

COOPERATIVE HARRIS HAWKS OPTIMIZATION-ADAPTIVE TABU SEARCH: A NOVEL COOPERATIVE METAHEURISTIC ALGORITHM FOR GLOBAL OPTIMIZATION

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ABSTRACT. *This paper proposes a novel cooperative metaheuristic algorithm, named the cooperative Harris hawks optimization-adaptive tabu search (cooperative HHO-ATS algorithm), which combines Harris hawks optimization (HHO), one of the most efficient population-based metaheuristic algorithms, and adaptive tabu search (ATS), one of the most powerful trajectory-based metaheuristic algorithms, for global optimization. The proposed cooperative HHO-ATS consists of two states that operate sequentially as a serial-heterogeneous search algorithm. It begins by exploring feasible solutions over the entire search space using the exploration property of HHO. Subsequently, the exploitation property of ATS is employed to search for the global optimum. The search performance of the proposed cooperative HHO-ATS is evaluated against 20 benchmark functions encompassing nonlinear, symmetric, asymmetric, and multi-modal functions. The results obtained by the cooperative HHO-ATS are compared with those obtained by ATS and HHO. In addition, its performance is evaluated against simulated annealing (SA), tabu search (TS), cuckoo search (CS), grey wolf optimizer (GWO), and the cooperative FPA-ATS algorithm. Experimental results indicate that the proposed cooperative HHO-ATS significantly outperforms both HHO and ATS, as well as the other selected mainstream metaheuristic algorithms, in terms of convergence speed and success rate.*

Keywords: Cooperative HHO-ATS algorithm, Harris hawks optimization, Adaptive tabu search, Metaheuristic algorithm, Global optimization

1. Introduction. Metaheuristic optimization methods have played a central role in solving complex combinatorial and numerical optimization problems for several decades [1-4]. The effectiveness of these methods relies on two complementary mechanisms: exploration, which promotes global search diversity, and exploitation, which intensifies the search within promising regions to refine candidate solutions. Based on their search strategies, metaheuristic algorithms are commonly classified into trajectory-based approaches, which emphasize exploitation through single-solution evolution, and population-based approaches, which enhance exploration by maintaining multiple candidate solutions simultaneously [1-4].

Among trajectory-based metaheuristics, adaptive tabu search (ATS) has been widely recognized for its strong local search efficiency. The ATS algorithm was developed by Sujitjorn et al. in 2006 [5] as an extension of the conventional tabu search (TS) method originally introduced by Glover in 1989 [6,7]. With its adaptive mechanisms, ATS has been successfully applied to a variety of engineering optimization problems, including system

modeling and identification, controller synthesis, power system optimization, and signal processing applications. Moreover, the theoretical convergence of ATS has been rigorously proved through probabilistic analysis [8-10].

In contrast, the Harris hawks optimization (HHO) proposed by Heidari et al. in 2019 [11] is one of the most powerful population-based, nature-inspired metaheuristics. HHO algorithm mimics the cooperative hunting behavior of Harris hawks (*Parabuteo unicinctus*), employing cooperative strategies to search for and capture prey, and their pursuit behaviors dynamically adapt to different situations and the escape patterns of the prey. The HHO has been widely applied to addressing a variety of optimization problems [12-14], particularly in engineering, computer science, and medical science. In engineering applications, HHO has been used for manufacturing system design, solar and photovoltaic systems, and power systems. In computer science, its applications include data mining and processing, image thresholding, networking and distributed systems, and software engineering problems. Also, HHO has also been applied to medical science problems such as drug design and discovery and mammogram image classification.

Recent advances in metaheuristic research emphasize the necessity of integrating exploration and exploitation in a balanced manner to enhance overall search efficiency. In this context, Talbi [15] introduced a new paradigm for combinatorial optimization that highlights the role of cooperation among metaheuristic algorithms. Under this paradigm, metaheuristic frameworks are categorized into parallel, hybrid, and cooperative approaches [15-19].

A hybrid metaheuristic combines two or more optimization techniques within a single framework to exploit their complementary strengths and improve search performance [20,21]. However, hybrid metaheuristics often increase computational complexity, parameter-tuning requirements, and implementation difficulty due to the tight integration of multiple components [22]. In contrast, cooperative metaheuristics involve multiple optimization algorithms, or multiple instances of the same algorithm, that collaborate through information exchange during the search process [23,24]. By preserving the individual search mechanisms of participating algorithms, cooperative approaches can achieve a better balance between exploration and exploitation, reduce premature convergence, and improve robustness across different problem instances. Their distributed structure also naturally supports parallelization and scalability, making them suitable for large-scale optimization problems [23,24].

Furthermore, cooperative metaheuristics represent a more recent development than conventional parallel and hybrid metaheuristics. Based on algorithmic composition and execution strategy, they can be classified into serial-homogeneous, parallel-homogeneous, serial-heterogeneous, and parallel-heterogeneous categories [16].

For parallel-homogeneous cooperative algorithms, such as parallel simulated annealing (PSA) [25], cooperative coevolutionary genetic algorithms (CCGA) [26], parallel tabu search (PTS) [27], cooperative particle swarm optimization (CPSO) [28], parallel cuckoo search (PCS) [29], and parallel grey wolf optimizer (PGWO) [30], as well as parallel-heterogeneous cooperative algorithms [31], implementation requires high-cost parallel processors or multi-core CPUs in order to achieve true parallel execution of the algorithms. Implementing parallel cooperative algorithms on a single-CPU platform results only in pseudo-parallel execution rather than genuine parallelism. In the case of serial- and parallel-homogeneous cooperative algorithms, where identical algorithmic components are employed across all cooperative units, the overall search behaviour tends to be biased toward either exploration or exploitation. Consequently, such approaches may lack a well-balanced search dynamic between global exploration and local exploitation. Accordingly, this study focuses on the development of serial-heterogeneous cooperative

algorithms. This framework can be effectively implemented on low-cost single-CPU platforms while maintaining computational practicality. More importantly, by integrating heterogeneous algorithmic components, this approach is capable of achieving a more appropriate and well-balanced trade-off between exploration and exploitation within the optimization process. According to the literature, a significant example of a serial-heterogeneous cooperative algorithm is the cooperative flower pollination algorithm combined with ATS, known as the cooperative FPA-ATS algorithm [32].

Therefore, a serial-heterogeneous cooperative framework is selected in this paper because the constituent algorithms can maintain their independent search behaviors while benefiting from continuous information sharing. This framework not only enhances search diversity and convergence reliability but also avoids some of the increased complexity and parameter-tuning difficulties commonly associated with tightly coupled hybrid metaheuristics. A novel cooperative HHO-ATS algorithm is proposed in this paper by leveraging the complementary characteristics of the ATS and HHO algorithms. For the proposed strategy, the HHO is initially employed to perform global exploration and identify a high-quality candidate solution. This solution is subsequently used as the starting point for the ATS algorithm, which exploits its strong local search capability to accelerate convergence toward the global optimum. The effectiveness of the proposed cooperative HHO-ATS is performed through global optimization against 20 benchmark functions. The results obtained by the cooperative HHO-ATS will be compared with those obtained by ATS and HHO, as well as mainstream metaheuristic algorithms and an existing cooperative algorithm. This paper consists of five sections. After an introduction presented in Section 1, the rest of the paper is arranged as follows. The ATS, HHO, and the cooperative HHO-ATS algorithms are described in Section 2. The benchmark functions selected for global optimization are detailed in Section 3. The experimental results and discussion are provided in Section 4. Finally, the conclusions and future research are given in Section 5.

2. ATS, HHO and HHO-ATS Algorithms. The ATS and HHO algorithms are firstly described. Then, the proposed cooperative HHO-ATS algorithm is illustrated in this section.

2.1. ATS algorithm. The ATS algorithm is an extension of the original TS using a stochastic iterative neighborhood strategy as the trajectory-based metaheuristics [5,8-10]. In each iteration, feasible solutions within a predefined search radius are randomly generated to form the neighborhood set. The ATS is composed of three principal components, i.e., tabu list (TL), adaptive-radius (AR) mechanism, and backtracking (BT) mechanism.

The TL serves as a short-term memory structure that stores previously visited solutions to prevent cycling and enhance search diversification. The AR mechanism dynamically adjusts the search radius to enhance search intensification. By reducing the search radius, the algorithm concentrates on a narrower region of the solution space. This accelerates convergence toward high-quality solutions and speeds up the search process to hit the global optimum. When the search becomes trapped in a local optimum, the BT mechanism is then activated. This mechanism randomly selects previously visited solutions recorded in the TL to diversify the search trajectory and guide the algorithm toward unexplored regions of the solution space. With TL, AR, and BT mechanisms, the ATS algorithm seeks to improve the probability of attaining the global optimum. The procedure of ATS can be summarized by the pseudo code illustrated in Figure 1.

According to Figure 1, f denotes the objective function, $\mathbf{x} = (x_1, \dots, x_d)$ represents the candidate solution in a d -dimensional search space, d is the problem dimension, R stands for the search radius, α denotes the weighting factor associated with the AR mechanism,

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ATS algorithm
Initialization:
- Objective function  $f(\mathbf{x})$ ,  $\mathbf{x} = (x_1, \dots, x_d)^T$ , and search spaces
- Search radius  $R$ , weighting factor  $\alpha$ , ( $0 < \alpha < 1$ ), and TL
- Iteration  $t=1$  and maximum iteration  $t_{\max}$  as TC
- Randomly generate the initial best solution  $\mathbf{x}^*$ 
  and evaluate  $f(\mathbf{x}^*)$ 
while ( $t \leq t_{\max}$  or TC are not met)
- Randomly  $N$  neighborhood solutions around  $\mathbf{x}^*$  within  $R$ 
- Evaluate all  $N$  solutions via  $f(\mathbf{x}^*)$ 
- Let one of the best solutions among  $N$  be  $\mathbf{x}$ 
  if1 ( $f(\mathbf{x}) < f(\mathbf{x}^*)$ )
  - Update  $\mathbf{x}^* = \mathbf{x}$ 
  end if1
  - Store  $\mathbf{x}$  in TL
  - Activate AR mechanism using  $R = \alpha R$ 
  if2 (Local entrapment occur)
  - Invoke BT mechanism by randomly selecting  $\mathbf{x}'$ 
    (one of the solutions stored in TL)
  - Update  $\mathbf{x}^* = \mathbf{x}'$ 
  end if2
- Update  $t = t+1$ 
end while
Report the best solution  $\mathbf{x}^*$ 

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FIGURE 1. Pseudo code of ATS

t is the iteration counter, and $t_{\max}$ specifies the maximum number of iterations, serving as the termination criterion (TC). Furthermore, $\mathbf{x}^*$ represents the current best solution identified during the search process. The set N denotes the neighborhoods comprising feasible candidate solutions generated at each iteration, and $\mathbf{x} \in N$ corresponds to the best solution within N . The notation $\mathbf{x}'$ indicates one of the previously visited solutions stored in the TL and potentially utilized during the BT mechanism. For further details about ATS, readers can refer to [5,8-10].

2.2. HHO algorithm. The HHO is a nature-inspired metaheuristic developed from the cooperative predation strategies of Harris hawks as the population-based metaheuristics [11]. During hunting process, hawks employ coordinated and adaptive hunting tactics to locate, track, and capture prey under dynamic environmental conditions. HHO consists of three main phases, i.e., exploration phase, transition from exploration to exploitation, and exploitation phase [11-14].

During the exploration phase, the positional update mechanism of the hawks is formulated as expressed in (1), where $X(t)$ denotes the position vector of a hawk at iteration t , and $X(t+1)$ represents its updated position in the subsequent iteration. The parameters r_1 , r_2 , r_3 , r_4 , and q are independently generated random variables following a uniform distribution within $[0, 1]$, and are updated at each iteration. LB and UB correspond to the lower and upper bounds of the search spaces, respectively. Furthermore, $X_{rand}(t)$ denotes a randomly selected hawk from the current population, while $X_{prey}(t)$ represents the position of the prey, interpreted as the best candidate solution identified thus far. The population mean position, $X_m(t)$, can be calculated as (2), where $X_i(t)$ indicates the

position vector of the i -th hawk at iteration t , and N_h denotes the total number of hawks in the population.

$$X(t+1) = \begin{cases} X_{rand}(t) - r_1 |X_{rand}(t) - 2r_2 X(t)|, & q \geq 0.5 \\ [X_{prey}(t) - X_m(t)] - r_3 [LB + r_4(UB - LB)], & q < 0.5 \end{cases} \quad (1)$$

$$X_m(t) = \frac{1}{N_h} \sum_{i=1}^{N_h} X_i(t) \quad (2)$$

For the transition from the exploration to the exploitation phase, the behavioral shift of the hawks from global searching to active pursuit is governed by the escaping energy of the prey, denoted by E , as formulated in (3), where T represents the maximum number of iterations, while E_0 denotes the initial escaping energy of the prey. The parameter E_0 is randomly generated within $[-1, 1]$ at each iteration to simulate unpredictable prey behavior. When $E_0 \in (0, 1]$, the prey is assumed to be gaining strength, whereas $E_0 \in [-1, 0)$ indicates that the prey is gradually weakening.

According to (3), the magnitude of E decreases progressively as the iteration counter increases, thereby enforcing a time-varying transition mechanism. Specifically, when $|E| \geq 1$, the algorithm emphasizes exploration, typically in the early search stages. Conversely, when $|E| < 1$, the search process shifts toward exploitation in the later stages, intensifying the refinement of promising candidate solutions.

$$E = 2E_0 \left(1 - \frac{t}{T}\right) \quad (3)$$

In the exploitation phase, the interaction between the hawks' chasing tactics and the prey's escape responses is mathematically modelled to emulate the surprise pounce mechanism observed in nature. It can be divided into four strategies in this phase, i.e., soft besiege, hard besiege, soft besiege with progressive rapid dives, and hard besiege with progressive rapid dives.

The soft besiege strategy is activated when $|E| \geq 0.5$ and $r \geq 0.5$, indicating that although the prey retains moderate escaping energy, it is insufficient to successfully evade the hawks due to progressive energy depletion. The updated positions of hawks can be modelled as stated in (4), where $\Delta X(t)$ denotes the difference between the positions of the preys and the current location in iteration t as stated in (5), and $J = 2(1 - r_5)$ is the random jump of the prey throughout the escaping procedure with the random number r_5 within $[0, 1]$.

$$X(t+1) = \Delta X(t) - E |J X_{prey}(t) - X(t)| \quad (4)$$

$$\Delta X(t) = X_{prey}(t) - X(t) \quad (5)$$

The hard besiege strategy is triggered when $|E| < 0.5$ and $r \geq 0.5$, indicating that the prey's escaping energy has significantly diminished and it is physically exhausted, thereby losing its ability to evade the hawks effectively. The updated positions of hawks can be modelled as expressed in (6).

$$X(t+1) = X_{prey}(t) - E |\Delta X(t)| \quad (6)$$

The soft besiege with progressive rapid dives is activated when $|E| \geq 0.5$ and $r < 0.5$, indicating that the prey still possesses sufficient escaping energy to evade immediate capture. Under this circumstance, the hawks initially establish a soft encirclement and subsequently perform rapid dive movements to simulate a surprise pounce mechanism. In this strategy, updated positions of hawks can be modelled as stated in (7), where F denotes the objective function, Y and Z represent next moves of hawks, which can be

calculated using (8) and (9), respectively. From (9), D is the dimension of problem, S is a random vector by size $1 \times D$, and LF is the Lévy-flight function, which can be calculated using (10), where u and v stand for random values within $[0, 1]$, β denotes a default constant set to 1.5, σ stands for the standard deviation, and Γ is the standard Gamma function.

$$X(t+1) = \begin{cases} Y, & F(Y) < F(X(t)) \\ Z, & F(Z) < F(X(t)) \end{cases} \quad (7)$$

$$Y = X_{prey}(t) - E |JX_{prey}(t) - X(t)| \quad (8)$$

$$Z = Y + S \times LF(D) \quad (9)$$

$$LF(X) = 0.01 \frac{u \times \sigma}{|v|^{1/\beta}}, \quad \sigma = \left[\frac{\Gamma(1+\beta) \times \sin(\pi\beta/2)}{\Gamma[(1+\beta)/2] \times \beta \times 2^{(\beta-1)/2}} \right]^{1/\beta} \quad (10)$$

The hard besiege with progressive rapid dives is activated when $|E| < 0.5$ and $r < 0.5$, reflecting a scenario in which the prey's escaping energy is critically low and it is incapable of effective evasion. In this strategy, updated positions of hawks can be modelled as expressed in (7), but Y will be calculated using (11), while $X_m(t)$ is obtained by (2).

$$Y = X_{prey}(t) - E |JX_{prey}(t) - X_m(t)| \quad (11)$$

The procedure of HHO can be summarized by the pseudo code shown in Figure 2. For more details about HHO, readers can refer to [11-14].

2.3. Cooperative HHO-ATS algorithm. The proposed cooperative HHO-ATS algorithm is designed as a serial-heterogeneous cooperative metaheuristic framework that combines the complementary strengths of HHO and ATS to achieve an effective balance between exploration and exploitation. The motivation behind this cooperation is based on the observation that HHO exhibits strong global exploration capability [11-14], whereas ATS is highly effective in local exploitation and solution refinement [5,8-10]. By integrating both algorithms sequentially, each method can operate in the search stage where its strengths are most beneficial.

In the proposed framework, HHO is employed during the first phase as the global search engine. Starting from a population of randomly generated candidate solutions, HHO explores diverse regions of the search space using its exploration-oriented hunting strategies and dynamic position updating mechanisms. This process enhances population diversity, improves search coverage, and reduces the risk of premature convergence to local optima. Throughout the search process, HHO continuously evaluates candidate solutions and maintains the best feasible solution found so far, referred to as the elite solution. The primary objective of this phase is to identify a high-quality solution located within a promising region of the search space rather than achieving complete convergence.

The transition from HHO to ATS is governed by a predefined switching iteration (SI). Once the current iteration reaches SI, the elite solution obtained from HHO is transferred to ATS and used as its initial solution. This mechanism allows ATS to inherit the search knowledge accumulated during the global exploration stage and begin its search from a promising region rather than from a randomly generated solution.

In the second phase, ATS performs intensive local exploitation around the elite solution provided by HHO. By using TL, AR, and BT mechanisms, ATS systematically refines the solution while avoiding cycling and repeated exploration of previously visited regions. The adaptive search mechanism enables ATS to improve solution quality efficiently and accelerate convergence toward the global optimum. Since ATS starts from a high-quality

HHO algorithm**Initialization:**

- Objective function $F(\mathbf{x})$, $\mathbf{x} = (x_1, \dots, x_d)^T$, and search spaces
- Population size of hawks N_h
- Randomly generate population X_i ($i = 1, 2, \dots, N_h$)
- Iteration $t = 1$, and maximum iterations T as TC

while ($t \leq T$ or TC are not met)

- Calculate $F(X_i)$ of all N_h hawks
- Set X_{prey} as the current best location

for (each hawk (X_i))

- Update E_0 and jump strength J using
 $E_0 = 2 \cdot \text{rand}() - 1$, $J = 2[1 - \text{rand}()]$
- Update the energy E using (3)

if1 ($|E| \geq 1$), % *Exploration phase*

- Update positions of hawks using (1)

end if1

if2 ($|E| < 1$), % *Exploitation phase*

if ($r \geq 0.5$ and $|E| \geq 0.5$), % *Soft besiege*

- Update positions of hawks using (4)

else if ($r \geq 0.5$ and $|E| < 0.5$), % *Hard besiege*

- Update positions of hawks using (6)

else if ($r < 0.5$ and $|E| \geq 0.5$), % *Soft besiege with progressive rapid dives*

- Update positions of hawks using (7),
when Y using (8)

else if ($r < 0.5$ and $|E| < 0.5$), % *Hard besiege with progressive rapid dives*

- Update positions of hawks using (7),
when Y using (11)

end if2

end for

- Update X_{prey}
- $t = t + 1$

end while

- Report the best solution X_{prey}

FIGURE 2. Pseudo code of HHO

solution generated by HHO, computational effort can be focused on local refinement rather than locating promising search regions.

The cooperation between HHO and ATS can therefore be regarded as a division-of-labor strategy. HHO is responsible for diversification and global exploration, while ATS focuses on intensification and local exploitation. This complementary interaction enables the algorithm to benefit from both broad search coverage and accurate solution refinement, thereby improving convergence speed, solution quality, and robustness compared with the standalone implementation of either HHO or ATS.

A key parameter in the proposed framework is the SI, which determines the balance between exploration and exploitation. For highly multimodal optimization problems containing numerous local optima, a larger SI value is generally preferred to allocate more iterations to global exploration. In contrast, for near-unimodal problems with smoother

search landscapes, a smaller SI value is typically sufficient, allowing earlier activation of the local exploitation phase. In general applications where prior knowledge of the search landscape is unavailable, SI may be set to approximately 50% of the maximum number of iterations.

The overall procedure of the proposed cooperative HHO-ATS algorithm can be summarized as follows. First, HHO as illustrated in Figure 2 performs global exploration from the 1st iteration to SI, and identifies the best elite feasible solution. Next, the elite solution is transferred to ATS as the initial solution. Finally, ATS as illustrated in Figure 1 conducts adaptive neighborhood-based local exploitation from the SI+1 iteration until the maximum number of iterations is reached. The best solution obtained throughout the optimization process is reported as the final optimal solution. The complete procedure of the proposed cooperative HHO-ATS algorithm is presented in the pseudo code shown in Figure 3.

3. Benchmark Functions. To evaluate its performance, the proposed cooperative HHO-ATS algorithm is tested against 20 well-established benchmark functions for global minimization. These test functions are selected from [33-36] to represent diverse characteristics and landscapes, including nonlinearity, symmetric and asymmetric properties, and multi-modality. The details of the 20 selected benchmark functions are given as follows.

1) Ackley function is expressed as $f_1(\mathbf{x})$ in (12), where D is the dimension of problem. With nonlinear, symmetric, and multi-modal properties, it possesses many local minima. Its global minimum is located at $\mathbf{x}^* = (0, \dots, 0)$ with $f_1(\mathbf{x}^*) = 0$.

2) Alpine function is stated as $f_2(\mathbf{x})$ in (13). It possesses nonlinear, symmetric, and multi-modal properties. With many local minima, its global minimum is located at $\mathbf{x}^* = (0, \dots, 0)$ with $f_2(\mathbf{x}^*) = 0$.

3) Bohachevsky function is represented as $f_3(\mathbf{x})$ in (14). It possesses nonlinear, asymmetric, and multi-modal properties. It has many local minima, and its global minimum is located at $\mathbf{x}^* = (0, \dots, 0)$ with $f_3(\mathbf{x}^*) = 0$.

4) Bukin function is expressed as $f_4(\mathbf{x})$ in (15). It has nonlinear, asymmetric, and multi-modal properties. With many local minima, its global minimum is located at $\mathbf{x}^* = (-10, 1)$ with $f_4(\mathbf{x}^*) = 0$.

5) De Jong function is stated as $f_5(\mathbf{x})$ in (16). With nonlinear, asymmetric, and multi-modal properties, it possesses many local minima. Its global minimum is located at $\mathbf{x}^* = (-32, -32)$ with $f_5(\mathbf{x}^*) = 0.9980$.

6) Drop-wave function is expressed as $f_6(\mathbf{x})$ in (17). It has nonlinear, symmetric, and multi-modal properties. With many local minima, its global minimum is located at $\mathbf{x}^* = (0, \dots, 0)$ with $f_6(\mathbf{x}^*) = -1$.

7) Easom function is represented as $f_7(\mathbf{x})$ in (18). With nonlinear, symmetric, and multi-modal properties, it has many local minima. Its global minimum is located at $\mathbf{x}^* = (\pi, \dots, \pi)$ with $f_7(\mathbf{x}^*) = -1$.

8) Eggholder function is given as $f_8(\mathbf{x})$ in (19). It possesses nonlinear, asymmetric, and multi-modal properties with many local minima. Its global minimum is located at $\mathbf{x}^* = (512, 404.2319)$ with $f_8(\mathbf{x}^*) = -959.6407$.

9) Griewank function is stated as $f_9(\mathbf{x})$ in (20). It has nonlinear, symmetric, and multi-modal properties with many local minima. Its global minimum is located at $\mathbf{x}^* = (0, \dots, 0)$ with $f_9(\mathbf{x}^*) = 0$.

10) Lévy function is represented as $f_{10}(\mathbf{x})$ in (21). It possesses nonlinear, symmetric, and multi-modal properties with many local minima. Its global minimum is located at $\mathbf{x}^* = (1, \dots, 1)$ with $f_{10}(\mathbf{x}^*) = 0$.

Cooperative HHO-ATS algorithm**Initialization:****For HHO:**

- Population size of hawks N_h
- Randomly generate population X_i ($i = 1, 2, \dots, N_h$)

For ATS:

- Search radius R , weighting factor α , ($0 < \alpha < 1$), and TL

For General:

- Objective function $f(\mathbf{x}) = F(\mathbf{x})$, $\mathbf{x} = (x_1, \dots, x_d)^T$
- Search spaces, iteration $t=1$ and maximum iteration $t_{\max}$ as TC
- SI for switching HHO to ATS

while1 ($t \leq$ SI or TC are not met) % HHO (First phase)

- Calculate $F(X_i)$ of all N_h hawks and set X_{prev} as the current best location

for (each hawk (X_i))

- Update E_0 and jump strength J using $E_0 = 2rand()-1$, $J = 2[1-rand()]$
- Update the energy E using (3)

if1 ($|E| \geq 1$), % *Exploration phase*

- Update positions of hawks using (1)

end if1**if2** ($|E| < 1$), % *Exploitation phase***if** ($r \geq 0.5$ and $|E| \geq 0.5$), % *Soft besiege*

- Update positions of hawks using (4)

else if ($r \geq 0.5$ and $|E| < 0.5$), % *Hard besiege*

- Update positions of hawks using (6)

else if ($r < 0.5$ and $|E| \geq 0.5$), % *Soft besiege with progressive rapid dives*

- Update positions of hawks using (7), when Y using (8)

else if ($r < 0.5$ and $|E| < 0.5$), % *Hard besiege with progressive rapid dives*

- Update positions of hawks using (7), when Y using (11)

end if2**end for**

- Update X_{prev}

- $t = t + 1$

end while1

- Set $\mathbf{x}^* = X_{prev}$ as the initial solution of ATS

- Randomly generate the initial best solution $\mathbf{x}^*$ and evaluate $f(\mathbf{x}^*)$

while2 ($t \leq t_{\max}$ or TC are not met) % ATS (Second phase)

- Randomly N neighborhood solutions around $\mathbf{x}^*$ within R and evaluate all N solutions via $f(\mathbf{x}^*)$

- Let one of the best solutions among N be $\mathbf{x}$

if3 ($f(\mathbf{x}) < f(\mathbf{x}^*)$)

- Update $\mathbf{x}^* = \mathbf{x}$

end if3

- Store $\mathbf{x}$ in TL and activate AR mechanism using $R = \alpha R$

if4 (Local entrapment occur)

- Invoke BT mechanism by randomly selecting $\mathbf{x}'$ (one of the solutions stored in TL)

- Update $\mathbf{x}^* = \mathbf{x}'$

end if4

- Update $t = t+1$

end while2

Report the best solution $\mathbf{x}^*$

FIGURE 3. Pseudo code of cooperative HHO-ATS algorithm

11) Michaelwicz function is expressed as $f_{11}(\mathbf{x})$ in (22). It has nonlinear, asymmetric, and multi-modal properties with many local minima. Its global minimum is located at $\mathbf{x}^* = (2.203190, 1.57049)$ with $f_{11}(\mathbf{x}^*) = -1.8013$.

12) Peaks function is given as $f_{12}(\mathbf{x})$ in (23). It has nonlinear, asymmetric, and multi-modal properties. Its global minimum is located at $\mathbf{x}^* = (0.2283, -1.6255)$ with $f_{12}(\mathbf{x}^*) = -6.5511$.

13) Rastrigin function is represented as $f_{13}(\mathbf{x})$ in (24). It possesses nonlinear, symmetric, and multi-modal properties with many local minima. Its global minimum is located at $\mathbf{x}^* = (0, \dots, 0)$ with $f_{13}(\mathbf{x}^*) = 0$.

14) Salomon function is stated as $f_{14}(\mathbf{x})$ in (25). It has nonlinear, symmetric, and multi-modal properties with many local minima. Its global minimum is located at $\mathbf{x}^* = (0, \dots, 0)$ with $f_{14}(\mathbf{x}^*) = 0$.

15) Schwefel function is given as $f_{15}(\mathbf{x})$ in (26). With nonlinear, asymmetric, and multi-modal properties, it has many local minima and its global minimum is located at $\mathbf{x}^* = (420.9687, \dots, 420.9687)$ with $f_{15}(\mathbf{x}^*) = 0$.

16) Shekel's fox-holes function is represented as $f_{16}(\mathbf{x})$ in (27). With nonlinear, asymmetric, and multi-modal properties, it has many local minima and its global minimum is located at $\mathbf{x}^* = (8.0241, 9.1465)$ with $f_{16}(\mathbf{x}^*) = -12.1190$.

17) Shubert function is given as $f_{17}(\mathbf{x})$ in (28). It has nonlinear, symmetric, and multi-modal properties with many local minima. Its 18 global minima are located at $\mathbf{x}^* = (-7.0835, 4.8580), (-7.0835, -7.7083), (-1.4251, -7.0835), (5.4828, 4.8580), (-1.4251, -0.8003), (4.8580, 5.4828), (-7.7083, -7.0835), (-7.0835, -1.4251), (-7.7083, -0.8003), (-7.7083, 5.4828), (-0.8003, -7.7083), (-0.8003, -1.4251), (-0.8003, 4.8580), (-1.4251, 5.4828), (5.4828, -7.7083), (4.8580, -7.0835), (5.4828, -1.4251),$ and $(4.8580, -0.8003)$ with $f_{17}(\mathbf{x}^*) = -186.7309$.

18) Sinusoid function is stated as $f_{18}(\mathbf{x})$ in (29). It possesses nonlinear, asymmetric, and multi-modal properties with many local minima. Its global minimum is located at $\mathbf{x}^* = (9.039, 8.668)$ with $f_{18}(\mathbf{x}^*) = -18.5547$.

19) Yang-1 function is expressed as $f_{19}(\mathbf{x})$ in (30). It has nonlinear, symmetric, and multi-modal properties with many local minima. Its global minimum is located at $\mathbf{x}^* = (0, \dots, 0)$ with $f_{19}(\mathbf{x}^*) = -1$.

20) Yang-2 function is given as $f_{20}(\mathbf{x})$ in (31). It possesses nonlinear, symmetric, and multi-modal properties, with many local minima. Its global minimum is located at $\mathbf{x}^* = (0, \dots, 0)$ with $f_{20}(\mathbf{x}^*) = 0$.

The two-dimensional landscapes of the 20 selected benchmark functions are depicted in Figure 4.

$$f_1(\mathbf{x}) = -20 \exp \left[-\frac{1}{5} \sqrt{\frac{1}{D} \sum_{i=1}^D x_i^2} \right] - \exp \left[\frac{1}{D} \sum_{i=1}^D \cos(2\pi x_i) \right] + 20 + e, \quad -30 \leq x_i \leq 30 \quad (12)$$

$$f_2(\mathbf{x}) = \sum_{i=1}^D |x_i \sin(x_i) + 0.1x_i|, \quad -10 \leq x_i \leq 10 \quad (13)$$

$$f_3(\mathbf{x}) = x_1^2 + 2x_2^2 - 0.3 \cos(3\pi x_1) - 0.4 \cos(4\pi x_2) + 0.7, \quad -50 \leq x_i \leq 50 \quad (14)$$

$$f_4(\mathbf{x}) = 100 \sqrt{|x_2 - 0.01x_1^2|} + 0.01 |x_1 + 10|, \quad -15 \leq x_1 \leq -5, \quad -3 \leq x_2 \leq 3 \quad (15)$$

$$f_5(\mathbf{x}) = \left[\frac{1}{500} + \sum_{j=1}^{25} \left\{ \frac{1}{j + (x_1 - a_{1j})^6 + (x_2 - a_{2j})^6} \right\} \right]^{-1}, \quad -40 \leq x_i \leq 40, \quad \left. \begin{aligned} a_{ij} = & \begin{pmatrix} -32 & -16 & 0 & 16 & 32 & -32 & \cdots & 0 & 16 & 32 \\ -32 & -32 & -32 & -32 & -32 & -16 & \cdots & 32 & 32 & 32 \end{pmatrix} \end{aligned} \right\} \quad (16)$$

$$f_6(\mathbf{x}) = -\frac{1 + \cos\left(12\sqrt{x_1^2 + x_2^2}\right)}{0.5(x_1^2 + x_2^2) + 2}, \quad -5.12 \leq x_i \leq 5.12 \quad (17)$$

$$f_7(\mathbf{x}) = -\cos(x_1)\cos(x_2)\exp\left[-(x_1 - \pi)^2 - (x_2 - \pi)^2\right], \quad -10 \leq x_i \leq 10 \quad (18)$$

$$f_8(\mathbf{x}) = -(x_2 + 47)\sin\sqrt{|0.5x_1 + x_2 + 47|} - x_1\sin\sqrt{|x_1 - x_2 - 47|}, \\ -512 \leq x_i \leq 512 \quad (19)$$

$$f_9(\mathbf{x}) = \sum_{i=1}^D \frac{x_i^2}{4000} - \prod_{i=1}^D \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1, \quad -100 \leq x_i \leq 100 \quad (20)$$

$$f_{10}(\mathbf{x}) = \sin^2(3\pi x_1) + (x_1 - 1)^2 [1 + \sin^2(3\pi x_2)] + (x_2 - 1)^2 [1 + \sin^2(3\pi x_2)], \\ -10 \leq x_i \leq 10 \quad (21)$$

$$f_{11}(\mathbf{x}) = -\sin(x_1)\sin^{20}\left(\frac{x_1^2}{\pi}\right) - \sin(x_2)\sin^{20}\left(\frac{2x_2^2}{\pi}\right), \quad 0 \leq x_i \leq \pi \quad (22)$$

$$f_{12}(\mathbf{x}) = 3(1 - x_1)^2 \exp[-x_1^2 - (x_2 + 1)^2] - 10\left(\frac{x_1}{5} - x_1^3 - x_2^5\right) \exp[-x_1^2 - x_2^2] \\ - \frac{1}{3} \exp[-(x_1 + 1)^2 - x_2^2], \quad -4 \leq x_i \leq 4 \quad (23)$$

$$f_{13}(\mathbf{x}) = 10D + \sum_{i=1}^D [x_i^2 - 10\cos(2\pi x_i)], \quad -5.12 \leq x_i \leq 5.12 \quad (24)$$

$$f_{14}(\mathbf{x}) = 1 - \cos\left(2\pi\sqrt{x_1^2 + x_2^2}\right) - 0.1\sqrt{x_1^2 + x_2^2}, \quad -100 \leq x_i \leq 100 \quad (25)$$

$$f_{15}(\mathbf{x}) = 418.9829D - \sum_{i=1}^D x_i \sin\left(\sqrt{|x_i|}\right), \quad -500 \leq x_i \leq 500 \quad (26)$$

$$f_{16}(\mathbf{x}) = -\sum_{j=1}^{30} \left[\frac{1}{c_j + (x_1 - a_{1j})^2 + (x_2 - a_{2j})^2} \right], \quad 0 \leq x_i \leq 10 \quad (27)$$

$$a_{ij} = \begin{pmatrix} 9.6810 & 9.4000 & 8.0250 & \cdots & 9.4960 & 4.1380 \\ 0.6670 & 2.0410 & 9.1520 & \cdots & 4.8300 & 2.5620 \end{pmatrix},$$

$$c_j = (0.806, 0.517, 0.100, 0.908, \dots, 0.608, 0.326)$$

$$f_{17}(\mathbf{x}) = \prod_{i=1}^n \left(\sum_{j=1}^5 j \cos((j+1)x_i + j) \right), \quad -10 \leq x_i \leq 10 \quad (28)$$

$$f_{18}(\mathbf{x}) = x_1 \sin(4x_1) + 1.1x_2 \sin(2x_2), \quad 0 \leq x_i \leq 10 \quad (29)$$

$$f_{19}(\mathbf{x}) = \left\{ \left[\sum_{i=1}^D \sin^2(x_i) \right] - \exp\left(-\sum_{i=1}^D x_i^2\right) \right\} \times \exp\left(-\sum_{i=1}^D \sin^2 \sqrt{|x_i|}\right), \\ -10 \leq x_i \leq 10 \quad (30)$$

$$f_{20}(\mathbf{x}) = \left(\sum_{i=1}^D |x_i| \right) \cdot \exp\left(-\sum_{i=1}^D \sin x_i^2\right), \quad -2\pi \leq x_i \leq 2\pi \quad (31)$$

4. Results and Discussion. To evaluate its effectiveness and convergence behavior, the proposed cooperative HHO-ATS algorithm is evaluated on 20 benchmark functions

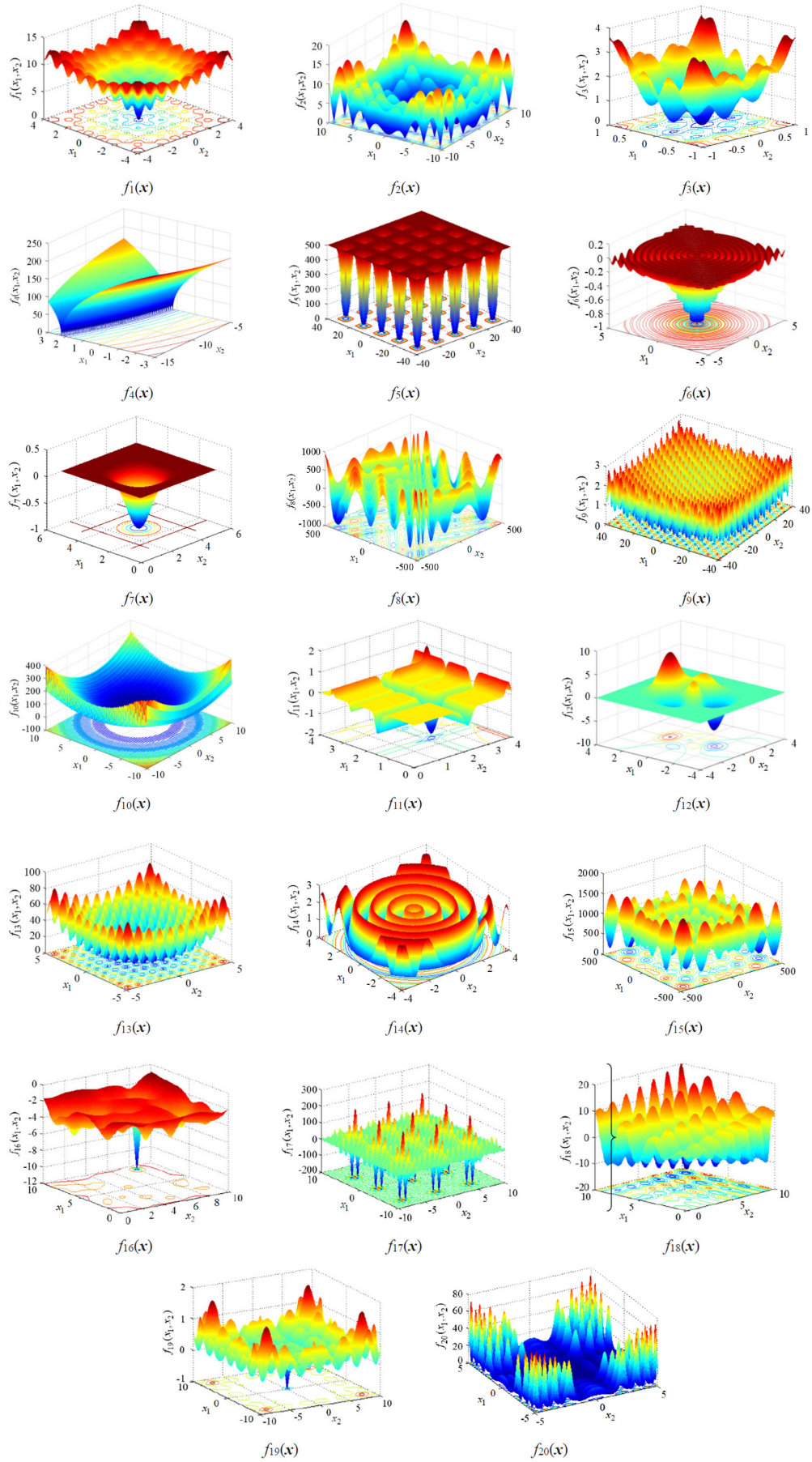


FIGURE 4. Two-dimensional landscapes of 20 selected benchmark functions

described in the previous section. Its search performance is firstly compared with those of the original HHO and ATS algorithms.

In this study, ATS, HHO, and cooperative HHO-ATS algorithms are implemented in MATLAB (R2018b) for coding and executed on a computer with an Intel(R) Core(TM) i5-3470 processor operating at 3.60 GHz and 4 GB of RAM. The searching parameters of ATS are selected according to the recommendations in [5] and fine-tuned through preliminary testing on the adopted benchmark functions, as summarized in Table 1, where N denotes the number of neighborhood members, R represents the initial search radius (expressed as a percentage of the search space), and $J_{\max}$ indicates the maximum number of solution cycling iterations required to activate the BT mechanism. Likewise, the HHO parameter settings follow the guidelines in [11] and are further adjusted based on initial experimental observations. Accordingly, the population size of hawks is fixed at $N_h = 40$ for all test problems. Based on these preliminary analyses of both constituent algorithms, the searching parameter configuration of the cooperative HHO-ATS algorithm is determined and summarized in Table 2, where N_h stands for the population size of hawks and SI denotes the switching iteration.

For each benchmark function, 100 independent trials are conducted to ensure statistical reliability of the results. The termination criteria for all algorithms are defined by two TC: TC-1, the objective function value $f(\mathbf{x})$ satisfies a predefined tolerance level ($\varepsilon \leq 10^{-15}$), or TC-2, the maximum number of iterations ($t_{\max} = 100$) is reached. Meeting the first condition (TC-1) indicates successful convergence to the desired precision, whereas reaching the second condition (TC-2) signifies that the algorithm fails to attain the specified tolerance within the allowed number of iterations.

The comparative results of ATS, HHO, and the proposed cooperative HHO-ATS algorithms are summarized in Table 3. The evaluation results reported in Table 3 are expressed in the form $AE \pm SD$ (SR%), where AE denotes the average number of function evaluations, SD represents the standard deviation, and SR indicates the percentage of success rates. The AE reflects the computational effort required by the algorithm; a lower AE corresponds to shorter search time. The SD measures performance stability, with smaller values indicating greater robustness across independent runs.

As reported in Table 3, the ATS algorithm cannot achieve 100% SR on any benchmark functions. The highest SR obtained by ATS is 98% for function f_{12} , while the lowest is 19% for function f_4 . Across all 20 benchmark functions, the average SR of ATS is 66.90%. In contrast, HHO achieved a 100% of SR on four benchmark functions (f_3 , f_{11} , f_{12} , and f_{18}). Its lowest SR is 64%, observed on function f_4 . When averaged over all 20 benchmark functions, HHO attained a mean SR of 86.35%.

For the proposed cooperative HHO-ATS algorithm, a 100% SR is consistently achieved across all benchmark functions. Considering the overall performance in terms of AE, SD, and SR, the proposed cooperative HHO-ATS clearly outperforms the standalone HHO and ATS algorithms. It demonstrates superior capability in locating global optimal solutions across all tested benchmark functions. Moreover, it achieves higher reliability, reduced average function evaluations, and lower variability.

The convergence characteristics of ATS, HHO, and the proposed cooperative HHO-ATS algorithms are depicted in Figure 5, when evaluated on the 20 benchmark functions under the TC-2 setting only. This configuration is intentionally selected to clearly demonstrate the convergence behavior of those three algorithms. Based on the results across all 20 benchmark functions, ATS exhibits slower convergence than HHO, whereas the proposed cooperative HHO-ATS achieves faster convergence than both HHO and ATS. This indicates that the proposed cooperative HHO-ATS provides faster convergence than HHO and

TABLE 1. Searching parameters of ATS

Functions	Searching parameters					
	N	R (%)	AR			BT
			Stage-I	Stage-II	Stage-III	$J_{\max}$
$f_1(\mathbf{x})$	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}9$, $R = R \times 1\text{e-}2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}11$ $R = R \times 1\text{e-}3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}13$ $R = R \times 1\text{e-}4$	20
$f_2(\mathbf{x})$	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}7$, $R = R \times 1\text{e-}2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}9$ $R = R \times 1\text{e-}3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}11$ $R = R \times 1\text{e-}4$	15
$f_3(\mathbf{x})$	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}7$, $R = R \times 1\text{e-}2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}9$ $R = R \times 1\text{e-}3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}11$ $R = R \times 1\text{e-}4$	25
$f_4(\mathbf{x})$	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}9$, $R = R \times 1\text{e-}2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}11$ $R = R \times 1\text{e-}3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}13$ $R = R \times 1\text{e-}4$	10
$f_5(\mathbf{x})$	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}7$, $R = R \times 1\text{e-}2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}9$ $R = R \times 1\text{e-}3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}11$ $R = R \times 1\text{e-}4$	20
$f_6(\mathbf{x})$	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}8$, $R = R \times 1\text{e-}2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}10$ $R = R \times 1\text{e-}3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}12$ $R = R \times 1\text{e-}4$	15
$f_7(\mathbf{x})$	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}8$, $R = R \times 1\text{e-}2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}10$ $R = R \times 1\text{e-}3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}12$ $R = R \times 1\text{e-}4$	25
$f_8(\mathbf{x})$	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}7$, $R = R \times 1\text{e-}2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}9$ $R = R \times 1\text{e-}3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}11$ $R = R \times 1\text{e-}4$	15
$f_9(\mathbf{x})$	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}7$, $R = R \times 1\text{e-}2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}9$ $R = R \times 1\text{e-}3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}11$ $R = R \times 1\text{e-}4$	15
$f_{10}(\mathbf{x})$	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}8$, $R = R \times 1\text{e-}2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}10$ $R = R \times 1\text{e-}3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}12$ $R = R \times 1\text{e-}4$	10
$f_{11}(\mathbf{x})$	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}7$, $R = R \times 1\text{e-}2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}9$ $R = R \times 1\text{e-}3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}11$ $R = R \times 1\text{e-}4$	20
$f_{12}(\mathbf{x})$	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}7$, $R = R \times 1\text{e-}2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}9$ $R = R \times 1\text{e-}3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}11$ $R = R \times 1\text{e-}4$	20
$f_{13}(\mathbf{x})$	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}7$, $R = R \times 1\text{e-}2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}9$ $R = R \times 1\text{e-}3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}11$ $R = R \times 1\text{e-}4$	25
$f_{14}(\mathbf{x})$	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}8$, $R = R \times 1\text{e-}2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}10$ $R = R \times 1\text{e-}3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}12$ $R = R \times 1\text{e-}4$	15
$f_{15}(\mathbf{x})$	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}7$, $R = R \times 1\text{e-}2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}9$ $R = R \times 1\text{e-}3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}11$ $R = R \times 1\text{e-}4$	20
$f_{16}(\mathbf{x})$	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}9$, $R = R \times 1\text{e-}2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}11$ $R = R \times 1\text{e-}3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}13$ $R = R \times 1\text{e-}4$	10
$f_{17}(\mathbf{x})$	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}8$, $R = R \times 1\text{e-}2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}10$ $R = R \times 1\text{e-}3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}12$ $R = R \times 1\text{e-}4$	15
$f_{18}(\mathbf{x})$	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}7$, $R = R \times 1\text{e-}2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}9$ $R = R \times 1\text{e-}3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}11$ $R = R \times 1\text{e-}4$	25
$f_{19}(\mathbf{x})$	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}8$, $R = R \times 1\text{e-}2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}10$ $R = R \times 1\text{e-}3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}12$ $R = R \times 1\text{e-}4$	20
$f_{20}(\mathbf{x})$	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}8$, $R = R \times 1\text{e-}2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}10$ $R = R \times 1\text{e-}3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1\text{e-}12$ $R = R \times 1\text{e-}4$	10

ATS, respectively. These results confirm that the proposed cooperative HHO-ATS exhibits superior convergence performance to that of ATS and HHO in global optimization.

To further compare the proposed cooperative HHO-ATS algorithm with mainstream metaheuristic algorithms and an existing cooperative algorithm, the experimental evaluation on the 20 benchmark functions presented in Section 3 is extended to include

TABLE 2. Searching parameters of the cooperative HHO-ATS

Functions	Searching parameters							
	HHO	ATS						SI
	N_h	N	R (%)	AR			BT	
				Stage-I	Stage-II	Stage-III	$J_{\max}$	
$f_1(\mathbf{x})$	40	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-9$ $R = R \times 1e-2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-11$ $R = R \times 1e-3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-13$ $R = R \times 1e-4$	20	50
$f_2(\mathbf{x})$	40	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-7$ $R = R \times 1e-2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-9$ $R = R \times 1e-3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-11$ $R = R \times 1e-4$	15	65
$f_3(\mathbf{x})$	40	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-7$ $R = R \times 1e-2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-9$ $R = R \times 1e-3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-11$ $R = R \times 1e-4$	25	50
$f_4(\mathbf{x})$	40	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-9$ $R = R \times 1e-2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-11$ $R = R \times 1e-3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-13$ $R = R \times 1e-4$	10	60
$f_5(\mathbf{x})$	40	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-7$ $R = R \times 1e-2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-9$ $R = R \times 1e-3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-11$ $R = R \times 1e-4$	20	55
$f_6(\mathbf{x})$	40	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-8$ $R = R \times 1e-2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-10$ $R = R \times 1e-3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-12$ $R = R \times 1e-4$	15	55
$f_7(\mathbf{x})$	40	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-8$ $R = R \times 1e-2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-10$ $R = R \times 1e-3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-12$ $R = R \times 1e-4$	25	60
$f_8(\mathbf{x})$	40	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-7$ $R = R \times 1e-2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-9$ $R = R \times 1e-3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-11$ $R = R \times 1e-4$	15	60
$f_9(\mathbf{x})$	40	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-7$ $R = R \times 1e-2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-9$ $R = R \times 1e-3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-11$ $R = R \times 1e-4$	15	65
$f_{10}(\mathbf{x})$	40	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-8$ $R = R \times 1e-2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-10$ $R = R \times 1e-3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-12$ $R = R \times 1e-4$	10	65
$f_{11}(\mathbf{x})$	40	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-7$ $R = R \times 1e-2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-9$ $R = R \times 1e-3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-11$ $R = R \times 1e-4$	20	60
$f_{12}(\mathbf{x})$	40	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-7$ $R = R \times 1e-2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-9$ $R = R \times 1e-3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-11$ $R = R \times 1e-4$	20	55
$f_{13}(\mathbf{x})$	40	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-7$ $R = R \times 1e-2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-9$ $R = R \times 1e-3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-11$ $R = R \times 1e-4$	25	55
$f_{14}(\mathbf{x})$	40	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-8$ $R = R \times 1e-2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-10$ $R = R \times 1e-3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-12$ $R = R \times 1e-4$	15	65
$f_{15}(\mathbf{x})$	40	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-7$ $R = R \times 1e-2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-9$ $R = R \times 1e-3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-11$ $R = R \times 1e-4$	20	60
$f_{16}(\mathbf{x})$	40	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-9$ $R = R \times 1e-2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-11$ $R = R \times 1e-3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-13$ $R = R \times 1e-4$	10	65
$f_{17}(\mathbf{x})$	40	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-8$ $R = R \times 1e-2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-10$ $R = R \times 1e-3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-12$ $R = R \times 1e-4$	15	65
$f_{18}(\mathbf{x})$	40	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-7$ $R = R \times 1e-2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-9$ $R = R \times 1e-3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-11$ $R = R \times 1e-4$	25	60
$f_{19}(\mathbf{x})$	40	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-8$ $R = R \times 1e-2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-10$ $R = R \times 1e-3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-12$ $R = R \times 1e-4$	20	60
$f_{20}(\mathbf{x})$	40	30	20	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-8$ $R = R \times 1e-2$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-10$ $R = R \times 1e-3$	$f(\mathbf{x}) < f(\mathbf{x}^*) + 1e-12$ $R = R \times 1e-4$	10	65

trajectory-based metaheuristics, i.e., simulated annealing (SA) [37] and TS [6,7], population-based metaheuristics, i.e., cuckoo search (CS) [38,39] and grey wolf optimizer (GWO) [40], and the cooperative FPA-ATS algorithm [32]. The search parameters of these algorithms are configured according to the recommendations provided in their original publications and those of ATS and cooperative HHO-ATS algorithms to ensure a fair comparison, as described below.

For SA [37], the initial temperature is set to $T_0 = 100$, while the final temperature is set to $T_F = 1 \times 10^{-5}$. Boltzmann's constant is $K = 1 \times 10^{-4}$, and the cooling factor is

TABLE 3. Results of ATS, HHO and cooperative HHO-ATS algorithms

Functions	Algorithms		
	ATS	HHO	Cooperative HHO-ATS
$f_1(\mathbf{x})$ ($D = 16$)	2.5108e+4±5.1813e+3(54%)	1.6474e+4±2.2546e+3(78%)	5.6585e+3±4.5318e+1(100%)
$f_2(\mathbf{x})$ ($D = 16$)	2.1467e+4±4.3054e+3(67%)	1.4380e+4±2.0591e+3(85%)	4.3107e+3±2.8646e+1(100%)
$f_3(\mathbf{x})$ ($D = 2$)	7.7923e+3±6.8210e+2(89%)	2.1575e+3±7.1728e+1(100%)	6.2184e+2±5.0143e+1(100%)
$f_4(\mathbf{x})$ ($D = 2$)	2.8719e+4±6.5787e+3(19%)	2.2279e+4±4.3709e+3(64%)	8.7416e+3±1.2905e+2(100%)
$f_5(\mathbf{x})$ ($D = 2$)	1.9584e+4±3.2655e+3(72%)	4.2417e+3±1.6364e+2(95%)	4.6680e+2±3.3027e+1(100%)
$f_6(\mathbf{x})$ ($D = 2$)	2.6854e+4±5.2309e+3(47%)	1.9186e+4±2.3797e+3(67%)	7.3152e+3±6.1209e+1(100%)
$f_7(\mathbf{x})$ ($D = 2$)	7.8912e+3±6.9188e+2(85%)	2.1974e+3±3.0655e+2(96%)	6.5202e+2±4.9134e+1(100%)
$f_8(\mathbf{x})$ ($D = 2$)	2.2814e+4±4.4213e+3(65%)	1.6583e+4±2.1898e+3(83%)	4.1048e+3±7.6041e+1(100%)
$f_9(\mathbf{x})$ ($D = 10$)	2.3740e+4±4.5125e+3(58%)	1.8174e+4±2.2993e+3(81%)	3.6495e+3±3.7126e+1(100%)
$f_{10}(\mathbf{x})$ ($D = 2$)	2.7169e+4±5.4630e+3(42%)	2.1390e+4±2.3986e+3(65%)	8.9193e+3±2.1204e+2(100%)
$f_{11}(\mathbf{x})$ ($D = 2$)	7.9129e+3±7.1082e+2(85%)	1.9985e+3±2.7404e+1(100%)	5.5197e+2±5.8960e+1(100%)
$f_{12}(\mathbf{x})$ ($D = 2$)	5.1704e+3±2.1436e+2(98%)	1.5837e+3±6.5356e+1(100%)	7.5224e+2±5.9712e+1(100%)
$f_{13}(\mathbf{x})$ ($D = 16$)	7.8616e+3±6.9704e+2(82%)	2.2054e+3±2.9138e+2(96%)	3.5668e+2±4.8153e+1(100%)
$f_{14}(\mathbf{x})$ ($D = 2$)	2.6290e+4±5.3021e+3(51%)	1.7286e+4±2.2951e+3(84%)	5.7202e+3±6.5589e+1(100%)
$f_{15}(\mathbf{x})$ ($D = 10$)	4.0118e+4±3.1317e+3(69%)	2.5613e+4±2.1107e+3(86%)	6.4081e+3±6.7042e+1(100%)
$f_{16}(\mathbf{x})$ ($D = 2$)	1.8621e+4±3.1464e+3(74%)	2.1819e+3±2.6542e+2(97%)	7.3275e+2±4.8240e+1(100%)
$f_{17}(\mathbf{x})$ ($D = 2$)	2.3172e+4±4.5280e+3(63%)	1.9482e+4±2.3206e+3(82%)	4.5590e+3±5.1078e+1(100%)
$f_{18}(\mathbf{x})$ ($D = 2$)	7.9286e+3±6.9978e+2(84%)	3.1045e+3±4.1508e+1(100%)	3.6156e+2±3.1283e+1(100%)
$f_{19}(\mathbf{x})$ ($D = 16$)	2.1953e+4±3.1586e+3(68%)	1.6403e+4±2.0989e+3(86%)	5.9847e+3±2.9635e+1(100%)
$f_{20}(\mathbf{x})$ ($D = 16$)	2.2267e+4±4.4018e+3(66%)	1.9607e+4±2.3174e+3(82%)	6.2061e+3±3.2842e+1(100%)

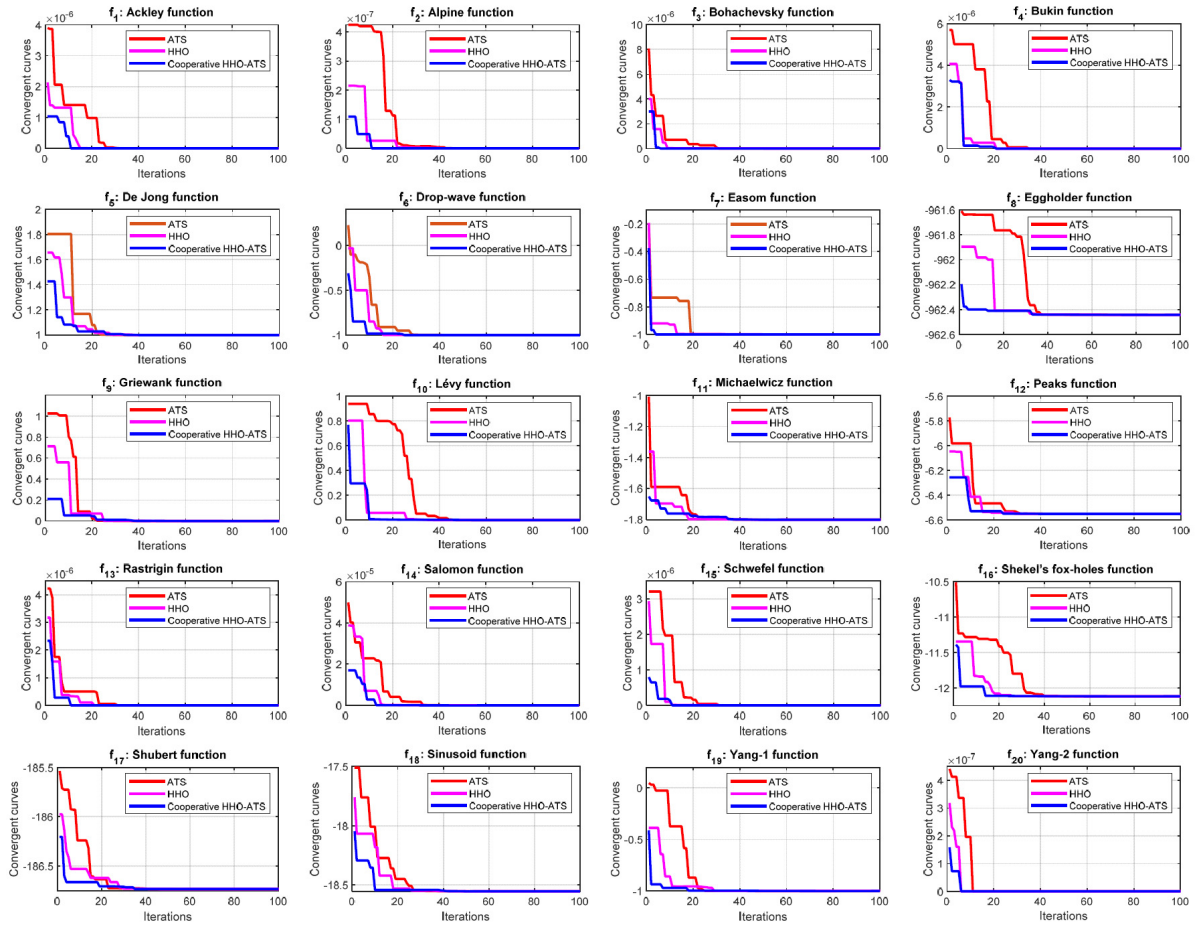


FIGURE 5. Convergence curves of ATS, HHO, and the cooperative HHO-ATS over 20 benchmark functions

$\alpha = 0.08$. The maximum number of iterations for reaching equilibrium is $NE = 80$. The maximum number of iterations, Max_Iter , is set to 100, corresponding to TC-2 of the cooperative HHO-ATS algorithm.

For TS [6,7], the search radius R and the number of neighborhoods N are set to the same values as those used in ATS (without the AR and BT mechanisms) in Table 1. The maximum number of iterations, t_{max} , is also set to 100, corresponding to TC-2 of the cooperative HHO-ATS algorithm.

For CS [38,39], the number of nests is set to $n = 40$, which is fixed to the same value as N_h in the cooperative HHO-ATS algorithm. The discovery rate of alien eggs is $p_a = 0.25$. The maximum number of generations, Max_Gen , is set to 100, corresponding to TC-2 of the cooperative HHO-ATS algorithm.

For GWO [40], the number of wolves is set to $n = 40$, which is fixed to the same value as N_h in the cooperative HHO-ATS algorithm. The maximum number of generations, Max_Gen , is set to 100, corresponding to TC-2 of the cooperative HHO-ATS algorithm.

For the cooperative FPA-ATS algorithm [32], the ATS parameters are set as shown in Table 2. In the FPA component, the number of flowers is set to $n = 40$, which is fixed to the same value as N_h in the cooperative HHO-ATS algorithm. The switching probability between local and global pollination is $p_a = 0.25$. The maximum number of generations, Max_Gen , is also set to 100, corresponding to TC-2 of the cooperative HHO-ATS algorithm.

All algorithms are evaluated through 100 independent trials for each benchmark function. In addition, both TC-1 and TC-2 are employed as TC. The results obtained by SA, TS, CS, GWO, and the cooperative FPA-ATS algorithm are compared with those of the proposed cooperative HHO-ATS algorithm as summarized in Table 4.

As shown in Table 4, the SA algorithm cannot achieve 100% SR on any benchmark functions. The highest SR obtained by SA is 71% for function f_{12} , while the lowest is 11% for function f_4 . Across all 20 benchmark functions, the average SR of SA is 45.95%. Similarly, the TS algorithm cannot achieve 100% SR on any benchmark functions. The highest SR obtained by TS is 74% for function f_{12} , while the lowest is 13% for function f_4 . Across all 20 benchmark functions, the average SR of TS is 48.90%. In contrast, CS algorithm achieved a 100% of SR on three benchmark functions (f_3 , f_{12} , and f_{18}). Its lowest SR is 53%, observed on function f_4 . When averaged over all 20 benchmark functions, CS attained a mean SR of 79.45%. GWO algorithm achieved a 100% of SR on four benchmark functions (f_3 , f_{11} , f_{12} , and f_{18}). Its lowest SR is 58%, observed on function f_4 . When averaged over all 20 benchmark functions, GWO attained a mean SR of 82.90%. For the cooperative FPA-ATS algorithm, it achieved a 100% of SR on fourteen benchmark functions (f_1 - f_3 , f_5 , f_7 , f_8 , f_{11} - f_{16} , f_{18} , and f_{19}). Its lowest SR is 97%, observed on function f_4 . When averaged over all 20 benchmark functions, the cooperative FPA-ATS algorithm attained a mean SR of 99.55%. For the proposed cooperative HHO-ATS algorithm, a 100% SR is consistently achieved across all benchmark functions. Compared with SA, TS, CS, GWO, and the cooperative FPA-ATS algorithms in terms of AE, SD, and SR, as presented in Table 4, the proposed cooperative HHO-ATS algorithm demonstrated the best overall performance. It achieved a 100% SR while attaining lower AE and SD values across all 20 benchmark functions, consistently locating the global optimum in every case.

5. Conclusions. A novel serial-heterogeneous cooperative metaheuristic optimization algorithm, namely the cooperative HHO-ATS algorithm, has been proposed in this paper for global optimization. The proposed approach integrates the strong global exploration capability of HHO with the intensive local exploitation capability of ATS through a sequential

TABLE 4. Results of SA, TS, CS, GWO, cooperative FPA-ATS and cooperative HHO-ATS algorithms

Functions	Algorithms		
	SA	TS	CS
$f_1(\mathbf{x})$ ($D = 16$)	3.4967e+4±7.2843e+3(35%)	3.1486e+4±6.8782e+3(38%)	2.6890e+4±2.7606e+3(70%)
$f_2(\mathbf{x})$ ($D = 16$)	3.1084e+4±5.9891e+3(42%)	2.8096e+4±5.6061e+3(45%)	2.1574e+4±2.8315e+3(78%)
$f_3(\mathbf{x})$ ($D = 2$)	8.6820e+3±8.9035e+2(63%)	8.4290e+3±7.7454e+2(66%)	2.6285e+3±8.7147e+1(100%)
$f_4(\mathbf{x})$ ($D = 2$)	3.5452e+4±7.8850e+3(11%)	3.1518e+4±7.1992e+3(13%)	2.8986e+4±5.3430e+3(53%)
$f_5(\mathbf{x})$ ($D = 2$)	2.8804e+4±4.6049e+3(53%)	2.5603e+4±4.1686e+3(57%)	5.1367e+3±2.2058e+2(86%)
$f_6(\mathbf{x})$ ($D = 2$)	3.2436e+4±6.7872e+3(22%)	2.8920e+4±6.1255e+3(25%)	2.6780e+4±2.8786e+3(58%)
$f_7(\mathbf{x})$ ($D = 2$)	9.0103e+3±7.6504e+2(64%)	8.5731e+3±7.2047e+2(67%)	2.8921e+3±3.7310e+2(87%)
$f_8(\mathbf{x})$ ($D = 2$)	3.3217e+4±5.2518e+3(45%)	2.8042e+4±5.5826e+3(47%)	2.1516e+4±2.8174e+3(73%)
$f_9(\mathbf{x})$ ($D = 10$)	2.9614e+4±5.8930e+3(38%)	2.7457e+4±5.2598e+3(41%)	2.5013e+4±2.9903e+3(71%)
$f_{10}(\mathbf{x})$ ($D = 2$)	3.2376e+4±6.3678e+3(23%)	2.9171e+4±5.9492e+3(26%)	2.7985e+4±2.8497e+3(57%)
$f_{11}(\mathbf{x})$ ($D = 2$)	8.4778e+3±7.9361e+2(62%)	8.2624e+3±7.5951e+2(65%)	2.6182e+3±3.2378e+1(98%)
$f_{12}(\mathbf{x})$ ($D = 2$)	5.9084e+3±3.2010e+2(71%)	5.6605e+3±2.7873e+2(74%)	1.5837e+3±6.5356e+1(100%)
$f_{13}(\mathbf{x})$ ($D = 16$)	8.5642e+3±7.5637e+2(60%)	8.2047e+3±7.2869e+2(63%)	2.9622e+3±3.5761e+2(85%)
$f_{14}(\mathbf{x})$ ($D = 2$)	3.3087e+4±6.3543e+3(31%)	2.9406e+4±5.9613e+3(33%)	2.4676e+4±2.9198e+3(76%)
$f_{15}(\mathbf{x})$ ($D = 10$)	4.8910e+4±3.9241e+3(48%)	4.6580e+4±3.7245e+3(52%)	3.1218e+4±2.7174e+3(81%)
$f_{16}(\mathbf{x})$ ($D = 2$)	2.7195e+4±3.8739e+3(53%)	2.2594e+4±3.4897e+3(57%)	2.8890e+3±3.1402e+2(88%)
$f_{17}(\mathbf{x})$ ($D = 2$)	2.9083e+4±4.9426e+3(42%)	2.6803e+4±4.7950e+3(46%)	2.6517e+4±2.8053e+3(75%)
$f_{18}(\mathbf{x})$ ($D = 2$)	8.5940e+3±7.5352e+2(63%)	8.2574e+3±7.3056e+2(65%)	3.7165e+3±4.7971e+1(100%)
$f_{19}(\mathbf{x})$ ($D = 16$)	2.7869e+4±3.8011e+3(47%)	2.5892e+4±3.5510e+3(50%)	2.4136e+4±2.6067e+3(77%)
$f_{20}(\mathbf{x})$ ($D = 16$)	2.9103e+4±5.0673e+3(46%)	2.6386e+4±4.8079e+3(48%)	2.5208e+4±2.7135e+3(76%)
Functions	Algorithms		
	GWO	Cooperative FPA-ATS	Cooperative HHO-ATS
$f_1(\mathbf{x})$ ($D = 16$)	2.4512e+4±2.5018e+3(73%)	6.7605e+3±6.0873e+1(100%)	5.6585e+3±4.5318e+1(100%)
$f_2(\mathbf{x})$ ($D = 16$)	1.8905e+4±2.4672e+3(81%)	5.9828e+3±4.9180e+1(100%)	4.3107e+3±2.8646e+1(100%)
$f_3(\mathbf{x})$ ($D = 2$)	2.3626e+3±8.2049e+1(100%)	8.6964e+2±8.7352e+1(100%)	6.2184e+2±5.0143e+1(100%)
$f_4(\mathbf{x})$ ($D = 2$)	2.5642e+4±4.8015e+3(58%)	9.9108e+3±3.1046e+2(97%)	8.7416e+3±1.2905e+2(100%)
$f_5(\mathbf{x})$ ($D = 2$)	4.6880e+3±1.9536e+2(90%)	6.0371e+2±5.8264e+1(100%)	4.6680e+2±3.3027e+1(100%)
$f_6(\mathbf{x})$ ($D = 2$)	2.3517e+4±2.6180e+3(62%)	9.8103e+3±8.6505e+1(99%)	7.3152e+3±6.1209e+1(100%)
$f_7(\mathbf{x})$ ($D = 2$)	2.6053e+3±3.3742e+2(92%)	8.4712e+2±6.8189e+1(100%)	6.5202e+2±4.9134e+1(100%)
$f_8(\mathbf{x})$ ($D = 2$)	1.8784e+4±2.4621e+3(78%)	6.5767e+3±9.2758e+1(100%)	4.1048e+3±7.6041e+1(100%)
$f_9(\mathbf{x})$ ($D = 10$)	2.1283e+4±2.4865e+3(76%)	5.7089e+3±5.6871e+1(99%)	3.6495e+3±3.7126e+1(100%)
$f_{10}(\mathbf{x})$ ($D = 2$)	2.5907e+4±2.6730e+3(61%)	1.2306e+4±4.0536e+2(98%)	8.9193e+3±2.1204e+2(100%)
$f_{11}(\mathbf{x})$ ($D = 2$)	2.3340e+3±2.9416e+1(100%)	7.1434e+2±8.0602e+1(100%)	5.5197e+2±5.8960e+1(100%)
$f_{12}(\mathbf{x})$ ($D = 2$)	1.5837e+3±6.5356e+1(100%)	9.4050e+2±8.5468e+1(100%)	7.5224e+2±5.9712e+1(100%)
$f_{13}(\mathbf{x})$ ($D = 16$)	2.4912e+3±3.2524e+2(92%)	5.9201e+2±6.6074e+1(100%)	3.5668e+2±4.8153e+1(100%)
$f_{14}(\mathbf{x})$ ($D = 2$)	2.1808e+4±2.4607e+3(81%)	7.6732e+3±8.7175e+1(100%)	5.7202e+3±6.5589e+1(100%)
$f_{15}(\mathbf{x})$ ($D = 10$)	2.8475e+4±2.4613e+3(84%)	8.5994e+3±8.8290e+1(100%)	6.4081e+3±6.7042e+1(100%)
$f_{16}(\mathbf{x})$ ($D = 2$)	2.6903e+3±2.8156e+2(92%)	9.6046e+2±7.0563e+1(100%)	7.3275e+2±4.8240e+1(100%)
$f_{17}(\mathbf{x})$ ($D = 2$)	2.2776e+4±2.6074e+3(78%)	7.6187e+3±7.5346e+1(99%)	4.5590e+3±5.1078e+1(100%)
$f_{18}(\mathbf{x})$ ($D = 2$)	3.4180e+3±4.4752e+1(100%)	6.5955e+2±5.7618e+1(100%)	3.6156e+2±3.1283e+1(100%)
$f_{19}(\mathbf{x})$ ($D = 16$)	1.9812e+4±2.3570e+3(82%)	8.2460e+3±6.0814e+1(100%)	5.9847e+3±2.9635e+1(100%)
$f_{20}(\mathbf{x})$ ($D = 16$)	2.2934e+4±2.6485e+3(78%)	9.2573e+3±5.8785e+1(99%)	6.2061e+3±3.2842e+1(100%)

cooperation mechanism. By combining these complementary search behaviors, the cooperative HHO-ATS algorithm achieves a more effective balance between exploration and exploitation, thereby improving convergence efficiency and search robustness. Experimental evaluations on 20 selected benchmark optimization functions demonstrated that the proposed cooperative HHO-ATS algorithm consistently outperformed the standalone ATS and HHO algorithms, as well as other mainstream metaheuristic algorithms, including SA, TS, CS, GWO, and the cooperative FPA-ATS algorithm, in terms of convergence behavior and solution quality. The results indicate that the cooperative framework successfully exploits the strengths of both constituent algorithms while mitigating their individual

weaknesses. Despite its promising performance, the proposed algorithm still possesses certain limitations. The effectiveness of the cooperative HHO-ATS algorithm depends on the selection of the SI, which controls the transition between the exploration and exploitation phases. An inappropriate SI value may lead to an imbalance between global search and local refinement, potentially affecting optimization performance. In addition, the current implementation employs a fixed switching strategy, which may not be equally effective for all optimization landscapes. Future research may focus on developing adaptive switching mechanisms capable of automatically adjusting the transition point according to the search progress and landscape characteristics. Furthermore, the applicability of the proposed cooperative HHO-ATS algorithm will be further extended to complex real-world industrial engineering problems, including vehicle routing, production planning, logistics and supply chain management, job shop scheduling, and assembly line balancing. Such extensions are expected to further validate its scalability, adaptability, and practical effectiveness in solving large-scale and highly constrained optimization problems.

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