

COOPERATIVE CONTROL OF MIXED VEHICLE PLATOON UNDER EXTERNAL DISTURBANCE

JIANXUN DONG¹, WENJU DU^{1,*}, BINBIN PENG² AND JIANGANG ZHANG³

¹School of Traffic and Transportation

³School of Mathematics and Physics

Lanzhou Jiaotong University

No. 88, West Anning Road, Anning District, Lanzhou 730070, P. R. China

2663161646@qq.com; zhangjg7715776@126.com

*Corresponding author: duwenju@mail.lzjtu.cn

²College of Management and Economics

Tianjin University

No. 92, Weijin Road, Nankai District, Tianjin 300072, P. R. China

pengbinbin@tju.edu.cn

Received November 2025; revised March 2026

ABSTRACT. *In order to reveal the influence of various uncertainties and disturbances in real driving environment on the cooperative control of mixed vehicle platoon, this paper studies the cooperative control of mixed vehicle platoon under external disturbance. Firstly, for the mixed vehicle platoon composed of multiple human-driven vehicle (HDV) and connected autonomous vehicle (CAV) randomly, we constructed a mixed vehicle platoon considering uncertain external disturbance. Secondly, a mixed vehicle platoon adaptive controller based on heterogeneous multi-agent pinning consensus and an adaptive controller considering driver reaction delay and vehicle communication delay are proposed, and the stability of mixed vehicle platoon system under external disturbance is studied. Finally, the numerical simulation experiments are designed to analyze the effects of the range of the disturbance, the location of the disturbance, the topology structure, the number of vehicles before and after the CAV can obtain information and driver reaction delay and vehicle communication delay on the stability of mixed vehicle platoon.*

Keywords: Intelligent transportation, Mixed vehicle platoon, Cooperative control, Stability, External disturbance

1. Introduction. The continuous rise in car ownership has led to numerous issues, such as traffic congestion, accidents, environmental pollution, and high energy consumption. Meanwhile, the emergence of connected vehicle technologies and autonomous vehicles offers a promising solution to these challenges. As a typical application of intelligent networked vehicles, vehicle platoon cooperative control enables all vehicles in the same lane to automatically adjust their motion state according to the state information of the surrounding vehicles, so as to achieve the same speed of all vehicles and achieve the desired queue geometry [1]. The vehicle platoon control is a typical distributed cooperative motion control method, which has obvious advantages in alleviating traffic congestion, improving traffic efficiency, enhancing driving safety and improving fuel economy, and it is a hot spot in the field of modern automobile research [1-4].

With the increasing popularity of intelligent networked vehicles, it is possible to coordinate the driving of mixed vehicle platoon composed of HDV and CAV. At present, many

scholars have carried out a lot of research on the problem of mixed vehicle platoon modeling and mixed vehicle platoon collaborative control, and have achieved a series of fruitful results [5-9]. Li and Wang [10] analyzed the car-following behavior of human-driven vehicles, semi-intelligent vehicles and autonomous vehicles in a mixed longitudinal platoon, and established a longitudinal platoon following model with a random mixture of multiple vehicles. Bian et al. [11] considered the heterogeneous third-order vehicle model, deduced the sufficient and necessary conditions of system stability, and obtained the relationship between the front time of heterogeneous vehicles and the inertial delay and control gain of the vehicle power system. Zheng et al. [12] studied the influence mechanism of CAV on traffic flow and traffic disturbance on closed ring roads, and theoretically analyzed the controllability and observability of mixed vehicle platoon system. Li et al. [13] proposed a distributed nonlinear vehicle platoon vertical controller based on a third-order model for heterogeneous vehicle platoon control under communication delay environment. Yang et al. [14] studied the mixed vehicle platoon collaborative control method of human-driven vehicles and intelligent networked vehicles, and analyzed the influence of rear vehicle information on platoon stability, tracking performance and fuel economy. Bian et al. [15] designed a platoon cooperative control method considering the information of the front and rear vehicles for a mixed vehicle platoon composed of human-driven vehicle and connected autonomous vehicle, and studied the stability of the mixed vehicle platoon. Du et al. [16] focused on mixed vehicle platoon with driver reaction delay and communication delay, introducing a longitudinal adaptive platoon controller based on the pinning consensus theory of heterogeneous multi-agent system, thereby resolving the cooperative control challenge in heterogeneous nonlinear vehicle platoon systems. However, the vast majority of present researches on cooperative control of mixed vehicle platoon are confined to fixed platoon formations or scenarios involving only a CAV, rendering such approaches inadequate for application in real-world driving environments, so it is necessary to study the control problem of mixed vehicle platoon composed of multiple HDV and CAV.

In addition, there are various uncertainties and disturbances in the real driving environment, which makes the realization of vehicle platoon cooperative control a challenging problem. Therefore, in recent years, many scholars have studied the collaborative control of vehicle platoon considering uncertainty and interference. For heterogeneous vehicle platoon with communication delay and parameter uncertainty, Gao et al. [17] proposed a distributed H_∞ control strategy to ensure robust stability, string stability and tracking ability of vehicle platoon. Zheng et al. [18] studied the robustness and distributed H_∞ controller synthesis of vehicle platoon system with undirected topological connections. Aiming at the uncertainty of vehicle dynamics model, Li et al. [19] proposed a decoupling distributed H_∞ control method to balance the robust stability, interference suppression and string stability of vehicle platoon. Chen et al. [20] established a collaborative driving model of homogeneous vehicles near traffic lights under V2X conditions, considering vehicle dynamics and external interference. Based on this model, they designed a distributed H_∞ robust controller, which not only ensured that vehicles could track speed changes according to traffic signal information, but also improved speed tracking performance and reduced fuel consumption. Wang et al. [21] proposed a distributed adaptive H_∞ control scheme to suppress the influence of uncertain vehicle dynamics, external interference and heterogeneous time-varying communication delay on vehicle queues, and at the same time ensure the robust stability and string stability of vehicle platoon. Elahi et al. [22] proposed a distributed H_∞ controller for vehicle platoon with communication data packet loss and communication time delay to ensure string stability of vehicle platoon under disturbance and model uncertainty. Wang et al. [23] proposed a collaborative control algorithm for the mixed vehicle platoon to solve the problem of weak robustness of the mixed vehicle

platoon, and studied the robustness of the algorithm in reducing the uncertainty of the platoon. Li et al. [24] addressed scenarios where vehicle platoon is interrupted by external heterogeneous vehicles performing lane changes, proposing a distributed backward control scheme based on an adaptive integral sliding mode platoon controller while considering communication delay in networked environment. This solution effectively tackles stability issue caused by parameter uncertainties in third-order vehicle node dynamic model. Zhang and Shi [25] adopted a novel adaptive event-triggered mechanism to investigate platoon control under semi-Markov switching topologies considering unknown external disturbance and communication burden during vehicle operation. Xu et al. [26] proposed an event-triggered quantized stabilization control method, achieving vehicle platoon system stability under communication delay, data packet loss and external environmental disturbance. Ma et al. [27] transformed the nonlinear vehicle model into a linear data model and proposed a data-driven distributed control algorithm for vehicle platoon. To sum up, most of the existing mixed vehicle platoon control only considers the string stability, under the condition that the leader vehicle is disturbed. Very few studies have taken account of the impact of external disturbances on the stability of the mixed vehicle platoon system when vehicles at arbitrary positions or multiple vehicles in the platoon are disturbed, making it difficult to apply to actual road traffic environments. Therefore, considering the platoon driving scenario consisting of multiple HDV and CAV and the effect of external disturbance for the cooperative control of mixed vehicle platoon needs more in-depth research.

Based on this, compared to previous studies on cooperative control of the mixed vehicle platoon, this paper constructs a mixed vehicle platoon system model considering the presence of external disturbances, specifically for platoon randomly composed of multiple HDVs and CAVs. Furthermore, through numerical simulation experiments, it analyzes in detail the impact of the disturbance range and disturbance position on the stability of the mixed vehicle platoon system. The results indicate that both the range and the position of the disturbance have a significant impact on the stability of the mixed vehicle platoon system. Specifically, a larger disturbance range leads to a more complex dynamic response of the system and deteriorates its stability. Furthermore, the more forward the position of the disturbed HDV, the worse the system stability becomes. In contrast, when the disturbance acts directly on the CAV, the system exhibits better stability due to the strong anti-disturbance capability of the CAV enabled by the controller designed in this study. Meanwhile, considering that HDV mainly relies on the driver's experience for control and is the uncontrollable part, while CAV can be controlled by the designed controller and belongs to the controllable part in the mixed vehicle platoon. Therefore, the paper adopted a pinning control strategy and proposed an adaptive controller based on heterogeneous multi-agent pinning consistency for the mixed vehicle platoon. And by imposing control on the CAV in the mixed vehicle platoon, the purpose of controlling the entire platoon is achieved.

The paper is organized as follows. In Section 2, the description of mixed vehicle platoon composed of HDV and CAV is given. The car-following models for HDV and CAV are constructed and the adaptive controller is proposed in Section 3. In Section 4, the mixed vehicle platoon model is conducted and the stability of mixed vehicle platoon system under external disturbance is studied. The cooperative control problem for mixed vehicle platoon considering time delays under external disturbance is studied in Section 5. Numerical simulation experiments are given to show the validity of the controllers and the effects of the range of the disturbance, the location of the disturbance, the topology structure, the number of vehicles before and after the CAV can obtain information and driver reaction

delay and vehicle communication delay on the stability of mixed vehicle platoon in Section 6. In Section 7, we conclude the paper.

2. Problem Description. As shown in Figure 1, the research object of this paper is a mixed vehicle platoon composed of $N + 1$ vehicles. Except for the lead vehicle, the platoon contains N_1 HDVs and N_2 CAVs. Due to the installation of advanced communication equipment, the CAV in the platoon can achieve information transmission and sharing beyond the line-of-sight range relying on V2V technology. Meanwhile, as they are equipped with various sensors, the CAV can also obtain the information of the adjacent HDVs in front of and behind them. Unlike CAV, the HDV can only rely on the driver's perception to obtain the status information of the adjacent vehicles ahead, but cannot obtain the information of vehicles beyond the line-of-sight range. Considering that vehicles may be affected by uncertain external disturbance when driving on the road, such as road construction, bad weather and traffic accidents, which seriously affect driving safety, therefore, it is of great practical significance to study the stability of the mixed vehicle platoon system under external disturbance conditions.

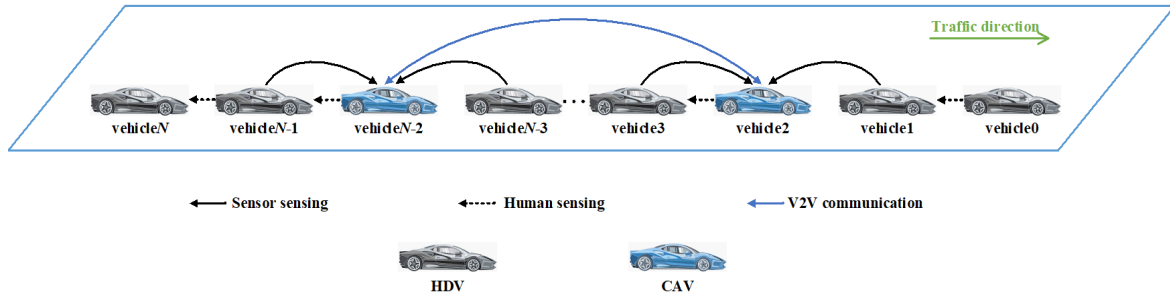


FIGURE 1. Mixed vehicle platoon driving scenario

The communication topology structure of the mixed vehicle platoon is described by algebraic graph theory. The communication topology between the following vehicles in the platoon is abstracted as a connected graph $\mathbf{G} = \{\mathbf{V}, \mathbf{E}, \mathbf{A}\}$, where $\mathbf{V} = \{v_1, v_2, \dots, v_N\}$ represents the set of vehicles in the platoon and $\mathbf{E} \subseteq \mathbf{V} \times \mathbf{V}$ represents the ordered edge set between the vehicles. And matrix $\mathbf{A} = (a_{ij})_{N \times N}$ represents the communication situation of the vehicle platoon. If vehicle i has a communication connection with vehicle j , then $a_{ij} = 1$; otherwise, $a_{ij} = 0$. Meanwhile, assuming that $\mathbf{G}$ has no self-loop, that is $a_{ii} = 0$ ($i = 1, 2, \dots, N$), the Laplacian matrix $\mathbf{L} = (l_{ij})_{N \times N}$ of graph $\mathbf{G}$ can be obtained as

$$l_{ij} = \begin{cases} -a_{ij} & i \neq j \\ \sum_{j=1, j \neq i}^N a_{ij} & i = j \end{cases} \quad (1)$$

3. Model Analysis and Controller Design.

3.1. The model of HDV. The HDV in the mixed vehicle platoon consists of two parts: the lead vehicle and the following vehicle. The lead vehicle only needs to transfer its own status information (such as velocity and position) to at least one following vehicle in the platoon to ensure that this information is transmitted directly or indirectly through the platoon, and its operating status is not affected by other vehicles in the platoon. Therefore, in order to focus on studying the stability of the entire mixed vehicle platoon when uncertain external disturbance acts on the following vehicles, the dynamic characteristics of the lead vehicle are characterized by the following simple form

$$\begin{cases} \dot{x}_0(t) = v_0(t) \\ \dot{v}_0(t) = 0 \end{cases} \quad (2)$$

where $x_0(t)$, $v_0(t)$ are respectively the position and velocity of the lead vehicle at time t .

In order to describe the car-following characteristics for the HDV in the following vehicle, many types of car-following models for HDV have been proposed in previous studies, such as stimulus-response models, psycho-physiological models, safety distance models, and neural network models. In 1995, the optimal-velocity model (OVM) proposed by Bando et al. [28] belongs to a typical stimulus-response type of model. The driver makes an appropriate response by perceiving the distance between his own vehicle and the vehicle in front. That is, when the driver senses that the distance from the vehicle in front is gradually decreasing, he slows down, and accelerates when the distance from the vehicle in front gradually increases. The OVM can accurately and intuitively describe the car-following behavior of HDV. Due to its excellent mathematical and physical properties such as analyzability and ease of simulation, it has been widely applied and studied since its proposal [5,29]. The dynamic model of HDV is as follows:

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = \mu_1[V(h_i(t)) - v_i(t)] + \mu_2[v_{i-1}(t) - v_i(t)] + \varphi_i(t) \end{cases} \quad (3)$$

where $x_i(t)$, $v_i(t)$ are respectively the position and velocity of the vehicle i at time t , $h_i(t) = x_{i-1}(t) - x_i(t)$ represents headway distance of vehicle $i - 1$ to vehicle i at time t , $\varphi_i(t)$ is the uncertain external disturbance subjected to the vehicle i , μ_1 , μ_2 are respectively the driver's sensitivity coefficients to the expected velocity error and the speed error of following the vehicle in front, and $V(h_i(t))$ is the expected velocity function of the vehicle, which is usually represented by the following piecewise function:

$$V(h_i(t)) = \begin{cases} 0, & h_i(t) \leq h_{st} \\ F(h_i(t)), & h_{st} < h_i(t) < h_{go} \\ v_{\max}, & h_i(t) \geq h_{go} \end{cases} \quad (4)$$

where h_{st} , h_{go} respectively represent the preset minimum and maximum headway distances between two adjacent vehicles, $v_{\max}$ is the preset maximum velocity of HDV, and $F(h_i(t))$ is a continuous and monotonically increasing function and exists in multiple forms. In this paper, the following typical nonlinear form is adopted to describe the car-following behavior of HDV:

$$F(h_i(t)) = \frac{v_{\max}}{2} \left[1 - \cos \left(\frac{\pi(h_i(t) - h_{st})}{h_{go} - h_{st}} \right) \right] \quad (5)$$

From the classic OVM (4) and (5), it can be seen that the vehicle stops running when the actual headway distance between two adjacent vehicles is less than or equal to h_{st} , the vehicle travels at a speed of $v_{\max}$ when the actual headway distance between two adjacent vehicles is greater than or equal to h_{go} , and the expected driving velocity of the vehicle satisfies $F(h_i(t))$ when the actual headway distance between two adjacent vehicles is greater than h_{st} but less than h_{go} . The graph of the expected velocity function of the vehicle in the OVM model is shown in Figure 2.

3.2. The model of CAV. The longitudinal dynamic characteristics of CAV in the mixed vehicle platoon are characterized by the second-order integrator model. Considering the influence of uncertain external disturbance, the dynamic model of CAV is as follows:

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t) + \varphi_i(t) \end{cases} \quad (6)$$

where $u_i(t)$ is the control input.

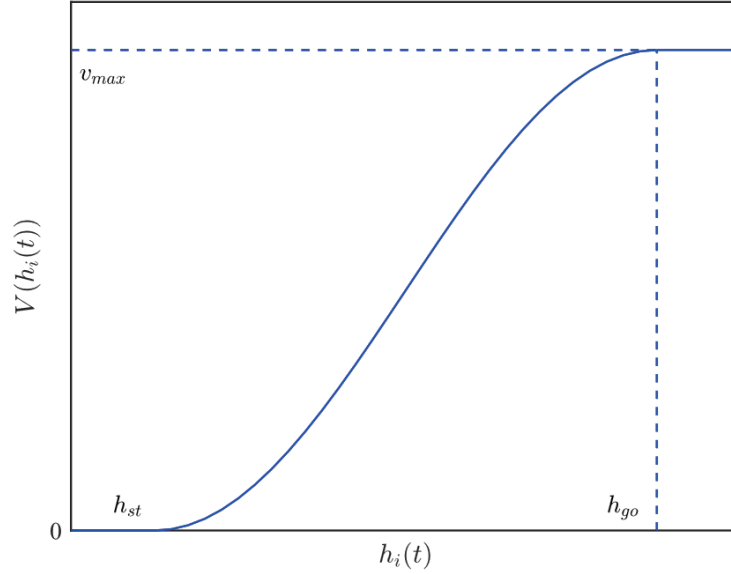


FIGURE 2. Desired velocity function of OVM

3.3. Controller design. The control objective of the mixed vehicle platoon is that all vehicles maintain the same desired velocity $v^* = V(h^*)$ and have a constant headway distance h^* , namely

$$\begin{cases} \lim_{t \rightarrow +\infty} \|h_i(t) - h^*\| = 0 \\ \lim_{t \rightarrow +\infty} \|v_i(t) - v^*\| = 0 \end{cases} \quad (7)$$

Considering that the CAV can obtain the state information of multiple vehicles located in front of and behind it and based on [15], this paper proposes a controller design method for a mixed vehicle platoon suitable for the bidirectional multi-vehicle following topology containing multiple CAVs. The consistency controller is designed as follows:

$$\begin{cases} u_i(t) = \sum_{j=1}^N a_{ij} [\alpha d_i(t) (V(\bar{h}_{i,j}(t)) - v_i(t)) + \beta(v_j(t) - v_i(t))] \\ \dot{d}_i(t) = e_i^T(t) e_i(t) \end{cases} \quad (8)$$

where $d_i(t)$ is the adaptive control gain, a time-varying parameter whose value is determined by the differential equation $\dot{d}_i(t) = e_i^T(t) e_i(t)$. $e_i(t) = [e_{h_i(t)}, e_{v_i(t)}]^T$, where $e_{h_i(t)} = h_i(t) - h^*$ is the headway distance error, and $e_{v_i(t)} = v_i(t) - v^*$ is the velocity error. $\bar{h}_{i,j}(t) = \frac{x_j(t) - x_i(t)}{i-j}$ is the average headway distance between vehicle i and vehicle j , and $\alpha, \beta > 0$ are the control parameters that cooperate with $d_i(t)$ to dynamically adjust the controller response, thereby maintaining the stability of the mixed vehicle platoon system under uncertain external disturbance, with their values being determined based on the stability condition derived from subsequent theoretical analysis. It should be noted that the form of the expected velocity function of the vehicle is as follows:

$$V(\bar{h}_{i,j}(t)) = \begin{cases} V_+(\bar{h}_{i,j}(t)), & j < i \\ V_-(\bar{h}_{i,j}(t)), & j > i \end{cases} \quad (9)$$

where the expression of $V_+(\bar{h}_{i,j}(t))$ is consistent with Equation (4), $V_-(\bar{h}_{i,j}(t))$ is the inverse OVM which is proposed to characterize the characteristic that the CAV in the mixed vehicle platoon can obtain the state information of the vehicles behind it, and its expression is as follows:

$$V_-(\bar{h}_{i,j}(t)) = \begin{cases} v_{\max}, & \bar{h}_{i,j}(t) \leq h_{st} \\ F(\bar{h}_{i,j}(t)), & h_{st} < \bar{h}_{i,j}(t) < h_{go} \\ 0, & \bar{h}_{i,j}(t) \geq h_{go} \end{cases} \quad (10)$$

where

$$F(\bar{h}_{i,j}(t)) = \frac{v_{\max}}{2} \left[1 + \cos \left(\frac{\pi (\bar{h}_{i,j}(t) - h_{st})}{h_{go} - h_{st}} \right) \right] \quad (11)$$

and $F(\bar{h}_{i,j}(t))$ is a continuous and monotonically decreasing function.

The graph of $V_-(\bar{h}_{i,j}(t))$ is shown in Figure 3. Combining Equation (10) and Equation (11), it is not difficult to see that the CAV in mixed vehicle platoon travels at a speed of $v_{\max}$ when the actual average headway distance between it and the vehicle behind it is less than or equal to h_{st} , the vehicle stops running when the actual average headway distance between it and the vehicle behind it is greater than or equal to h_{go} , and the expected driving velocity of the vehicle satisfies $F(\bar{h}_{i,j}(t))$ when the actual average headway distance between it is greater than h_{st} but less than h_{go} .

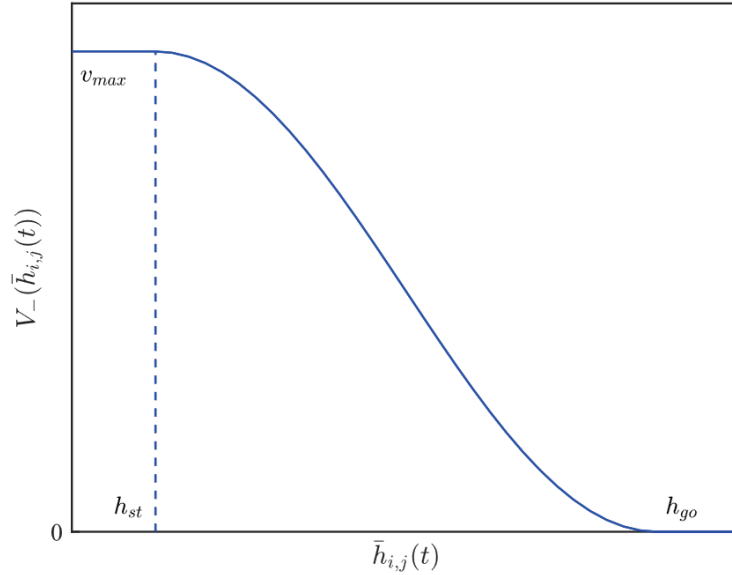


FIGURE 3. Desired velocity function of inverse OVM

Therefore, it can be seen from Equation (9) that in the mixed vehicle platoon, the strategy of CAV following the vehicle in front and the vehicle behind is opposite. That is, $V(\bar{h}_{i,j}(t)) = V_+(\bar{h}_{i,j}(t))$ when CAV acquires the information of the vehicle in front of it, and $V(\bar{h}_{i,j}(t)) = V_-(\bar{h}_{i,j}(t))$ when CAV acquires the information of the vehicle behind it.

4. Stability Analysis of Mixed Vehicle Platoon. In order to subsequently analyze the stability of the mixed vehicle platoon under disturbance condition, the error of the vehicle state information is defined as follows:

$$\begin{cases} e_{x_i}(t) = x_i(t) - x_i^*(t) \\ e_{h_i}(t) = h_i(t) - h^* \\ e_{v_i}(t) = v_i(t) - v^* \end{cases} \quad (12)$$

where $x_i^*(t) = v^*t - ih^*$ is the ideal position of vehicle i in the mixed vehicle platoon at time t .

Since this paper focuses on studying the stability of the mixed vehicle platoon system when uncertain external disturbance acts on the following vehicles, the number set of the following vehicles in the mixed vehicle platoon is defined as $\mathbf{S}$, where the number set of the following HDVs is $\mathbf{H}_r$ and the number set of the following CAVs is $\mathbf{S}/\mathbf{H}_r$. Then the state of the following vehicles in the mixed vehicle platoon can be defined as

$$\mathbf{X}(t) = [x_1(t), x_2(t), \dots, x_N(t)]^T, \quad \mathbf{V}(t) = [v_1(t), v_2(t), \dots, v_N(t)]^T$$

where

$$x_i(t) = \begin{cases} x_i^r(t), & i \in \mathbf{H}_r \\ x_i^c(t), & i \in \mathbf{S}/\mathbf{H}_r \end{cases}, \quad v_i(t) = \begin{cases} v_i^r(t), & i \in \mathbf{H}_r \\ v_i^c(t), & i \in \mathbf{S}/\mathbf{H}_r \end{cases}$$

and $x_i^r(t)$, $x_i^c(t)$ represent the positions of the i -th HDV and the i -th CAV serving as the following vehicle in the mixed vehicle platoon at time t , $v_i^r(t)$, $v_i^c(t)$ represent the velocities of the i -th HDV and the i -th CAV serving as the following vehicle in the mixed vehicle platoon at time t .

According to the above definition, the dynamic model of HDV serving as the following vehicle in the mixed vehicle platoon can be written as

$$\begin{cases} \dot{x}_i^r(t) = v_i^r(t) \\ \dot{v}_i^r(t) = \mu_1 [V(h_i^r(t)) - v_i^r(t)] + \mu_2 [v_{i-1}^r(t) - v_i^r(t)] + \varphi_i^r(t) \end{cases} \quad (13)$$

Linearizing $V(h_i^r(t))$ at (h^*, v^*) in Equation (13), we can obtain

$$V(h_i^r(t)) = V(h^*) + \dot{V}(h^*) (h_i^r(t) - h^*) \quad (14)$$

and substituting Equation (14) into Equation (13), we have

$$\begin{cases} \dot{x}_i^r(t) = v_i^r(t) \\ \dot{v}_i^r(t) = \mu_1 [V(h^*) - v_i^r(t) + \dot{V}(h^*) (h_i^r(t) - h^*)] + \mu_2 [v_{i-1}^r(t) - v_i^r(t)] + \varphi_i^r(t) \end{cases} \quad (15)$$

Further, by writing the state of the vehicle in Equation (15) in the form of an error, we obtain

$$\begin{cases} \dot{e}_{x_i(t)}^r = e_{v_i(t)}^r \\ \dot{e}_{v_i(t)}^r = \mu_1 \dot{V}(h^*) e_{h_i(t)}^r + \mu_2 e_{v_{i-1}(t)}^r - (\mu_1 + \mu_2) e_{v_i(t)}^r + \varphi_i^r(t) \end{cases} \quad (16)$$

For the convenience of subsequent stability analysis, Equation (16) is further written in the following form:

$$\begin{cases} \dot{e}_{x_i(t)}^r = e_{v_i(t)}^r \\ \dot{e}_{h_i(t)}^r = e_{v_{i-1}(t)}^r - e_{v_i(t)}^r \\ \dot{e}_{v_i(t)}^r = \mu_1 \dot{V}(h^*) e_{h_i(t)}^r + \mu_2 e_{v_{i-1}(t)}^r - (\mu_1 + \mu_2) e_{v_i(t)}^r + \varphi_i^r(t) \end{cases} \quad (17)$$

Next, we define the following vector

$$\begin{aligned} \mathbf{e}_{x(t)}^r &= [e_{x_1(t)}^r, e_{x_2(t)}^r, \dots, e_{x_{N_1(t)}}^r]^T, & \mathbf{e}_{h(t)}^r &= [e_{h_1(t)}^r, e_{h_2(t)}^r, \dots, e_{h_{N_1(t)}}^r]^T, \\ \mathbf{e}_{v(t)}^r &= [e_{v_1(t)}^r, e_{v_2(t)}^r, \dots, e_{v_{N_1(t)}}^r]^T, & \boldsymbol{\varphi}_r(t) &= [\varphi_1^r(t), \varphi_2^r(t), \dots, \varphi_{N_1(t)}^r(t)]^T. \end{aligned}$$

Then Equation (17) can be further written as

$$\begin{cases} \dot{\mathbf{e}}_{x(t)}^r = \mathbf{e}_{v(t)}^r \\ \dot{\mathbf{e}}_{h(t)}^r = \mathbf{Q}_1 \mathbf{e}_{v(t)}^r \\ \dot{\mathbf{e}}_{v(t)}^r = \mu_1 \dot{V}(h^*) \mathbf{e}_{h(t)}^r + \mathbf{M} \mathbf{e}_{v(t)}^r + \boldsymbol{\varphi}_r(t) \end{cases} \quad (18)$$

where

$$\mathbf{Q}_1 = \begin{bmatrix} -1 & 0 & 0 & \cdots & 0 \\ 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -1 \end{bmatrix}_{N_1 \times N_1},$$

$$\mathbf{M} = \begin{bmatrix} -(\mu_1 + \mu_2) & 0 & 0 & \cdots & 0 \\ \mu_2 & -(\mu_1 + \mu_2) & 0 & \cdots & 0 \\ 0 & \mu_2 & -(\mu_1 + \mu_2) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -(\mu_1 + \mu_2) \end{bmatrix}_{N_1 \times N_1}.$$

Similarly, the dynamic model of CAV serving as the following vehicle in the mixed vehicle platoon can be written as

$$\begin{cases} \dot{x}_i^c(t) = v_i^c(t) \\ \dot{v}_i^c(t) = u_i(t) + \varphi_i^c(t) \end{cases} \quad (19)$$

By adding the consistency controller (8), we can obtain

$$\begin{cases} \dot{x}_i^c(t) = v_i^c(t) \\ \dot{v}_i^c(t) = \sum_{j=1}^N a_{ij} [\alpha d_i(t) (V(\bar{h}_{i,j}(t)) - v_i^c(t)) + \beta(v_j(t) - v_i^c(t))] + \varphi_i^c(t) \end{cases} \quad (20)$$

Linearizing $V(\bar{h}_{i,j}(t))$ at (h^*, v^*) in Equation (20), we can get

$$V(\bar{h}_{i,j}(t)) = V(h^*) + \dot{V}(h^*) (\bar{h}_{i,j}(t) - h^*) \quad (21)$$

and substituting Equation (21) into Equation (20), we have

$$\begin{cases} \dot{x}_i^c(t) = v_i^c(t) \\ \dot{v}_i^c(t) = \sum_{j=1}^N a_{ij} [\alpha d_i(t) (V(h^*) - v_i^c(t) + \dot{V}(h^*) (\bar{h}_{i,j}(t) - h^*)) \\ + \beta(v_j(t) - v_i^c(t))] + \varphi_i^c(t) \end{cases} \quad (22)$$

Because of $\bar{h}_{i,j}(t) = \frac{x_j(t) - x_i(t)}{i-j}$, Equation (22) can be further written as

$$\begin{cases} \dot{x}_i^c(t) = v_i^c(t) \\ \dot{v}_i^c(t) = \sum_{j=1}^N a_{ij} \left[\alpha d_i(t) \left(V(h^*) - v_i^c(t) + \dot{V}(h^*) \left(\frac{x_j(t) - x_i^c(t)}{i-j} - h^* \right) \right) \right. \\ \left. + \beta(v_j(t) - v_i^c(t)) \right] + \varphi_i^c(t) \end{cases} \quad (23)$$

The state of the vehicle in Equation (23) is written in the form of error as follows:

$$\begin{cases} \dot{e}_{x_i(t)} = e_{v_i(t)} \\ \dot{e}_{v_i(t)} = \sum_{j=1}^N a_{ij} \left[\alpha d_i(t) \left(-e_{v_i(t)} + \dot{V}(h^*) \left(\frac{e_{x_j(t)} - e_{x_i(t)}}{i-j} \right) \right) + \beta(e_{v_j(t)} - e_{v_i(t)}) \right] \\ + \varphi_i^c(t) \end{cases} \quad (24)$$

Further simplification yields

$$\begin{cases} \dot{e}_{x_i}^c = e_{v_i}^c \\ \dot{e}_{v_i}^c = -\alpha d_i(t) \sum_{j=1}^N a_{ij} e_{v_j}^c + \alpha \dot{V}(h^*) d_i(t) \sum_{j=1}^N \frac{a_{ij}}{i-j} (e_{x_j}(t) - e_{x_i}^c(t)) \\ \quad + \beta \sum_{j=1}^N a_{ij} (e_{v_j}(t) - e_{v_i}^c(t)) + \varphi_i^c(t) \end{cases} \quad (25)$$

and for the convenience of subsequent stability analysis, Equation (25) is further written in the following form:

$$\begin{cases} \dot{e}_{x_i}^c = e_{v_i}^c \\ \dot{e}_{h_i}^c = e_{v_{i-1}}^c - e_{v_i}^c \\ \dot{e}_{v_i}^c = -\alpha d_i(t) \sum_{j=1}^N a_{ij} e_{v_j}^c + \alpha \dot{V}(h^*) d_i(t) \sum_{j=1}^N \frac{a_{ij}}{i-j} (e_{x_j}(t) - e_{x_i}^c(t)) \\ \quad + \beta \sum_{j=1}^N a_{ij} (e_{v_j}(t) - e_{v_i}^c(t)) + \varphi_i^c(t) \end{cases} \quad (26)$$

Let $\eta_{ij} = \frac{a_{ij}}{i-j}$, and we define

$$\begin{aligned} \mathbf{e}_{x(t)}^c &= [e_{x_1}^c(t), e_{x_2}^c(t), \dots, e_{x_{N_2}}^c(t)]^T, & \mathbf{e}_{h(t)}^c &= [e_{h_1}^c(t), e_{h_2}^c(t), \dots, e_{h_{N_2}}^c(t)]^T, \\ \mathbf{e}_{v(t)}^c &= [e_{v_1}^c(t), e_{v_2}^c(t), \dots, e_{v_{N_2}}^c(t)]^T, & \boldsymbol{\varphi}_c(t) &= [\varphi_1^c(t), \varphi_2^c(t), \dots, \varphi_{N_2}^c(t)]^T. \end{aligned}$$

Then Equation (26) can be further written as

$$\begin{cases} \dot{\mathbf{e}}_{x(t)}^c = \mathbf{e}_{v(t)}^c \\ \dot{\mathbf{e}}_{h(t)}^c = \mathbf{Q}_2 \mathbf{e}_{v(t)}^c \\ \dot{\mathbf{e}}_{v(t)}^c = -\alpha \mathbf{d}(t) [\mathbf{A}^+ \mathbf{e}_{v(t)}^c + \dot{V}(h^*) \boldsymbol{\Phi} \mathbf{e}_{x(t)}] - \beta \mathbf{L} \mathbf{e}_{v(t)} + \boldsymbol{\varphi}_c(t) \end{cases} \quad (27)$$

where

$$\begin{aligned} \mathbf{d}(t) &= \text{diag} \{d_1(t), d_2(t), \dots, d_{N_2}(t)\}, & \mathbf{Q}_2 &= \begin{bmatrix} -1 & 0 & 0 & \cdots & 0 \\ 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -1 \end{bmatrix}_{N_2 \times N_2}, \\ \mathbf{A}^+ &= \begin{bmatrix} \sum_{j=1}^N a_{1j} & 0 & 0 & \cdots & 0 \\ 0 & \sum_{j=1}^N a_{2j} & 0 & \cdots & 0 \\ 0 & 0 & \sum_{j=1}^N a_{3j} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \sum_{j=1}^N a_{N_2j} \end{bmatrix}_{N_2 \times N_2}, \end{aligned}$$

$$\Phi = \begin{bmatrix} \sum_{j=1, j \neq 1}^N \eta_{1j} & -\eta_{12} & -\eta_{13} & \cdots & -\eta_{1N} \\ -\eta_{21} & \sum_{j=1, j \neq 2}^N \eta_{2j} & -\eta_{23} & \cdots & -\eta_{2N} \\ -\eta_{31} & -\eta_{32} & \sum_{j=1, j \neq 3}^N \eta_{3j} & \cdots & -\eta_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\eta_{N_2 1} & -\eta_{N_2 2} & -\eta_{N_2 3} & \cdots & \sum_{j=1, j \neq N_2}^N \eta_{N_2 j} \end{bmatrix}_{N_2 \times N},$$

and $\mathbf{L}$ is the Laplace matrix of the system.

Therefore, the controlled model for the mixed vehicle platoon system can be expressed as

$$\begin{cases} \dot{\mathbf{e}}_{x(t)}^r = \mathbf{e}_{v(t)}^r \\ \dot{\mathbf{e}}_{x(t)}^c = \mathbf{e}_{v(t)}^c \\ \dot{\mathbf{e}}_{h(t)}^r = \mathbf{Q}_1 \mathbf{e}_{v(t)}^r \\ \dot{\mathbf{e}}_{h(t)}^c = \mathbf{Q}_2 \mathbf{e}_{v(t)}^c \\ \dot{\mathbf{e}}_{v(t)}^r = \mu_1 \dot{V}(h^*) \mathbf{e}_{h(t)}^r + \mathbf{M} \mathbf{e}_{v(t)}^r + \boldsymbol{\varphi}_r(t) \\ \dot{\mathbf{e}}_{v(t)}^c = -\alpha \mathbf{d}(t) [\mathbf{A}^+ \mathbf{e}_{v(t)}^c + \dot{V}(h^*) \Phi \mathbf{e}_{x(t)}] - \beta \mathbf{L} \mathbf{e}_{v(t)} + \boldsymbol{\varphi}_c(t) \end{cases} \quad (28)$$

Let $\mathbf{e}(t) = \left[\left(\mathbf{e}_{x(t)}^r \right)^T, \left(\mathbf{e}_{x(t)}^c \right)^T, \left(\mathbf{e}_{h(t)}^r \right)^T, \left(\mathbf{e}_{h(t)}^c \right)^T, \left(\mathbf{e}_{v(t)}^r \right)^T, \left(\mathbf{e}_{v(t)}^c \right)^T \right]^T$, $\boldsymbol{\varphi}(t) = [\boldsymbol{\varphi}_r^T(t), \boldsymbol{\varphi}_c^T(t)]^T$, and then the system (28) can be written as

$$\dot{\mathbf{e}}(t) = (\Xi_1 + \Xi_2) \mathbf{e}(t) + \Xi_3 \boldsymbol{\varphi}(t) \quad (29)$$

where

$$\Xi_1 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_{N_1 \times N_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_{N_2 \times N_2} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{Q}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{Q}_2 \\ \mathbf{0} & \mathbf{0} & \mu_1 \dot{V}(h^*) \mathbf{I}_{N_1 \times N_1} & \mathbf{0} & \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\alpha \mathbf{d}(t) \mathbf{A}^+ \end{bmatrix}_{3N \times 3N},$$

$$\Xi_2 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\alpha \dot{V}(h^*) \mathbf{d}(t) \Phi & \mathbf{0} & -\beta \mathbf{L} \end{bmatrix}_{3N \times 3N}, \quad \Xi_3 = ([\mathbf{0}, \mathbf{0}, \mathbf{I}_{N \times N}]_{N \times 3N})^T.$$

Theorem 4.1. Consider the mixed vehicle platoon system under external disturbance (28). If there exist $\alpha, \beta > 0$, such that

$$\begin{bmatrix} \Xi_1 + \Xi_2 & \Xi_3 \\ \mathbf{0} & \mathbf{0} \end{bmatrix} < 0 \quad (30)$$

then the system (28) can realize asymptotic stability.

Proof: Design the following Lyapunov function:

$$W(t) = \frac{1}{2} \mathbf{e}^T(t) \mathbf{e}(t) \quad (31)$$

and then differentiating $W(t)$ along (29), we have

$$\dot{W}(t) = \mathbf{e}^T(t)(\Xi_1 + \Xi_2)\mathbf{e}(t) + \mathbf{e}^T(t)\Xi_3\boldsymbol{\varphi}(t)$$

Let $\boldsymbol{\Theta}(t) = [\mathbf{e}^T(t), \boldsymbol{\varphi}^T(t)]^T$, and we have

$$\dot{W}(t) = \boldsymbol{\Theta}^T(t) \begin{bmatrix} \Xi_1 + \Xi_2 & \Xi_3 \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \boldsymbol{\Theta}(t) \quad (32)$$

If condition (30) is held, then $\dot{W}(t) < 0$. Based on Lyapunov stability theory, the mixed vehicle platoon system under external disturbance (28) is asymptotically stable.

5. Cooperative Control of Mixed Vehicle Platoon with Time Delays.

5.1. Controller design and model construction. In practical driving scenarios, information exchange between vehicles is not instantaneous, and time delays are crucial factors affecting vehicle following behavior and traffic flow stability. Therefore, building upon the aforementioned system modeling and stability analysis, this section investigates the impact of driver reaction delay and vehicle communication delay on system stability. Assuming homogeneous delays, a consensus controller accounting for time delays is designed as follows:

$$\begin{cases} u_i(t) = \sum_{j=1}^N a_{ij} [\alpha d_i(t) (V(\bar{h}_{i,j}(t - \tau_2)) - v_i(t - \tau_2)) + \beta(v_j(t - \tau_2) - v_i(t - \tau_2))] \\ \dot{d}_i(t) = \mathbf{e}_i^T(t - \tau_2) \mathbf{e}_i(t - \tau_2) \end{cases} \quad (33)$$

where τ_2 is the vehicle communication delay.

Substituting Equation (33) into Equation (6), we have

$$\begin{cases} \dot{x}_i^c(t) = v_i^c(t) \\ \dot{v}_i^c(t) = \sum_{j=1}^N a_{ij} [\alpha d_i(t) (V(\bar{h}_{i,j}(t - \tau_2)) - v_i^c(t - \tau_2)) + \beta(v_j(t - \tau_2) - v_i^c(t - \tau_2))] + \varphi_i^c(t) \end{cases} \quad (34)$$

Linearizing $V(\bar{h}_{i,j}(t - \tau_2))$ at (h^*, v^*) in Equation (34), we can obtain

$$\begin{cases} \dot{x}_i^c(t) = v_i^c(t) \\ \dot{v}_i^c(t) = \sum_{j=1}^N a_{ij} [\alpha d_i(t) (V(h^*) - v_i^c(t - \tau_2) + \dot{V}(h^*) (\bar{h}_{i,j}(t - \tau_2) - h^*)) + \beta(v_j(t - \tau_2) - v_i^c(t - \tau_2))] + \varphi_i^c(t) \end{cases} \quad (35)$$

Based on the definitions of the error and $\bar{h}_{i,j}(t - \tau_2) = \frac{x_j(t - \tau_2) - x_i(t - \tau_2)}{i - j}$, and consistent with Section 4, Equation (35) can be rewritten as

$$\begin{cases} \dot{e}_{x_i}^c(t) = e_{v_i}^c(t) \\ \dot{e}_{v_i}^c(t) = \sum_{j=1}^N a_{ij} \left[\alpha d_i(t) \left(-e_{v_i}^c(t - \tau_2) + \dot{V}(h^*) \left(\frac{e_{x_j}(t - \tau_2) - e_{x_i}^c(t - \tau_2)}{i - j} \right) \right) + \beta \left(e_{v_j}(t - \tau_2) - e_{v_i}^c(t - \tau_2) \right) \right] + \varphi_i^c(t) \end{cases} \quad (36)$$

To facilitate subsequent stability analysis, Equation (36) can be further expressed in the following form:

$$\begin{cases} \dot{e}_{x_i}^c(t) = e_{v_i}^c(t) \\ \dot{e}_{h_i}^c(t) = e_{v_{i-1}}^c(t) - e_{v_i}^c(t) \\ \dot{e}_{v_i}^c(t) = -\alpha d_i(t) \sum_{j=1}^N a_{ij} e_{v_i}^c(t-\tau_2) + \alpha \dot{V}(h^*) d_i(t) \sum_{j=1}^N \frac{a_{ij}}{i-j} (e_{x_j}(t-\tau_2) - e_{x_i}^c(t-\tau_2)) \\ \quad + \beta \sum_{j=1}^N a_{ij} (e_{v_j}(t-\tau_2) - e_{v_i}^c(t-\tau_2)) + \varphi_i^c(t) \end{cases} \quad (37)$$

We define

$$\begin{aligned} \mathbf{e}_{x(t-\tau_2)}^c &= \left[e_{x_1(t-\tau_2)}^c, e_{x_2(t-\tau_2)}^c, \dots, e_{x_{N_2}(t-\tau_2)}^c \right]^T, \\ \mathbf{e}_{v(t-\tau_2)}^c &= \left[e_{v_1(t-\tau_2)}^c, e_{v_2(t-\tau_2)}^c, \dots, e_{v_{N_2}(t-\tau_2)}^c \right]^T. \end{aligned}$$

Thus, Equation (37) can be written in the following form:

$$\begin{cases} \dot{\mathbf{e}}_{x(t)}^c = \mathbf{e}_{v(t)}^c \\ \dot{\mathbf{e}}_{h(t)}^c = \mathbf{Q}_2 \mathbf{e}_{v(t)}^c \\ \dot{\mathbf{e}}_{v(t)}^c = -\alpha \mathbf{d}(t) \left[\mathbf{A}^+ \mathbf{e}_{v(t-\tau_2)}^c + \dot{V}(h^*) \boldsymbol{\Phi} \mathbf{e}_{x(t-\tau_2)}^c \right] - \beta \mathbf{L} \mathbf{e}_{v(t-\tau_2)}^c + \boldsymbol{\varphi}_c(t) \end{cases} \quad (38)$$

Similarly, considering driver reaction delay, the dynamic model of HDV in the mixed platoon can be written as follows:

$$\begin{cases} \dot{x}_i^r(t) = v_i^r(t) \\ \dot{v}_i^r(t) = \mu_1 [V(h_i^r(t-\tau_1)) - v_i^r(t-\tau_1)] + \mu_2 [v_{i-1}^r(t-\tau_1) - v_i^r(t-\tau_1)] + \varphi_i^r(t) \end{cases} \quad (39)$$

where τ_1 is the driver reaction delay.

Linearizing $V(h_i^r(t-\tau_1))$ at (h^*, v^*) in Equation (39), we can obtain

$$\begin{cases} \dot{x}_i^r(t) = v_i^r(t) \\ \dot{v}_i^r(t) = \mu_1 \left[V(h^*) - v_i^r(t-\tau_1) + \dot{V}(h^*) (h_i^r(t-\tau_1) - h^*) \right] \\ \quad + \mu_2 [v_{i-1}^r(t-\tau_1) - v_i^r(t-\tau_1)] + \varphi_i^r(t) \end{cases} \quad (40)$$

Consistent with Section 4, transform the vehicle states in Equation (40) into error form

$$\begin{cases} \dot{e}_{x_i}^r(t) = e_{v_i}^r(t) \\ \dot{e}_{v_i}^r(t) = \mu_1 \dot{V}(h^*) e_{h_i(t-\tau_1)}^r + \mu_2 e_{v_{i-1}(t-\tau_1)}^r - (\mu_1 + \mu_2) e_{v_i(t-\tau_1)}^r + \varphi_i^r(t) \end{cases} \quad (41)$$

To facilitate subsequent stability analysis, Equation (41) can be further expressed in the following form:

$$\begin{cases} \dot{e}_{x_i}^r(t) = e_{v_i}^r(t) \\ \dot{e}_{h_i}^r(t) = e_{v_{i-1}(t)}^r - e_{v_i}^r(t) \\ \dot{e}_{v_i}^r(t) = \mu_1 \dot{V}(h^*) e_{h_i(t-\tau_1)}^r + \mu_2 e_{v_{i-1}(t-\tau_1)}^r - (\mu_1 + \mu_2) e_{v_i(t-\tau_1)}^r + \varphi_i^r(t) \end{cases} \quad (42)$$

We define $\mathbf{e}_{h(t-\tau_1)}^r = \left[e_{h_1(t-\tau_1)}^r, e_{h_2(t-\tau_1)}^r, \dots, e_{h_{N_1}(t-\tau_1)}^r \right]^T$, $\mathbf{e}_{v(t-\tau_1)}^r = \left[e_{v_1(t-\tau_1)}^r, e_{v_2(t-\tau_1)}^r, \dots, e_{v_{N_1}(t-\tau_1)}^r \right]^T$; thus, Equation (42) can be written in the following form:

$$\begin{cases} \dot{\mathbf{e}}_{x(t)}^r = \mathbf{e}_{v(t)}^r \\ \dot{\mathbf{e}}_{h(t)}^r = \mathbf{Q}_1 \mathbf{e}_{v(t)}^r \\ \dot{\mathbf{e}}_{v(t)}^r = \mu_1 \dot{V}(h^*) \mathbf{e}_{h(t-\tau_1)}^r + \mathbf{M} \mathbf{e}_{v(t-\tau_1)}^r + \boldsymbol{\varphi}_r(t) \end{cases} \quad (43)$$

Therefore, the controlled model of the mixed vehicle platoon system considering time delays under external disturbance can be expressed as

$$\begin{cases} \dot{\mathbf{e}}_{x(t)}^r = \mathbf{e}_{v(t)}^r \\ \dot{\mathbf{e}}_{x(t)}^c = \mathbf{e}_{v(t)}^c \\ \dot{\mathbf{e}}_{h(t)}^r = \mathbf{Q}_1 \mathbf{e}_{v(t)}^r \\ \dot{\mathbf{e}}_{h(t)}^c = \mathbf{Q}_2 \mathbf{e}_{v(t)}^c \\ \dot{\mathbf{e}}_{v(t)}^r = \mu_1 \dot{V}(h^*) \mathbf{e}_{h(t-\tau_1)}^r + \mathbf{M} \mathbf{e}_{v(t-\tau_1)}^r + \boldsymbol{\varphi}_r(t) \\ \dot{\mathbf{e}}_{v(t)}^c = -\alpha \mathbf{d}(t) [\mathbf{A}^+ \mathbf{e}_{v(t-\tau_2)}^c + \dot{V}(h^*) \boldsymbol{\Phi} \mathbf{e}_{x(t-\tau_2)}] - \beta \mathbf{L} \mathbf{e}_{v(t-\tau_2)} + \boldsymbol{\varphi}_c(t) \end{cases} \quad (44)$$

5.2. System stability analysis. Let $\mathbf{e}(t) = \left[\left(\mathbf{e}_{x(t)}^r \right)^\top, \left(\mathbf{e}_{x(t)}^c \right)^\top, \left(\mathbf{e}_{h(t)}^r \right)^\top, \left(\mathbf{e}_{h(t)}^c \right)^\top, \left(\mathbf{e}_{v(t)}^r \right)^\top, \left(\mathbf{e}_{v(t)}^c \right)^\top \right]^\top$, $\boldsymbol{\varphi}(t) = [\boldsymbol{\varphi}_r^\top(t), \boldsymbol{\varphi}_c^\top(t)]^\top$, and then the system (44) can be written as

$$\dot{\mathbf{e}}(t) = \boldsymbol{\Xi}_4 \mathbf{e}(t) + \boldsymbol{\Xi}_5 \mathbf{e}(t - \tau_1) + (\boldsymbol{\Xi}_6 + \boldsymbol{\Xi}_2) \mathbf{e}(t - \tau_2) + \boldsymbol{\Xi}_3 \boldsymbol{\varphi}(t) \quad (45)$$

where

$$\begin{aligned} \boldsymbol{\Xi}_4 &= \begin{bmatrix} 0 & 0 & 0 & 0 & \mathbf{I}_{N_1 \times N_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{I}_{N_2 \times N_2} \\ 0 & 0 & 0 & 0 & \mathbf{Q}_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{Q}_2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{3N \times 3N}, \\ \boldsymbol{\Xi}_6 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\alpha \mathbf{d}(t) \mathbf{A}^+ \end{bmatrix}_{3N \times 3N}, \\ \boldsymbol{\Xi}_5 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu_1 \dot{V}(h^*) \mathbf{I}_{N_1 \times N_1} & 0 & \mathbf{M} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{3N \times 3N} \end{aligned}$$

Lemma 5.1. [30] For any vector $\mathbf{a}, \mathbf{b} \in \mathbf{R}^n$ and any positive definite matrix $\mathbf{X} \in \mathbf{R}^{n \times n}$, there is

$$-2\mathbf{a}^\top \mathbf{b} \leq \varpi \mathbf{a}^\top \mathbf{X} \mathbf{a} + \mathbf{b}^\top \mathbf{X}^{-1} \mathbf{b},$$

in particular, when $\mathbf{X} = \varpi \mathbf{I}$, there is

$$-2\mathbf{a}^\top \mathbf{b} \leq \varpi \mathbf{a}^\top \mathbf{a} + \varpi^{-1} \mathbf{b}^\top \mathbf{b}.$$

Theorem 5.1. If there exists $\vartheta_1, \vartheta_2 > 0$ such that

$$\begin{bmatrix} \boldsymbol{\Xi}_4 + \frac{5}{4} (\sqrt{\vartheta_1} + \sqrt{\vartheta_2}) \mathbf{I}_{3N \times 3N} & \boldsymbol{\Xi}_3 \\ \mathbf{0} & \mathbf{0} \end{bmatrix} < 0 \quad (46)$$

then the mixed vehicle platoon system considering time delays (44) is asymptotically stable.

Proof: Take the Lyapunov-Krasovskii as follows:

$$W(t) = \frac{1}{2} \mathbf{e}^T(t) \mathbf{e}(t) + \sqrt{\vartheta_1} \int_{t-\tau_1}^t \mathbf{e}^T(\theta) \mathbf{e}(\theta) d\theta + \sqrt{\vartheta_2} \int_{t-\tau_2}^t \mathbf{e}^T(\theta) \mathbf{e}(\theta) d\theta \quad (47)$$

Then we get the derivative of $W(t)$ along Equation (45) as follows:

$$\begin{aligned} \dot{W}(t) &= \mathbf{e}^T(t) \dot{\mathbf{e}}(t) + \sqrt{\vartheta_1} [\mathbf{e}^T(t) \mathbf{e}(t) - \mathbf{e}^T(t - \tau_1) \mathbf{e}(t - \tau_1)] \\ &\quad + \sqrt{\vartheta_2} [\mathbf{e}^T(t) \mathbf{e}(t) - \mathbf{e}^T(t - \tau_2) \mathbf{e}(t - \tau_2)] \\ &= \mathbf{e}^T(t) [\mathbf{\Xi}_4 \mathbf{e}(t) + \mathbf{\Xi}_5 \mathbf{e}(t - \tau_1) + (\mathbf{\Xi}_6 + \mathbf{\Xi}_2) \mathbf{e}(t - \tau_2) + \mathbf{\Xi}_3 \boldsymbol{\varphi}(t)] + \sqrt{\vartheta_1} \mathbf{e}^T(t) \mathbf{e}(t) \\ &\quad - \sqrt{\vartheta_1} \mathbf{e}^T(t - \tau_1) \mathbf{e}(t - \tau_1) + \sqrt{\vartheta_2} \mathbf{e}^T(t) \mathbf{e}(t) - \sqrt{\vartheta_2} \mathbf{e}^T(t - \tau_2) \mathbf{e}(t - \tau_2) \quad (48) \\ &= \mathbf{e}^T(t) \left[\mathbf{\Xi}_4 + \sqrt{\vartheta_1} \mathbf{I}_{3N \times 3N} + \sqrt{\vartheta_2} \mathbf{I}_{3N \times 3N} \right] \mathbf{e}(t) + \mathbf{e}^T(t) \mathbf{\Xi}_5 \mathbf{e}(t - \tau_1) \\ &\quad + \mathbf{e}^T(t) (\mathbf{\Xi}_6 + \mathbf{\Xi}_2) \mathbf{e}(t - \tau_2) - \sqrt{\vartheta_1} \mathbf{e}^T(t - \tau_1) \mathbf{e}(t - \tau_1) \\ &\quad - \sqrt{\vartheta_2} \mathbf{e}^T(t - \tau_2) \mathbf{e}(t - \tau_2) + \mathbf{e}^T(t) \mathbf{\Xi}_3 \boldsymbol{\varphi}(t) \end{aligned}$$

According to Lemma 5.1, we have

$$\begin{aligned} \mathbf{e}^T(t) \mathbf{\Xi}_5 \mathbf{e}(t - \tau_1) &\leq \frac{\sqrt{\vartheta_1}}{4} \mathbf{e}^T(t) \mathbf{e}(t) + \frac{1}{\sqrt{\vartheta_1}} \mathbf{e}^T(t - \tau_1) \mathbf{\Xi}_5^T \mathbf{\Xi}_5 \mathbf{e}(t - \tau_1) \\ &\leq \frac{\sqrt{\vartheta_1}}{4} \mathbf{e}^T(t) \mathbf{e}(t) + \sqrt{\vartheta_1} \mathbf{e}^T(t - \tau_1) \mathbf{e}(t - \tau_1) \end{aligned}$$

and

$$\begin{aligned} \mathbf{e}^T(t) (\mathbf{\Xi}_6 + \mathbf{\Xi}_2) \mathbf{e}(t - \tau_2) &\leq \frac{\sqrt{\vartheta_2}}{4} \mathbf{e}^T(t) \mathbf{e}(t) \\ &\quad + \frac{1}{\sqrt{\vartheta_2}} \mathbf{e}^T(t - \tau_2) (\mathbf{\Xi}_6 + \mathbf{\Xi}_2)^T (\mathbf{\Xi}_6 + \mathbf{\Xi}_2) \mathbf{e}(t - \tau_2) \\ &\leq \frac{\sqrt{\vartheta_2}}{4} \mathbf{e}^T(t) \mathbf{e}(t) + \sqrt{\vartheta_2} \mathbf{e}^T(t - \tau_2) \mathbf{e}(t - \tau_2), \end{aligned}$$

where ϑ_1 and ϑ_2 are respectively the largest eigenvalues of $\mathbf{\Xi}_5^T \mathbf{\Xi}_5$ and $(\mathbf{\Xi}_6 + \mathbf{\Xi}_2)^T (\mathbf{\Xi}_6 + \mathbf{\Xi}_2)$.

Substituting the results back into Equation (48), we obtain

$$\dot{W}(t) \leq \mathbf{e}^T(t) \left[\mathbf{\Xi}_4 + \frac{5}{4} (\sqrt{\vartheta_1} + \sqrt{\vartheta_2}) \mathbf{I}_{3N \times 3N} \right] \mathbf{e}(t) + \mathbf{e}^T(t) \mathbf{\Xi}_3 \boldsymbol{\varphi}(t) \quad (49)$$

Let $\boldsymbol{\Theta}(t) = [\mathbf{e}^T(t), \boldsymbol{\varphi}^T(t)]^T$, and then Equation (49) can be further written in the following form:

$$\dot{W}(t) \leq \boldsymbol{\Theta}^T(t) \begin{bmatrix} \mathbf{\Xi}_4 + \frac{5}{4} (\sqrt{\vartheta_1} + \sqrt{\vartheta_2}) \mathbf{I}_{3N \times 3N} & \mathbf{\Xi}_3 \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \boldsymbol{\Theta}(t) \quad (50)$$

At this point, if condition (46) is satisfied, then $\dot{W}(t) < 0$. Therefore, according to Lyapunov stability theory, it can be concluded that the mixed vehicle platoon system considering time delays under external disturbance (44) can achieve asymptotic stability.

6. Numerical Simulations. Considering a mixed vehicle platoon randomly composed of 8 HDVs and CAVs, without loss of generality, the positions of all vehicles except the lead vehicle in the platoon are randomly set $\mathbf{H}_r = \{1, 3, 4, 5, 7\}$, $\mathbf{S}/\mathbf{H}_r = \{2, 6\}$, and the information topology is shown in Figure 4.

The simulation experiment was carried out in the Matlab environment, and the simulation parameters were set as follows: $\alpha = 0.5$, $\beta = 0.2$, $h_{st} = 5$ m, $h_{go} = 35$ m, $v_{\max} = 30$

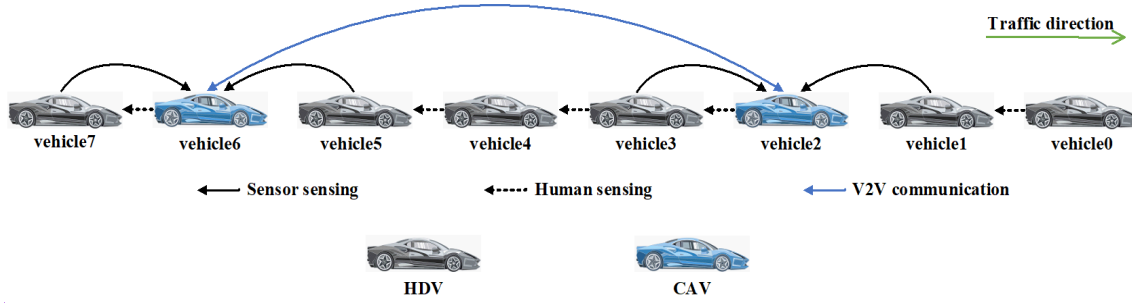


FIGURE 4. Information topology of a mixed vehicle platoon system

$\text{m}\cdot\text{s}^{-1}$, $\mu_1 = 0.6$, $\mu_2 = 0.9$ [8], and the desired velocity $v^* = V(h^*) = 15 \text{ m}\cdot\text{s}^{-1}$, the constant headway distance $h^* = 20 \text{ m}$. Furthermore, according to Equations (5), (9) and (11), it can be obtained:

$$\dot{V}(h^*) = \begin{cases} \pi/2, & j < i \\ -\pi/2, & j > i \end{cases}$$

Without loss of generality, the initial headway distance and the initial velocity of the vehicles in the mixed vehicle platoon are randomly taken within a certain range, as shown in Table 1.

TABLE 1. Initial state values of the vehicles

Initial state	The value
Headway distance $\{h_1, h_2, \dots, h_7\}$	$\{17, 16, 22, 12, 18, 15, 20\}$
Velocity $\{v_0, v_1, \dots, v_7\}$	$\{15, 17, 12, 16, 12, 22, 23, 15\}$

In order to illustrate that the control protocol proposed in this paper is effective for the mixed vehicle platoon affected by uncertain external disturbance, the following stability analysis must be conducted. The evolution curves of the headway distance and velocity in the mixed vehicle platoon when the following vehicles are not subject to uncertain external disturbance are shown in Figure 5. As can be seen from Figure 5, the headway distance and velocity gradually tend to the constant headway distance $h^* = 20 \text{ m}$ and desired velocity $v^* = 15 \text{ m}\cdot\text{s}^{-1}$ from the randomly given initial values under the action of

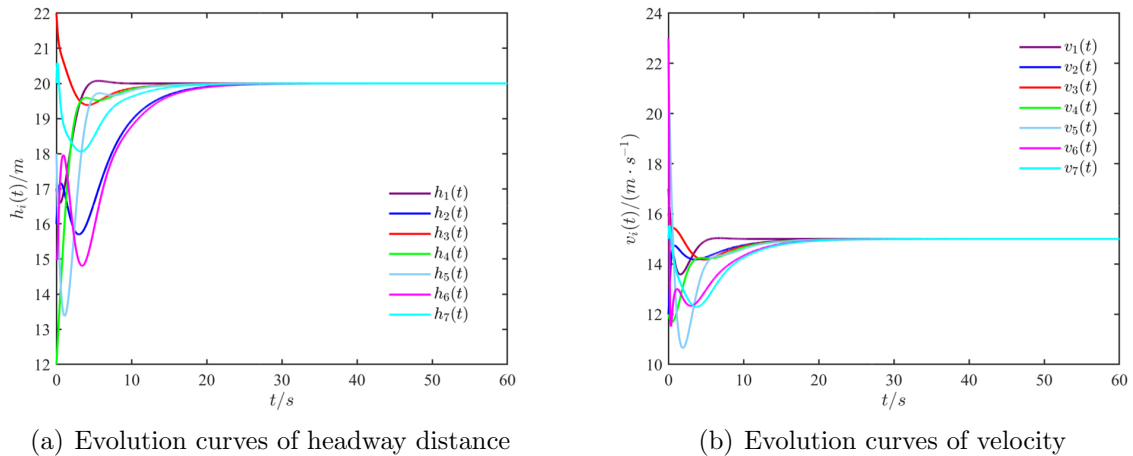


FIGURE 5. (color online) The evolution curves of headway distance and velocity for the following vehicles in a mixed platoon without disturbance

the controller. This indicates that the controller designed in this paper can achieve the inner-vehicle stability of the mixed vehicle platoon.

Suppose the first, third and fifth vehicles are affected by external disturbance $5 \sin(2t) + 5 \cos(2t)$ at time $30 \text{ s} \leq t \leq 35 \text{ s}$, and the evolution curves of the headway distance and velocity under the condition of external disturbance are shown in Figure 6. As can be seen from Figure 6, although the external disturbance applied to the first, third and fifth vehicles have a negative impact on the stability of the mixed vehicle platoon, the mixed vehicle platoon can still return to its original stable state in a short time under the action of the controller. This indicates that the controller designed in this paper has a significant inhibitory effect on the external disturbance.

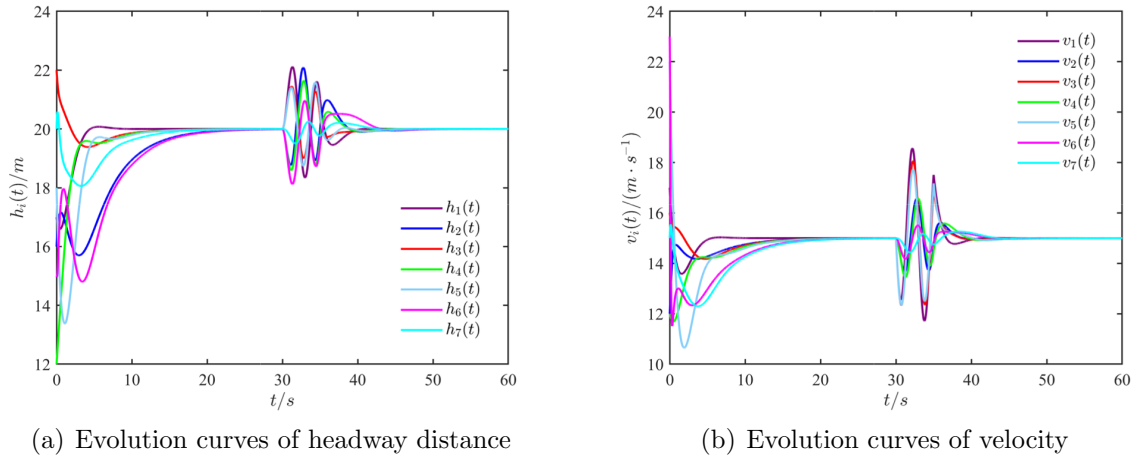


FIGURE 6. (color online) The evolution curves of headway distance and velocity for following vehicles in a mixed platoon with disturbance

6.1. The influence of the range of disturbance on stability. In order to comparatively analyze the time when the mixed vehicle platoon system tends to stabilize under different circumstances, the mean values of velocity errors and the mean values of the headway distance errors between adjacent two vehicles are defined as follows:

$$\begin{cases} \bar{V}(t) = \sum_{i=1}^N (v_i(t) - v^*) / N, \\ \bar{H}(t) = \sum_{i=1}^N (h_i(t) - h^*) / N, \end{cases} \quad (51)$$

where $\bar{V}(t)$ is the mean value of the velocity error, and $\bar{H}(t)$ is the mean value of the headway distance error.

In order to study the influence of the range of disturbance on stability, this section comparatively analyzes the disturbance situations of the first vehicle, the first, third, fifth vehicles and all the following vehicles in the mixed vehicle platoon. The evolution curves of the mean values of the headway distance error $\bar{H}(t)$ and the velocity error $\bar{V}(t)$ corresponding to the above three situations are shown in Figure 7. It can be seen from Figure 7 that the convergence speed of the mean error evolution curve is the fastest when only one vehicle is disturbed, followed by three vehicles. And the convergence speed of the mean error evolution curve is the slowest when all the following vehicles are disturbed. Meanwhile, it can be seen that when all the following vehicles are disturbed, the oscillation

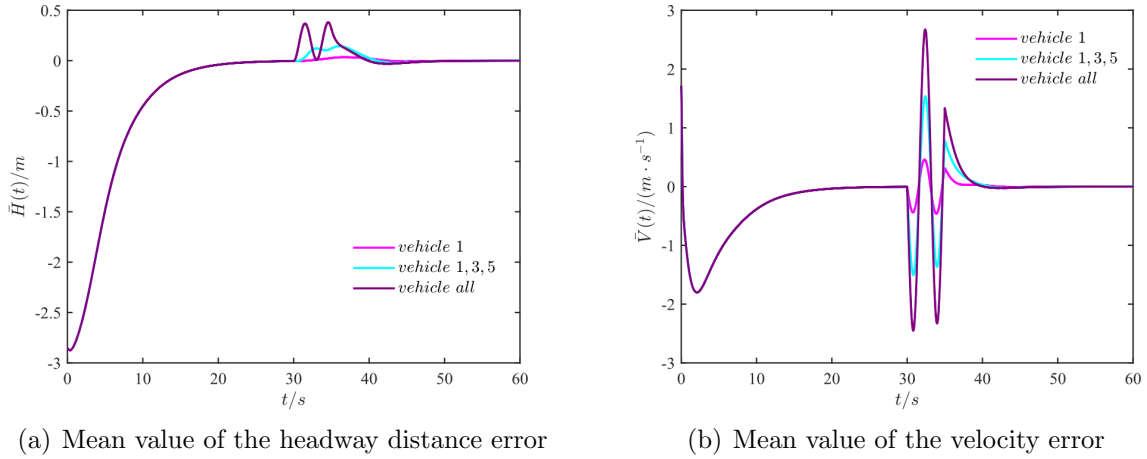


FIGURE 7. (color online) The evolution curves of the mean error under different disturbance ranges

amplitude of the average error evolution curve is the largest, followed by 3 vehicles, and the smallest is 1 vehicle. This indicates that the range of the disturbance effect has a significant impact on the stability of the mixed vehicle platoon system. Specifically, when the range of the disturbance effect is large, the dynamic response of the system will be more complex, and the interaction between vehicles will intensify, resulting in a slower convergence speed of the error mean evolution curves and generating larger oscillations. When the range of disturbance is small, the recovery ability of the system is strong. Therefore, the convergence speed of the error mean evolution curves is fast and the oscillation amplitude is small.

6.2. The influence of the position of disturbance on stability. In order to study the influence of the position of disturbance on stability, this section respectively compares and analyzes the disturbance situations of HDV located in front of CAV and HDV located behind CAV in the mixed vehicle platoon. In order to make the results of the simulation experiment easier to compare and analyze, we choose a relatively simple disturbance $10 \sin(0.2\pi t)$, $30 \text{ s} \leq t \leq 35 \text{ s}$ and select a mixed vehicle platoon containing only one CAV at the same time. The communication topology is shown in Figure 8.

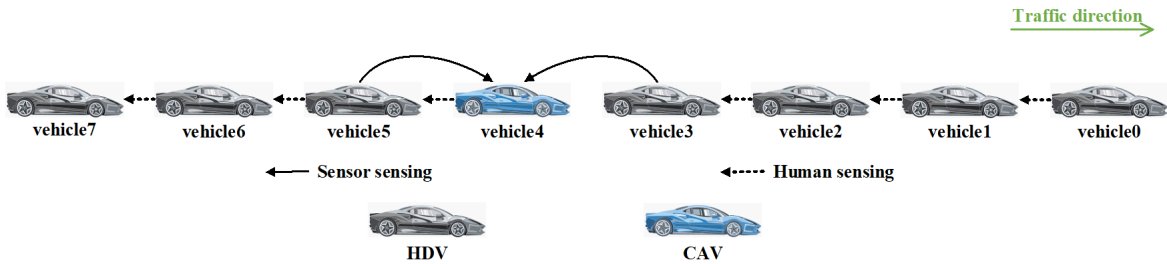


FIGURE 8. Information topology of mixed vehicle platoon system containing one CAV

First, the evolution curves of the mean values of the headway distance error and the velocity error of vehicles 1, 2 and 3 when they are respectively disturbed are compared and analyzed, and the results are shown in Figure 9. As can be seen from Figure 9, the convergence speed of the mean error evolution curve of the third vehicle in the mixed vehicle platoon is the fastest when it is disturbed, followed by the second vehicle, and

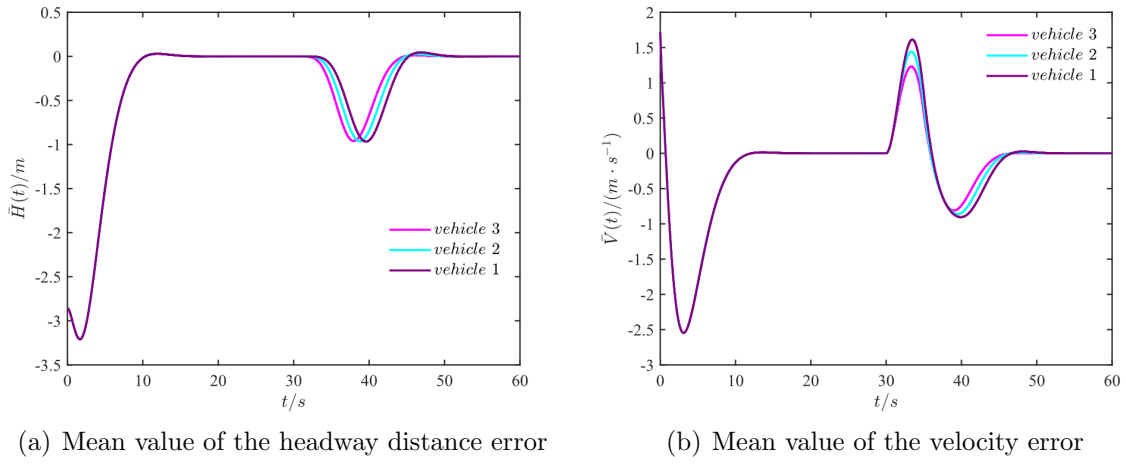


FIGURE 9. (color online) The evolution curves of the mean error in the case of HDV at different positions in front of CAV under the disturbance

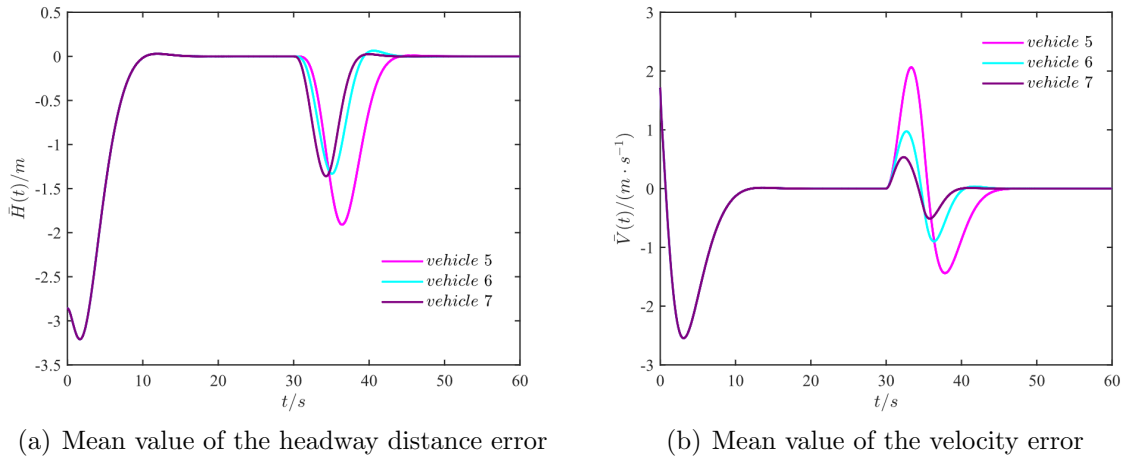


FIGURE 10. (color online) The evolution curves of the mean error in the case of HDV at different positions behind CAV under the disturbance

the convergence speed of the mean error evolution curve of the first vehicle is the slowest when it is disturbed. Secondly, the evolution curves of the mean values of the headway distance error and the velocity error of vehicles 5, 6 and 7 when they are respectively disturbed are compared and analyzed, and the results are shown in Figure 10. It can be seen from Figure 10 that the convergence speed of the mean error evolution curve of the 7th vehicle in the mixed vehicle platoon is the fastest when it is disturbed, followed by the 6th vehicle, and the convergence speed of the mean error evolution curve of the 5th vehicle is the slowest when it is disturbed. To sum up, the location of the disturbance effect will have a significant impact on the stability of the mixed vehicle platoon system. When the disturbance acts on the HDV located in front of the CAV, the change in the state of the HDV will affect all the vehicles behind. The disturbance will spread backward vehicle by vehicle, and each subsequent vehicle needs to adjust to respond to the state change of the vehicle in front, resulting in an expansion of the influence range of the disturbance and an increase in the adjustment time of the entire platoon. When the disturbance acts on the HDV located behind the CAV, the affected range is relatively small, and the vehicles

in front of it are almost unaffected. Therefore, the error adjustment time of the entire platoon is relatively fast.

Next, in order to conduct a comparative analysis with the situation where the disturbance acts on the HDV in the mixed vehicle platoon, we will study the situation where the disturbance acts on the CAV. The evolution curves of the mean values of the headway distance error and the velocity error are shown in Figure 11. As can be seen from Figure 11, when CAV in the mixed vehicle platoon is disturbed, the evolution curve of the mean error is almost close to 0 within the range of the disturbance and the subsequent time period. This indicates that compared with HDV, CAV in the mixed vehicle platoon has a stronger anti-disturbance ability, and it also further confirms the effectiveness of the controller proposed in this paper.

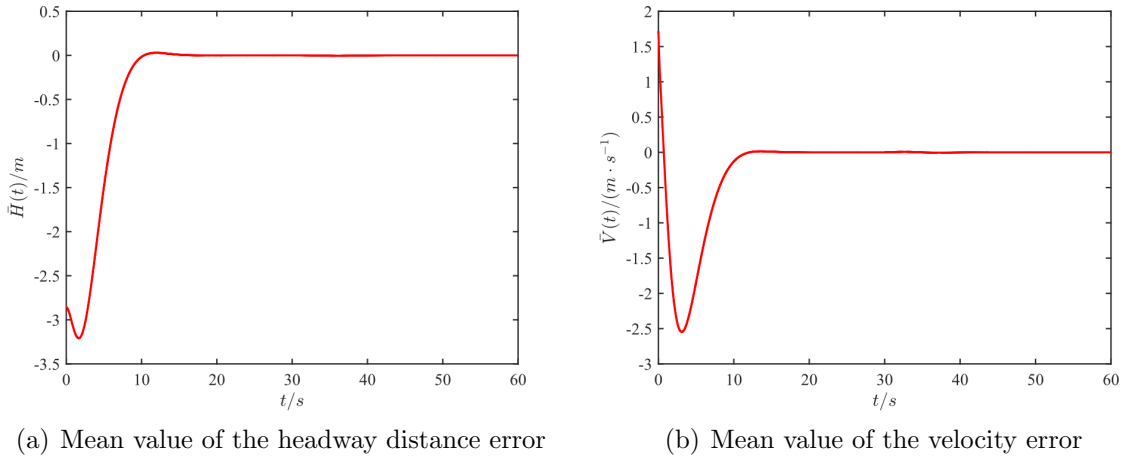


FIGURE 11. The evolution curves of the mean error under the condition of disturbance acting on CAV

Finally, we noticed that when the disturbance acted on the fifth vehicle in the mixed vehicle platoon, that is, the first HDV located behind the CAV, the oscillation amplitudes of the evolution curves corresponding to the mean values of the headway distance error and the velocity error were the largest. It can be seen more clearly from Figure 12 that this is obviously an abnormal phenomenon. This is because the CAV in the mixed vehicle platoon can obtain the state information from the fifth vehicle, and the fifth vehicle is located after the CAV. Therefore, when the disturbance acts on the fifth vehicle, not only the remaining HDVs behind it will be disturbed, but also the CAV in front of it will be affected by the disturbance, resulting in continuous changes in its state. This leads to the situation that when the disturbance acts on the fifth vehicle in the mixed vehicle platoon, the remaining HDVs behind the fifth vehicle will not only be affected by the state change of the fifth vehicle, but also by the state change of the CAV, causing the influence generated by the disturbance to superimpose and further resulting in greater fluctuations in the evolution curve of the mean error.

6.3. The influence of different topological structures on stability. This section analyzes the influence of unidirectional topology and bidirectional topology on the stability of the mixed vehicle platoon system. The geometric configuration of the mixed vehicle platoon as shown in Figure 8 is selected and it is assumed that the disturbance only acts on the first following vehicle in the mixed vehicle platoon. The corresponding simulation experiment results are shown in Figures 13 and 14. Figure 13 shows the evolution curves

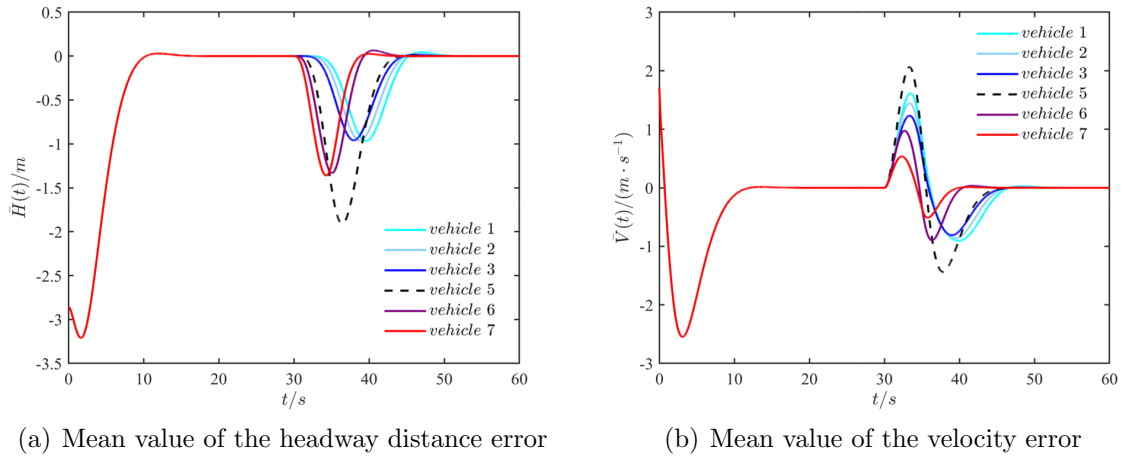


FIGURE 12. (color online) The evolution curves of the mean error under the case of HDV at different positions of the mixed vehicle platoon under the disturbance

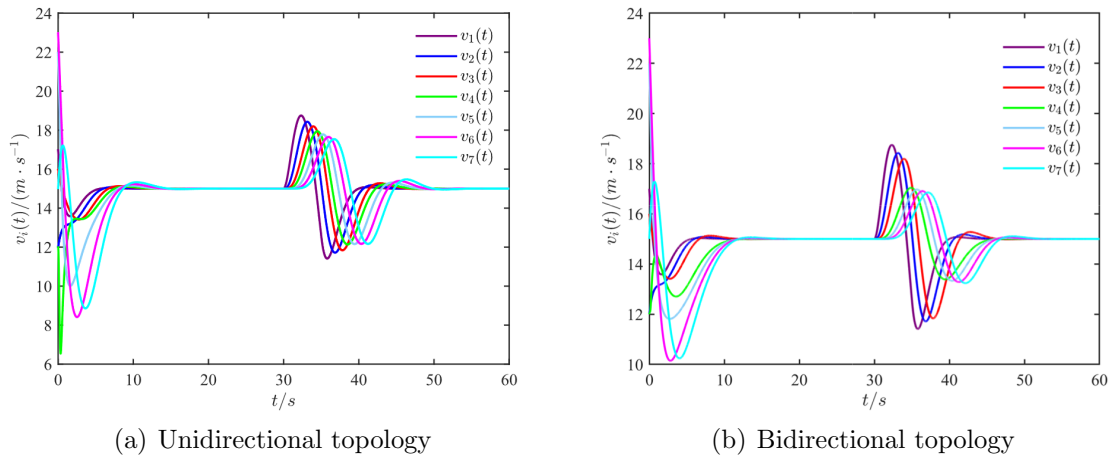


FIGURE 13. (color online) The evolution curves of velocity under different topological structures of the mixed vehicle platoon

of velocity under different topological structures of the mixed vehicle platoon. It can be seen that when the convergence speed is almost the same, compared with the unidirectional topology, the oscillation amplitude of the vehicle speed evolution curve under the bidirectional topology is generally smaller. It can be clearly seen from Figure 13(b) that the disturbance acting on the first following vehicle has significantly weakened when passing through the CAV in the mixed vehicle platoon, which once again confirms that the CAV in the mixed vehicle platoon has a strong anti-disturbance ability.

Figure 14 shows the evolution curves of the mean values of the headway distance error and the velocity error corresponding to the two topologies. According to Figure 14, compared with the unidirectional topology, the oscillation amplitude of the error mean evolution curves in the bidirectional topology is smaller when the convergence speed is almost the same, which is consistent with the conclusion obtained in Figure 13. Therefore, under the condition that other factors are the same, compared with the unidirectional topology, the bidirectional topology has a stronger ability to resist disturbance and the stability of the mixed vehicle platoon system is better.

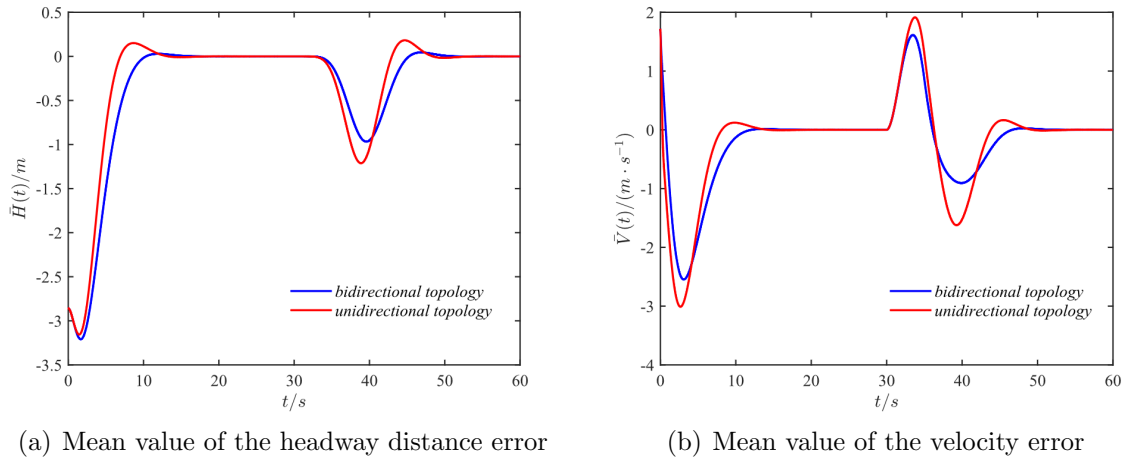


FIGURE 14. (color online) The evolution curves of the mean error under different topological structures

6.4. The influence of multiple front and rear vehicles on stability. This section analyzes the influence of the number of front and rear vehicles that can obtain information from CAV on the stability of the mixed vehicle platoon system. The geometric configuration of the mixed vehicle platoon as shown in Figure 8 is selected and it is assumed that the disturbance only acts on the first following vehicle. The evolution curves of the mean values of the headway distance error and the velocity error are shown in Figure 15. As can be seen from Figure 15, the convergence speed of the error mean evolution curve is the fastest when CAV can obtain the information of three vehicles at the front and three at the back, followed by two vehicles each in the front and back. When CAV can obtain the information of one vehicle at the front and one at the back, the convergence speed of the error mean evolution curve is the slowest and the oscillation amplitude is the largest. This indicates that the greater the number of front and rear vehicles for which the CAV can obtain information, the stronger the ability of the mixed vehicle platoon to resist disturbance, and the better the system stability.

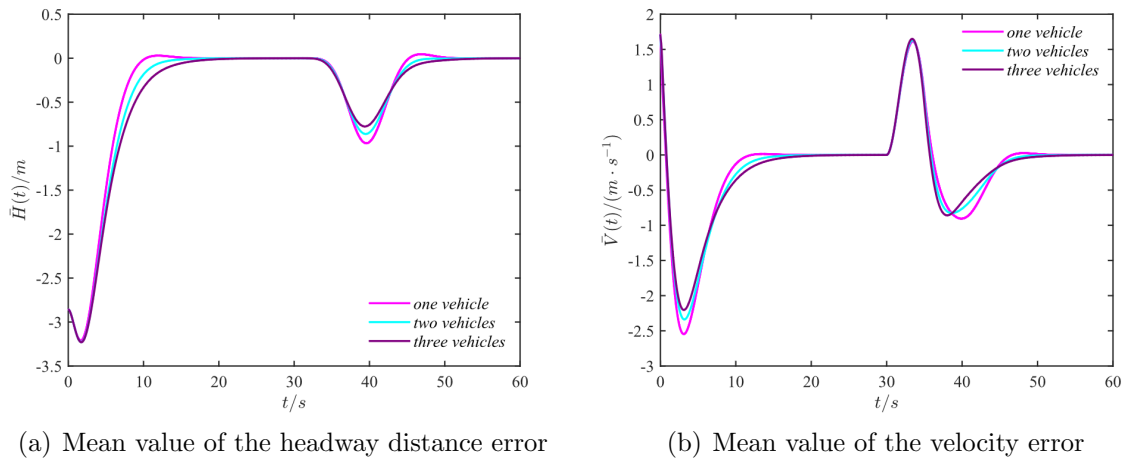


FIGURE 15. (color online) The evolution curves of the mean error under different numbers of front and rear vehicles

6.5. The influence of time delays on stability. This section analyzes the influence of driver reaction delay and vehicle communication delay on the stability of mixed vehicle platoon under external disturbance. Numerical simulations are conducted with three different delays configurations $\tau_1 = 0.1$, $\tau_2 = 0.01$, $\tau_1 = 0.2$, $\tau_2 = 0.02$ and $\tau_1 = 0.5$, $\tau_2 = 0.05$ while maintaining identical parameters and simulation conditions. Using configuration $\tau_1 = 0.5$, $\tau_2 = 0.05$, Figure 16 illustrates the evolution curves of headway distance and velocity between two adjacent vehicles in the mixed vehicle platoon considering time delays under external disturbance. As can be seen from Figure 16, compared to the no delay scenario shown in Figure 6, the oscillation amplitude significantly increases and the convergence time substantially prolongs after disturbance occurrence. This demonstrates that time delays adversely affect the stability of mixed vehicle platoon under external disturbance. Nevertheless, under the action of the consensus controller in this work, the system ultimately regains stability, confirming that the proposed controller considering delays maintains the strong anti-disturbance ability.

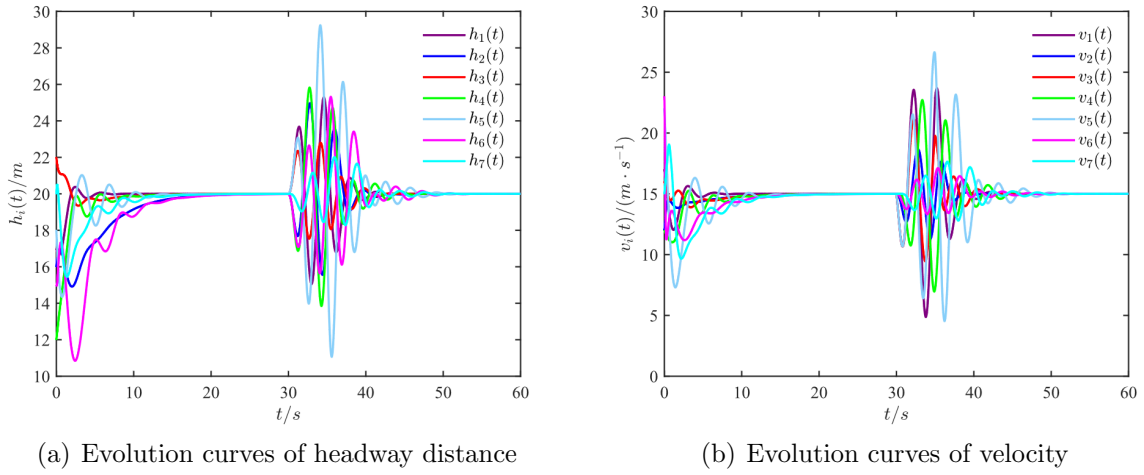


FIGURE 16. (color online) The evolution curves of headway distance and velocity between two adjacent vehicles in the mixed vehicle platoon considering time delays under external disturbance

The evolution curves of the mean values of the headway distance error $\bar{H}(t)$ and the velocity error $\bar{V}(t)$ are plotted for the three delays configurations $\tau_1 = 0.1$, $\tau_2 = 0.01$, $\tau_1 = 0.2$, $\tau_2 = 0.02$, $\tau_1 = 0.5$, $\tau_2 = 0.05$ and the no delay scenario, as shown in Figure 17. The results demonstrate that as driver reaction delay and vehicle communication delay progressively increase, the convergence speed of the mean error curves gradually slows down while the oscillation amplitude progressively enlarges. This further confirms the adverse effect of time delays on system stability, while verifying that the proposed controller can effectively suppress the influence of both driver reaction delay and vehicle communication delay on stability of the mixed vehicle platoon system.

7. Conclusion. Based on the heterogeneous multi-agent pinning consensus control theory, this paper studies the cooperative control problem of the mixed vehicle platoon randomly composed of HDV and CAV under the action of external disturbance. Firstly, a mixed vehicle platoon model considering external disturbance was constructed, and an adaptive controller based on heterogeneous multi-agent pinning consensus and an adaptive controller considering driver reaction delay and vehicle communication delay were proposed. Secondly, based on numerical simulation experiments, the influences of the range of

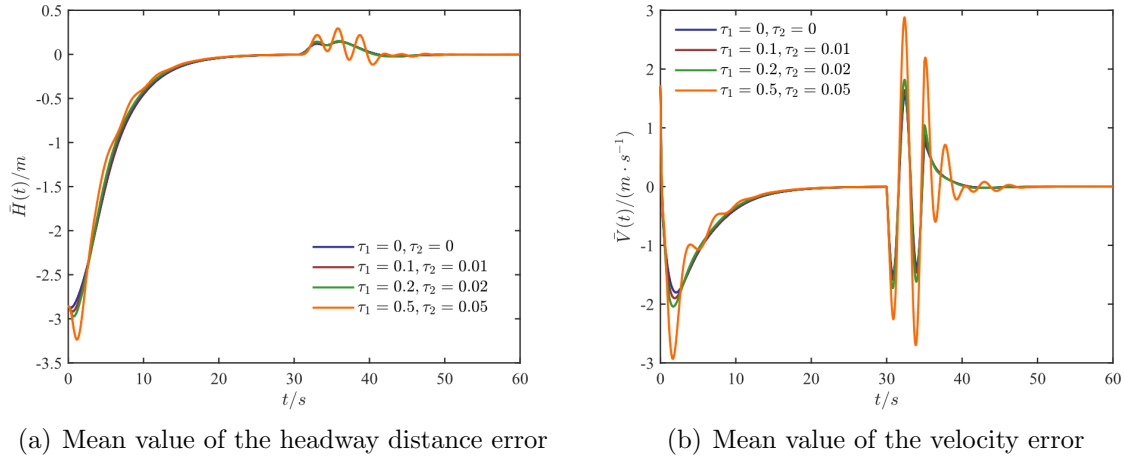


FIGURE 17. (color online) The evolution curves of the mean error in the mixed vehicle platoon considering time delays under external disturbance

the disturbance effect, the position of the disturbance effect, the topological structure, the number of front and rear vehicles with obtainable information from CAV and driver reaction delay and vehicle communication delay on the stability of the mixed vehicle platoon system were analyzed in detail. The research results show that: 1) For the mixed vehicle platoon randomly composed of multiple HDVs and CAVs under the action of external disturbance, the adaptive controller based on heterogeneous multi-agent pinning consensus designed in this paper can effectively act on the platoon, enabling all vehicles in the mixed vehicle platoon to travel in an orderly manner in platoon mode. 2) The larger the range of the disturbance effect is, the more complex the dynamic response of the system is and the worse the stability is. The further forward the position of the disturbed HDV is, the worse the stability of the system will be. And due to the strong anti-disturbance ability of the CAV, the stability of the system is better when the disturbance acts directly on the CAV. 3) Compared with unidirectional topology, bidirectional topology has a stronger ability to resist disturbance, and the stability of the mixed vehicle platoon system is better. 4) The more the number of front and rear vehicles for which CAV can obtain information, the stronger the ability of the mixed vehicle platoon to resist disturbance and the better the system stability. 5) Driver reaction delay and vehicle communication delay have adverse effect on the stability of the mixed vehicle platoon system under external disturbance. The proposed controller considering time delays in this paper effectively suppresses the influence of both types of delays on the system stability, while maintaining the strong anti-disturbance ability even within time-delay system.

This study has the following limitations. 1) In modeling the mixed vehicle platoon system, the car-following models used for HDV and CAV are deterministic models derived from previous researches. However, in real-world traffic environment, vehicle motion states are influenced by various factors, resulting in many unknown variables. Therefore, using deterministic models to characterize car-following behavior has certain limitation. 2) This study only considers homogeneous driver reaction time delay and communication time delay. However, the actual driving environment is a multi-delay and dynamic delay system composed of multiple interacting factors. Consequently, the consideration of time delays in this study also has limitation. Since this study focuses on the impact of disturbance on the stability of mixed vehicle platoon system and focuses on how to design a reasonable and efficient controller to enable the system to achieve asymptotic consensus

under disturbance, it does not delve deeply into issues of model uncertainty, multi-delay, and time-varying delay. In future research, we will introduce uncertainty system modeling method to construct the mixed vehicle platoon system model that more closely resembles reality. This will enable further investigation into the cooperative control of mixed vehicle platoon, thereby enhancing the practical applicability of the theoretical research. Additionally, we will design a consensus controller that better reflects real-world conditions based on multi-delay and time-varying delay, further improving the adaptability and robustness of the mixed vehicle platoon system in complex real-world traffic scenario.

Acknowledgment. This work is supported by the National Natural Science Foundation of China (No. 72361018), Natural Science Foundation of Gansu Province (Nos. 25JRRA160, 23JRR858), Joint Innovation Fund Project of Lanzhou Jiaotong University and Tianjin University (No. LH2023006). The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

REFERENCES

- [1] Y. Zheng, S. E. Li, K. Q. Li and L. Y. Wang, Stability margin improvement of vehicular platoon considering undirected topology and asymmetric control, *IEEE Transactions on Control Systems Technology*, vol.24, no.5, pp.1253-1265, 2015.
- [2] A. Ghasemi, R. Kazemi and S. Azadi, Stable decentralized control of a platoon of vehicles with heterogeneous information feedback, *IEEE Transactions on Vehicular Technology*, vol.62, no.9, pp.4299-4308, 2013.
- [3] F. Gao, D. Dang, S. Huang and S. Li, Decoupled robust control of vehicular platoon with identical controller and rigid information flow, *International Journal of Automotive Technology*, vol.18, no.1, pp.157-164, 2017.
- [4] S. Feng, Y. Zhang, S. E. Li, Z. Cao, H. X. Liu and L. Li, String stability for vehicular platoon control: Definitions and analysis methods, *Annual Reviews in Control*, vol.47, pp.81-97, 2019.
- [5] L. Zhang and G. Oros, Motif-based design for connected vehicle systems in presence of heterogeneous connectivity structures and time delays, *IEEE Transactions on Intelligent Transportation Systems*, vol.17, no.6, pp.1638-1651, 2016.
- [6] H. Chehardoli and A. Ghasemi, Formation control of longitudinal vehicular platoons under generic network topology with heterogeneous time delays, *Journal of Vibration and Control*, vol.25, no.3, pp.655-665, 2019.
- [7] S. Li, Q. Li and H. Wang, String stability of mixed platoon under cooperative adaptive cruise control, *Journal of Transportation Systems Engineering and Information Technology*, vol.21, no.4, pp.82-89, 2021.
- [8] M. Hu, L. Bu and H. Qin, Modeling and cooperative control of mixed vehicle platoon under multi-time delay, *Automotive Engineering*, vol.44, no.9, pp.1359-1371, 2022.
- [9] W. Du, Stability analysis and control of a car-following model in a connected and autonomous environment, *International Journal of Innovative Computing, Information and Control*, vol.19, no.3, pp.671-686, 2023.
- [10] F. Li and Y. Wang, Cooperative adaptive cruise control for string stable mixed traffic: Benchmark and human-centered design, *IEEE Transactions on Intelligent Transportation Systems*, vol.18, no.12, pp.3473-3485, 2017.
- [11] Y. Bian, Y. Zheng, E. Ren, S. E. Li, J. Wang and K. Li, Reducing time headway for platooning of connected vehicles via V2V communication, *Transportation Research Part C: Emerging Technologies*, vol.102, pp.87-105, 2019.
- [12] Y. Zheng, J. Wang and K. Li, Smoothing traffic flow via control of autonomous vehicles, *IEEE Internet of Things Journal*, vol.7, no.5, pp.3882-3896, 2020.
- [13] Y. Li, C. He, H. Zhu and T. Zheng, Nonlinear longitudinal control for heterogeneous connected vehicle platoon in the presence of communication delays, *Acta Automatica Sinica*, vol.47, no.12, pp.2841-2856, 2021.
- [14] Y. Yang, Y. Bian, M. Hu, C. Du, B. Xu and Z. Qin, Cooperative control of vehicular platoon in mixed traffic environment, *Control and Information Technology*, vol.1, pp.9-16, 2021.

- [15] Y. Bian, Y. Yang, M. Hu, C. Du, B. Xu and Z. Qin, Study on the stability of mixed vehicular platoon based on bidirectional multiple-vehicle following topologies, *China Journal of Highway and Transport*, vol.35, no.3, pp.66-77, 2022.
- [16] W. Du, C. Ma and J. Zhang, Cooperative control of mixed vehicle platoon based on pinning consensus of heterogeneous multi-agent system, *IET Intelligent Transport Systems*, vol.18, no.12, pp.2485-2501, 2024.
- [17] F. Gao, S. E. Li, Y. Zheng and D. Kum, Robust control of heterogeneous vehicular platoon with uncertain dynamics and communication delay, *IET Intelligent Transport Systems*, vol.10, no.7, pp.503-513, 2016.
- [18] Y. Zheng, S. E. Li, K. Li and W. Ren, Platooning of connected vehicles with undirected topologies: Robustness analysis and distributed H-infinity controller synthesis, *IEEE Transactions on Intelligent Transportation Systems*, vol.19, no.5, pp.1353-1364, 2018.
- [19] S. E. Li, F. Gao, K. Li, L.-Y. Wang, K. You and D. Cao, Robust longitudinal control of multi-vehicle systems – A distributed H-infinity method, *IEEE Transactions on Intelligent Transportation Systems*, vol.19, no.9, pp.2779-2788, 2018.
- [20] D. Chen, M. Zhao, D. Sun, L. Zheng, S. Jin and J. Chen, Robust H_∞ control of cooperative driving system with external disturbances and communication delays in the vicinity of traffic signals, *Physica A: Statistical Mechanics and Its Applications*, vol.542, 123385, 2020.
- [21] J. Wang, H. Zheng, J. Guo, J. Fan and K. Li, Distributed adaptive robust H_∞ control of intelligent-connected electric vehicles platooning subject to communication delay, *IET Intelligent Transport Systems*, vol.15, no.5, pp.699-711, 2021.
- [22] A. Elahi, A. Alfi and H. Modares, H_∞ consensus of homogeneous vehicular platooning systems with packet dropout and communication delay, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol.52, no.6, pp.3680-3691, 2022.
- [23] Y. Wang, S. Lin, Y. Wang, B. De Schutter and J. Xu, Robustness analysis of platoon control for mixed types of vehicles, *IEEE Transactions on Intelligent Transportation Systems*, vol.24, no.1, pp.331-340, 2023.
- [24] Y. Li, X. Wang, L. Huang, X. Huang and X. Hu, Adaptive longitudinal control of vehicle platoon based on combination spacing policy, *China Journal of Highway and Transport*, vol.37, no.2, pp.239-252, 2024.
- [25] T. Zhang and T. Shi, Vehicle platooning control based on adaptive event-triggered under random communication topologies, *Journal of Hangzhou Dianzi University (Natural Sciences)*, vol.45, no.3, pp.29-41, 2025.
- [26] L. Xu, T. Wang and G. Yin, Event-triggered quantized control of vehicle platooning system under disturbances and communication delay, *Journal of Southeast University (Natural Science Edition)*, vol.54, no.6, pp.1551-1560, 2024.
- [27] Y. Ma, W. Che, C. Deng and Z. Wu, Data-driven distributed vehicle platoon control for heterogeneous nonlinear vehicle systems, *IEEE Transactions on Intelligent Transportation Systems*, vol.25, no.3, pp.2373-2381, 2024.
- [28] M. Bando, K. Hasebe, A. Nakayama, A. Shibata and Y. Sugiyama, Dynamical model of traffic congestion and numerical simulation, *Physical Review E*, vol.51, no.2, 1035, 1995.
- [29] J. I. Ge and G. Orosz, Dynamics of connected vehicle systems with delayed acceleration feedback, *Transportation Research Part C: Emerging Technologies*, vol.46, pp.46-64, 2014.
- [30] Y. S. Moon, P. Park, W. H. Kwon and Y. S. Lee, Delay-dependent robust stabilization of uncertain state-delayed systems, *International Journal of Control*, vol.74, no.14, pp.1447-1455, 2001.

Author Biography



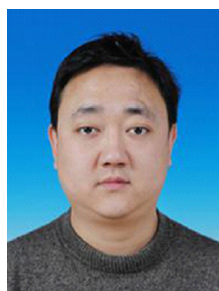
Jianxun Dong received the B.Sc. degree in Traffic and Transportation from Lanzhou Jiaotong University, China, 2023. He is currently a postgraduate at the School of Traffic and Transportation, Lanzhou Jiaotong University, China. His main research interests include intelligent connected vehicle platoon control.



Wenju Du received the B.Sc. degree in Mathematics and Applied Mathematics from Hexi University, China, 2011; the M.Sc. degree in Applied Mathematics from Lanzhou Jiaotong University, China, 2014; the Ph.D. degree in Transportation Planning and Management from Lanzhou Jiaotong University, China, in 2019. He is currently an associate professor at the School of Traffic and Transportation, Lanzhou Jiaotong University, China. His main research interests include the traffic flow modeling and intelligent connected vehicle platoon control. He has published over 20 papers on well-known journals. He is in charge of one project funded by the National Natural Science Foundation of China, one project funded by the Natural Science Foundation of Gansu Province, etc.



Binbin Peng received the B.Sc. degree in Management and a M.Sc. degree in Engineering from Tianjin University, China, in 2011 and 2014, respectively; the Ph.D. from the University of Chinese Academy of Sciences, China, in 2018. He is currently an associate professor at the College of Management and Economics, Tianjin University, China. His main research fields include energy and environmental policies, low-carbon economy and management, energy conservation and emission reduction in industries, air pollution prevention and control, and the development of renewable energy, etc.



Jiangang Zhang received the B.S. degree in School of Mathematics and Statistics of Lanzhou University, China, in 2001; M.S. degree in School of Mathematics and Physics and Ph.D. degree in School of Traffic and Transportation of Lanzhou Jiaotong University, China, in 2005 and 2011, respectively. He is currently a professor at the School of Mathematics and Physics, Lanzhou Jiaotong University, China. His research interests mainly include nonlinear dynamical system analysis and control, stability analysis and control of stochastic dynamical systems, etc. He has published over 60 high-level academic papers. As the person in charge, he has completed a number of national and provincial scientific research projects.