

AN APPLICATION OF INTERVAL-VALUED INTUITIONISTIC FUZZY SETS FOR MEDICAL DIAGNOSIS OF HEADACHE

JEONG YONG AHN¹, KYUNG SOO HAN¹, SUN YOUNG OH² AND CHONG DEUK LEE³

¹Department of Statistical Informatics (Institute of Applied Statistics)

²Department of Neurology, School of Medicine

³Division of Electronic Engineering
Chonbuk National University

Jeonju, Korea

{ jyahn; kshan; ohsun; cdlee1008 }@jbnu.ac.kr

Received January 2010; revised May 2010

ABSTRACT. *This study is to propose a new approach for medical diagnosis using the distance between interval-valued intuitionistic fuzzy sets. For this purpose, we developed an interview chart with interval fuzzy degrees based on the relation between symptoms and diseases (three types of headache), and utilized the interval-valued intuitionistic fuzzy weighted arithmetic average operator to aggregate fuzzy information from the symptoms. In addition, we proposed a measure based on distance between interval-valued intuitionistic fuzzy sets for medical diagnosis. The proposed method is illustrated by a numerical example.*

Keywords: Diagnosis measure, Aggregate operator, Interview chart, Interval-valued fuzzy sets

1. **Introduction.** A major task of medical science is to diagnose diseases. It, however, is not a direct and simple task at all, because the information available to the physician about his patient and about medical relationships in general is inherently uncertain [1]. To improve the problem, there have proposed many approaches and theories such as fuzzy set theory and rough set theory [10, 20, 35].

Fuzzy set theory makes it possible to define the inexact medical information as fuzzy sets, therefore, it can be utilized for modeling the diagnostic process. An application of fuzzy set on medical science fields already proposed by Zadeh in 1969 [36] and Sanchez [23] invented a fully developed relationships modelling theory of symptoms and diseases using fuzzy sets. Since Atanassov [4] introduced the concept of intuitionistic fuzzy sets, fuzzy set theory has been utilized in many approaches to model the diagnostic process [1, 3, 11, 14, 27, 33, 34].

However, the approaches have some drawbacks. First, some researches such as De *et al.* [11] and Ahn *et al.* [3] applied the max-min-max composition rule to determine the disease of patients. The main problem of the method using the max-min-max compositions is the loss of information because the composition neglects in fact most values except for extreme ones. Second, a disease in general is presented through many symptoms and the symptoms significantly associated with the disease. Therefore, we need to aggregate the symptoms. This is not considered in many studies. In addition, most researches for medical diagnosis don't use the interval data.

To solve these problems, we propose a new approach for medical diagnosis using the distance between interval-valued intuitionistic fuzzy sets. The features and advantages of the approach are as follows: First, it makes a diagnosis by aggregating the information

of many symptoms. Second, it uses the distance of interval data to reduce the loss of information. Third, we developed an interview chart for preliminary diagnosis. Therefore, the approach can be easily applied in practice through a computer program module.

As an extension of our previous studies [3, 15] the proposed method is applied to several patients according to the types of headache. Headache, one of the most common reasons for neurological consultation, is a condition of pain in the head and sometimes neck or upper back pain may also be interpreted as headache. There are two categories of headache: primary and secondary headache. Primary headache is not associated with other diseases. Examples of primary headache are migraine, tension and cluster headache. Secondary headache is caused by associated diseases.

For medical diagnosis of headache, we develop an interview chart with interval fuzzy degrees based on the relation among symptoms and three types of headache (migraine, tension and cluster). Second, we use the interval-valued intuitionistic fuzzy weighted arithmetic average operator to aggregate fuzzy information from the symptoms. Last, we propose a measure based on distance between interval-valued intuitionistic fuzzy sets for medical diagnosis. At the end the practicality of the diagnosis method is illustrated by a numerical example.

2. Preliminaries.

2.1. Fuzzy sets. Since Zadeh [35] introduced fuzzy sets (FS) in 1965, many approaches [13, 21, 22, 26] and theories [4, 5, 7, 28] treating imprecision and uncertainty have been proposed. Some of these theories, such as intuitionistic fuzzy sets (IFS), interval-valued fuzzy sets (IVFS), and interval-valued intuitionistic fuzzy sets (IVIFS), are extensions of FS theory and the others try to handle imprecision and uncertainty in different ways [19].

The concept of IFS has been introduced by Atanassov [4] as a generalization concept of FS. Since the first public statement of this notion was made in 1983, IFS has become a popular topic of investigation in the FS community [7, 25]. Later, Turksen [28] introduced the concept of IVFS and Atanassov and Gargov [5] introduced the concept of IVIFS, which is a generalization of the IFS. The fundamental characteristic of the IVFS and IVIFS is that the values of its membership and nonmembership function are intervals rather than exact numbers. Let us review the basic concepts of FS.

Definition 2.1. (FS) Let X is a set (space), with a generic element of X denoted by x , that is $X = \{x\}$. Then a FS is defined as Equation (1).

$$A = \{\langle x, \mu_A(x) \rangle \mid x \in X\} \quad (1)$$

where $\mu_A : X \rightarrow [0, 1]$ is the membership function of the FS A , $\mu_A(x) \in [0, 1]$ is the degree of membership of the element x to the set A .

Definition 2.2. (IFS) For a set X , an IFS A in the sense of Atanassov is given by Equation (2).

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\} \quad (2)$$

where the functions $\mu_A : X \rightarrow [0, 1]$ and $\nu_A : X \rightarrow [0, 1]$, with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$, $\forall x \in X$.

The numbers, $\mu_A(x) \in [0, 1]$ and $\nu_A(x) \in [0, 1]$, denote the degree of membership and the degree of non-membership of the element x to the set A , respectively. For each IFS A in X , the amount $\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x))$ is called the degree of indeterminacy (hesitation part), which may cater to membership value, non-membership value or both.

Definition 2.3. (IVFS) Let $[I]$ be the set of all closed subintervals of the interval $[0, 1]$ and $M = [M_L, M_U] \in [I]$, where M_L and M_U are the lower extreme and the upper extreme, respectively. For a set X , an IVFS A is given by Equation (3).

$$A = \{\langle x, M_A(x) \rangle \mid x \in X\} \quad (3)$$

where the function $M_A : X \rightarrow [I]$ defines the degree of membership of an element x to A , and $M_A(x) = [M_{AL}(x), M_{AU}(x)]$ is called an interval-valued fuzzy number.

Definition 2.4. (IVIFS) For a set X , an IVIFS A is an object having the form Equation (4).

$$A = \{\langle x, M_A(x), N_A(x) \rangle \mid x \in X\} \quad (4)$$

where $M_A : X \rightarrow [I]$ and $N_A : X \rightarrow [I]$ represent the degree of membership and non-membership, $0 \leq \sup(M_A(x)) + \sup(N_A(x)) \leq 1$, $\forall x \in X$. $M_A(x) = [M_{AL}(x), M_{AU}(x)]$ and $N_A(x) = [N_{AL}(x), N_{AU}(x)]$, so $A = ([M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)])$.

2.2. Application of fuzzy sets in medical diagnosis. The fuzzy sets have been utilized in several different approaches to model the diagnostic process [6, 9, 24, 33]. In this section, we present an application of IFS theory in Sanchez's approach for medical diagnosis [23]. He represented the physician's medical knowledge as a fuzzy relation between symptoms and diseases. The approach was elaborated by Adlassnig [1] and applied in many studies such as [11, 14].

Let $S = \{S_1, \dots, S_m\}$, $D = \{D_1, \dots, D_n\}$ and $P = \{P_1, \dots, P_q\}$ denote the sets of symptoms, diseases and patients, respectively. Two fuzzy relations (FR) Q and R are defined as Equation (5) and Equation (6).

$$Q = \{\langle (p, s), \mu_Q(p, s), \nu_Q(p, s) \rangle \mid (p, s) \in P \times S\} \quad (5)$$

$$R = \{\langle (s, d), \mu_R(s, d), \nu_R(s, d) \rangle \mid (s, d) \in S \times D\} \quad (6)$$

where $\mu_Q(p, s)$ and $\nu_Q(p, s)$ indicate the degrees for patient's symptoms, i.e., the degrees are the relationship between patient and symptoms (**patient's degrees**). In other words, $\mu_Q(p, s)$ indicates the degree to which the symptom s appears in patient p , and $\nu_Q(p, s)$ indicates the degree to which the symptom s does not appear in patient p . Similarly, $\mu_R(s, d)$ and $\nu_R(s, d)$ are the relationship between symptoms and diseases (**confirmability degrees**), i.e., $\mu_R(s, d)$ is the degree to which symptom s confirms the presence of disease d , and $\nu_R(s, d)$ the degree to which the symptom s does not confirm the presence of disease d , respectively. Note that Q is defined on the set $P \times S$ and R on the set $S \times D$. The composition T of R and Q ($T = R \circ Q$) for diagnosis of disease describes the state of patients in terms of disease as a FR from P to D given by the membership function Equation (7) and non-membership function Equation (8).

$$\mu_T(p, d) = \max_s \{\min[\mu_Q(p, s), \mu_R(s, d)]\} \quad (7)$$

$$\nu_T(p, d) = \min_s \{\max[\nu_Q(p, s), \nu_R(s, d)]\} \quad (8)$$

for all $p \in P$ and $d \in D$.

3. The Proposed Approach. In this section, we introduce the proposed approach for medical diagnosis. The approach is divided into four stages:

- stage 1: Collect the patient's degrees and confirmability degrees for patient's symptoms. Confirmability degrees are presented in the interview chart developed in this study. Patient's degrees are assigned by a physician.
- stage 2: Calculate the IIFWAA of the patient's degrees and confirmability degrees, respectively, using an aggregate operator.
- stage 3: Calculate the distance using the IIFWAA calculated in stage 2.

TABLE 1. Interview chart for migraine items

No	Items(Symptoms)	IF degree					
		migraine		tension		cluster	
		μ_{RC}	ν_{RC}	μ_{RC}	ν_{RC}	μ_{RC}	ν_{RC}
M1	Positive family history...	[0.5, 0.6]	[0.2, 0.3]	[0.2, 0.3]	[0.4, 0.6]	[0.2, 0.3]	[0.5, 0.6]
M2	At least five attacks...	[0.7, 0.8]	[0.1, 0.2]	[0.1, 0.2]	[0.6, 0.7]	[0.1, 0.2]	[0.6, 0.7]
M3	Headache lasting...	[0.5, 0.6]	[0.2, 0.3]	[0.3, 0.4]	[0.4, 0.6]	[0.1, 0.3]	[0.3, 0.5]
:	:	:	:	:	:	:	:
M23	Concurrent with...	[0.6, 0.8]	[0.1, 0.2]	[0.1, 0.2]	[0.6, 0.7]	[0.2, 0.3]	[0.6, 0.7]

- stage 4: Determine the disease of patient based on the distance.

3.1. Interview chart. A diagnosis procedure usually starts off with an interview of patient and doctor [18]. Therefore, the screening method using questionnaire is helpful in diagnosis of headache and interview chart is a leading part.

In our earlier work [2], we developed an interview chart for preliminary diagnosis of headache, where the qualitative data from the interview chart were obtained and then quantified by dual scaling. However, the method has some problems such as loss of information and insufficient use of physician's knowledge.

In the next study [3], an extended version of our previous interview chart has been implemented. In the chart, we reformed the fuzzy degrees and added some composite symptoms. The chart consisted of 22, 17 and 14 items (symptoms) for the three types of headache (migraine, tension and cluster), respectively. The chart was investigated by 5 physicians. We estimated headache labels of patients using the information obtained from the chart. Two interview charts above had an exact number in $[0, 1]$ as the membership/non-membership degrees.

In this study, we developed an improved interview chart, an interval-valued version of the interview chart developed in our previous studies, based on physician's knowledge. The chart consists of 23($M1 \sim M23$), 17($T1 \sim T17$) and 15($C1 \sim C15$) items for the three types of headache, respectively. Table 1 is the interview chart for migraine type. Each item has confirmability degrees with the relation among symptoms and the three types of headache, and has an interval-value in $[0, 1]$ as the degrees. In the chart, 7 items ($M21 \sim M23$, $T16 \sim T17$, $C14 \sim C15$) are composite symptoms. Composite symptom is a meaningful item for diagnosis of headache. For example, if a patient simultaneously has symptoms M5, M8 and M15, he/she has a composite symptom and the symptoms are displayed in the composite item M22. In the improved chart, two composite items (M23 and C15) are added in the items of previous version [15].

3.2. An aggregate operator. The chart developed in this study has 23 symptoms for the migraine type of headache and the symptoms significantly associated with the type. Likewise, the symptoms for tension and cluster significantly associated with their types, respectively. Therefore, it is general that some symptoms appear simultaneously and compositely from a patient. For example, a patient might have the symptoms $M3 \sim M6$, $C1 \sim C3$, simultaneously. In this case, we need to aggregate the interval-valued intuitionistic fuzzy information corresponding to the degrees for patient's symptoms and confirmability degrees.

Up to now, many operators have been proposed for aggregating information [8, 30, 31]. Two of the most common operators for aggregating arguments are the weighted averaging operator and the ordered weighted averaging operators.

In this study, we utilize the interval-valued intuitionistic fuzzy weighted arithmetic average (IIFWAA) operator developed by Xu [32] to aggregate fuzzy information from the symptoms. It is defined as follows.

Definition 3.1. (IIFWAA Operator) Let $A = \{(x_i, M_A(x_i), N_A(x_i)) \mid (i = 1, 2, \dots, n)\}$ be a collection of interval-valued intuitionistic fuzzy values. Then, an IIFWAA operator is defined as Equation (9).

$$\begin{aligned} IIFWAA(A) = & ([1 - \prod_{i=1}^n (1 - M_{AL}(x_i))^{\omega_i}, 1 - \prod_{i=1}^n (1 - M_{AU}(x_i))^{\omega_i}], \\ & [\prod_{i=1}^n (N_{AL}(x_i))^{\omega_i}, \prod_{i=1}^n (N_{AU}(x_i))^{\omega_i}]) \end{aligned} \quad (9)$$

where $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ be the weight vectors of A . In addition, $\omega_i > 0$ and $\sum_{i=1}^n \omega_i = 1$.

In this study, we use $\omega = (1/n, 1/n, \dots, 1/n)$. The aggregation result IIFWAA is still IVIFS and is not very sensitive to A . For medical diagnosis, we first calculate IIFWAA from degrees of the interview chart and then use a measure based on distance between IVIFS.

3.3. A distance measure. In diagnosis with IFS data, we generally determine the diagnostic labels of patient p for any disease d such that both inequalities $0.5 < \mu_T(p, d)$ and $\nu_T(p, d) < 0.5$ are satisfied. However, as mentioned above, the max-min-max composition rule is affected by only extreme values. As a result, the diagnosis approach using the measures, $\mu_T(p, d)$ and $\nu_T(p, d)$, leads to quite conservative results. For example, let use the confirmability membership degrees ($\mu_R(s, d)$) for a patient are 0.7, 0.4 and 0.7. Then the diagnosis measure $\mu_T(p, d)$ based on max-min-max composition is 0.4. However, if the symptoms are significantly associated with a disease, it is reasonable that the diagnosis measure has a value above 0.4.

As an alternative to the max-min-max composition, other measurements such as similarity and distance between IFS have attracted many researchers [12, 16, 17, 29, 37]. Szmidt and Kacprzyk [27] proposed the distance measures between IFS for medical diagnosis. Park *et al.* [19] proposed new distance measures between IVFS. In this study, we propose a measure based on distance between IVIFS. In the measure, we consider the hesitate part to modify Park's distances. The measure, the normalized Hamming distance considering the hesitate part, is defined as follows.

Definition 3.2. (Distance Measure) For any two IVIFS $A = \{(x_i, M_A(x_i), N_A(x_i)) \mid (i = 1, 2, \dots, n)\}$ and $B = \{(x_i, M_B(x_i), N_B(x_i)) \mid (i = 1, 2, \dots, n)\}$, the normalized Hamming distance considering the hesitate part is defined as Equation (10).

$$\begin{aligned} l_h(A, B) = & (1/4n) \sum [|M_{AL}(x_i) - M_{BL}(x_i)| + |M_{AU}(x_i) - M_{BU}(x_i)| + |N_{AL}(x_i) - N_{BL}(x_i)| \\ & + |N_{AU}(x_i) - N_{BU}(x_i)| + |H_{AL}(x_i) - H_{BL}(x_i)| + |H_{AU}(x_i) - H_{BU}(x_i)|] \end{aligned} \quad (10)$$

where H is the hesitate part.

4. Illustrative Example. In this section, we present an example to illustrate medical diagnosis process. For medical diagnosis of headache, the example uses the patient's degrees $\langle M_Q(p, s), N_Q(p, s) \rangle$ assigned by a physician, and confirmability degree $\langle M_R(s, d), N_R(s, d) \rangle$ indicated in the interview chart.

Let us consider patient P_1 . P_1 's symptoms are (M5, M8, M12, M15, M18, M19) of migraine, (T3, T6, T10) of tension headache, and (C4, C11) of cluster headache. P_1 simultaneously has symptoms M5, M8 and M15 (the symptoms are displayed in the composite symptom M22), therefore, the symptoms of migraine are represented in (M12,

TABLE 2. Patient P_1 's degrees: $\langle M_Q(P_1, s), N_Q(P_1, s) \rangle$

symptom	M12	M18	M19	M22	T3	T6	T10	C4	C11
M_Q	[0.5, 0.6]	[0.5, 0.6]	[0.4, 0.6]	[0.7, 0.8]	[0.6, 0.7]	[0.5, 0.7]	[0.4, 0.6]	[0.5, 0.6]	[0.5, 0.7]
N_Q	[0.2, 0.3]	[0.1, 0.3]	[0.1, 0.2]	[0.1, 0.2]	[0.1, 0.2]	[0.2, 0.3]	[0.2, 0.4]	[0.1, 0.2]	[0.2, 0.3]

TABLE 3. Confirmability degrees: $\langle M_R(s, d), N_R(s, d) \rangle$

symptom	migraine		tension		cluster	
	M_R	N_R	M_R	N_R	M_R	N_R
M12	[0.6, 0.7]	[0.1, 0.2]	[0.2, 0.3]	[0.5, 0.6]	[0.1, 0.3]	[0.4, 0.6]
M18	[0.6, 0.7]	[0.2, 0.3]	[0.2, 0.4]	[0.4, 0.6]	[0.4, 0.6]	[0.1, 0.2]
M19	[0.5, 0.6]	[0.1, 0.2]	[0.1, 0.2]	[0.6, 0.7]	[0.3, 0.4]	[0.3, 0.5]
M22	[0.7, 0.8]	[0.1, 0.2]	[0.1, 0.2]	[0.6, 0.8]	[0.1, 0.2]	[0.7, 0.8]
T3	[0.3, 0.4]	[0.4, 0.5]	[0.6, 0.7]	[0.1, 0.2]	[0.2, 0.3]	[0.5, 0.6]
T6	[0.2, 0.4]	[0.4, 0.6]	[0.6, 0.7]	[0.1, 0.3]	[0.1, 0.3]	[0.5, 0.6]
T10	[0.2, 0.3]	[0.4, 0.5]	[0.5, 0.6]	[0.2, 0.3]	[0.1, 0.2]	[0.4, 0.6]
C4	[0.5, 0.6]	[0.2, 0.3]	[0.1, 0.2]	[0.6, 0.7]	[0.6, 0.7]	[0.1, 0.2]
C11	[0.2, 0.4]	[0.3, 0.5]	[0.3, 0.4]	[0.2, 0.3]	[0.5, 0.7]	[0.1, 0.3]

TABLE 4. Patient's degrees: (IIFWAA M_Q , IIFWAA N_Q)

Q	symptom M	symptom T	symptom C
P_1	([0.54, 0.66], [0.12, 0.24])	([0.51, 0.67], [0.16, 0.29])	([0.50, 0.65], [0.14, 0.24])

M18, M19, M22). The stages for medical diagnosis of the proposed approach are as follows:

- stage 1: Table 2 is the degrees for P_1 's symptoms assigned by a physician, and Table 3 is the confirmability degrees indicated in the interview chart.
- stage 2: Based on Table 2 and Table 3, Table 4 and Table 5 are calculated by applying IIFWAA operator Equation (9). For example, [0.61, 0.71], an IIFWAA M_R of Table 5, is calculated as follows: The confirmability membership degrees of the symptoms (M12, M18, M19, M22) are ([0.6, 0.7], [0.6, 0.7], [0.5, 0.6], [0.7, 0.8]) and $\omega = (1/4, 1/4, 1/4, 1/4)$. Then,

$$0.61 = 1 - \{(1 - 0.6)^{1/4}\} * \{(1 - 0.6)^{1/4}\} * \{(1 - 0.5)^{1/4}\} * \{(1 - 0.7)^{1/4}\}$$

$$0.71 = 1 - \{(1 - 0.7)^{1/4}\} * \{(1 - 0.7)^{1/4}\} * \{(1 - 0.6)^{1/4}\} * \{(1 - 0.8)^{1/4}\}$$

An IIFWAA N_R of Table 5, [0.12, 0.22], is calculated as follows: From the confirmability non-membership degrees ([0.1, 0.2], [0.2, 0.3], [0.1, 0.2], [0.1, 0.2]) of the symptoms (M5, M8, M18, M19),

$$0.12 = \{0.1^{1/4}\} * \{0.2^{1/4}\} * \{0.1^{1/4}\} * \{0.1^{1/4}\}$$

$$0.22 = \{0.2^{1/4}\} * \{0.3^{1/4}\} * \{0.2^{1/4}\} * \{0.2^{1/4}\}$$

- stage 3: Table 6 is calculated by applying Equation (10) in Table 4 and Table 5. For example, 0.16, the distance for migraine of Table 6, is calculated as follows:

$$0.16 = (1/12)[(|0.54 - 0.61| + \dots + |0.34 - 0.27|) + \dots + (|0.50 - 0.37| + \dots + |0.36 - 0.39|)]$$

- stage 4: The lowest distance points out a proper diagnosis. As a result, we can diagnose that patient P_1 suffers preferentially from migraine.

TABLE 5. Confirmability degrees: (IIFWAA M_R , IIFWAA N_R)

R	Migraine	Tension	Cluster
symptom M	([0.61, 0.71], [0.12, 0.22])	([0.15, 0.28], [0.52, 0.67])	([0.24, 0.40], [0.30, 0.47])
symptom T	([0.23, 0.37], [0.40, 0.53])	([0.57, 0.67], [0.13, 0.26])	([0.13, 0.27], [0.46, 0.60])
symptom C	([0.37, 0.51], [0.24, 0.39])	([0.21, 0.31], [0.35, 0.46])	([0.55, 0.70], [0.10, 0.24])

TABLE 6. Distance for P_1 's symptoms: l_h

T	Migraine	Tension	Cluster
P_1	0.16	0.26	0.24

5. Conclusion. In this paper, IVIFS theory has been applied to make a diagnosis of headache as a new approach on decision support practice in medicine. For medical diagnosis of headache, we developed an interview chart with interval fuzzy degrees based on the relation among symptoms and three types of headache. Second, we utilized the IIFWAA operator to aggregate interval-valued fuzzy information from the symptoms. Last, we proposed a measure based on distance between IVIFS for medical diagnosis. The result of the example indicates that it is possible to classify headache using our diagnosis method. We expect that the method will be improved to be an efficient tool for medical diagnosis and the physician's decision.

Acknowledgment. This paper was supported by research funds of Chonbuk National University in 2009.

REFERENCES

- [1] K. P. Adlassnig, Fuzzy set theory in medical diagnosis, *IEEE Trans. on Systems, Man and Cybernetics*, vol.16, pp.260-265, 1986.
- [2] J. Y. Ahn, Y. H. Kim and S. K. Kim, A fuzzy differential diagnosis of headache applying linear regression method and fuzzy classification, *IEICE Trans. on Information and Systems*, vol.E86-D, no.12, pp.2790-2793, 2003.
- [3] J. Y. Ahn, K. S. Mun, Y. H. Kim, S. Y. Oh and B. S. Han, A fuzzy method for medical diagnosis of headache, *IEICE Trans. on Information and Systems*, vol.E91-D, no.4, pp.1215-1217, 2008.
- [4] K. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, vol.20, pp.87-96, 1986.
- [5] K. Atanassov and G. Gargov, Interval-valued intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, vol.31, pp.343-349, 1989.
- [6] F. Aversa, E. Gronda, S. Pizzuti and C. Aragno, A fuzzy logic approach to decision support in medicine, *Proc. of the Conf. on Systemics, Cybernetics and Informatics*, 2002.
- [7] J. J. Buckley, *Fuzzy Probability and Statistics*, Springer, 2006.
- [8] T. Calvo, G. Mayor and R. Mesiar, *Aggregation Operators: New Trends and Applications*, Physica-Verlag, 2002.
- [9] T. Chaira and T. Chaira, Intuitionistic fuzzy sets: Application to medical image segmentation, *Studies in Computational Intelligence*, vol.85, pp.51-68, 2008.
- [10] W. Cui and D. I. Blockley, Interval provability theory for evidential support, *International Journal of Intelligent Systems*, vol.5, pp.183-192, 1990.
- [11] S. K. De, R. Biswas and A. R. Roy, An application of intuitionistic fuzzy sets in medical diagnosis, *Fuzzy Sets and Systems*, vol.117, pp.209-213, 2001.
- [12] X. Guo, H. Zhang and Z. Chang, Image thresholding algorithm based on image gradient and fuzzy set distance, *ICIC Express Letters*, vol.4, no.3(B), pp.1059-1064, 2010.
- [13] H. L. Hsu and B. Wu, An innovative approach on fuzzy correlation coefficient with interval data, *International Journal of Innovative Computing, Information and Control*, vol.6, no.3, pp.1049-1058, 2010.
- [14] P. R. Innocent and R. I. John, Computer aided fuzzy medical diagnosis, *Information Sciences*, vol.162, pp.81-104, 2004.

- [15] C. D. Lee, S. Y. Oh, H. M. Choi and J. Y. Ahn, A medical diagnosis based on interval-valued fuzzy sets, *Biomedical Engineering: Applications, Basis and Communications*, submitted, 2009.
- [16] D. Li and C. Cheng, New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions, *Pattern Recognition Letters*, vol.23, pp.221-225, 2002.
- [17] Z. Liang and P. Shi, Similarity measures on intuitionistic fuzzy sets, *Pattern Recognition Letters*, vol.24, pp.2687-2693, 2003.
- [18] S. Moein, S. A. Monadjemi and P. Moallem, A novel fuzzy-neural based medical diagnosis system, *World Academy of Science, Engineering and Technology*, vol.37, pp.157-161, 2008.
- [19] J. H. Park, K. M. Lim, J. S. Park and Y. C. Kwun, Distances between interval-valued intuitionistic fuzzy sets, *Journal of Physics: Conf. Series*, vol.96, 2008.
- [20] Z. Pawlak, *Rough Sets: Theoretical Aspects of Reasoning About Data*, Kluwer Academic Publishing, 1991.
- [21] Z. Pei, X. Liu and L. Zou, Extracting association rules based on intuitionistic fuzzy sets, *International Journal of Innovative Computing, Information and Control*, vol.6, no.6, pp.2567-2580, 2010.
- [22] T. Samatsu, K. Tachikawa and Y. Shi, GUI form for car retrieval systems using fuzzy theory, *ICIC Express Letters*, vol.2, no.3, pp.245-249, 2008.
- [23] E. Sanchez, Medical diagnosis and composite fuzzy relations, *Advances in Fuzzy Set Theory and Applications*, M. M. Gupta, R. K. Ragade and R. R. Yager (eds.), Elsevier Science Ltd, 1979.
- [24] C. Schuh, Fuzzy sets and their application in medicine, *Proc. of the North American Fuzzy Information Society*, pp.86-91, 2005.
- [25] R. Seising, A history of medical diagnosis using fuzzy relations, *Proc. of the Conf. on Fuzziness*, 2004.
- [26] T. S. Shih, J. S. Su and J. S. Yao, Fuzzy linear programming based on interval-valued fuzzy sets, *International Journal of Innovative Computing, Information and Control*, vol.5, no.8, pp.2081-2090, 2009.
- [27] E. Szmidt and J. Kacprzyk, Intuitionistic fuzzy sets in intelligent data analysis for medical diagnosis, *Lecture Notes in Computer Science*, vol.2074, pp.263-271, 2001.
- [28] B. Turksen, Interval valued fuzzy sets based on normal forms, *Fuzzy Sets and Systems*, vol.20, pp.191-210, 1986.
- [29] W. Wang and X. Xin, Distance measure between intuitionistic fuzzy sets, *Pattern Recognition Letters*, vol.26, pp.2063-2069, 2005.
- [30] Z. S. Xu and Q. L. Da, An overview of operators for aggregating information, *International Journal of the Intelligence Systems*, vol.18, pp.953-969, 2003.
- [31] Z. S. Xu, *Uncertain Multiple Attribute Decision Making: Methods and Applications*, Tsinghua University Press, 2004.
- [32] Z. S. Xu, Methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision making, *Control and Decision*, vol.22, pp.215-219, 2007.
- [33] K. Yamada, Diagnosis under compound effects and multiple causes by means of the conditional causal possibility approach, *Fuzzy Sets and Systems*, vol.145, pp.183-212, 2004.
- [34] J. F. Yao and J. S. Yao, Fuzzy decision making for medical diagnosis based on fuzzy number and compositional rule of inference, *Fuzzy Sets and Systems*, vol.120, pp.351-366, 2001.
- [35] L. A. Zadeh, Fuzzy sets, *Information and Control*, vol.8, pp.338-353, 1965.
- [36] L. A. Zadeh, Biological applications of the theory of fuzzy sets and systems, *Proc. of an International Symposium on Biocybernetics of the Central Nervous System*, pp.199-206, 1969.
- [37] W. Zeng, F. Yu, X. Yu, H. Chen and S. Wu, Entropy of intuitionistic fuzzy set based on similarity measure, *International Journal of Innovative Computing, Information and Control*, vol.5, no.12, pp.4737-4744, 2009.