V	10.13	2.05	1.23	4.92	2.95	250	0	II	8.92	2.18	1.31	-0.75	250	0	1.14
500	I	100.00	5.00	5.00	—	—	12	0	I	69.81	4.71	4.71	—	17.19	0	1.43
	V	12.70	1.78	1.78	3.80	3.80	250	0	II	10.58	1.84	1.84	-0.86	250	0	1.20
700	I	140.00	5.00	7.00	—	—	8.57	0	I	97.74	4.71	6.60	—	12.28	0	1.43
	II	14.97	1.64	2.29	3.21	4.49	250	0	II	11.84	1.64	2.30	-0.89	250	0	1.26
900	I	180.00	5.00	9.00	—	—	6.67	0	I	125.66	4.71	8.49	—	9.55	0	1.43
	II	17.09	1.54	2.77	2.83	5.09	250	0	II	12.87	1.51	2.72	-0.88	250	0	1.33
1100	I	220.00	5.00	11.00	—	—	5.45	0	I	153.59	4.71	10.37	—	7.81	0	1.43
	II	19.09	1.47	3.24	2.56	5.63	250	0	II	13.76	1.41	3.10	-0.86	250	0	1.39

Table 8 for RIF indicates the same behavior as Table 6 for CM and NM. Table 8 for EIF shows the same behavior as Table 6 for CM and NM, except that y_0 (absolute value) tends to increase to $P = 900$ kN, and then equals.

Table 9 for RIF indicates the same behavior as Table 6 for CM and NM. Table 9 for EIF shows the same behavior as Table 6 for CM and NM, except that y_0 (absolute value) tends to increase.

Table 10 for RIF indicates the same behavior as Table 6 for CM and NM. Table 10 for EIF shows the same behavior as Table 6 for CM and NM, except that y_0 (absolute value) tends to increase.

Table 11 for RIF indicates that when P increases for CM: a and b are equal and tend to decrease; $A_{\min\text{RIF}}$ tends to decrease, and for NM, a and b are equal and tend to decrease to $P = 1200$ kN and then increase; a_1 and b_1 are equal and tend to increase; $A_{\min\text{RIF}}$ tends

TABLE 8. Study 4 ($P = 600$ kN and $M_y = 700$ kN-m)

M_x (kN-m)	Case	$A_{\min\text{RIF}}$ (m ²)	a (m)	b (m)	a_1 (m)	b_1 (m)	σ_1 (kN/m ²)	σ_3 (kN/m ²)	Case	$A_{\min\text{EIF}}$ (m ²)	a (m)	b (m)	y_0 (m)	$\sigma_{\max 1}$ (kN/m ²)	$\sigma_{\min 2}$ (kN/m ²)	$A_{\min\text{RIF}} / A_{\min\text{EIF}}$
100	I	28.00	7.00	1.00	—	—	42.86	0	I	19.55	6.60	0.94	—	61.39	0	1.43
	V	7.60	3.65	0.52	10.30	1.47	250	0	II	6.92	3.93	0.56	-0.37	250	0	1.10
300	I	84.00	7.00	3.00	—	—	14.29	0	I	58.64	6.60	2.83	—	20.46	0	1.43
	V	11.72	2.61	1.12	5.80	2.49	250	0	II	9.98	2.72	1.17	-0.71	250	0	1.17
500	I	140.00	7.00	5.00	—	—	8.57	0	I	97.74	6.60	4.71	—	12.28	0	1.43
	II	14.97	2.29	1.64	4.49	3.21	250	0	II	11.84	2.30	1.64	-0.89	250	0	1.26
700	I	196.00	7.00	7.00	—	—	6.12	0	I	136.83	6.60	6.60	—	8.77	0	1.43
	II	17.90	2.12	2.12	3.79	3.79	250	0	II	13.24	2.05	2.05	-0.97	250	0	1.35
900	I	252.00	7.00	9.00	—	—	4.76	0	I	175.93	6.60	8.49	—	6.82	0	1.43
	II	20.64	2.00	2.58	3.35	4.30	250	0	II	14.40	1.89	2.43	-0.99	250	0	1.43
1100	I	308.00	7.00	11.00	—	—	3.90	0	I	215.02	6.60	10.37	—	5.58	0	1.43
	II	23.26	1.92	3.02	3.03	4.76	250	0	II	15.40	1.77	2.77	-0.99	250	0	1.51

TABLE 9. Study 5 ($P = 600$ kN and $M_y = 900$ kN-m)

M_x (kN-m)	Case	$A_{\min\text{RIF}}$ (m ²)	a (m)	b (m)	a_1 (m)	b_1 (m)	σ_1 (kN/m ²)	σ_3 (kN/m ²)	Case	$A_{\min\text{EIF}}$ (m ²)	a (m)	b (m)	y_0 (m)	$\sigma_{\max 1}$ (kN/m ²)	$\sigma_{\min 2}$ (kN/m ²)	$A_{\min\text{RIF}} / A_{\min\text{EIF}}$
100	I	36.00	9.00	1.00	—	—	33.33	0	I	25.13	8.49	0.94	—	47.75	0	1.43
	V	8.30	4.32	0.48	11.57	1.29	250	0	II	7.53	4.64	0.52	-0.34	250	0	1.10
300	I	108.00	9.00	3.00	—	—	11.11	0	I	75.40	8.49	2.83	—	15.92	0	1.43
	V	13.17	3.14	1.05	6.57	2.19	250	0	II	10.86	3.22	1.07	-0.68	250	0	1.21
500	I	180.00	9.00	5.00	—	—	6.67	0	I	125.66	8.49	4.71	—	9.55	0	1.43
	II	17.09	2.77	1.54	5.09	2.83	250	0	II	12.87	2.72	1.51	-0.88	250	0	1.33
700	I	252.00	9.00	7.00	—	—	4.76	0	I	175.93	8.49	6.60	—	6.82	0	1.43
	II	20.64	2.57	2.00	4.30	3.35	250	0	II	14.40	2.43	1.89	-0.99	250	0	1.43
900	I	324.00	9.00	9.00	—	—	3.70	0	I	226.19	8.49	8.49	—	5.31	0	1.43
	II	23.98	2.45	2.45	3.79	3.79	250	0	II	15.66	2.23	2.23	-1.05	250	0	1.53
1100	I	396.00	9.00	11.00	—	—	3.03	0	I	276.46	8.49	10.37	—	4.34	0	1.43
	II	27.19	2.36	2.88	3.43	4.20	250	0	II	16.74	2.09	2.55	-1.08	250	0	1.62

TABLE 10. Study 6 ($P = 600$ kN and $M_y = 1100$ kN-m)

M_x (kN-m)	Case	$A_{\min\text{RIF}}$ (m ²)	a (m)	b (m)	a_1 (m)	b_1 (m)	σ_1 (kN/m ²)	σ_3 (kN/m ²)	Case	$A_{\min\text{EIF}}$ (m ²)	a (m)	b (m)	y_0 (m)	$\sigma_{\max 1}$ (kN/m ²)	$\sigma_{\min 2}$ (kN/m ²)	$A_{\min\text{RIF}} / A_{\min\text{EIF}}$
100	I	44.00	11.00	1.00	—	—	27.27	0	I	30.72	10.37	0.94	—	39.07	0	1.43
	V	8.95	4.96	0.45	12.72	1.16	250	0	II	8.05	5.31	0.48	-0.32	250	0	1.11
300	I	132.00	11.00	3.00	—	—	9.09	0	I	92.15	10.37	2.83	—	13.02	0	1.43
	II	14.53	3.65	1.00	7.27	1.98	250	0	II	11.61	3.68	1.00	-0.65	250	0	1.25
500	I	220.00	11.00	5.00	—	—	5.45	0	I	153.59	10.37	4.71	—	7.81	0	1.43
	II	19.09	3.24	1.47	5.63	2.56	250	0	II	13.76	3.10	1.41	-0.86	250	0	1.39
700	I	308.00	11.00	7.00	—	—	3.90	0	I	215.02	10.37	6.60	—	5.58	0	1.43
	II	23.26	3.02	1.92	4.76	3.03	250	0	II	15.40	2.78	1.77	-0.99	250	0	1.51
900	I	396.00	11.00	9.00	—	—	3.03	0	I	276.46	10.37	8.49	—	4.34	0	1.43
	II	27.19	2.88	2.36	4.20	3.43	250	0	II	16.74	2.55	2.09	-1.08	250	0	1.62
1100	I	484.00	11.00	11.00	—	—	2.48	0	I	337.90	10.37	10.37	—	33.55	0	1.43
	II	30.96	2.78	2.78	3.79	3.79	250	0	II	17.90	2.39	2.39	-1.12	250	0	1.73

to decrease to $P = 1200$ kN and then increases. Table 11 for EIF indicates that when P increases for CM: a and b are equal and tend to decrease; $A_{\min\text{EIF}}$ tends to decrease, and for NM, a and b are equal and remain constant up to $P = 1200$ kN and then increase; y_0 (absolute value) and $A_{\min\text{EIF}}$ are equal to $P = 1000$ kN and then tend to increase. Here, at $P = 2000$ kN, the maximum value of $\sigma_{\max} = 250$ kN/m² is not reached in Case I for RIF (fully compressed area), but it is reached in Case I for EIF (partially compressed area). Therefore, $A_{\min}$ for RIF is governed by the partially compressed area, and for EIF it is governed by the fully compressed area at $P = 2000$ kN.

TABLE 11. Study 7 ($M_x = 700$ kN-m and $M_y = 700$ kN-m)

P (kN)	Case	$A_{\min RIF}$ (m ²)	a (m)	b (m)	a_1 (m)	b_1 (m)	σ_1 (kN/m ²)	σ_3 (kN/m ²)	Case	$A_{\min EIF}$ (m ²)	a (m)	b (m)	y_0 (m)	$\sigma_{\max 1}$ (kN/m ²)	$\sigma_{\min 2}$ (kN/m ²)	$A_{\min RIF} / A_{\min EIF}$
200	I	1764.00	21.00	21.00	—	—	0.23	0	I	1231.50	19.80	19.80	—	0.33	0	1.43
	II	65.54	4.05	4.05	2.19	2.29	250	0	II	13.24	2.05	2.05	-0.97	250	0	4.95
400	I	441.00	10.50	10.50	—	—	1.81	0	I	307.88	9.90	9.90	—	2.60	0	1.43
	II	25.49	2.52	2.52	3.10	3.10	250	0	II	13.24	2.05	2.05	-0.97	250	0	1.93
600	I	196.00	7.00	7.00	—	—	6.12	0	I	136.83	6.60	6.60	—	8.77	0	1.43
	II	17.90	2.12	2.12	3.79	3.79	250	0	II	13.24	2.05	2.05	-0.97	250	0	1.35
800	I	110.25	5.25	5.25	—	—	14.51	0	I	76.97	4.95	4.95	—	20.79	0	1.43
	V	15.52	1.97	1.97	4.39	4.39	250	0	II	13.24	2.05	2.05	-0.97	250	0	1.17
1000	I	70.56	4.20	4.20	—	—	28.34	0	I	49.26	3.96	3.96	—	40.60	0	1.43
	V	14.69	1.92	1.92	4.96	4.96	250	0	II	13.24	2.05	2.05	-0.97	250	0	1.11
1200	I	49.00	3.50	3.50	—	—	48.98	0	I	34.21	3.30	3.30	—	70.16	0	1.43
	V	14.53	1.91	1.91	5.54	5.54	250	0	II	13.25	2.05	2.05	-1.04	250	0	1.10
1400	I	36.00	3.00	3.00	—	—	77.78	0	I	25.13	2.83	2.83	—	111.41	0	1.43
	V	14.74	1.92	1.92	6.12	6.12	250	0	II	13.49	2.07	2.07	-1.43	250	0	1.09
1600	I	27.56	2.62	2.62	—	—	116.10	0	I	19.24	2.47	2.47	—	166.30	0	1.43
	V	15.16	1.95	1.95	6.72	6.72	250	0	II	13.97	2.11	2.11	-1.77	250	0	1.09
1800	I	21.78	2.33	2.33	—	—	165.31	0	I	15.20	2.20	2.20	—	236.79	0	1.43
	V	15.70	1.98	1.98	7.31	7.31	250	0	II	14.81	2.17	2.17	-2.09	250	0	1.06
2000	I	17.64	2.10	2.10	—	—	226.76	0	I	15.20	2.20	2.20	—	250	13.15	1.16
	V	16.32	2.02	2.02	7.92	7.92	250	0	II	16.33	2.28	2.28	-2.24	250	0	1.00

Table 12 for RIF indicates that when M_x increases for CM: a remains constant; b and $A_{\min RIF}$ tend to increase, and for NM, a tends to decrease to $M_x = 500$ kN-m and then is equal; b_1 tends to increase to $M_x = 500$ kN-m and then is equal; b and $A_{\min RIF}$ tend to increase. Table 12 for EIF indicates that when M_x increases for CM: a remains constant; b and $A_{\min EIF}$ tend to increase, and for NM, a remains constant; y_0 (absolute value) tends to decrease to $M_x = 1000$ kN-m and then increases; b and $A_{\min EIF}$ tend to increase.

TABLE 12. Study 8 ($P = 600$ kN and $M_y = 0$ kN-m)

M_x (kN-m)	Case	$A_{\min RIF}$ (m ²)	a (m)	b (m)	a_1 (m)	b_1 (m)	σ_1 (kN/m ²)	σ_2 (kN/m ²)	Case	$A_{\min EIF}$ (m ²)	a (m)	b (m)	y_0 (m)	$\sigma_{\max 1}$ (kN/m ²)	$\sigma_{\min 2}$ (kN/m ²)	$A_{\min RIF} / A_{\min EIF}$
250	Y-I	3.93	0.50	1.96	—	—	250	55.53	Y-I	3.98	0.50	2.53	—	250	51.58	0.99
	Y-II	4.80	0.96	1.25	—	2.50	250	0	Y-II	4.86	0.50	3.09	-3.02	250	0	0.99
500	Y-I	5.00	0.50	2.50	—	—	240	0	Y-I	5.24	0.50	3.33	—	229.18	0	0.95
	Y-II	4.87	0.50	2.43	—	4.80	250	0	Y-II	4.94	0.50	3.15	-2.97	250	0	0.99
750	Y-I	7.50	0.50	3.75	—	—	160	0	Y-I	7.85	0.50	5.00	—	152.79	0	0.96
	Y-II	5.70	0.50	2.85	—	4.80	250	0	Y-II	5.77	0.50	3.67	-2.54	250	0	0.99
1000	Y-I	10.00	0.50	5.00	—	—	120	0	Y-I	10.47	0.50	6.67	—	114.59	0	0.96
	Y-II	6.53	0.50	3.27	—	4.80	250	0	Y-II	6.57	0.50	4.18	-2.16	250	0	0.99
1250	Y-I	12.50	0.50	6.25	—	—	96	0	Y-I	13.09	0.50	8.33	—	91.67	0	0.95
	Y-II	7.37	0.50	3.68	—	4.80	250	0	Y-II	7.34	0.50	4.67	-2.20	250	0	1.00
1500	Y-I	15.00	0.50	7.50	—	—	80	0	Y-I	15.71	0.50	10.00	—	76.39	0	0.95
	Y-II	8.20	0.50	4.10	—	4.80	250	0	Y-II	8.04	0.50	5.12	-2.41	250	0	1.02

Table 13 for RIF indicates that when P increases for CM: a remains constant; b and $A_{\min RIF}$ tend to decrease to $P = 750$ kN and then tend to increase, and for NM, a is equal to $P = 500$ kN and then tends to increase; b_1 tends to increase to $P = 500$ kN, remains constant to $P = 750$ kN, and then tends to decrease; b tends to decrease; $A_{\min RIF}$ tends to decrease to $P = 500$ kN and then tends to increase. Table 13 for EIF indicates that when P increases for CM: a remains constant; b and $A_{\min EIF}$ tend to decrease to $P = 750$ kN and then tend to increase, and for NM, a remains constant; y_0 (absolute value), b and $A_{\min EIF}$ tend to increase.

Table 14 for RIF indicates that when M_y increases for CM: b remains constant; a and $A_{\min RIF}$ tend to increase, and for NM, b tends to decrease to $M_y = 500$ kN-m and then is

TABLE 13. Study 9 ($M_x = 500$ kN-m and $M_y = 0$ kN-m)

P (kN)	Case	$A_{\min RIF}$ (m ²)	a (m)	b (m)	a_1 (m)	b_1 (m)	σ_1 (kN/m ²)	σ_2 (kN/m ²)	Case	$A_{\min EIF}$ (m ²)	a (m)	b (m)	y_0 (m)	$\sigma_{\max 1}$ (kN/m ²)	$\sigma_{\min 2}$ (kN/m ²)	$A_{\min RIF} / A_{\min EIF}$
250	Y-I	12.00	0.50	6.00	—	—	41.67	0	Y-I	12.57	0.50	8.00	—	39.79	0	0.95
	Y-II	5.33	0.50	2.67	—	2.00	250	0	Y-II	4.64	0.50	2.96	-1.39	250	0	1.15
500	Y-I	6.00	0.50	3.00	—	—	166.67	0	Y-I	6.28	0.50	4.00	—	159.15	0	0.96
	Y-II	4.67	0.50	2.33	—	4.00	250	0	Y-II	4.73	0.50	3.01	-2.15	250	0	0.99
750	Y-I	5.27	0.50	2.64	—	—	250	34.36	Y-I	5.35	0.50	3.41	—	250	30.42	0.99
	Y-II	6.00	0.75	2.00	—	4.00	250	0	Y-II	6.02	0.50	3.83	-3.81	250	0	1.00
1000	Y-I	6.00	0.50	3.00	—	—	250	83.33	Y-I	6.07	0.50	3.86	—	250	79.48	0.99
	Y-II	8.00	1.33	1.50	—	3.00	250	0	Y-II	8.00	0.50	5.10	-5.09	250	0	1.00
1250	Y-I	6.77	0.50	3.39	—	—	250	119.17	Y-I	6.84	0.50	4.35	—	250	115.62	0.99
	Y-II	10.00	2.08	1.20	—	2.40	250	0	Y-II	10.01	0.50	6.37	-6.36	250	0	1.00
1500	Y-I	7.58	0.50	3.79	—	—	250	145.64	Y-I	7.64	0.50	4.87	—	250	142.47	0.99
	Y-II	12.00	3.00	1.00	—	2.00	250	0	Y-II	12.01	0.50	7.65	-7.63	250	0	1.00

TABLE 14. Study 10 ($P = 600$ kN and $M_x = 0$ kN-m)

M_y (kN-m)	Case	$A_{\min RIF}$ (m ²)	a (m)	b (m)	a_1 (m)	b_1 (m)	σ_1 (kN/m ²)	σ_2 (kN/m ²)	Case	$A_{\min EIF}$ (m ²)	a (m)	b (m)	y_0 (m)	$\sigma_{\max 1}$ (kN/m ²)	$\sigma_{\min 2}$ (kN/m ²)	$A_{\min RIF} / A_{\min EIF}$
250	X-I	3.93	1.96	0.50	—	—	250	55.53	X-I	3.98	2.53	0.50	—	250	51.58	0.99
	X-II	4.80	1.25	0.96	2.50	—	250	0	X-II	4.86	3.09	0.50	-3.02	250	0	0.99
500	X-I	5.00	2.50	0.50	—	—	240	0	X-I	5.24	3.33	0.50	—	229.18	0	0.95
	X-II	4.87	2.43	0.50	4.80	—	250	0	X-II	4.94	3.15	0.50	-2.97	250	0	0.99
750	X-I	7.50	3.75	0.50	—	—	160	0	X-I	7.85	5.00	0.50	—	152.79	0	0.96
	X-II	5.70	2.85	0.50	4.80	—	250	0	X-II	5.77	3.67	0.50	-2.54	250	0	0.99
1000	X-I	10.00	5.00	0.50	—	—	120	0	X-I	10.47	6.67	0.50	—	114.59	0	0.96
	X-II	6.53	3.27	0.50	4.80	—	250	0	X-II	6.57	4.18	0.50	-2.16	250	0	0.99
1250	X-I	12.50	6.25	0.50	—	—	96	0	X-I	13.09	8.33	0.50	—	91.67	0	0.95
	X-II	7.37	3.68	0.50	4.80	—	250	0	X-II	7.34	4.67	0.50	-2.20	250	0	1.00
1500	X-I	15.00	7.50	0.50	—	—	80	0	X-I	15.71	10.00	0.50	—	76.39	0	0.95
	X-II	8.20	4.10	0.50	4.80	—	250	0	X-II	8.04	5.12	0.50	-2.41	250	0	1.02

TABLE 15. Study 11 ($M_x = 0$ kN-m and $M_y = 500$ kN-m)

P (kN)	Case	$A_{\min RIF}$ (m ²)	a (m)	b (m)	a_1 (m)	b_1 (m)	σ_1 (kN/m ²)	σ_2 (kN/m ²)	Case	$A_{\min EIF}$ (m ²)	a (m)	b (m)	x_0 (m)	$\sigma_{\max 1}$ (kN/m ²)	$\sigma_{\min 2}$ (kN/m ²)	$A_{\min RIF} / A_{\min EIF}$
250	X-I	12.00	6.00	0.50	—	—	41.67	0	X-I	12.57	8.00	0.50	—	39.79	0	0.95
	X-II	5.33	2.67	0.50	2.00	—	250	0	X-II	4.64	2.96	0.50	-1.39	250	0	1.15
500	X-I	6.00	3.00	0.50	—	—	166.67	0	X-I	6.28	4.00	0.50	—	159.15	0	0.96
	X-II	4.67	2.33	0.50	4.00	—	250	0	X-II	4.73	3.01	0.50	-2.15	250	0	0.99
750	X-I	5.27	2.64	0.50	—	—	250	34.36	X-I	5.35	3.41	0.50	—	250	30.42	0.99
	X-II	6.00	2.00	0.75	4.00	—	250	0	X-II	6.02	3.83	0.50	-3.81	250	0	1.00
1000	X-I	6.00	3.00	0.50	—	—	250	83.33	X-I	6.07	3.86	0.50	—	250	79.48	0.99
	X-II	8.00	1.50	1.33	3.00	—	250	0	X-II	8.00	5.10	0.50	-5.09	250	0	1.00
1250	X-I	6.77	3.39	0.50	—	—	250	119.17	X-I	6.84	4.35	0.50	—	250	115.62	0.99
	X-II	10.00	1.20	2.08	2.40	—	250	0	X-II	10.01	6.37	0.50	-6.36	250	0	1.00
1500	X-I	7.58	3.79	0.50	—	—	250	145.64	X-I	7.64	4.87	0.50	—	250	142.47	0.99
	X-II	12.00	1.00	3.00	2.00	—	250	0	X-II	12.01	7.65	0.50	-7.63	250	0	1.00

equal; a_1 tends to increase to $M_y = 500$ kN-m and then is equal; a and $A_{\min RIF}$ tend to increase. Table 14 for EIF indicates that when M_y increases for CM: b remains constant; a and $A_{\min EIF}$ tend to increase, and for NM, b remains constant; y_0 (absolute value) tends to decrease to $M_y = 1000$ kN-m and then increases; a and $A_{\min EIF}$ tend to increase.

Table 15 for RIF indicates that when P increases for CM: b remains constant; a and $A_{\min RIF}$ tend to decrease to $P = 750$ kN and then tend to increase, and for NM, b is equal to $P = 500$ kN and then tends to increase; a_1 tends to increase to $P = 500$ kN, remains constant to $P = 750$ kN, and then tends to decrease; a tends to decrease; $A_{\min RIF}$ tends to decrease to $P = 500$ kN and then tends to increase. Table 15 for EIF indicates that

when P increases for CM: b remains constant; a and $A_{\min\text{EIF}}$ tend to decrease to $P = 750$ kN and then tend to increase, and for NM, b remains constant; y_0 (absolute value), a and $A_{\min\text{EIF}}$ tend to increase.

Figure 7 indicates the $A_{\min}$ for RIF and EIF between the CM and NM subjected to biaxial bending, and shows the following.

1) $A_{\min}$ is smaller for EIF than RIF for both CM and NM in all biaxial bending examples, except for NM at $P = 600$ kN, $M_x = 100$ kN-m and $M_y = 100$ kN-m (see Figure 7(a)).

2) $A_{\min}$ is smaller for EIF than RIF for CM with a ratio of EIF = 0.70 RIF in all examples, except in Figure 7(a) which has a ratio of EIF = 0.93 RIF at $M_x = 100$ kN-m, and in Figure 7(g) which has a ratio of EIF = 0.86 RIF at $P = 2000$ kN.

3) $A_{\min}$ is smaller for EIF than RIF for NM, except in Figure 7(a) with a smaller ratio of EIF = 0.92 RIF at $M_x = 300$ kN-m (see Figure 7(a)) and $M_x = 100$ kN-m (see Figure 7(b)), and with a larger ratio of EIF = 0.20 RIF at $P = 2000$ kN (see Figure 7(g)). Figure 7(a) at $M_x = 100$ kN-m shows that RIF = 0.98 EIF, and Figure 7(g) indicates that RIF = EIF.

4) When the maximum pressure of $\sigma_{\max} = 250$ kN/m² is reached in CM, $A_{\min}$ occurs for RIF and EIF for both in Case I, and when the maximum pressure of $\sigma_{\max} = 250$ kN/m² is not reached in CM, $A_{\min}$ occurs for RIF in Case II or V and for EIF in Case II.

5) $A_{\min}$ is the smallest for NM with respect to CM for RIF with a smaller difference of NM = 0.93 CM at $P = 2000$ kN (see Figure 7(g)) and with a larger difference of NM = 0.04 CM at $P = 200$ kN (see Figure 7(g)), and for CM with respect to NM for RIF with a larger difference of CM = 0.96 NM at $M_x = 100$ kN-m (see Figure 7(a)).

6) $A_{\min}$ is the smallest for NM with respect to CM for EIF with a smaller difference of NM = 0.97 CM at $P = 1800$ kN (see Figure 7(g)) and with a larger difference of NM = 0.01 CM at $P = 200$ kN (see Figure 7(g)), and for CM with respect to NM for EIF with a smaller difference of CM = 0.88 NM at $M_x = 100$ kN-m (see Figure 7(a)) and with a larger difference of CM = 0.93 NM at $P = 2000$ kN (see Figure 7(g)).

7) When the moments are equal, a square footing appears for RIF and a circular footing for EIF.

8) When moments increase, $A_{\min}$ increases for CM and NM, but the $A_{\min\text{RIF}}/A_{\min\text{EIF}}$ ratio is the same for CM and increases for NM (see Figures 7(a), 7(b), 7(c), 7(d), 7(e), 7(f)).

9) When the load increases, $A_{\min}$ decreases for CM and NM, but the $A_{\min\text{RIF}}/A_{\min\text{EIF}}$ ratio is the same for CM and decreases for NM (see Figure 7(g)).

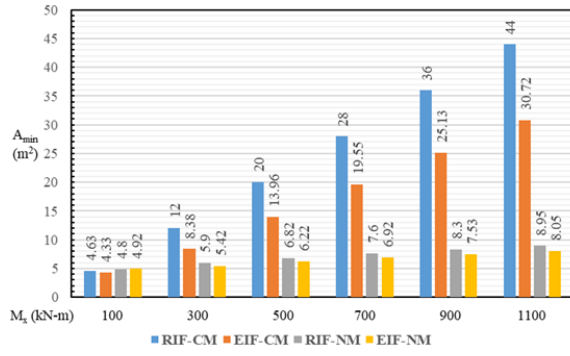
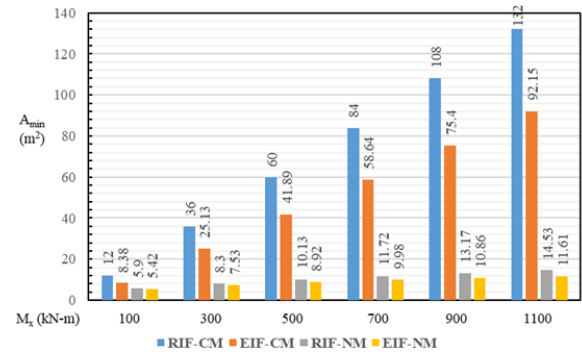
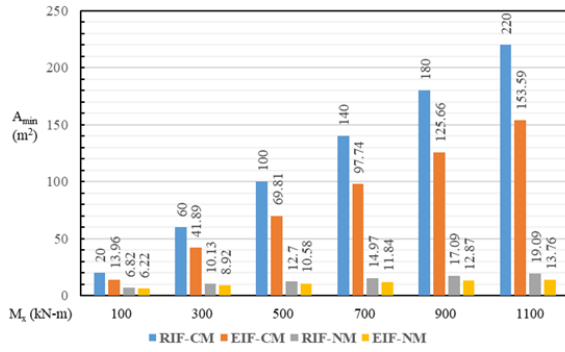
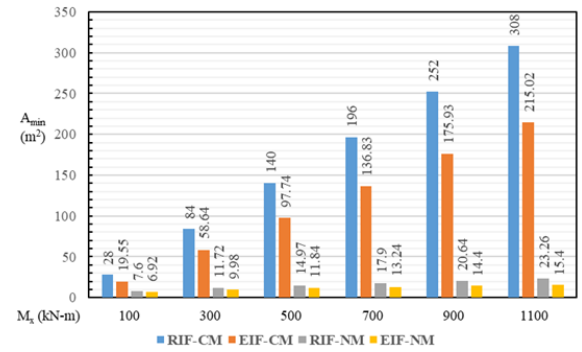
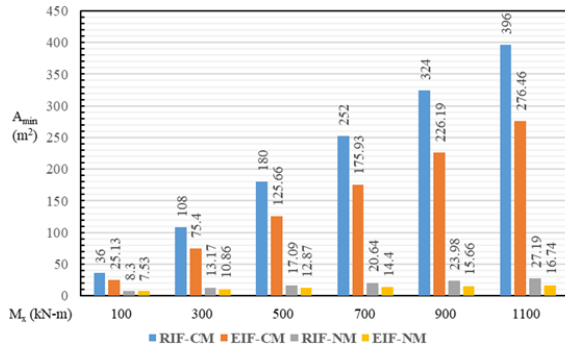
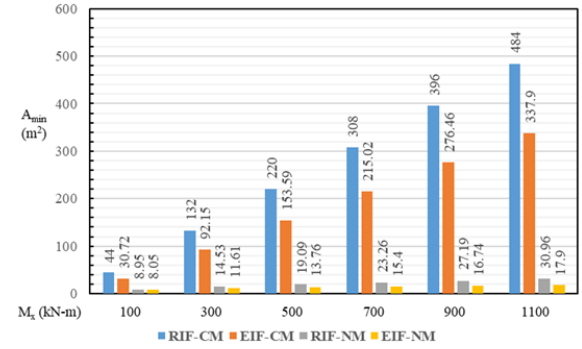
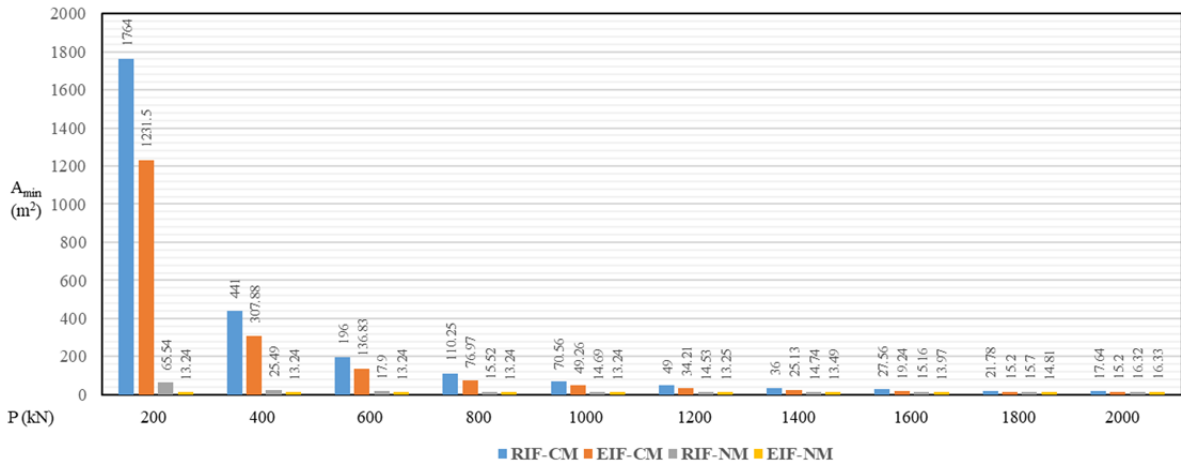
Figure 8 indicates the $A_{\min}$ for RIF and EIF between the CM and NM subjected to uniaxial bending (Axial load is on the Y-axis), and shows the following:

1) $A_{\min}$ is smaller for RIF than EIF in CM for all examples with a smaller difference of RIF = 0.99 EIF at $M_x = 250$ kN-m (see Figure 8(a)) and $P = 500, 750, 1000, 1250, 1500$ kN (see Figure 8(b)), and with a larger difference of RIF = 0.95 EIF at $M_x = 500, 750, 1000$ kN-m (see Figure 8(a)) and $P = 250$ kN (see Figure 8(b)).

2) $A_{\min}$ is smaller for EIF than RIF in NM appears with a ratio of EIF = 0.87 RIF at $P = 250$ kN (see Figure 8(b)), and $A_{\min}$ is smaller for RIF than EIF in NM appears with a ratio of RIF = 0.99 EIF at $M_x = 250, 500, 750, 1000$ kN-m (see Figure 8(a)) and $P = 500$ kN (see Figure 8(b)).

3) When the maximum pressure of $\sigma_{\max} = 250$ kN/m² is reached in CM, $A_{\min}$ occurs for RIF and EIF in Case Y-I, and when the maximum pressure of $\sigma_{\max} = 250$ kN/m² is not reached in CM, $A_{\min}$ occurs for RIF and EIF in Case Y-II.

4) $A_{\min}$ is smaller for NM than CM for RIF with a smaller difference of NM = 0.97 CM at $M_x = 500$ kN-m (see Figure 8(a)) and with a larger difference of NM = 0.44 CM

(a) $P = 600$ kN and $M_y = 100$ kN-m(b) $P = 600$ kN and $M_y = 300$ kN-m(c) $P = 600$ kN and $M_y = 500$ kN-m(d) $P = 600$ kN and $M_y = 700$ kN-m(e) $P = 600$ kN and $M_y = 900$ kN-m(f) $P = 600$ kN and $M_y = 1100$ kN-m(g) $M_x = 700$ kN-m and $M_y = 700$ kN-mFIGURE 7. Comparison of $A_{\min}$ between RIF and EIF subjected to biaxial bending

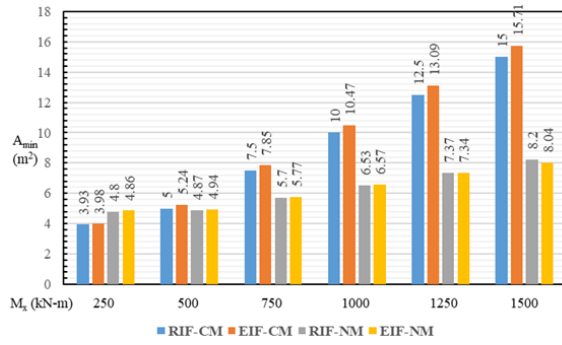
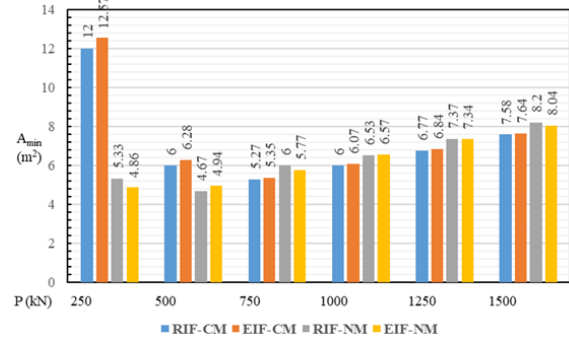

 (a) $P = 600$ kN and $M_y = 0$ kN-m

 (b) $M_x = 500$ kN-m and $M_y = 0$ kN-m

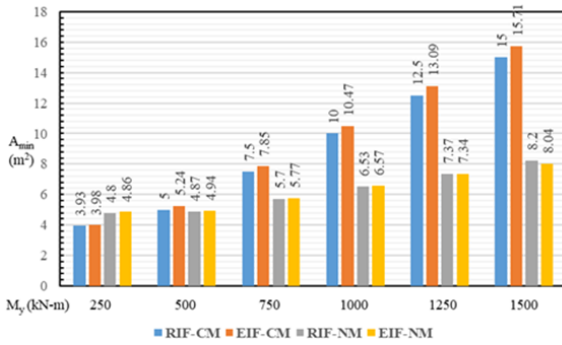
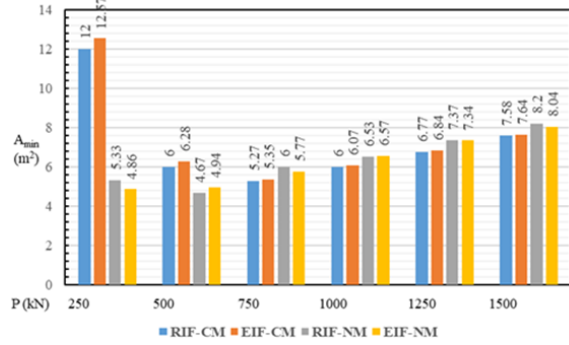
 FIGURE 8. Comparison of $A_{\min}$ between RIF and EIF subjected to uniaxial bending when P is on the Y-axis

 (a) $P = 600$ kN and $M_x = 0$ kN-m

 (b) $M_x = 0$ kN-m and $M_y = 500$ kN-m

 FIGURE 9. Comparison of $A_{\min}$ between RIF and EIF subjected to uniaxial bending when P is on the X-axis

at $P = 250$ kN (see Figure 8(b)), and $A_{\min}$ is smaller for CM than NM for RIF with a smaller difference of CM = 0.82 NM at $M_x = 250$ kN-m (see Figure 8(a)) and with a larger difference of CM = 0.63 NM at $P = 1500$ kN (see Figure 8(b)).

5) $A_{\min}$ is smaller for NM than CM for EIF with a smaller difference of NM = 0.94 CM at $M_x = 500$ kN-m (see Figure 8(a)) and with a larger difference of NM = 0.37 CM at $P = 250$ kN (see Figure 8(b)), and $A_{\min}$ is smaller for CM than NM for EIF with a smaller difference of CM = 0.89 NM at $P = 750$ kN (see Figure 8(b)) and with a larger difference of CM = 0.64 NM at $P = 1500$ kN (see Figure 8(b)).

6) When moment increases, $A_{\min}$ increases for CM and NM (see Figure 8(a)).

7) When the load increases, $A_{\min}$ decreases for CM to $P = 750$ kN and then tends to increase for RIF and EIF, and $A_{\min}$ decreases for NM in RIF at $P = 500$ kN and then tends to increase and for NM in EIF tends to increase (see Figure 8(b)).

Figure 9 indicates the $A_{\min}$ for RIF and EIF between the CM and NM subjected to uniaxial bending (Axial load is on the X-axis), and shows results similar to Figure 8. Since the loads and moments are the same for both studies, the only difference is that in Figure 8 there are moments about the X-axis and no moments about the Y-axis, while in Figure 9 there are moments about the Y-axis and no moments about the X-axis.

5. Conclusions. This research presents four comparative studies of minimum ground contact areas for RIF and EIF. The comparative studies are the followings: 1) The CM

and NM for RIF; 2) The CM and NM for EIF; 3) The CM for RIF and EIF; 4) The NM for RIF and EIF.

For RIF, the CM is Case I (area is fully compressed), and the NM is the smallest of Cases II, III, IV, and V (area is partially compressed). For EIF, the CM is Case I (area is fully compressed), and the new model is Case II (area is partially compressed).

The main contributions presented in this research are as the following.

1) The EIF under biaxial bending requires less area than the RIF in the NM except when the maximum pressure of $\sigma_{\max} = 250 \text{ kN/m}^2$ is reached in CM. This means that when the maximum value of $\sigma_{\max} = 250 \text{ kN/m}^2$ is reached at the CM, it is not necessary to continue with the NM.

2) The $A_{\min}$ for biaxial bending in the CM indicates a great saving of up to 30.19% using the EIF compared to the RIF. To justify this saving, $\sigma_{\max 1}$ reaches the maximum value of 47.75 kN/m^2 for EIF and the maximum value of 33.33 kN/m^2 for RIF; therefore, EIF generates a larger $\sigma_{\max 1}$ and logically a smaller area (see Table 5).

3) The $A_{\min}$ for biaxial bending in the NM shows a great saving of up to 79.80% using the EIF compared to the RIF. To justify this saving, the corresponding values are substituted into Equation (35) and $P = 1162.14 \text{ kN}$ is obtained for EIF, so the same area appears in $P = 200, 400, 600, 800$ and 1000 kN (see Table 11).

4) The $A_{\min}$ for uniaxial bending in the CM indicates a saving of up to 4.58% using the RIF compared to the EIF. To justify this saving, $\sigma_{\max 1}$ reaches the maximum value of 240 kN/m^2 for RIF and the maximum value of 229.18 kN/m^2 for EIF; therefore, RIF generates a larger $\sigma_{\max 1}$ and logically a smaller area (see Tables 12 and 14).

5) The $A_{\min}$ for uniaxial bending in the NM shows a saving of up to 12.95% using the EIF compared to the RIF. To justify this saving, the corresponding values are substituted into Equations (45) or (48) and $P = 408.04 \text{ kN}$ is obtained for EIF; therefore, it is greater than $P = 200 \text{ kN}$ (see Tables 13 and 15).

6) When the moments are equal in the CM and NM, a square footing appears for RIF and a circular footing for EIF.

7) When the moments increase for biaxial bending in CM and NM, $A_{\min \text{RIF}}$ and $A_{\min \text{EIF}}$ increase.

8) When the moments increase for biaxial bending, the difference of $A_{\min \text{RIF}}$ versus $A_{\min \text{EIF}}$ tends to increase in NM, while for CM it is constant except when it reaches maximum pressure.

9) When loads increase for biaxial bending, $A_{\min \text{RIF}}$ and $A_{\min \text{EIF}}$ decrease in CM, while for NM $A_{\min \text{RIF}}$ and $A_{\min \text{EIF}}$ tend to decrease to a certain value and then increase.

10) When loads increase for biaxial bending, the difference of $A_{\min \text{RIF}}$ versus $A_{\min \text{EIF}}$ tends to increase in NM, while for CM it is constant except when it reaches maximum pressure.

The main advantage of this study is that it directly impacts the cost of constructing the footing, since by presenting a smaller contact area with the ground, it generates a smaller volume of footing fill.

The suggestion for the next investigation is a comparative study for the complete design of RIF and EIF taking into account that the contact area with the ground is partially compressed.

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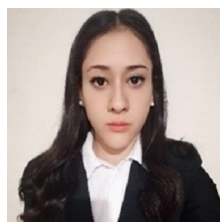
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