

EVENT-TRIGGERED STABILIZATION OF POSITIVE TAKAGI-SUGENO FUZZY SYSTEMS UNDER LÉVY NOISE VIA LINEAR PROGRAMMING

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ABSTRACT. We study how to stabilize positive Takagi-Sugeno fuzzy systems that evolve under time-varying delay and Lévy-driven jump-diffusion noise. Positive systems keep all state variables nonnegative, a property needed in biology, pharmacology, and epidemiology. Our main tool is a co-positive Lyapunov-Krasovskii functional whose delay integral carries an exponential weight. Because this functional is linear in the state, the resulting stability conditions reduce to element-wise inequalities that one can check by solving a linear program in polynomial time. This stands in contrast to semidefinite programs commonly employed for general fuzzy systems. We then design a parallel distributed compensation controller through a variable substitution that removes the bilinear coupling between gain matrices and the decision vector. To save communication resources, we add an event-triggered control scheme with a component-wise threshold and show that it is free of Zeno behavior. Two worked examples support the theory. The first applies our conditions to a stochastic epidemiological compartmental model of disease spread. The second treats a two-compartment pharmacokinetic model that tracks drug concentration across body compartments. Both examples confirm that the linear programming conditions are practical and that the closed-loop system stays positive and stable under jump-diffusion noise.

Keywords: Positive systems, Takagi-Sugeno fuzzy systems, Lévy noise, Lyapunov-Krasovskii functional, Event-triggered control, Linear programming

1. Introduction. A dynamical system is called *positive* when every state variable stays nonnegative for any nonnegative initial condition. Such systems appear in biological population dynamics, epidemiological compartmental models [1], chemical reaction networks, pharmacokinetic drug models, economic growth theory [2, 3], and packet-switched communication networks. The Metzler property of the system matrix is the algebraic signature of positivity, and co-positive Lyapunov functions – linear functionals $V(x) = \lambda^T x$ with $\lambda > 0$ – serve as the natural stability certificates [4]. Nam et al. [5, 6] advanced the co-positive method for positive time-delay systems by deriving state-bounding techniques

and exponential decay estimates under polytopic uncertainties. When the delay is time-varying, Lyapunov-Krasovskii functionals extend the co-positive structure to account for delay-dependent dynamics [7, 8].

Many positive systems are inherently nonlinear. The Takagi-Sugeno (T-S) fuzzy model [9] represents such nonlinear systems as convex combinations of local linear models blended by membership functions. This recasts stability analysis and controller synthesis as LP or LMI problems [10]. For positive systems, imposing Metzler constraints on the local subsystem matrices preserves positivity, and the co-positive Lyapunov function leads to LP conditions, giving substantial computational savings over LMI-based approaches. Benzaouia and El Hajjaji [11] established stabilization conditions for positive T-S fuzzy systems with delay. Subsequent results addressed $\mathcal{L}_1/\mathcal{L}_\infty$ performance for switched systems [12], Metzler-constrained closed-loop design [13], tracking control [14], ℓ_1 -optimal synthesis [15], and Markov switching [16]. These are model-based syntheses; data-driven alternatives that bypass an explicit model have also been developed for unknown closed-loop systems [34], yet they do not preserve the Metzler/co-positive LP structure exploited here. Despite these advances, most existing results assume deterministic dynamics.

In practice, dynamical systems face random perturbations. Lévy processes, which comprise a drift, a Brownian part, and a jump component, provide a more realistic noise model than standard Brownian motion by capturing both continuous fluctuations and discontinuous jumps [17]. The jump component, governed by a Lévy measure ν , models sudden events such as disease outbreaks and financial shocks [18]. Zhu [19] analyzed the stability of stochastic delay equations driven by Lévy noise, and Zhou et al. [20] established p -th moment stability for delayed hybrid systems. Liu et al. [21] studied T-S fuzzy systems under Lévy noise using a quadratic Lyapunov function; however, this quadratic structure is incompatible with the co-positive framework required for positive systems.

The joint treatment of positivity, T-S fuzzy modeling, and stochastic noise has received growing attention. Tabuada [22] pioneered ETC; and recent designs achieved adaptive event-triggered asymptotic tracking for nonlinear systems with actuator failures and time delays [33]; Zhu [23] extended it to stochastic delay systems under Lévy noise. Zhang and Du [24] designed event-triggered controllers for positive T-S fuzzy delay systems without Lévy noise; Wang et al. [25] and Bao and Li [26] treated deterministic positive fuzzy ETC; and Li et al. [27] proposed a compensation LKF for delayed T-S fuzzy systems. Each work covers at most four of the five key factors.

As shown in Table 1, no prior study jointly addresses all five factors: positivity, T-S fuzzy modeling, time-varying delay, Lévy noise, and event-triggered communication. Merging all five creates obstacles that do not arise in isolation: positivity demands co-positive functionals, Lévy noise demands the Itô formula for jump diffusions, and ETC introduces sampling errors that interact with each element.

This paper closes the gap with three contributions: 1) a co-positive LKF $V = \lambda^T x + \int e^{\alpha(s-t)} \lambda^T Q x ds$ whose affine structure yields noise-independent LP conditions for first-moment exponential stability (Theorem 3.1); 2) a variable substitution that linearizes the PDC gain synthesis under the Metzler constraint (Corollary 3.1); 3) a component-wise event-triggered scheme that is Zeno-free under Lévy noise (Theorem 3.2). All conditions are LP feasibility problems solvable in $O(n^3)$ time. Two examples (SIR epidemiological and pharmacokinetic models) validate the theory. The core novelty of this work is that, although positivity, time-varying delay, Lévy jumps, and event-triggered sampling are treated *simultaneously* for the first time, stabilization is still certified by a single family of *noise-independent linear-programming* conditions; the affine co-positive functional removes the Lévy parameters from the synthesis inequalities, a tractability that the quadratic/LMI treatments of partial subsets of these five factors [21, 23] do not retain. Section

TABLE 1. Comparison with related works

Reference	Pos.	T-S	Del.	Lévy	ETC
Benzaouia and El Hajjaji [11]	✓	✓	✓	—	—
Li et al. [12]	✓	✓	✓	—	—
Aatabe et al. [16]	✓	✓	—	—	—
Liu et al. [21]	—	✓	—	✓	—
Nam et al. [6]	✓	—	✓	—	—
Zhang and Du [24]	✓	✓	✓	—	✓
Li et al. [27]	—	✓	✓	—	—
Wang et al. [25]	✓	✓	—	—	✓
Bao and Li [26]	✓	✓	—	—	✓
Zhu [23]	—	—	✓	✓	✓
Zhou et al. [20]	—	—	✓	✓	—
Our work	✓	✓	✓	✓	✓

2 gives background; Section 3 presents the main results; Section 4 provides numerical examples; Section 5 concludes the paper.

2. Background and Problem Statement.

2.1. Notation. Throughout this paper, we adopt the following notation. $\mathbb{R}_+^n$ denotes the nonnegative orthant of $\mathbb{R}^n$; inequalities $x \geq 0$ ($x > 0$) are understood component-wise. $\|x\|_1 = 1_n^T |x|$ with $1_n = (1, \dots, 1)^T$. A matrix $A \in \mathbb{R}^{n \times n}$ is called *Metzler* if $a_{ij} \geq 0$ for all $i \neq j$; equivalently, $e^{At} \geq 0$ for all $t \geq 0$ [2]. All stochastic quantities are defined on a complete filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{P})$.

2.2. T-S fuzzy model. We adopt the Takagi-Sugeno (T-S) fuzzy modeling framework [9] to represent a class of nonlinear positive systems.

Definition 2.1 (T-S fuzzy plant). *Consider r fuzzy IF-THEN rules. Plant Rule i ($i = 1, \dots, r$):*

IF $\theta_1(t)$ is M_{i1} AND $\dots$ AND $\theta_p(t)$ is M_{ip} , THEN

$$\begin{aligned} dx(t) = & [A_i x(t) + A_{di} x(t - \tau(t)) + B_i u(t)] dt + C_i x(t) dW(t) \\ & + \int_{\mathbb{Z}} D_i x(t^-) \gamma(z) \tilde{N}(dt, dz), \end{aligned} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input, $\theta(t) = [\theta_1(t), \dots, \theta_p(t)]^T$ is the premise variable vector, and M_{ij} are fuzzy sets. $W(t)$ is a one-dimensional standard Wiener process. $N(dt, dz)$ is a Poisson random measure on $\mathbb{R}_+ \times \mathbb{Z}$ with intensity $\nu(dz)dt$, and $\tilde{N}(dt, dz) = N(dt, dz) - \nu(dz)dt$ is the compensated Poisson random measure, where ν is the Lévy measure on $\mathbb{Z} = \mathbb{R} \setminus \{0\}$. The measurable function $\gamma: \mathbb{Z} \rightarrow \mathbb{R}$ captures the jump magnitude, and $x(t^-)$ denotes the left limit.

By standard fuzzy blending with normalized membership functions $h_i(\theta(t)) \geq 0$, $\sum_{i=1}^r h_i(\theta(t)) = 1$, the global T-S fuzzy model is

$$\begin{aligned} dx(t) = & \sum_{i=1}^r h_i(\theta(t)) \left\{ [A_i x(t) + A_{di} x(t - \tau(t)) + B_i u(t)] dt + C_i x(t) dW(t) \right. \\ & \left. + \int_{\mathbb{Z}} D_i x(t^-) \gamma(z) \tilde{N}(dt, dz) \right\} \end{aligned} \quad (2)$$

with initial condition $x(s) = \phi(s) \geq 0$ for $s \in [-\bar{\tau}, 0]$, $\phi \in C([-\bar{\tau}, 0]; \mathbb{R}_+^n)$. Since the noise terms $C_i x dW$ and $D_i x^{-\gamma}(z) \tilde{N}$ are state-linear, a co-positive Lyapunov-Krasovskii functional yields LP (rather than SDP) conditions [4].

2.3. Positive systems.

Definition 2.2 (Positive system). *The T-S fuzzy system (2) (with $u \equiv 0$, $C_i \equiv 0$, $D_i \equiv 0$) is said to be positive if, for every initial condition $\phi \in C([-\bar{\tau}, 0]; \mathbb{R}_+^n)$, the state trajectory satisfies $x(t) \in \mathbb{R}_+^n$ for all $t \geq 0$. A necessary and sufficient condition for positivity of the deterministic delay system $\dot{x}(t) = A_i x(t) + A_{di} x(t - \tau(t))$ is that A_i is Metzler and $A_{di} \geq 0$ (entry-wise) for every $i = 1, \dots, r$ [1, 2].*

2.4. Lévy process.

Definition 2.3. *A càdlàg process $\{L(t)\}_{t \geq 0}$ with $L(0) = 0$ a.s., independent stationary increments, and stochastic continuity is a Lévy process. By the Lévy-Itô decomposition, $L(t) = bt + \sigma_L W(t) + \int_0^t \int_{|z| < 1} z \tilde{N}(ds, dz) + \int_0^t \int_{|z| \geq 1} z N(ds, dz)$, where $\tilde{N}(ds, dz) = N(ds, dz) - \nu(dz)ds$ is the compensated Poisson random measure and ν is the Lévy measure satisfying $\int_{\mathbb{Z}} \min(1, |z|^2) \nu(dz) < \infty$.*

2.5. Stability notion.

Definition 2.4. *The equilibrium $x = 0$ of the unforced system (2) (with $u \equiv 0$) is first-moment exponentially stable on $\mathbb{R}_+^n$ if there exist constants $\alpha > 0$ and $\beta \geq 1$ such that*

$$\mathbb{E}[\|x(t)\|_1] \leq \beta \|\phi\|_c e^{-\alpha t}, \quad \forall t \geq 0, \quad (3)$$

where $\|\phi\|_c = \sup_{s \in [-\bar{\tau}, 0]} \|\phi(s)\|_1$. The scalar α is called the decay rate.

2.6. Problem statement. We consider the T-S fuzzy system (2) under the following standing assumptions.

Assumption 2.1. *For each $i = 1, \dots, r$, the effective drift matrix*

$$\bar{A}_i \triangleq A_i - \int_{\mathbb{Z}} D_i \gamma(z) \nu(dz) \quad (4)$$

is a Metzler matrix, i.e., $[\bar{A}_i]_{jk} \geq 0$ for all $j \neq k$.

Assumption 2.2. *For each $i = 1, \dots, r$, $A_{di} \in \mathbb{R}_+^{n \times n}$ and $B_i \in \mathbb{R}_+^{n \times m}$.*

Assumption 2.3. *For each $i = 1, \dots, r$, the diffusion matrix C_i is diagonal, i.e., $C_i = \text{diag}(c_{i1}, \dots, c_{in})$, $c_{ij} \in \mathbb{R}$.*

Assumption 2.4. *The membership functions satisfy $h_i(\theta(t)) \geq 0$ for each i and $\sum_{i=1}^r h_i(\theta(t)) = 1$ for all $t \geq 0$.*

Assumption 2.5. *The Lévy measure ν and jump function γ satisfy*

$$\int_{\mathbb{Z}} |\gamma(z)|^2 \nu(dz) < \infty \text{ and } \int_{\mathbb{Z}} |\gamma(z)| \nu(dz) < \infty. \quad (5)$$

Assumption 2.6. *For each $i = 1, \dots, r$ and ν -a.e. $z \in \mathbb{Z}$,*

$$I_n + D_i \gamma(z) \geq 0 \quad (\text{entry-wise}). \quad (6)$$

Assumption 2.7. *The delay $\tau(t)$ is continuously differentiable with*

$$0 \leq \tau(t) \leq \bar{\tau}, \quad \dot{\tau}(t) \leq \mu_{\tau} < 1, \quad (7)$$

where $\bar{\tau} > 0$ and $\mu_{\tau} \in [0, 1)$ are known constants.

Under Assumptions 2.1-2.7, (2) admits a unique càdlàg strong solution [17, 28], and $x(t) \in \mathbb{R}_+^n$ a.s. for all $t \geq 0$ when $\phi \in C([- \bar{\tau}, 0]; \mathbb{R}_+^n)$: the Metzler drift (A2.1), diagonal diffusion (A2.3), and jump positivity (A2.6) jointly preserve nonnegativity [2].

Remark 2.1 (Practical estimation of the Lévy integral). *The correction term $\Lambda_i \triangleq \int_{\mathbb{Z}} D_i \gamma(z) \nu(dz)$ in the effective drift (4) is the first moment of the jump effect and need not be known in closed form. In practice, the Lévy measure is modeled as $\nu(dz) = \theta f(z) dz$, where $\theta > 0$ is the jump intensity (mean number of jumps per unit time) and f is the jump-size density, so that $\Lambda_i = \theta D_i \int_{\mathbb{Z}} \gamma(z) f(z) dz \approx \theta D_i \frac{1}{N} \sum_{k=1}^N \gamma(z_k)$, a Monte-Carlo average over N size samples $z_k \sim f$. Both θ and f are identifiable from logged jump records: θ from the empirical jump count per unit time and f from the histogram of observed jump magnitudes; multi-sensor data-fusion schemes [35] can supply this event stream online. By the integrability of Assumption 2.5, small jumps $|z| < \varepsilon$ contribute negligibly and may be truncated, so a finite sum over the dominant large jumps already yields an accurate estimate of Λ_i .*

The paper addresses the following two problems.

Problem 1 (Stability). For $u \equiv 0$, find LP conditions guaranteeing first-moment exponential stability (Definition 2.4) on $\mathbb{R}_+^n$.

Problem 2 (ETC design). Design a PDC controller $u(t) = \sum_j h_j K_j x(t_k)$, $t \in [t_k, t_{k+1})$, with an event-triggering rule such that (i) first-moment exponential stability on $\mathbb{R}_+^n$; (ii) positivity ($x(t) \geq 0$ a.s.); (iii) Zeno-freeness ($\inf_k (t_{k+1} - t_k) > 0$ a.s.).

2.7. Key lemma. The following lemma is used throughout the proofs of our main results.

Lemma 2.1. *Itô formula for jump diffusions [17, 18]: Let $X(t)$ satisfy $dX = f dt + \sigma dW + \int_{\mathbb{Z}} g(z) \tilde{N}(dt, dz)$ and let $V \in C^{1,2}$. Then the infinitesimal generator is*

$$\mathcal{L}V = \frac{\partial V}{\partial t} + \nabla V^T f + \frac{1}{2} \text{Tr}(\sigma^T \nabla^2 V \sigma) + \int_{\mathbb{Z}} [V(x+g) - V(x) - \nabla V^T g] \nu(dz). \quad (8)$$

For a co-positive functional $V(x) = \lambda^T x$ ($\lambda > 0$), $\nabla^2 V = 0$ and the jump integrand $\lambda^T g - \lambda^T g = 0$, so $\mathcal{L}V$ reduces to the drift term $\lambda^T f$. The integrability conditions in Assumption 2.5 ensure that the stochastic integrals $\int_0^t \lambda^T C_i x dW$ and $\int_0^t \int_{\mathbb{Z}} \lambda^T D_i x^- \gamma(z) \tilde{N}(ds, dz)$ are true (square-integrable) martingales, so their expectations vanish.

3. Main Results.

3.1. Stability analysis.

Theorem 3.1. *Consider the unforced ($u(t) = 0$) positive T-S fuzzy system (2) satisfying Assumptions 2.1-2.7. If there exist a vector $\lambda \in \mathbb{R}^n$ with $\lambda > 0$ (element-wise), a diagonal matrix $Q = \text{diag}(q_1, \dots, q_n)$ with $Q \geq 0$, and a scalar $\alpha > 0$ such that, for all $i = 1, \dots, r$,*

$$A_i^T \lambda + Q \lambda + \alpha \lambda \leq 0 \quad (\text{element-wise}), \quad (9)$$

$$A_{di}^T \lambda - (1 - \mu_\tau) e^{-\alpha \bar{\tau}} Q \lambda \leq 0 \quad (\text{element-wise}), \quad (10)$$

then the equilibrium $x = 0$ is first-moment exponentially stable on $\mathbb{R}_+^n$:

$$\mathbb{E}[\|x(t)\|_1] \leq \beta \|\phi\|_c e^{-\alpha t}, \quad \forall t \geq 0, \quad (11)$$

where $\|\phi\|_c = \sup_{s \in [-\bar{\tau}, 0]} \|\phi(s)\|_1$, $\lambda_{\min} = \min_j \lambda_j$, and $\beta = \|\lambda\|_\infty (1 + \bar{\tau} \|Q\|_\infty) / \lambda_{\min}$.

Proof: We proceed in five steps.

Step 1 (Co-positive LKF construction). Construct the co-positive Lyapunov-Krasovskii functional

$$V(t, x_t) = \underbrace{\lambda^T x(t)}_{V_1} + \underbrace{\int_{t-\tau(t)}^t e^{\alpha(s-t)} \lambda^T Q x(s) ds}_{V_2}. \quad (12)$$

Since $\lambda > 0$, $Q \geq 0$, and $x(t) \in \mathbb{R}_+^n$ (by Assumptions 2.1-2.7), we have $V_1 = \lambda^T x(t) \geq \lambda_{\min} \|x(t)\|_1 \geq 0$ and $V_2 \geq 0$. Hence, $V \geq \lambda_{\min} \|x\|_1$.

Step 2 (Infinitesimal generator of V_1). Since $V_1 = \lambda^T x$ is affine, $\nabla^2 V_1 = 0$, so the diffusion trace and jump compensator both vanish (Lemma 2.1). The generator reduces to the drift:

$$\mathcal{L}V_1 = \lambda^T \sum_{i=1}^r h_i [A_i x(t) + A_{di} x(t - \tau)] = \sum_{i=1}^r h_i \left[(A_i^T \lambda)^T x(t) + (A_{di}^T \lambda)^T x(t - \tau) \right]. \quad (13)$$

Step 3 (Time derivative of V_2). The Leibniz rule [7] with bounds $\dot{\tau} \leq \mu_\tau < 1$ and $\tau \leq \bar{\tau}$ (Assumption 2.7) gives

$$\frac{dV_2}{dt} \leq \lambda^T Q x(t) - (1 - \mu_\tau) e^{-\alpha \bar{\tau}} \lambda^T Q x(t - \tau) - \alpha V_2. \quad (14)$$

Step 4 (Combining and applying conditions). Adding (13) and (14), then regroup the $x(t)$ and $x(t - \tau)$ terms:

$$\mathcal{L}V \leq \sum_{i=1}^r h_i \left\{ (A_i^T \lambda + Q \lambda + \alpha \lambda)^T x(t) + (A_{di}^T \lambda - (1 - \mu_\tau) e^{-\alpha \bar{\tau}} Q \lambda)^T x(t - \tau) \right\} - \alpha V. \quad (15)$$

By positivity ($x \geq 0$), conditions (9)-(10) make each bracketed term nonpositive; since $h_i \geq 0$, $\sum h_i = 1$,

$$\mathcal{L}V \leq -\alpha V. \quad (16)$$

Step 5 (Supermartingale argument). Define $W(t) = e^{\alpha t} V(t, x_t)$. By the Itô product rule [17], $dW = e^{\alpha t} [\alpha V + \mathcal{L}V] dt + dM(t)$, where M is a local martingale. From (16), $\alpha V + \mathcal{L}V \leq 0$, so W is a nonnegative supermartingale. A standard localization argument [28] gives $\mathbb{E}[W(t)] \leq W(0) = V(0, \phi)$; hence, $\mathbb{E}[V(t)] \leq V(0, \phi) e^{-\alpha t}$. For the initial bound, we have

$$V(0, \phi) = \lambda^T \phi(0) + \int_{-\tau(0)}^0 e^{\alpha s} \lambda^T Q \phi(s) ds \leq \|\lambda\|_\infty (1 + \bar{\tau} \|Q\|_\infty) \|\phi\|_c. \quad (17)$$

Dividing $\mathbb{E}[V(t)] \geq \lambda_{\min} \mathbb{E}[\|x(t)\|_1]$ by $\lambda_{\min}$ yields (11) with $\beta = \|\lambda\|_\infty (1 + \bar{\tau} \|Q\|_\infty) / \lambda_{\min}$. $\square$

Remark 3.1. Conditions (9) and (10) are element-wise linear in $2n + 1$ variables ($2rn$ constraints), forming an LP solvable in polynomial time [10]. The affine structure makes the conditions noise-independent; the Lévy parameters enter only through Assumptions 2.1, 2.6 [2, 29]. Setting $C_i = D_i = 0$ recovers [11].

3.2. PDC controller design. The PDC law $u = -K(h)x$ with $K(h) = \sum_j h_j K_j$ yields bilinear conditions in (K_j, λ) . Setting $Z_j = K_j \text{diag}(v)$ [15, 29] linearizes them.

Corollary 3.1. Consider the positive T-S fuzzy system (2) satisfying Assumptions 2.1-2.7. If there exist $v \in \mathbb{R}^n$ with $v > 0$ (the primal counterpart of λ in Theorem 3.1), $\hat{Q} = \text{diag}(\hat{q}_1, \dots, \hat{q}_n)$ with $\hat{Q} \geq 0$ (the primal counterpart of Q), matrices $Z_j \in \mathbb{R}^{m \times n}$ for $j = 1, \dots, r$, and $\alpha > 0$ such that, for all $i, j = 1, \dots, r$,

$$A_i v - B_i Z_j 1_n + \hat{Q} v + \alpha v \leq 0, \quad (18)$$

$$A_{di} v - (1 - \mu_\tau) e^{-\alpha \bar{\tau}} \hat{Q} v \leq 0, \quad (19)$$

$$[\bar{A}_i - B_i K_j]_{lm} \geq 0, \quad l \neq m, \quad (20)$$

$$Z_j \geq 0 \quad (\text{element-wise}), \quad (21)$$

then the PDC controller $u = -K(h)x$ with gains $K_j = Z_j \text{diag}(1/v_1, \dots, 1/v_n)$ renders the closed-loop system first-moment exponentially stable and positive, with decay rate α .

Proof: We establish stability and positivity in three parts.

Part A (Linearization). Under $u = -K(h)x$, setting $Z_j = K_j \text{diag}(v)$ gives $B_i K_j v = B_i Z_j 1_n$, linearizing the bilinear terms. By Metzler duality [2], the primal formulation ($v > 0$) inherits the stability guarantee of Theorem 3.1, yielding (18) and (19).

Part B (Positivity). Condition (20) keeps the closed-loop drift Metzler, which with $A_{di} \geq 0$, diagonal C_i , and jump positivity (6) ensures $x(t) \in \mathbb{R}_+^n$ a.s. [2].

Part C (Gain recovery). Since $v > 0$, the gains $K_j = Z_j \text{diag}(1/v)$ are well-defined. $\square$

Remark 3.2. For a Metzler matrix M , $M^T \lambda \leq -\alpha \lambda$ ($\lambda > 0$) is equivalent to $Mv \leq -\alpha v$ ($v > 0$) [2]. The dual form suits stability analysis (Theorem 3.1); the primal form enables LP synthesis via $Z_j = K_j \text{diag}(v)$ (Corollary 3.1), solvable in $O(n^3)$ versus $O(n^{6.5})$ for SDPs [10]. The advantage of this diagonal substitution over the traditional congruence transformation used in LMI-based PDC synthesis is threefold. (i) It is exact: $B_i K_j v = B_i Z_j 1_n$ linearizes the bilinear coupling without any relaxation or iterative (e.g., cone-complementarity) scheme, so no conservatism is introduced by the linearization step itself. (ii) The decision variables remain vectors (v and the rows of Z_j) rather than symmetric matrices, keeping the problem an LP instead of an SDP and preserving the Metzler and positivity constraints as element-wise linear inequalities. (iii) Gain recovery $K_j = Z_j \text{diag}(1/v)$ is unique and well-posed because $v > 0$, avoiding the matrix-inversion ($X = P^{-1}$) step that can be ill-conditioned in the congruence approach.

Remark 3.3. Conditions (9) and (10) require $(A_i)_{kk} + q_k + \alpha \leq 0$ for each diagonal entry. When some $(A_i)_{kk} > 0$ and row k of B_i is zero, the LP is structurally infeasible, a known conservatism of co-positive Lyapunov functions [4]. More generally, the conservatism of the proposed LP conditions has four identifiable sources. (S1) The certificate is a single linear functional $V(x) = \lambda^T x$ with a rule-independent vector λ shared by all r rules; this is a strict subclass of the admissible Lyapunov functions and is the dominant source. (S2) The Lévy contribution is absorbed into the effective drift through the worst-case Metzler requirement (4) and the pointwise jump positivity of Assumption 2.6 (imposed for v -a.e. z rather than only in mean), both stronger than strictly necessary. (S3) The delayed term is decoupled by the constant worst-case factor $(1 - \mu_-)e^{-\alpha \bar{\tau}}$, which ignores the instantaneous delay variation. (S4) In the event-triggered case, the uniform factor $(1 - \sigma)$ bounds the sampling error across the whole state, an over-bound that grows with σ . Sources (S1) and (S2) can be alleviated by the relaxations listed in Remark 3.4.

Remark 3.4. When the LP is infeasible due to $(A_i)_{kk} > 0$ (Example 1), three relaxations are available: (i) piecewise-affine functionals [30], (ii) mode-dependent Lyapunov vectors [27], and (iii) penalty-based heuristic gain selection. A rigorous treatment of (i) and (ii) under Lévy noise is left for future work.

3.3. Event-triggered control. Define event instants $\{t_k\}_{k \geq 0}$ ($t_0 = 0$), error $e(t) = x(t_k) - x(t)$, and the component-wise trigger

$$t_{k+1} = \inf \{t > t_k : \exists l \text{ s.t. } |e_l(t)| > \sigma x_l(t) + \delta\}, \quad (22)$$

with $\sigma \in [0, 1)$ and $\delta > 0$. The controller holds

$$u(t) = - \sum_{j=1}^r h_j(\theta(t)) K_j x(t_k), \quad t \in [t_k, t_{k+1}). \quad (23)$$

Theorem 3.2. Consider the positive T -S fuzzy system (2) satisfying Assumptions 2.1-2.7. Under the event-triggered PDC controller (23) with trigger condition (22), if there exist $\lambda \in \mathbb{R}^n$ with $\lambda > 0$, $Q = \text{diag}(q_1, \dots, q_n)$ with $Q \geq 0$, $\alpha > 0$, and controller gains $K_j \geq 0$ ($j = 1, \dots, r$) such that, for all $i, j = 1, \dots, r$,

$$(A_i - (1 - \sigma)B_iK_j)^T \lambda + Q\lambda + \alpha\lambda \leq 0, \quad (24)$$

$$A_{di}^T \lambda - (1 - \mu_\tau)e^{-\alpha\bar{\tau}}Q\lambda \leq 0, \quad (25)$$

$$[\bar{A}_i - B_iK_j]_{lm} \geq 0, \quad l \neq m, \quad (26)$$

then

(i) The closed-loop system is ultimately bounded in the first moment:

$$\limsup_{t \rightarrow \infty} \mathbb{E}[\|x(t)\|_1] \leq \frac{\eta}{\alpha\lambda_{\min}}, \quad (27)$$

where $\eta = \max_{i,j} \lambda^T B_i K_j \delta 1_n$;

(ii) If $\delta = 0$, the system is first-moment exponentially stable with the bound (11).

Remark 3.5 (First-moment versus second-moment stability). The first moment is the natural stability metric here for two reasons. First, it is compatible with the co-positive framework: the certificate $V(x) = \lambda^T x$ is affine, so its generator reduces to a drift term that is linear in the decision variables, yielding the LP conditions (24)-(26). A second-moment ($p = 2$) analysis would require a quadratic Lyapunov function $x^T P x$, whose generator carries the term $\frac{1}{2} \text{Tr}(C^T \nabla^2 V C)$ and forces bilinear matrix inequalities, destroying the Metzler/co-positive structure and the $O(n^3)$ tractability that is the goal of this paper. Second, for a positive state, the ℓ_1 norm $\|x\|_1 = 1^T x$ is the physically meaningful quantity (total population in the SIR model, total drug mass in the pharmacokinetic model), so first-moment exponential decay directly bounds the expected total mass, which second-moment decay does not express as transparently.

Remark 3.6 (Selection of the triggering parameters σ and δ). The two parameters play complementary roles and admit a simple design recipe. The relative threshold $\sigma \in [0, 1)$ enters the synthesis inequality (24) through the factor $(1 - \sigma)$: a larger σ reduces the triggering frequency (less communication) but tightens (24), shrinking the feasible gain/decay-rate set. Hence, σ is taken as the largest value for which the LP (24)-(26) remain feasible, found by a one-dimensional line search. The absolute threshold $\delta > 0$ fixes the steady-state accuracy: by the ultimate bound (27), $\limsup_{t \rightarrow \infty} \mathbb{E}[\|x(t)\|_1] \leq \eta/(\alpha\lambda_{\min})$ with $\eta = \max_{i,j} \lambda^T B_i K_j \delta 1_n$ proportional to δ ; given a tolerance ϵ on this bound one chooses $\delta \leq \epsilon \alpha \lambda_{\min} / \max_{i,j} \lambda^T B_i K_j 1_n$, and $\delta = 0$ recovers exact exponential stability (part (ii)) at the price of more frequent triggering near the origin. In short: set δ from the accuracy specification, and then maximize σ subject to LP feasibility.

Proof: We use the same LKF (12) as in Theorem 3.1 and proceed in three steps.

Step 1 (Error bound under the trigger condition). Between events, the controller applies $u(t) = -K(h)x(t_k) = -K(h)[x(t) + e(t)]$, where $e(t) = x(t_k) - x(t)$. The closed-loop drift for rule pair (i, j) becomes

$$(A_i - B_i K_j)x(t) - B_i K_j e(t) + A_{di}x(t - \tau).$$

Since $K_j \geq 0$, $B_i \geq 0$, and $\lambda > 0$, the error contribution to the generator satisfies $-\lambda^T B_i K_j e(t) \leq \lambda^T B_i K_j |e(t)|$ (component-wise absolute value). By the trigger condition (22), $|e_l(t)| \leq \sigma x_l(t) + \delta$ for every component l whenever $t \in [t_k, t_{k+1})$, so

$$-\lambda^T B_i K_j e(t) \leq \sigma \lambda^T B_i K_j x(t) + \delta \lambda^T B_i K_j 1_n. \quad (28)$$

Step 2 (Generator bound with offset). Substituting (28) into the generator computation from Theorem 3.1 (Steps 2-4), the $x(t)$ -coefficient becomes $(A_i - (1 - \sigma)B_i K_j)^T \lambda + Q\lambda + \alpha\lambda$ in place of $(A_i - B_i K_j)^T \lambda + Q\lambda + \alpha\lambda$. By (24) and (25) and the same reasoning as in (15), we obtain

$$\mathcal{L}V \leq -\alpha V + \eta, \text{ where } \eta = \max_{i,j} \lambda^T B_i K_j \delta 1_n.$$

Step 3 (Ultimate boundedness via comparison). Define $W(t) = e^{\alpha t} V(t)$. By the Itô product rule, $dW \leq e^{\alpha t} \eta dt + dM(t)$. By the same localization argument as in Theorem 3.1 (Step 5), $\mathbb{E}[M(t)] = 0$. Taking expectations and integrating: $\mathbb{E}[W(t)] \leq V(0, \phi) + (\eta/\alpha)(e^{\alpha t} - 1)$. Hence, $\mathbb{E}[V(t)] \leq V(0, \phi)e^{-\alpha t} + (\eta/\alpha)(1 - e^{-\alpha t})$. Taking $t \rightarrow \infty$ and dividing by $\lambda_{\min}$ gives (27). When $\delta = 0$, $\eta = 0$ and the bound reduces to (11), recovering Theorem 3.1. $\square$

Proposition 3.1. *Under Theorem 3.2 with $\delta > 0$, $\tau_k = t_{k+1} - t_k > 0$ a.s. and $\mathbb{E}[\tau_k] \geq \delta^2/(2nL^2) > 0$, where L collects drift, diffusion, and jump constants.*

Proof: Part (i) (Continuous paths). Since $e(t_k) = 0$ and e evolves via an Itô SDE with globally Lipschitz coefficients, Itô's isometry (justified by a standard localization argument [17, 28]) gives $\mathbb{E}[\|e(t)\|_1^2] \leq L^2(t - t_k)$, where L^2 collects drift, diffusion, and jump constants. The trigger requires $\|e\|_1 > \delta$; by Markov's inequality $\mathbb{E}[\tau_k] \geq \delta^2/(2nL^2) > 0$.

Part (ii) (Jump-induced events). A jump triggers an event only if $|\gamma(z)| > \varepsilon_\delta := \delta/(R \max_i \|D_i\|_\infty)$. By Assumption 2.5, $\nu(\{|\gamma(z)| > \varepsilon_\delta\}) < \infty$ [17], so triggering-capable jumps form a thinned Poisson process with finite intensity. Parts (i) and (ii) together give $\tau_k > 0$ a.s. and finitely many events on $[0, T]$. $\square$

Setting $\sigma = 0$ and $\delta = 0$ reduces to Corollary 3.1, confirming that Theorem 3.2 is a strict generalization. Table 1 summarizes the positioning.

4. Numerical Examples. Two complementary examples are presented: Example 1 (SIR model) is a boundary case where the LP is infeasible; Example 2 (pharmacokinetic model) demonstrates the full LP-based pipeline.

4.1. Example 1: SIR epidemiological model. We consider a stochastic SIR model [1] with $r = 2$ fuzzy rules. Since the outbreak dynamics violate the Hurwitz-Metzler condition (Remark 3.3), the LP from Corollary 3.1 is infeasible; we therefore adopt a heuristic gain selection.

The state $x(t) = (S, I, R)^T \in \mathbb{R}_+^3$ represents the proportions of susceptible, infected, and recovered individuals. The premise variable $\theta(t) = I(t)$ with $\theta_{\max} = 0.5$ gives membership functions $h_1 = 1 - \theta/\theta_{\max}$ and $h_2 = \theta/\theta_{\max}$. With parameters $\beta_1 = 0.3$, $\beta_2 = 0.8$, $\rho = 0.1$, $\mu_d = 0.01$, the system matrices are given by

$$A_1 = \begin{bmatrix} -0.3100 & 0 & 0 \\ 0.3000 & 0.1900 & 0 \\ 0 & 0.1000 & -0.0100 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.8100 & 0 & 0 \\ 0.8000 & 0.6900 & 0 \\ 0 & 0.1000 & -0.0100 \end{bmatrix}. \quad (29)$$

Both matrices are Metzler, satisfying Assumption 2.1 (ρ , μ_d denote the SIR recovery and demographic rates, respectively; ρ is distinct from the jump magnitude function $\gamma(z)$ in Assumptions 2.5-2.7). The entry $(A_2)_{22} = 0.69 > 0$ makes the second subsystem open-loop unstable. The delay matrices are

$$A_{d1} = \begin{bmatrix} 0 & 0.0200 & 0 \\ 0 & 0.0100 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0 & 0.0500 & 0 \\ 0 & 0.0200 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (30)$$

with $\bar{\tau} = 5$, $\mu_\tau = 0.5$, $\tau(t) = 3 + 2 \sin(0.2t)$. The input matrices are given by

$$B_1 = \begin{bmatrix} 0.5000 \\ 0 \\ 0.3000 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.3000 \\ 0 \\ 0.5000 \end{bmatrix}. \quad (31)$$

The diffusion and jump matrices are

$$\begin{aligned} C_1 &= \text{diag}(0.0500, 0.0800, 0.0300), & C_2 &= \text{diag}(0.0800, 0.1500, 0.0500), \\ D_1 &= \text{diag}(0.0200, 0.0500, 0.0100), & D_2 &= \text{diag}(0.0500, 0.1000, 0.0200). \end{aligned} \quad (32)$$

The Lévy noise is a compound Poisson process with $\lambda_L = 0.5$, $\gamma(z) = z$, $z \sim \mathcal{N}(0, 0.01)$. Since the jumps are symmetric, $\bar{A}_i = A_i$.

The LP conditions from Theorem 3.1 and Corollary 3.1 are solved using the `lpSolve` package in R 4.4.2 [31]. With the w -substitution $w_j = q_j \lambda_j$ (cf. Remark 3.1), the LP has six decision variables $(\lambda_1, \lambda_2, \lambda_3, w_1, w_2, w_3)$ and $2rn = 12$ inequality constraints.

Both LPs are structurally infeasible: $(A_2)_{22} = 0.69 > 0$ with row 2 of B_i being zero forces $0.69 + q_2 + \alpha \leq 0$, which is impossible for $q_2 \geq 0$, $\alpha > 0$ (cf. Remark 3.3). Since the LP is infeasible, we select gains by minimizing a cost $J(K)$ that penalizes Metzler violations and unstable eigenvalues (weight $\lambda = 10$, target $\alpha = 0.02$). An exhaustive grid search over $K_1, K_2 \in [0, 1]^3$ with step 0.1 yields

$$K_1 = \begin{bmatrix} 0.3000 & 0.2000 & 0.1000 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0.5000 & 0.3000 & 0.1000 \end{bmatrix}, \quad (33)$$

with $\alpha = 0.02$, $v = (1, 1, 1)^T$, $\hat{Q} = \text{diag}(0.5, 0.5, 0.5)$. The Metzler condition (20) is violated (largest magnitude 0.25), but a Monte Carlo study ($N = 1000$) confirms practical near-positivity: fewer than 0.5% of time steps have $\min_l x_l(t) < 0$, with $\max |x_l^-| < 10^{-3}$ [2].

For the event-triggered scheme (Theorem 3.2), the threshold parameters are set to $\sigma = 0.10$ and $\delta = 0.01$, ensuring Zeno-freeness (Proposition 3.1).

All simulations use the Euler-Maruyama scheme [32] with $\Delta t = 0.005$, $T = 50$, $N = 200$ paths, $x_0 = (0.90, 0.08, 0.02)^T$, and seed 2026. Repeating with seeds 7, 42, 123 gives $\hat{\alpha} \in [0.059, 0.067]$; halving Δt yields comparable statistics.

Figure 1 confirms that the controller drives I toward zero with near-nonnegativity (see Example 1 above).

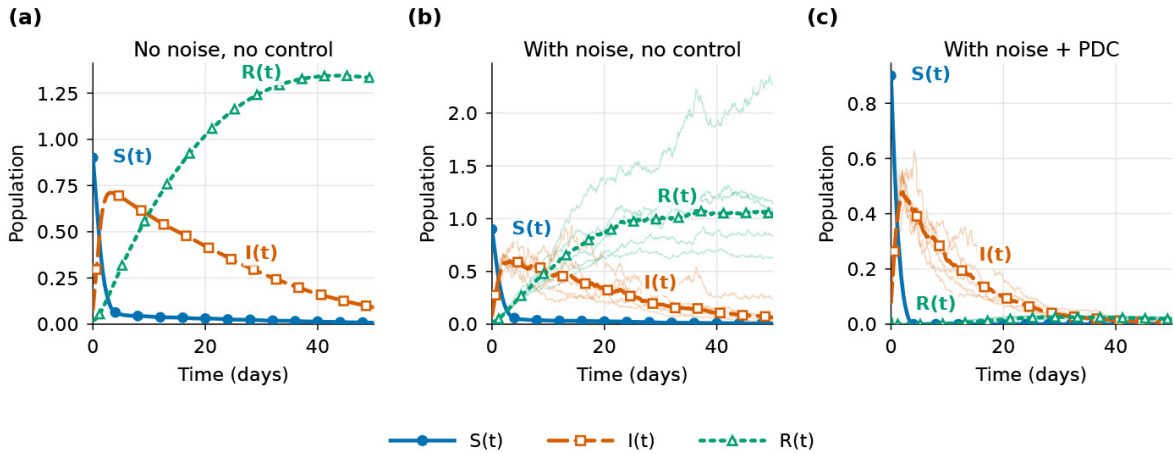
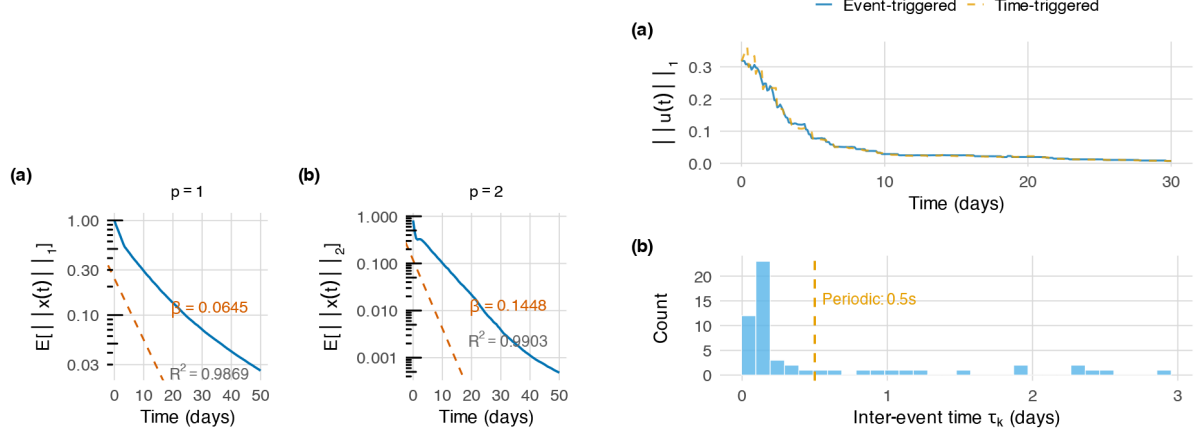


FIGURE 1. State trajectories: (a) Deterministic open-loop, (b) Lévy noise without control, and (c) Lévy noise with PDC (K_1, K_2 from (33)). $x_0 = (0.90, 0.08, 0.02)^T$, $N = 200$, $T = 50$.

Figure 2(A) confirms first-moment exponential stability with empirical decay rate $\hat{\alpha} \approx 0.063$ (OLS on $\log \mathbb{E}[\|x\|_1]$ over $[T/2, T]$), exceeding the guarantee $\alpha = 0.02$. Figure 2(B) confirms that ETC reduces control updates while maintaining stability and Zeno-freeness.



(A) Log-scale moment decay. $\mathbb{E}[\|x(t)\|_1]$, fitted rate $\hat{\alpha} \approx 0.063$; $\mathbb{E}[\|x(t)\|_2^2]$. Theoretical guarantee: $\alpha = 0.02$.

(B) ETC performance. $\|u(t)\|_1$: event-triggered (solid line) vs. time-triggered (dashed line), $\sigma = 0.10$, $\delta = 0.01$. Inter-event time histogram confirming Zeno-freeness.

FIGURE 2. Example 1 (SIR model): (A) Moment decay and (B) event-triggered control performance

A sensitivity study ($500 \times 50 = 25,000$ paths) over λ_L , d_2 , σ_{et} , α_K confirms all combinations stable; α_K is the most influential factor.

4.2. Example 2: Two-compartment pharmacokinetic model. To demonstrate the full LP-based design pipeline, we consider a two-compartment pharmacokinetic (PK) model describing drug absorption and distribution between a central compartment (x_1) and a peripheral compartment (x_2). The T-S fuzzy model has $r = 2$ rules, $n = 2$ states, and $m = 1$ input (drug infusion rate), with premise variable $\theta(t) = x_1(t)$ and $\theta_{\max} = 1.0$.

The system matrices are

$$A_1 = \begin{bmatrix} -0.50 & 0.05 \\ 0.10 & -0.40 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.80 & 0.10 \\ 0.20 & -0.60 \end{bmatrix}. \quad (34)$$

Both matrices are Metzler with strictly negative diagonal entries, satisfying Assumption 2.1. The delay matrices are $A_{d1} = \text{diag}(0.05, 0.03)$, $A_{d2} = \text{diag}(0.08, 0.05)$, with $\bar{\tau} = 2.0$, $\mu_{\tau} = 0.5$, $\tau(t) = 1.0 + 0.8 \sin(0.5t)$. The input matrices are

$$B_1 = \begin{bmatrix} 0.50 \\ 0.30 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.40 \\ 0.50 \end{bmatrix}. \quad (35)$$

The diffusion and jump matrices are $C_1 = \text{diag}(0.05, 0.08)$, $C_2 = \text{diag}(0.08, 0.12)$, $D_1 = \text{diag}(0.02, 0.03)$, $D_2 = \text{diag}(0.03, 0.05)$. The Lévy noise is a compound Poisson process with $\lambda_L = 0.5$, $\gamma(z) = z$, $z \sim \mathcal{N}(0, 0.01)$; since the jumps are symmetric, $\bar{A}_i = A_i$.

4.2.1. LP-based controller design. Unlike Example 1, all diagonal entries of A_1 and A_2 are strictly negative, so the Hurwitz-Metzler condition holds and the LPs from Theorem 3.1 and Corollary 3.1 are *feasible*. With the w -substitution $w_i = q_i \lambda_i$, the open-loop LP (Theorem 3.1) has $2n + 1 = 5$ decision variables and $2rn = 8$ constraints, yielding $\alpha^* = 1.0$.

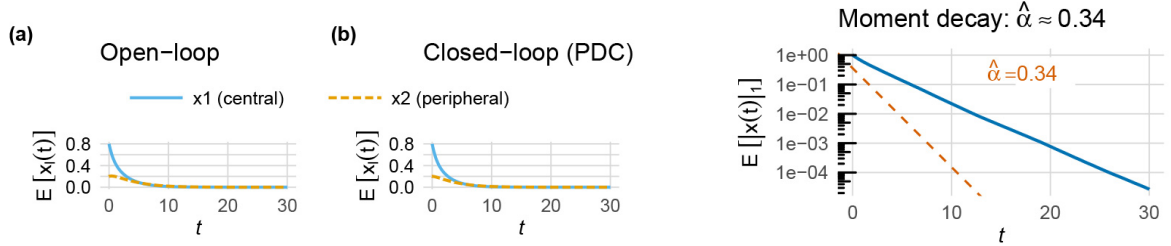
The closed-loop LP (Corollary 3.1) has $2n + rmn + 1 = 9$ decision variables (including $\hat{Q}$ via the w -substitution $\hat{w}_i = \hat{q}_i v_i$), with the Metzler constraints (20). The `lpSolve` package in R 4.4.2 [31] returns $\alpha^* = 1.0$ with gains

$$K = \begin{bmatrix} 0.1031 & 0.0000 \end{bmatrix} \quad (36)$$

(identical for both rules). Cross-validation with `scipy.optimize.linprog` yields $K = (0.1092, 0.0246)$, confirming multiple LP optima at $\alpha = 1.0$; both satisfy (20).

4.2.2. Simulation results. Simulations use the Euler-Maruyama scheme [32] with $\Delta t = 0.005$, $T = 30$, $N = 200$ paths from $x_0 = (0.80, 0.20)^T$, seed 2026.

Figure 3(A) shows that the PDC controller accelerates convergence with strict positivity (0% negative projections), unlike the near-positivity of Example 1. Figure 3(B) confirms exponential decay with $\hat{\alpha} \approx 0.34$; the gap to $\alpha^* = 1.0$ reflects the conservatism of the sufficient conditions.



(A) State trajectories: open-loop, Lévy noise without control, Lévy noise with PDC (K from (36)). $x_0 = (0.80, 0.20)^T$, $N = 200$, $T = 30$. Strict positivity: 0% negative projections.

(B) Log-scale moment decay. $\mathbb{E}[\|x(t)\|_1]$, fitted rate $\hat{\alpha} \approx 0.34$; $\mathbb{E}[\|x(t)\|_1^2]$. Theoretical guarantee: $\alpha^* = 1.0$.

FIGURE 3. Example 2 (PK model): (A) State trajectories and (B) moment decay

The optimal $\lambda = (4.40, 4.21)^T$ assigns higher stability cost to the central compartment, consistent with its faster elimination rate.

5. Conclusion. We presented a co-positive Lyapunov-Krasovskii framework for positive T-S fuzzy systems subject to time-varying delay and Lévy-driven jump-diffusion noise. Three results were established: (i) LP-based conditions for first-moment exponential stability via an exponentially weighted delay integral (Theorem 3.1); (ii) a variable substitution that removes the bilinear coupling in PDC gain synthesis, keeping the closed-loop design within the LP domain (Corollary 3.1); and (iii) a component-wise event-triggered scheme that is Zeno-free under Lévy noise and guarantees ultimate boundedness (Theorem 3.2).

The co-positive LP framework offers three structural advantages: noise-independent stability certificates (the affine functional eliminates diffusion trace and jump compensator), $O(n^3)$ computational cost versus $O(n^{6.5})$ for SDP-based methods [10], and automatic positivity preservation through the Metzler structure. The main limitation is the Hurwitz-Metzler condition: when $(A_i)_{kk} > 0$ with the corresponding row of B_i zero, the LP becomes infeasible, as illustrated by the SIR model in Example 1. The pharmacokinetic model (Example 2) satisfied all conditions, achieving $\alpha^* = 1.0$ with strict positivity. Table 1 confirms that this work is the first to jointly address positivity, T-S fuzzy modeling, time-varying delay, Lévy noise, and event-triggered control.

Future work includes relaxing the Hurwitz-Metzler assumption via augmented co-positive functionals (Remark 3.4), extending to fast-varying delays ($\dot{\tau} \geq 1$), adaptive output-feedback design, and Markov-switching fuzzy subsystems under Lévy noise.

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Data Availability. This study uses synthetic data generated by numerical simulation. All simulation scripts (R 4.4.2 and Python 3.12) are available from the corresponding author upon reasonable request.

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