

## COMMENTS ON JØSANG'S NORMAL COARSENING AND CONSENSUS OPERATOR

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**ABSTRACT.** *Definitions of two different ways of coarsening of basic belief assignments to opinions — the simple coarsening and the normal coarsening — are recalled in this contribution. A relation of results of combination of the normal opinions using the consensus operator to belief functions on an original  $n$ -element frame of discernment is examined. A questionable meaning of the normal coarsening is discussed.*

**Keywords:** Belief function, Coarsening, Combination of belief functions, The consensus operator

1. **Introduction.** Belief functions are one of the widely used formalisms for uncertainty representation and processing. They enable representation of incomplete and uncertain knowledge, belief updating and combination of evidence. Originally, the belief functions were introduced as a principal notion of Dempster-Shafer Theory (DST) or the Mathematical Theory of Evidence [11].

For combination of beliefs, DST uses Dempster's rule of combination. Under strict probabilistic assumptions its results are correct and probabilistically interpretable for any couple of belief functions. Nevertheless, these assumptions are rarely fulfilled in real applications. There are not so rare examples where the assumptions are not fulfilled and where the results of Dempster's rule are counter intuitive; thus, a rule with more intuitive results is required in such situations.

Hence a series of modifications of Dempster's rule was suggested and alternative approaches were created. Jøsang's 'Subjective Logic' is one of the alternative approaches to the belief function processing. To simplify their processing, all belief functions are transformed to 2-element frames of discernment, which consist of  $\emptyset \neq X \subsetneq \Theta$  and of  $\Theta \setminus X$ , i.e. the original frame of discernment  $\Theta$  is coarsened to frames consisting of two complementary parts  $\Theta^X = \{X, \Theta \setminus X\}$ . The idea of Jøsang's subjective logic is to keep an expected probability of  $X \subset \Theta$  from the original  $n$ -element frame. For this reason the so-called opinions are used. An opinion consists of belief on  $\Theta^X$  and of parameter  $a^X$  which indicates atomicity / relative cardinality of  $X$  in  $\Theta$  (Section 2).

Unfortunately, in full generality, the idea of the subjective logic does not work on general belief functions, which are constructed on coarsened frames of discernment in the usual way, as it was used in the original papers on Jøsang's subjective logic [6, 7]. Some of the problems were brought up also in [3]. Hence a different transformation<sup>1</sup> of belief

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<sup>1</sup>A new alternative to the normal coarsening — called the *smooth coarsening* has recently appeared [10]. It would be an interesting task for future research to analyse it. But it does not look like it can solve all the problems of the normal coarsening.