

SYNTHESIS DATA DRIVEN IDENTIFICATION AND CONTROL FOR INTERNAL MODEL STRUCTURE

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ABSTRACT. *This paper considers data driven synthesis analysis for internal model control strategy from both the theoretical perspective and industrial applications. Specifically, from the purpose of theoretical illustration, after internal model control structure is reviewed, one interesting equivalent property between internal model control and closed loop control is established, showing all theoretical results can be extended to more general closed loop case. Due to the existence of one necessary identified plant, the detailed modeling process for the identified plant is yielded to two explicit forms, i.e., nonparametric estimate as a linear rough value and nonlinear estimate on basis of multidimensional Taylor network structure. Based on the obtained identified plant, the feedforward controller and feedback filter are all devised through prediction error method and model reference method, respectively. Prediction error method centers around minimizing both input prediction and output prediction error, and model reference method is to achieve dual missions, i.e., perfect tracking and disturbance rejection. Roughly speaking, all theoretical results combine identification, optimization and control together into internal model control in order to complete the existing research. Finally, some simulations are given to prove our theoretical derivations about the identified plant and controller design from minimizing error, model mismatching and disturbance rejection.*

Keywords: Internal model control, Equivalence, Plant estimate, Prediction error, Model reference, Disturbance rejection

1. Introduction. Dynamical system theory provides a paradigm for modeling and studying phenomena that undergo spatial and temporal evolution. A dynamical system consists of three elements, i.e., a setting in which the dynamical behavior takes place, such as a term, topological space, manifold or locally compact metric space, a mathematical rule or dynamic which specifies the evolution of the system over time, and an initial condition from which the system starts at some initial time. During past century, dynamical system theory quickly spread from the study of astronomical bodies to modeling and studying numerous physical phenomena occurring in nature involving changes over time, and it has become one of the most important and fundamental fields of modern science and engineering. The application of dynamical systems has crossed interdisciplinary boundaries from chemistry to biochemistry to chemical kinetics, from medicine to biology to population genetics, from economics to sociology, and from physics to mechanics to engineering.

Then increasingly complex nature of engineering systems requiring feedback control to obtain a desired system behavior also gives rise to dynamical systems. Feedback control theory involves the analysis and synthesis of a feedback controller that manipulates system inputs to obtain a desired effect on the output of the system in the face of system uncertainty and system disturbance or noise. Furthermore, since most physical and engineering systems are inherently unclear, the resulting feedback dynamical system can exhibit a very rich dynamical behavior.

As in recent years, open loop control system has been replaced by closed loop control system gradually, which adds a feedback path to the input part, thus improving the performances, for example, trajectory tracking, disturbance or noise rejection, and output mismatching. Furthermore, as two controllers exist in the so-called closed loop control system, the control accuracy and other closed loop stability are also improved greatly. Based on the nice properties of closed loop control system, now closed loop control is more widely considered in both academia and practical industry than the original open loop control, and some adjustments are also proposed on the basis of the mature contributions to the closed loop control system. Internal model control, proposed in 1982, is a special closed loop control structure, due to its simple tracking, good regulation performance, strong robustness and ability to eliminate unmeasurable disturbances. Based on the structure of internal model control, its uniqueness lies in the fact that the feedback signal is not the direct output signal, but the difference signal or an error signal. This error signal corresponds to the difference between the output signals, obtained from the actual plant, and the identified plant, respectively. Here, for the sake of understanding, the output for the actual plant is named as actual output, and meanwhile, the output for the identified plant is the identified output. The error signal between the actual output and identified output is returned to the input part. Due to the appearance of the identified plant, firstly we need apply the physical modeling or system identification theory to constructing a mathematical model or equation as the identified plant, corresponding to the important modeling or identification process. It means within the detailed internal model control strategy, the first premise is to model the unknown plant; then the error, representing the identification accuracy, is returned to achieve the feedback function. When the identification accuracy is nice, i.e., zero error, internal model control is reduced to the common simple open loop control structure. Thus, the latter internal model controller is dependent on the identification accuracy of that unknown plant. However, external disturbance or noise cannot be avoided in practice, for example, closed loop input-output noise, process noise, and plant uncertainty, thus affecting the consequent identification process and internal model controller design process, so our goal in this paper is to combine identification and control into internal model control, while considering disturbance rejection and perfect tracking or model mismatching together to guarantee the achieved control accuracy.

Due to the fact that the number of references on internal model control is vast, here we only list some new contributions. [1] explores the use of control affine neural nonlinear autoregressive models for nonlinear system identification and model based control design, and [2] proposes a novel framework – neural internal model control with reinforcement learning based control to enhance robustness. A new adaptive based proportional control within the framework of internal model control is employed in various industries, including waster water treatment, water desalination and car factory [3]. Furthermore, data driven control is combined with internal model control [4], being model free, i.e., not requiring the mathematical model. [5] points out that data driven internal model control only requires the past input and output data to estimate the unknown system's behavior and then incorporates the estimate into a robust control scheme that compensates the unknown uncertainty and disturbance. Another key feature is the application

of a new adaptive internal model controller [6], avoiding the problem of overestimating the control gains. During these years, our team also has some contributions about internal model control, for example, combination with virtual reference feedback tuning and internal model control [7], recursive parameter identification for internal model control [8], optimal input signal design for internal model control [9], model structure validation [10], adaptive internal model control [11], and direct data driven internal model control [12]. Generally, internal model control corresponds to model-based control for closed loop control structure, i.e., first modeling and second control iteratively.

Based on above explanation and description on internal model control, this paper continues to do some contributions for internal model control from both points of plant identification and controller design while achieving dual missions, i.e., perfect tracking and disturbance rejection. More specifically, firstly after reviewing the detailed structure of internal model control and some preliminaries about system theory, then secondly, the equivalence between internal model control and closed loop control structure is proved to show the closed relations between the feedforward controller and feedback filter. As it is necessary to obtain the identified plant, thirdly we turn to internal model identification, i.e., applying system identification theory to getting a mathematical model, named the identified plant in nonparametric estimate and nonlinear estimate, respectively. By virtue of our identified plant, the explicit internal model controller is designed to achieve our dual missions, i.e., perfect tracking or model mismatching and disturbance rejection within two different circumstances. All above works concentrate on plant identification and controller design only through our own mathematical derivations about advanced control theory. Both plant identification and controller design communicate with each other to guarantee the idea zero tracking error and above two mentioned missions. After reviewing the internal model control structure, an interesting equivalent property between internal model control and closed loop control is established, showing all theoretical results can be extended to more general closed loop cases. Due to the existence of a necessary identified plant, the detailed modeling process for the identified plant is yielded to two explicit forms, i.e., the nonparametric estimate as a linear rough value and the nonlinear estimate on the basis of the multidimensional Taylor network structure. Based on the obtained identified plant, the feedforward controller and feedback filter are all devised through prediction error method and model reference method, respectively. Prediction error method centers around minimizing both input prediction and output prediction error, and model reference method is to achieve dual missions, i.e., perfect tracking and disturbance rejection. Roughly speaking, all theoretical results combine identification, optimization and control together into internal model control in order to complete the existing research.

Generally, new contributions of this paper are formulated as follows.

- 1) Equivalence is proven to show internal model control is a special closed loop control structure.
- 2) Nonparametric estimate and nonlinear estimate are proposed to identify the unknown plant, corresponding to its linearity of the nonlinear form.
- 3) Internal model controller is designed from dual aspects of perfect tracking and disturbance rejection through our mathematical derivations.

Due to two important parts within the internal model control structure, i.e., internal model and internal model control, two missions or goals always exist to be internal model identification and internal model controller design, discussed in our paper. Specifically, firstly the internal model is identified from the points of the nonparametric estimate and nonlinear estimate only through the left and right observed data, i.e., data driven identification. Secondly, the internal model controller is designed by data driven control too,

corresponding to the predictive error method from minimization of prediction error, and the model reference method for model mismatching or disturbance rejection. Generally, whatever internal model identification and internal model controller design, data driven strategy is proposed to achieve above two related parts, while combining identification, control and optimization.

2. Internal Model Control Principle. As all industrial plants operate within the framework of the closed loop system, replacing the single open loop system gradually, our considered internal model control structure, plotted in Figure 1, is also a special closed loop system, which returns an output error signal, not the direct output signal.

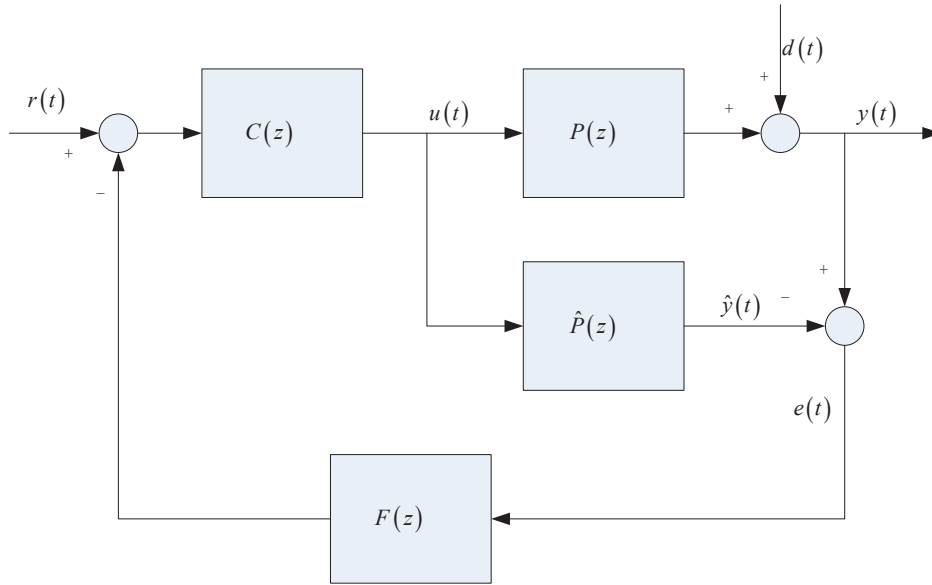


FIGURE 1. Internal model control structure

In Figure 1, $\{r(t), y(t)\}$ are the entire closed loop input-output, and $\{C(z), P(z)\}$ correspond to the feedforward controller and actual plant. $F(z)$ means the feedback filter, $u(t)$ is the output of feedforward controller $C(z)$. External noise $d(t)$ appears in an additive form.

The goal of controller design is to design $C(z)$ while satisfying one necessary optimality. However, as it requires some prior information about the plant $P(z)$, we need spend time to model the plant $P(z)$ by the approximate model or identified plant $\hat{P}(z)$. Observing Figure 1 again, when using the same signal $u(t)$ to excite the actual plant $P(z)$ and the identified plant $\hat{P}(z)$, respectively, their error signal $e(t)$ between two outputs $\{y(t), \hat{y}(t)\}$, being $e(t) = y(t) - \hat{y}(t)$, is returned to pass through the feedback filter $F(z)$. In addition, variable z means time delay operator.

Remark 2.1. Within the above internal model control structure, as the feedback signal $e(t)$ means an error signal, i.e., $e(t) = y(t) - \hat{y}(t)$, if the identified plant $\hat{P}(z)$ equals its actual plant $P(z)$, i.e., $P(z) = \hat{P}(z)$, then the error signal $e(t)$ is reduced to the external disturbance $d(t)$, for example,

$$e(t) = y(t) - \hat{y}(t) = P(z)u(t) + d(t) - \hat{P}(z)u(t) = d(t)$$

Within the case of no external disturbance $d(t) = 0$, and perfect identification $P(z) = \hat{P}(z)$, our considered internal model control structure is the simple open loop system, i.e., $e(t) = 0$.

From Figure 1, some existing mathematical relations hold very easily.

$$\begin{aligned} y(t) &= P(z)u(t) + d(t); \quad \hat{y}(t) = \hat{P}(z)u(t); \\ e(t) &= y(t) - \hat{y}(t) = P(z)u(t) + d(t) - \hat{P}(z)u(t) = [P(z) - \hat{P}(z)] u(t) + d(t) \end{aligned} \quad (1)$$

and

$$\begin{aligned} u(t) &= C(z)[r(t) - F(z)e(t)]; \\ y(t) &= P(z)C(z)r(t) - P(z)C(z)F(z) [P(z) - \hat{P}(z)] u(t) - P(z)C(z)F(z)d(t) \\ u(t) &= C(z)r(t) - C(z)F(z) [P(z) - \hat{P}(z)] u(t) - C(z)F(z)d(t) \end{aligned} \quad (2)$$

After simple but tedious computations, we have

$$y(t) = \frac{P(z)C(z)r(t)}{1 + C(z)F(z) [P(z) - \hat{P}(z)]} + \frac{1 - C(z)F(z)\hat{P}(z)}{1 + C(z)F(z) [P(z) - \hat{P}(z)]} d(t) \quad (3)$$

Error signal $e(t)$ is

$$e(t) = y(t) - \hat{y}(t) = \frac{C(z) [P(z) - \hat{P}(z)] r(t) + d(t)}{1 + C(z)F(z) [P(z) - \hat{P}(z)]} \quad (4)$$

Controller output $u(t)$ is

$$u(t) = \frac{C(z)r(t) - C(z)F(z)d(t)}{1 + C(z)F(z) [P(z) - \hat{P}(z)]} \quad (5)$$

Considering the above three Equations (3), (4) and (5), the identified plant $\hat{P}(z)$ appears in all physical relations, telling us before doing the controller design, firstly the identified plant $\hat{P}(z)$ is needed by virtue of system identification theory.

3. Equivalence. Before identifying the unknown plant $P(z)$, the equivalence between our considered internal model control structure in Figure 1 and the classical simple closed loop control structure, plotted in Figure 2, is proven.

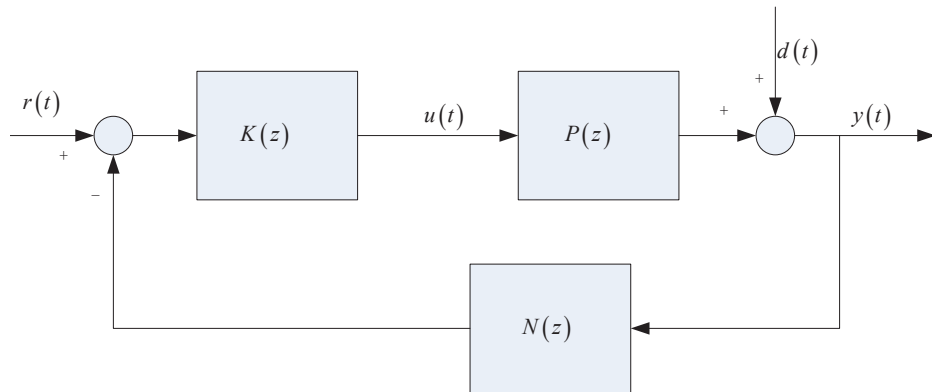


FIGURE 2. Classical closed loop control structure

In Figure 2, $\{K(z), N(z)\}$ are the feedforward controller and feedback controller, and other physical variables are similar to those in Figure 1.

The input-output relation in Figure 2 holds that

$$y(t) = P(z)u(t) + d(t) = P(z)K(z)r(t) - P(z)K(z)N(z)y(t) + d(t) \quad (6)$$

i.e.,

$$y(t) = \frac{P(z)K(z)r(t) + d(t)}{1 + P(z)K(z)N(z)} \quad (7)$$

Comparing closed loop output Equations (3) and (7), we let

$$\begin{aligned} \frac{P(z)C(z)}{1 + C(z)F(z) [P(z) - \hat{P}(z)]} &= \frac{P(z)K(z)}{1 + P(z)K(z)N(z)}; \\ \frac{1}{1 + P(z)K(z)N(z)} &= \frac{1 - C(z)F(z)\hat{P}(z)}{1 + C(z)F(z) [P(z) - \hat{P}(z)]} \end{aligned} \quad (8)$$

i.e.,

$$K(z) = \frac{C(z)}{1 + C(z)F(z) [P(z) - \hat{P}(z)] - C(z)P(z)N(z)} \quad (9)$$

Continuing to derive, we have the following relations:

$$\begin{aligned} 1 + P(z)K(z)N(z) &= \frac{1 + C(z)F(z) [P(z) - \hat{P}(z)]}{1 - C(z)F(z)\hat{P}(z)}; \quad K(z)N(z) = \frac{C(z)F(z)}{1 - C(z)F(z)\hat{P}(z)}; \\ N(z) &= \frac{F(z) + C(z)F^2(z) [P(z) - \hat{P}(z)]}{1 + C(z)F(z)\hat{P}(z) + C(z)N(z)P(z)} + F(z) \end{aligned} \quad (10)$$

From Equation (9), it holds that

$$K(z) = \frac{C(z)}{1 - C(z)F(z)\hat{P}(z)}; \quad N(z) = F(z) \quad (11)$$

The above Equation (11) shows the closed relation between $\{K(z), N(z)\}$ and $\{C(z), F(z)\}$, formulated in Figure 3. Moreover, based on their relations, the internal model

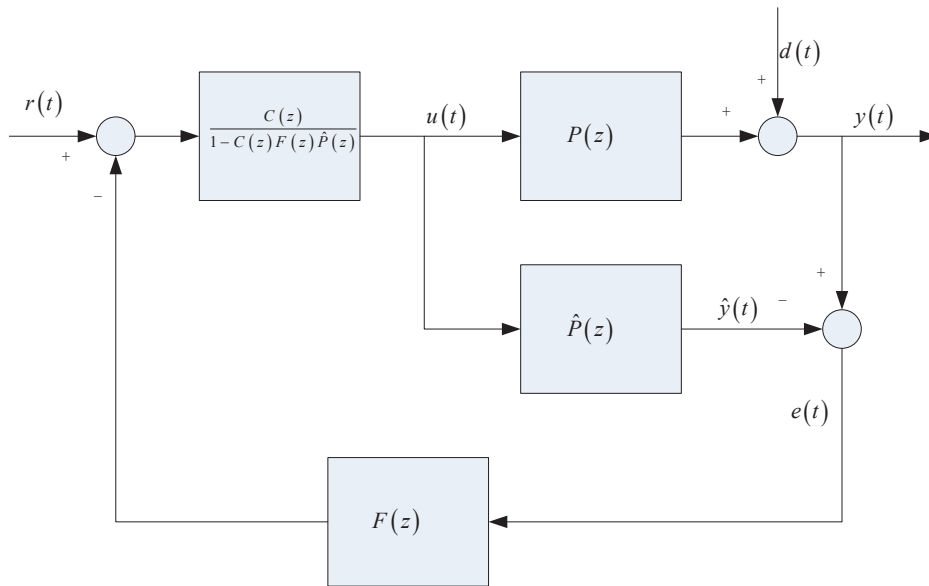


FIGURE 3. Equivalent relation between the internal model and the closed loop structure

control structure is equivalent to the classical closed loop control structure, only through some single substitution operation.

4. Internal Model Identification. Let us turn back to the internal model control structure in Figure 1; three problems are ongoing to solve, i.e., plant identification $\hat{P}(z)$, feedforward controller design $C(z)$ and feedback filter choice $F(z)$, covered in our own contributions.

4.1. Nonparametric estimate. To obtain a rough estimate for the plant $\hat{P}(z)$, we observe that

$$y(t) = P(z)u(t) + d(t); \quad y(t)u(t) = P(z)u^2(t) + u(t)d(t) \quad (12)$$

or

$$y^2(t) = \|P(z)\|^2 u^2(t) + 2P(z)u(t)d(t) + d^2(t) \quad (13)$$

Taking expectation operation on both sides, we have

$$\begin{aligned} \phi_{yu}(w) &= P(z)\phi_u(w) + \phi_{ud}(w); \quad \phi_y(w) = \|P(z)\|^2 \phi_u(w) + 2P(z)\phi_{ud}(w) + \sigma^2; \\ \phi_y(w) - P(z)\phi_u(w) &= P(z)\phi_{ud}(w) + \sigma^2 \end{aligned} \quad (14)$$

where $\{\phi_u(w), \phi_y(w), \phi_{ud}(w)\}$ are three spectrums for three data sequences $\{u(t), y(t), d(t)\}$, and $\sigma^2 = E[d^2(t)]$ means autocovariance of the external disturbance $d(t)$.

Then a rough estimate for the identified plant $\hat{P}(z)$ is yielded as follows:

$$\hat{P}(z) = \frac{\phi_u(w) - \sigma^2}{\phi_{ud}(w) + \phi_{yu}(w)} \quad (15)$$

Furthermore, collecting a group of input-output data $\{u(t), y(t)\}_{t=1}^N$, where N is the total number of data, the identified plant $\hat{P}(z)$ must satisfy the following data fitting problem, i.e.,

$$\hat{P}(z) = \arg \min_{P(z)} \frac{1}{N} \sum_{t=1}^N [y(t) - P(z)u(t)]^2 \quad (16)$$

After differentiating with respect to the unknown plant, it holds that

$$\frac{2}{N} \sum_{t=1}^N [y(t) - P(z)u(t)]u^T(t) = 0; \quad \frac{2}{N} \sum_{t=1}^N y(t)u^T(t) = \frac{2}{N} \sum_{t=1}^N P(z)u(t)u^T(t) \quad (17)$$

i.e.,

$$\hat{P}(z)\phi_u(w) = \phi_{yu}(w); \quad \hat{P}(z) = \frac{\phi_{yu}(w)}{\phi_u(w)} \quad (18)$$

Equation (15) or (18) gives a rough estimate for $\hat{P}(z)$ only through spectrum operations, related with input-output data $\{u(t), y(t), d(t)\}_{t=1}^N$.

Remark 4.1. Based on these two rough estimates $\hat{P}(z)$ in Equations (15) and (18), after exciting the whole closed loop system with one sine signal, and then collecting data sequences $\{u(t), y(t), d(t)\}_{t=1}^N$ or $\{u(t), y(t)\}_{t=1}^N$ to formulate their own power spectrums, the rough estimate $\hat{P}(z)$ is yielded only through the simple ratio and spectrum operations.

4.2. Nonlinear estimate. The above rough spectrum estimate for the identified plant $\hat{P}(z)$ is to extract an explicit form from the input-output data sequences $\{u(t), y(t)\}_{t=1}^N$, corresponding to nonparametric estimate. However, it is not an unbiased estimate, thus bringing additional work to do some improvements to guarantee its unbiased property.

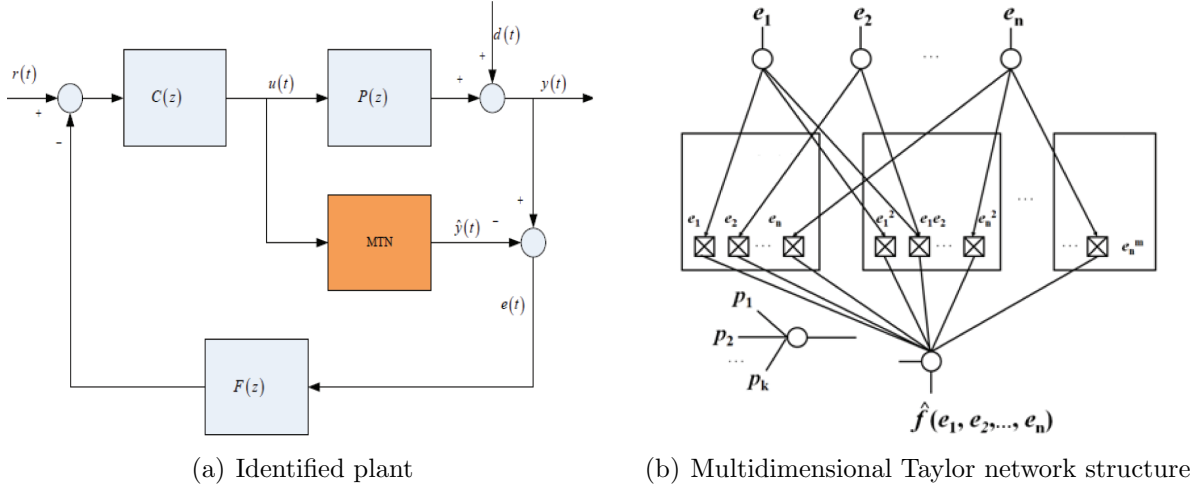


FIGURE 4. Nonlinear estimate based on MTN

It is well known that all natural phenomena are nonlinearities during our normal life, so it is necessary to model the unknown plant as a nonlinear estimate by virtue of the multidimensional Taylor network (MTN) structure, plotted in Figure 4.

Observing Figure 4(a), the identified plant $\hat{P}(z)$ is a nonlinear plant, i.e., $\hat{P}(z, u(t))$, and then the input-output relation of the identified plant $\hat{P}(z)$ is $\hat{y}(t) = \hat{P}(z, u(t))$, $t = 1, 2, \dots, N$. For analysis purpose to expand this nonlinearity $\hat{P}(z, u(t))$, a multidimensional Taylor network (MTN) structure is proposed to approximate the original nonlinear identified plant $\hat{P}(z, u(t))$. To the best of knowledge, multidimensional Taylor network is a new type of fixed structure controller based on Taylor series expansion. Its core concept is to use high order nonlinear expansion to achieve the accurate approximation and control accuracy. Multidimensional Taylor network gives a combination of zero order terms, first order terms and high order terms for different high order external input signals and chooses the weighted sum of these terms to represent the consequent control laws. A multidimensional Taylor network is seen in Figure 4(b), where three layers exist, i.e., input layer, hidden layer and output layer.

More precisely, input layer contains p inputs to receive external information, and hidden layer is responsible for processing the input information of entire network, composed of the combination of p inputs from the input layer. Output layer analyzes each layer to obtain the relationship among all nodes in each layer and the corresponding feedback mechanism. For our MTN approximation, we choose error $u(t)$, its differentiation $\dot{u}(t)$, and integration $\int u(t)$ as three input signals to excite MTN. Assume the maximal output power is 3, and the total number of terms in the expansion reaches 16, i.e., representing the nonlinear MTN output in an explicit form:

$$\begin{aligned}
 e_1(t) &= u(t); \quad e_2(t) = \dot{u}(t); \quad e_3(t) = \int u(t); \\
 u(t) &= \theta_1 e_1(t) + \theta_2 e_2(t) + \theta_3 e_3(t) + \theta_4 e_1(t)e_2(t) + \theta_5 e_1(t)e_3(t) + \theta_6 e_2(t)e_3(t) \\
 &\quad + \theta_7 e_1^2(t) + \theta_8 e_2^2(t) + \theta_9 e_3^2(t) + \theta_{10} e_1(t)e_2^2(t) + \theta_{11} e_1(t)e_3^2(t) + \theta_{12} e_2(t)e_3^2(t) \\
 &\quad + \theta_{13} e_1^3(t) + \theta_{14} e_2^3(t) + \theta_{15} e_3^3(t) + \theta_{16} e_1(t)e_2(t)e_3(t) = \varphi(t)^T \theta
 \end{aligned} \tag{19}$$

where the above nonlinear MTN output $\hat{y}(t)$ is also a regression form, the regressor $\varphi(t)$ and weight vector θ are defined as

$$\varphi(t) = [e_1(t) \quad e_2(t) \quad \cdots \quad e_1(t)e_2(t)e_3(t)]; \quad \theta = [\theta_1 \quad \theta_2 \quad \cdots \quad \theta_{16}] \tag{20}$$

Remark 4.2. Equation (19) tells us that although the MTN output is a nonlinear form, after reformulation, it is simplified into a linear relation with respect to the network weights θ and all nonlinear variables are stacked into the regressor vector $\varphi(t)$. Moreover, as all elements of the regressor vector $\varphi(t)$ are composed of $\{u(t), \dot{u}(t), \int u(t)\}$, based on data $\{u(t), y(t)\}_{t=1}^N$, variables $\{u(t), \varphi(t)\}_{t=1}^N$ are known or collected in priori, thus resulting in a linear least squares problem again.

To improve the approximation accuracy, an easy way is to adjust those unknown network weights, i.e., the vector θ through solving the following least squares problem.

$$\hat{\theta} = \arg \min_{\theta} J(\theta) = \arg \min_{\theta} \frac{1}{N} \sum_{t=1}^N [y(t) - \varphi(t)^T \theta]^2$$

$$\{u(t), y(t)\}_{t=1}^N \rightarrow \{y(t), \varphi(t)\}_{t=1}^N \text{ are all known} \quad (21)$$

For the above unconstrained optimization problem, the weight estimations are

$$\hat{\theta} = \left[\sum_{t=1}^N \varphi(t)^T \varphi(t) \right]^{-1} \left[\sum_{t=1}^N \varphi(t)^T y(t) \right] \quad (22)$$

Solution (22) is enough from an applied point of view. When to analyze its statistical property, straightforward differentiation gives

$$\frac{\partial J(\theta)}{\partial \theta} = -\frac{2}{N} \sum_{t=1}^N [y(t) - \varphi(t)^T \theta] \varphi(t); \quad \frac{\partial^2 J(\theta)}{\partial \theta^2} = \frac{2}{N} \sum_{t=1}^N \varphi(t)^T \varphi(t) \quad (23)$$

Then the recursive form is

$$\hat{\theta}_{k+1} = (1 - \gamma) \hat{\theta}_k + \gamma \left[\sum_{t=1}^N \varphi(t)^T \varphi(t) \right]^{-1} \left[\sum_{t=1}^N \varphi(t)^T y(t) \right] \quad (24)$$

Set a true weight vector θ^* as

$$\theta^* = \left[\sum_{t=1}^N \varphi(t)^T \varphi(t) \right]^{-1} \left[\sum_{t=1}^N \varphi(t)^T y(t) \right] \quad (25)$$

Then

$$\hat{\theta}_{k+1} = (1 - \gamma) \hat{\theta}_k + \gamma \theta^*; \quad \hat{\theta}_{k+1} - \theta^* = (1 - \gamma) (\hat{\theta}_k - \theta^*) \quad (26)$$

Set network weight error as $\tilde{\theta}_{k+1} = \hat{\theta}_{k+1} - \theta^*$. It holds that

$$\tilde{\theta}_{k+1} = \hat{\theta}_{k+1} - \theta^* = (1 - \gamma) (\hat{\theta}_k - \theta^*) = \dots = (1 - \gamma)^{k+1} \tilde{\theta}_0; \quad \tilde{\theta}_0 = \theta_0 - \theta^* \quad (27)$$

Finally, the statistical property of the least squares algorithm (26) is formulated in Theorem 4.1.

Theorem 4.1. Considering the problem of identifying the MTN weights θ , to guarantee the approximation accuracy of the least squares solution in Equation (26), stepsize γ must satisfy $0 < \gamma < 1$. Then we have $\lim_{t \rightarrow \infty} (1 - \gamma)^{k+1} = 0$ and $\tilde{\theta}_{k+1} = \hat{\theta}_{k+1} - \theta^* \rightarrow 0$, i.e., $\hat{\theta}_{k+1} \rightarrow \theta^*$.

5. Internal Model Controller Design. After applying the above two rough estimates $\hat{P}(z) = \frac{\phi_{yu}(w)}{\phi_u(w)}$ and $\hat{P}(z) = \frac{\phi_u(w) - \sigma^2}{\phi_{ud}(w) + \phi_{yu}(w)}$ and the nonlinear multidimensional Taylor network estimate $\hat{P}(z) = \varphi(t) \hat{\theta}$, we substitute them into the closed loop output response to get

$$y(t) = \frac{P(z)C(z)}{1 + C(z)F(z) [P(z) - \hat{P}(z)]} r(t) + \frac{1 - C(z)F(z)\hat{P}(z)}{1 + C(z)F(z) [P(z) - \hat{P}(z)]} d(t) \quad (28)$$

Set the identification error as

$$\Delta P = [P(z) - \hat{P}(z)] \quad (29)$$

Then Equation (28) is abbreviated as follows:

$$y(t) = \frac{P(z)C(z)}{1 + C(z)F(z)\Delta P} r(t) + \frac{1 - C(z)F(z)\hat{P}(z)}{1 + C(z)F(z)\Delta P} d(t) \quad (30)$$

Similarly

$$u(t) = \frac{C(z)}{1 + C(z)F(z)\Delta P} r(t) - \frac{C(z)F(z)}{1 + C(z)F(z)\Delta P} d(t) \quad (31)$$

Considering the problem of designing the feedforward controller $C(z)$ and feedback filter $F(z)$, prediction error method and model reference method are proposed to achieve zero error and perfect tracking, respectively.

5.1. Prediction error method. Prediction error method concerns on designing feedforward controller $C(z)$ and feedback filter $F(z)$ through minimizing both the output prediction error $e_1(t)$ and input prediction error $e_2(t)$. From the output Equation (20), its one step ahead output prediction $\hat{y}(t/t-1)$ is

$$\hat{y}(t/t-1) = \frac{P(z)C(z)}{1 - C(z)F(z)\hat{P}(z)} r(t) + \left[1 - \frac{1 + C(z)F(z)\Delta P}{1 - C(z)F(z)\hat{P}(z)} \right] y(t) \quad (32)$$

so one step ahead output prediction error $e_1(t)$ is computed as

$$e_1(t) = \frac{P(z)C(z)}{1 - C(z)F(z)\hat{P}(z)} r(t) - \frac{1 + C(z)F(z)\Delta P}{1 - C(z)F(z)\hat{P}(z)} y(t) \quad (33)$$

Similarly, from the input Equation (31), its one step ahead input prediction $\hat{u}(t/t-1)$ and one step ahead input prediction error $e_2(t)$ are derived as follows:

$$\begin{aligned} \hat{u}(t/t-1) &= -\frac{1}{F(z)} r(t) + \left[1 + \frac{1 + C(z)F(z)\Delta P}{C(z)F(z)} \right] u(t); \\ e_2(t) &= u(t) - \hat{u}(t/t-1) = -\frac{1}{F(z)} r(t) + \frac{1 + C(z)F(z)\Delta P}{C(z)F(z)} u(t) \end{aligned} \quad (34)$$

From the above two one step ahead prediction errors $e_1(t)$ and $e_2(t)$, $\{C(z), F(z)\}$ are devised to minimize the following total prediction error:

$$\begin{aligned} \{C(z), F(z)\} &= \arg \min_{\{C(z), F(z)\}} \frac{1}{N} \sum_{t=1}^N \left[\frac{P(z)C(z)}{1 - C(z)F(z)\hat{P}(z)} r(t) - \frac{1 + C(z)F(z)\Delta P}{1 - C(z)F(z)\hat{P}(z)} y(t) \right]^2 \\ &\quad + \frac{1}{N} \sum_{t=1}^N \left[\frac{1}{F(z)} r(t) - \frac{1 + C(z)F(z)\Delta P}{C(z)F(z)} u(t) \right]^2 \end{aligned} \quad (35)$$

Equation (35) corresponds to an unconstrained optimization problem; thus, lots of existing optimization algorithms can be applied directly, such as gradient algorithm, Newton algorithm, and interior point algorithm.

5.2. Model reference method. As prediction error method only minimizes both prediction errors without considering any other factors, in reality, two factors must be considered, i.e., model mismatching and disturbance rejection. Specifically, the reference model $M(z)$ is given as follows:

$$M(z) = \frac{b_1 z^{n-1} + \dots + b_{n-1} z + b_n}{z^n + a_1 z^{n-1} + \dots + a_{n-1} z + a_n} \quad (36)$$

which is a rational transfer function.

The goal of model mismatching is to guarantee the transfer function from input to output be equal to the given reference model $M(z)$, while rejecting the effect of the external disturbance $d(t)$, i.e., letting

$$\frac{P(z)C(z)}{1 + C(z)F(z)\Delta P} \rightarrow M(z); \quad \frac{1 - C(z)F(z)\hat{P}(z)}{1 + C(z)F(z)\Delta P} \rightarrow 0 \quad (37)$$

$\{C(z), F(z)\}$ are designed through minimizing the following cost function:

$$\{C(z), F(z)\} = \arg \min_{\{C(z), F(z)\}} \left\| \frac{P(z)C(z)}{1 + C(z)F(z)\Delta P} - M(z) \right\|^2 + \left\| \frac{1 - C(z)F(z)\hat{P}(z)}{1 + C(z)F(z)\Delta P} \right\|^2 \quad (38)$$

where notation $\|\cdot\|$ means the common Euclidean norm.

The second term in cost function (38) corresponds to disturbance rejection, i.e., guaranteeing zero. However, zero is an ideal case; in practice, we can let it be sufficiently small. Set a sufficiently small level as $0 < a \leq 0.5$, satisfying

$$\frac{1 - C(z)F(z)\hat{P}(z)}{1 + C(z)F(z)\Delta P} = a \text{ or } \leq 0.5 \quad (39)$$

i.e.,

$$1 + C(z)F(z)\Delta P = \frac{1 - C(z)F(z)\hat{P}(z)}{a} \quad (40)$$

Substituting Equation (40) into model mismatching problem, it holds that

$$aP(z)C(z) = M(z) \left[1 - C(z)F(z)\hat{P}(z) \right] = M(z) - M(z)C(z)F(z)\hat{P}(z) \quad (41)$$

and

$$C(z) = \frac{M(z)}{aP(z) + M(z)C(z)F(z)\hat{P}(z)} = \frac{M(z)}{(a + M(z)C(z)F(z))\hat{P}(z)} \quad (42)$$

where the following equities are used in above equation

$$\begin{aligned} \frac{1}{1 + C(z)F(z)\Delta P} &= \frac{a}{1 - C(z)F(z)\hat{P}(z)}; \quad 1 + C(z)F(z)\Delta P = \frac{1 - C(z)F(z)\hat{P}(z)}{a}; \\ \frac{P(z)C(z)}{1 + C(z)F(z)\Delta P} &= \frac{P(z)C(z)a}{1 - C(z)F(z)\hat{P}(z)} \end{aligned} \quad (43)$$

and

$$\begin{aligned} 1 - \frac{M(z)}{aP(z) + M(z)C(z)F(z)\hat{P}(z)} F(z)\hat{P}(z) &= \frac{aP(z)}{aP(z) + M(z)C(z)F(z)\hat{P}(z)}; \\ a + aC(z)F(z)\Delta P &= 1 - C(z)F(z)\hat{P}(z); \\ \left[a\Delta P + \hat{P}(z) \right] C(z)F(z) &= 1 - a; \quad C(z)F(z) = \frac{1 - a}{a\Delta P + \hat{P}(z)} \end{aligned} \quad (44)$$

i.e.,

$$C(z) = \frac{1-a}{a\Delta P + \hat{P}(z)} \frac{1}{F(z)} \quad (45)$$

Comparing Equations (42) and (45), we have

$$C(z)F(z) = \frac{M(z)F(z)}{aP(z) + M(z)F(z)\hat{P}(z)} = \frac{1-a}{a\Delta P + \hat{P}(z)} \quad (46)$$

An easy calculation shows that

$$\begin{aligned} M(z)F(z) [a\Delta P + \hat{P}(z)] &= (1-a)aP(z) + (1-a)M(z)F(z)\hat{P}(z); \\ M(z)F(z) [a\Delta P + a\hat{P}(z)] &= (1-a)aP(z); \quad M(z)F(z) = (1-a) \end{aligned} \quad (47)$$

It holds that

$$F(z) = \frac{1-a}{M(z)} \quad (48)$$

Substituting Equation (48) into Equation (42), we have

$$C(z) = \frac{M(z)}{aP(z) + M(z)F(z)\hat{P}(z)} = \frac{M(z)}{aP(z) + (1-a)\hat{P}(z)} = \frac{M(z)}{\hat{P}(z) + a\Delta P} \quad (49)$$

Generally, the above mathematical derivations are formulated in the following Theorem 5.1.

Theorem 5.1. *Considering the internal model control structure in Figure 1, feedforward control and feedback filter $\{C(z), F(z)\}$ are designed to satisfy dual missions, i.e., perfect tracking and disturbance rejection. Given the reference model $M(z)$ and a sufficiently small level $a \leq 0.5$, $\{C(z), F(z)\}$ are chosen as follows:*

$$C(z) = \frac{M(z)}{\hat{P}(z) + a\Delta P}; \quad F(z) = \frac{1-a}{M(z)} \quad (50)$$

where the identified plant $\hat{P}(z)$ is chosen from its rough estimate or nonlinear estimate in Section 4.

Furthermore, the corresponding internal model controller is replotted in Figure 5.

6. Simulation. As internal model control includes plant identification, parameter optimization and controller design, i.e., identification, optimization and control, Figure 1 tells us if the identified plant $\hat{P}(z)$ is equal to the actual value of the true plant $P(z)$, then internal model control is reduced to the classical open loop control, i.e., $e(t) = 0$ and $u(t) = C(z)r(t)$. Due to space limit, only the nonlinear estimate based on the multidimensional Taylor network structure is simulated, i.e., applying input-output data $\{u(t), y(t)\}_{t=1}^N$ of the nonlinear internal model to construct the multidimensional Taylor network with some identified network parameters.

During the entire identification process, the following plant model $P(z)$ is considered as the actual plant:

$$P(z) = \frac{(z-0.6)(z-0.8)(z-0.2)(z+0.4)(z+0.3)}{(z-1.2)(z-1.8)(z-2)(z-0.4)(z+0.6)} \quad (51)$$

Whatever the nonparametric estimate (18) or parametric estimate (24) is used to identify the above plant model, the collected input-output data are needed to construct power spectrums $\{\phi_{yu}(w), \phi_u(w)\}$ and regressor vector $\varphi(t)$. Fifty groups of input-output

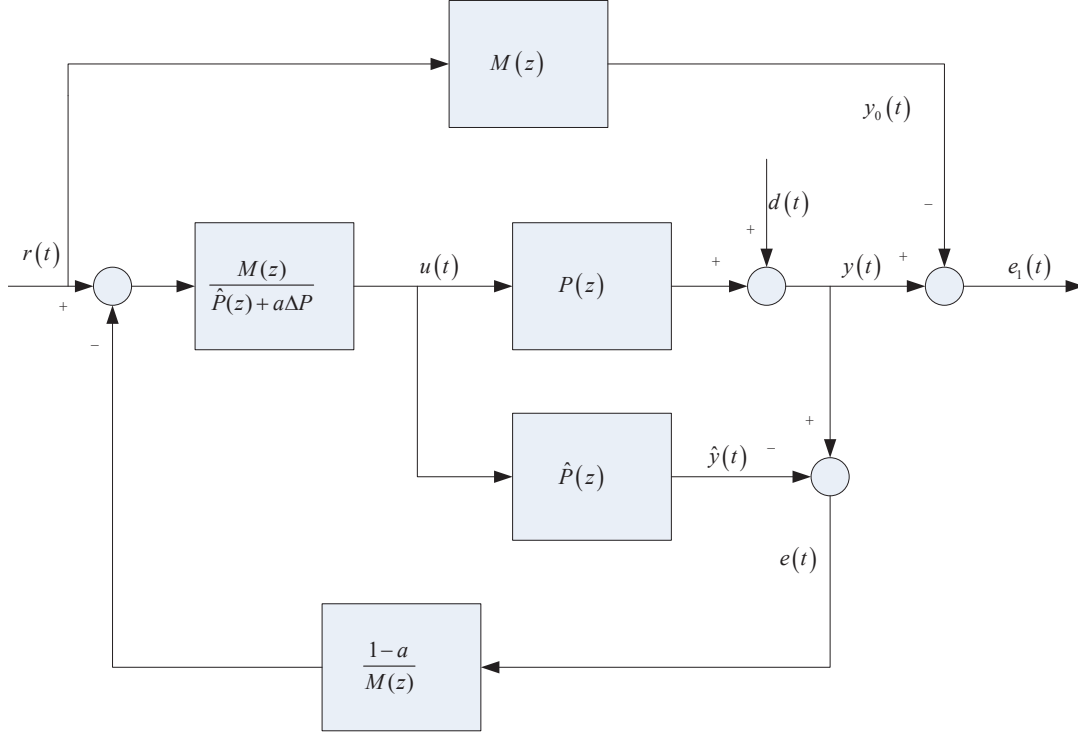


FIGURE 5. Our internal model controller

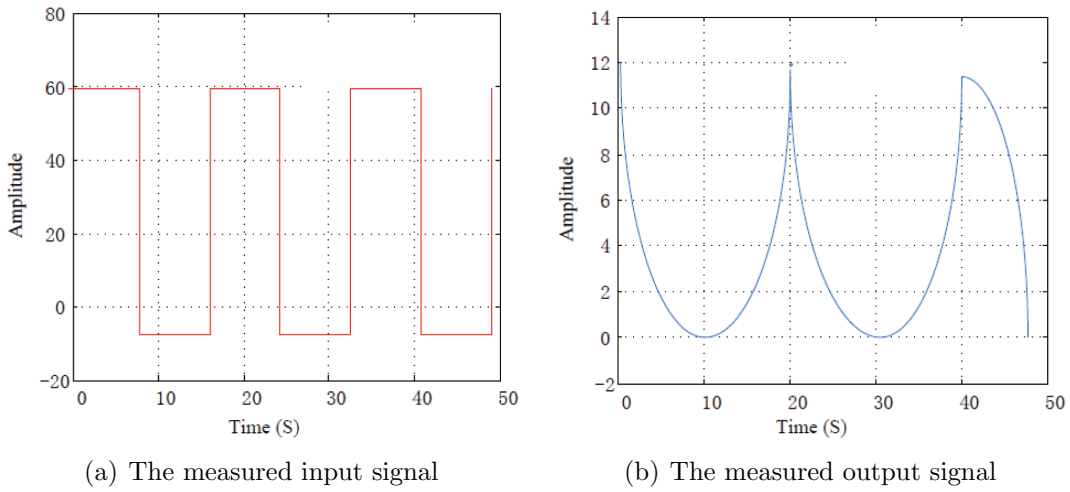


FIGURE 6. The measured input-output data

datasets $\{y(t), u(t)\}_{t=1}^{50}$ are measured through some physical devices and plotted in Figure 6, where the total number of groups is 50. Due to the space limitation, only the parametric estimate (22) is applied to identifying the plant model $P(z)$, describing as a rational transfer function form. To be convenient for using our proposed recursive parameter in Theorem 4.1, we formulate the plant parameters as a parameter vector θ , i.e.,

$$\begin{aligned} \theta &= [\theta_1 \ \theta_2 \ \theta_3 \ \theta_4 \ \theta_5 \ \theta_6 \ \theta_7 \ \theta_8 \ \theta_9 \ \theta_{10}] \\ &= [0.6 \ 0.8 \ 0.2 \ -0.4 \ -0.3 \ 1.2 \ 1.8 \ 2.0 \ 0.4 \ -0.6] \end{aligned} \quad (52)$$

and the corresponding regressor vector $\varphi(t)$ is defined as follows:

$$\begin{aligned}\varphi(t) &= \begin{bmatrix} \varphi_1(t) & \varphi_2(t) & \varphi_3(t) & \varphi_4(t) & \varphi_5(t) \\ \varphi_6(t) & \varphi_7(t) & \varphi_8(t) & \varphi_9(t) & \varphi_{10}(t) \end{bmatrix} \\ &= \begin{bmatrix} e_1(t) & e_2(t) & e_3(t) & e_1(t)e_2(t) & e_1(t)e_3(t) \\ e_2(t)e_3(t) & e_1^2(t) & e_2^2(t) & e_3^2(t) & e_1(t)e_2(t)e_3(t) \end{bmatrix}\end{aligned}$$

Based on the mentioned nonlinear plant, expressed by the MTN structure in Figure 4, the plant output signal $y(t)$ is yielded as

$$y(t) = \begin{bmatrix} e_1(t) & e_2(t) & \cdots & e_1(t)e_2(t)e_3(t) \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{10} \end{bmatrix}$$

where $\{e_1(t), e_2(t), e_3(t)\}$ are defined in Equation (19).

Rewrite the above plant output signal $y(t)$ as $y(t) = \varphi(t)\theta$. Then the classical least squares algorithm, similar to Equation (22), is applied to identifying these ten unknown parameters. Figure 7 and Figure 8 show the plant parameters' estimations and the consequent cost function in Equations (21) and (22), where we see each plant estimate will converge to its actual value respectively with iteration increase, and also the cost function is decreased to zero gradually.

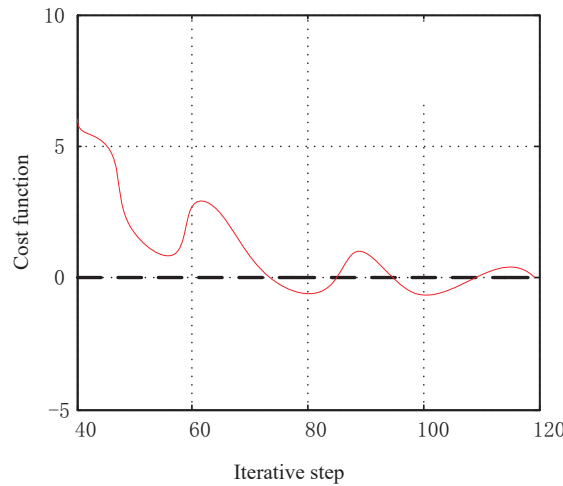


FIGURE 7. Cost function varying

7. Conclusion. Based on the existing contributions and our previous work on the internal model control strategy, this paper continues to complete the theoretical study and practical application. Firstly, the equivalence is derived to show internal model control is a special case of the classical closed loop structure, meaning our identification and control analysis are also suited for more general closed loop cases. Secondly, two explicit identification plants are derived to correspond to the rough linear nonparametric estimate and the other nonlinear estimate, respectively, where spectral knowledge, learning algorithm and multidimensional Taylor network are combined together. Thirdly, the internal model controller is designed by prediction error method and model reference method, corresponding to minimal error, perfect tracking and disturbance rejection. Fourthly, all our derived identification results and internal model controller are applied directly into the practical industry.

To the best of our knowledge, machine learning, deep learning and reinforcement learning are ongoing to affect our normal life; consequently, our future work will make the

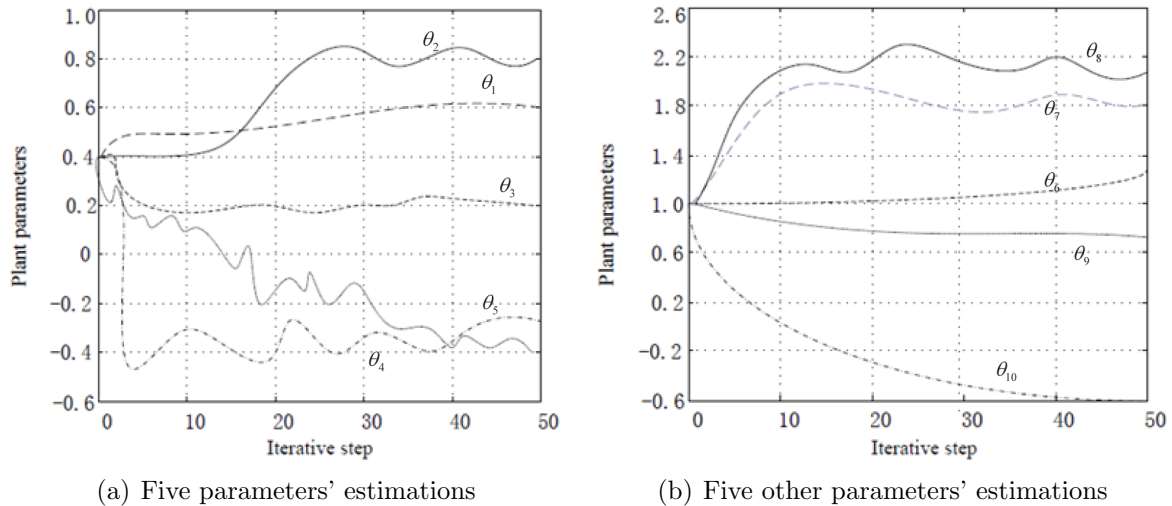


FIGURE 8. Plant parameters' estimations

controller and plant smarter in real world system through introducing other interesting concepts, for example, adaptation, safety, learning and intelligence.

REFERENCES

- [1] J. Xie, F. Bonassi and R. Scattolini, Learning control affine neural NARX models for internal model control design, *IEEE Transactions on Automation Science and Engineering*, vol.22, pp.8137-8149, 2024.
- [2] F. Gao, C. Yu, Y. Wang and Y. Wu, Neural internal model control: Learning a robust control policy via predictive error feedback, *IEEE Robotics and Automation Letters*, vol.10, no.7, pp.6848-6855, 2025.
- [3] A. Abushokor and S. M. Amrr, Model free adaptive time delay based estimation control for input saturated coupled tank system: Experimental validation, *IEEE Transactions on Automation Science and Engineering*, vol.22, pp.19340-19351, 2025.
- [4] V. K. Singh, S. Kamal, B. Bandyopadhyay, S. Ghosh and T. N. Dinh, Prescribed time optimal control of nonlinear dynamical systems with application to a coupled tank system, *IEEE Transactions on Automation Science and Engineering*, vol.22, no.3, pp.191-201, 2025.
- [5] R. Sarkar, A. Baberjec and E. Hashemi, Adaptive time delay control for cooperative platconing considering slip, *ACM Transactions on Cyber-Physical Systems*, vol.9, no.1, pp.1-22, 2025.
- [6] B. Lakshminarayanan, F. Dettu et al., Inverse supervised learning of controller tuning rules, *Automatica*, vol.178, no.2, Article ID 112356, 2025.
- [7] J. Wang and R. A. Ramirez-Mendoza, Finite sampled properties of virtual reference feedback tuning with two degrees of freedom controllers, *ISA Transactions*, vol.99, no.6, pp.37-49, 2020.
- [8] J. Wang, Y. Wang and R. A. Ramirez-Mendoza, Iterative identification and model predictive control: Theory and application, *International Journal of Innovative Computing, Information and Control*, vol.17, no.1, pp.93-109, 2021.
- [9] J. Wang, R. A. Ramirez-Mendoza and X. Tang, Robust analysis for data driven model predictive control, *Systems Science & Control Engineering*, vol.9, no.1, pp.393-404, 2021.
- [10] H. Zhu and J. Wang, Optimal data driven control tuning for unknown closed loop system, *International Journal of Innovative Computing, Information and Control*, vol.21, no.4, pp.903-917, 2025.
- [11] R. Wen, J. Wang, B. Zeng and Y. Wang, Synthesis performance analysis on data driven controller design for unknown closed loop system, *International Journal of Innovative Computing, Information and Control*, vol.21, no.4, pp.1081-1095, 2025.
- [12] J. Wang, R. A. Ramirez-Mendoza and R. Morales-Menendez, *Data Driven Strategy: Theory and Application*, CRC Press, Boca Raton, FL, USA, 2023.

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