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EUGENIC BACTERIAL MEMETIC ALGORITHM FOR FUZZY ROAD TRANSPORT TRAVELING SALESMAN PROBLEM

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Abstract. *The aim of the Traveling Salesman Problem (TSP) is to find the cheapest way of visiting all elements in a given set of cities (nodes) exactly once and returning to the starting point. In solutions presented in the literature costs of travel between nodes are based on Euclidean distances, the problem is symmetric and the costs are constant and crisp values. Practical application in road transportation and supply chains are often uncertain or fuzzy. The risk attitude depends on the features of the given operation. The model presented in this paper handles the fuzzy, time dependent nature of the TSP and also gives a solution for the asymmetric loss aversion by embedding the risk attitude into the fitness function of the eugenic bacterial memetic algorithm. Computational results are presented for different cases. The classical TSP is investigated along with a modified instance where some costs between the cities are described with fuzzy numbers. Two different techniques are proposed to evaluate the uncertainties in the fuzzy cost values. The time dependent version of the fuzzy TSP is also investigated and simulation experiences are presented.*
Keywords: Traveling salesman problem, Eugenic bacterial memetic algorithm, Time dependent fuzzy costs, Uncertainty management

1. Introduction.

The aim of the Traveling Salesman Problem (TSP) is to find the cheapest path reaching all elements in a given set of cities (nodes) where the cost of travel between each pair of them is given, including the return to the starting point. The TSP is a very good representative of a larger class of problems known as combinatorial optimization problems [2]. For its practical importance and the wide range of applications in practice many approaches, heuristic searches and algorithms have been suggested [4,10,12,30,38], while different extensions and variations of the original TSP and similar problems have been investigated [24,25,31]. The problem presented in the literature most frequently has the following features. Costs of travel between nodes (cities) are based on Euclidean distances, the problem is symmetric and the costs are constant. Since the original formulation of the problem states: the aim is to find the "cheapest" tour, thus the cost matrix that represents the distances between each pair must be determined by calculating the actual costs of the transportation processes. The costs of transportation consist of two main elements: costs proportional to transit distances (km) and costs proportional to transit times. Obviously, the physical distances can be considered as constant values in a given relation, however, transit times are subject to external factors [14] such as weather conditions, traffic circumstances, etc., so they should be treated as a time-dependent variable. On the other hand, in real road networks the actual distance between two points often alter from the Euclidean distance, furthermore occasionally some extra costs (e.g., ferriage, tunnel fare) can modify the distance-related variable costs. Considering these characteristics the original TSP should be reconstructed, so that realistic solutions can be developed. Furthermore, the actual costs are rarely constant and predictable, so fuzzy cost coefficients may be applied in order to represent uncertainty [1,21]. In modern logistics systems, uncertainty and inaccuracy are not tolerated due to the widespread just-in-time approach. Service time windows are considered [13], and the evaluation functions are modified in order to map the realistic circumstances [36]. Multi-objective models have been presented [16] dealing with a set of alternative solutions. In this paper, we embed the sensitivity for uncertainty and risk aversion into the evaluation process, so that the priority of accuracy in relation to the actual costs can be set in advance.

For giving a good approximate solution to the above-mentioned fuzzy road-transport TSP (FRTTSP) we suggest a eugenic bacterial memetic algorithm (EBMA) since that algorithm is suitable for global optimization of even non-linear, high-dimensional, multi-modal and discontinuous problems.

The structure of the paper is as follows. In Section 2, the classical TSP is discussed. Section 3 presents the eugenic bacterial memetic algorithm. In Section 4, the modified TSP is described and analyzed. Section 5 explains how the eugenic bacterial memetic algorithm can be used for solving the classical and the modified TSP. Section 6 presents a numerical example for the classical TSP and for the FRTTSP, too. Different FRTTSP versions are investigated. First, only fuzzy edges are used for some cost values. Second, different techniques are proposed to evaluate the fuzziness of the tour. Another instance is considered as well, in which the elements of the cost matrix depend on the time elapsed from the beginning of the given operation. Loss aversion of the decision maker is taken into consideration in the fitness function. Section 7 concludes the paper.

2. **Formulating and Solutions for the Classical TSP.**

In the case of the traveling salesman problem, the mathematical description can be a graph where each city is represented by a node and the edges are connecting pairs of cities (pairs of nodes) in the graph. A distance (or cost) is associated with every edge. A *complete* graph is a graph in which each pair of graph vertices are connected by an edge. A round-trip of the cities corresponds to a special subset of the lines when each city is visited exactly once. This is called a tour or a Hamiltonian cycle in graph theory.

To formulate the symmetric case with n nodes (cities) $c_{ij} = c_{ji}$ so a graph can be considered where there is only one (undirected) arc between every two nodes. Let $x_{ij} = \{0, 1\}$ be the decision variable ($i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, n$) and $x_{ij} = 1$, means that the arc connecting $node_i$ to $node_j$ is an element of the tour. Let

$$x_{ii} = 0 \quad (i = 1, 2, \ldots, n) \tag{1}$$

mean that no loops exist in the graph, i.e., no tour element is allowed from a node to itself. Furthermore,

$$\sum_{i=1}^{n}\sum_{j=1}^{n} x_{ij} = n \tag{2}$$

is the number of decision variables where $x_{ij} = 1$ is equal to n, and

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \forall j \in \{1, 2, \ldots, n\} \tag{3}$$

and

$$\sum_{j=1}^{n} x_{ij} = 1 \quad \forall i \in \{1, 2, \ldots, n\} \tag{4}$$

means that each column and row of the decision matrix has a single element with a value 1 (i.e., each city is visited exactly once). For assuring the close circuit, an additional constraint must be set: a permutation of nodes $(p_1, p_2, \ldots, p_n)$ has to be constructed. The goal is to minimize the total cost $C(p)$ that can be calculated as:

$$C(p) = \left(\sum_{i=1}^{n-1} c_{p_i, p_{i+1}} \right) + c_{p_n, p_1}. \tag{5}$$

For a symmetrical network there are $\frac{(n-1)!}{2}$ possible tours (because the degree of freedom is $(n-1)$, and tours describing the same sequence but in opposite directions are not considered different tours) and for asymmetric networks where $c_{ij} \neq c_{ji}$ the number of possible tours is $(n-1)!$. Some reduction can be done [17], however, it is clear that an exhaustive search is not possible in practice for a large n. Rather than enumerating all possibilities, successful algorithms for solving the TSP problem have been capable of eliminating most of the roundtrips without ever actually considering them. The main groups of search engines are [2,4,10,12,24,30,33,35,38]:

- Mixed-integer programming.
- Branch-and-bound method.
- Heuristic searches (local search algorithms, simulated annealing, neural networks, genetic algorithms, particle swarm optimization, ant colony optimization, artificial immune systems, etc.).

In this paper, a eugenic bacterial memetic algorithm (a special evolutionary technique) is proposed as a novel heuristic search for TSP.

3. Eugenic Bacterial Memetic Algorithms.

Nature inspired evolutionary optimization algorithms are often suitable for global optimization of even non-linear, high-dimensional, multi-modal and discontinuous problems. The original genetic algorithm was developed by Holland [18] and was based on the process of evolution of biological organisms. It used three operators: reproduction, crossover and mutation. Later, new kinds of evolutionary based techniques were proposed, which all imitated phenomena that can be found in nature.

Bacterial Evolutionary Algorithm (BEA) [27] is one of these techniques. BEA uses two operators; the *bacterial mutation* and the *gene transfer* operation. These new operators are based on the microbial evolution phenomenon. Bacteria share chunks of their genes rather than perform a neat crossover in chromosomes. The bacterial mutation operation optimizes the chromosome of one bacterium; the gene transfer operation allows the transfer of information between the bacteria in the population. Each bacterium represents a solution for the original problem. BEA has been applied to a wide range of problems, for instance, optimizing the fuzzy rule bases [9,27], feature selection [6] and combinatorial optimization problems [25].

Evolutionary algorithms are global searchers, however, in most cases they give only a quasi-optimal solution to the problem, because their convergence speed is low. Local search approaches can give a more accurate solution, however, they are searching for the solution only in the neighborhood of the search space. Local search approaches might be useful in improving the performance of the basic evolutionary algorithm, which may find the global optimum with sufficient precision in this combined way. Combinations of evolutionary and local-search methods are usually referred to as *memetic algorithms*

[26]. Memetic algorithms can be very useful tools for solving NP-hard problems similar to the TSP as shown in [19,32]. A new kind of memetic algorithm based on the bacterial approach is the bacterial memetic algorithm (BMA) proposed earlier by the authors [5].

The algorithm consists of four steps. First, an initial population with N_{ind} individuals has to be created. This can be done randomly or some other techniques can be considered, which will be presented in Subsection 5.5. Then, bacterial mutation, a local search and gene transfer are applied, until a stopping criterion is fulfilled.

The bacterial mutation is applied to each chromosome one by one. First, N_{clones} copies (clones) of the bacterium are generated, then a certain segment of the chromosome is randomly selected and the parameters of this selected segment are randomly changed in each clone (mutation). Next, all the clones and the original bacterium are evaluated and the best individual is selected, and it transfers the mutated segment into the other individuals. This process continues until all of the segments of the chromosome have been mutated and tested. At the end of this process, the clones are eliminated.

After the bacterial mutation operator some kind of local search is applied. This method depends on the given problem. For the TSP it is detailed in Section 5.

In the next step, the other evolutionary operation, gene transfer is applied, which allows the recombination of genetic information between two bacteria. First, the population must be divided into two halves. The better 50% of the bacteria are called the superior half, while the other bacteria are the inferior half. One bacterium is randomly chosen from the superior half, this will be the source bacterium and another is randomly chosen from the inferior half, this will be the destination bacterium. A segment from the source bacterium is chosen randomly and this segment will overwrite a segment of the destination bacterium or it will be added to the destination bacterium. This process is repeated N_{inf} times. The stopping condition is usually given by a predefined maximum generation number (N_{gen}). When N_{gen} is achieved then the algorithm ends otherwise it continues with the bacterial mutation step.

In the algorithm eugenic elements are used as well [34]. This eugenism reflects the humans will of decision, and puts some deterministic elements into the algorithm. Details will be given in Section 5.

The basic algorithm has four parameters: the number of generations (N_{gen}), the number of bacteria in the population (N_{ind}), the number of clones in the bacterial mutation (N_{clones}) and the number of infections (N_{inf}) in the gene transfer operation.

4. Variable Costs and Risk Aversion.

4.1. **Cost in real road networks.** The transportation system is a set of complex human, social, economic and political interactions, and traditionally, probability theory has been used for handling the obvious uncertainty, however, on the basis of recent research it is that possibility theory offers a more useful way of handling the uncertain situations that often arise in transportation analysis. The two classes of uncertainty in transportation are [21]:

- **Vagueness:** this is associated with the lack of clarity of the definition, such as the loss caused by a delivery delay in our model.
- **Ambiguity:** this is associated with the lack of clarity in information. In our proposed model, the predicted cost elements are ambiguous.

Considering real road transport networks, especially in city logistics, the actual circumstances and condition of the transit process are subject to not only the topography of the given network but to timing as well. Referring to the phenomenon of cyclic peak-hours and also to the weekly (monthly, yearly) periodicity of traffic on road, it can be stated that the unit cost of traveling is also a variable, and it can be described as a time series rather than a constant. The main reasons of that are the following:

- The actual cost of 1 km depends on the current fuel consumption, which is partly affected by the speed, however, the speed is externally determined by the current traffic.
- A relatively large proportion of transportation cost is the cost of labor (i.e., wages of drivers), which is calculated on driving time. In Europe, distance-based payment for commercial drivers are not allowed according to the European Agreement Concerning the Work of Crews of Vehicles Engaged in International Road Transport (AETR).
- The return on equity capital is a vital issue for haulage companies, since the transportation sector of the economy is a capital intensive one, meaning that the utilization of a vehicle in time is a crucial problem.

In addition, during long-distance shipments the drivers occasionally must stop for a rest (according to AETR) for a minimum of an 11 hour period (often overnight), and very often there are weekend traffic restrictions for heavy vehicles.

Considering the uncertainty of relevant data it can be stated, that one's estimated travel time by automobile between two points has possibilistic features due to the imprecision and perception of measurement. It can be seen that the circumstances and conditions are significantly changing in time that is the actual value of the cost matrix element c_{ij} should be subject to the timing of transit between $node_i$ and $node_j$ and an appropriate representation of imprecision can be done with use of fuzzy numbers. In this sense, geographical optimization alone is not appropriate, and the road transport operation has to be scheduled in time as well, since the actual cost of the trip from $node_i$ to $node_j$ is determined not only by the selection of edges.

4.2. **The modified TSP, the fuzzy road transport traveling salesman problem (FRTTSP).** To fulfill the requirements of realistic road transport processes, we propose the following modification to the classical TSP.

Since the overall target is to achieve the cheapest tour (in monetary terms), the constraint that each node (city) is visited *only* once is skipped. For practical reasons, the important constraint is to visit each node *at least* once, and a longer route can be cheaper, considering the total cost of distance covered and time elapsed.

Calculating with time-dependent cost coefficients that are not necessarily proportional to distances, a longer route may be the cheaper one under certain circumstances. In some cases, when a graph is mapping the real road network, there are some nodes that have only one connection to the rest, so when visiting that particular city one must return to a point that had been visited before. Thus, in the FRTTSP we eliminate restrictions (2) – (4).

If a significant improvement in traffic conditions can be expected that is a future value of a cost c_{ij} will be less than its present value, it is worth waiting (suspending the tour for a while) and continuing in the next step. In this case, obviously the cost of staying at one point must be calculated. In this sense, we eliminate restriction (1) as well. Very often in the solutions restriction (1) is fulfilled by selecting $c_{ii} = \infty$ formally (i.e., the graph edge is deleted in practice). In our case, c_{ii} is the cost of staying at $node_i$ in a given step, thus c_{ii} can be any non-negative finite value.

The permutation of nodes $(p_1, p_2, \ldots, p_n)$ is being modified as well. As a city may be visited several times, objective function (5) must be rewritten:

$$C(p) = \left(\sum_{i=1}^{T-1} c_{p_i, p_{i+1}} \right) + c_{p_T, p_1}, \tag{6}$$

where T is the number of visited cities in the tour, $n \leq T \leq n \cdot m$, and $m \in \mathrm{N}$ is a multiplier factor, and p_T is the city last visited. Objective function (6) is a generalized form of TSP, in the classical case $m = 1$.

We propose a model in which the distance (cost) of cities is also considered in calculating the elapsed time. This problem is modeled by introducing a new parameter called velocity (v). If v is high and the general tendency of costs is growing proportional to time (that is the traffic is getting heavier and heavier) then the faster traveling salesman may carry out the tour with lower costs than other salesman with lower velocity. The total cost of the tour has to be calculated as follows: each element of the cost matrix has the following form:

$$c_{ij}(t) = c_{ij} + a_{ij} \cdot f_{ij}(t), \tag{7}$$

where c_{ij} is the distance (cost) between $node_i$ and $node_j$ without any time dependency, $f_{ij}(t)$ can be any function expressing time dependency and a_{ij} is a constant value. First, we calculate the time needed for visiting the first city from the initial city:

$$t_{p_1,p_2} = \frac{c_{p_1,p_2}(0)}{v}, \tag{8}$$

where p_1 is the initial city, p_2 is the city visited first and v is the velocity of the traveling salesman. The calculation of time required to cover the distance between cities is vital, since the cost elements are time dependent, so the actual cost between two cities can be determined only if the total time elapsed is known, as it is shown in (7). The time dependency of the cost matrix is represented by a virtual distance value. If a cost element is growing then the required time is growing as well, since the v is constant. The cost between $node_{p_k}$ and $node_{p_{k+1}}$ will be:

$$c_{p_k,p_{k+1}}(t_{elapsed_k}) = c_{p_k,p_{k+1}} + a_{p_k,p_{k+1}} \cdot f_{p_k,p_{k+1}}(t_{elapsed_k}), \tag{9}$$

where

$$t_{elapsed_k} = \sum_{i=1}^{k-1} t_{p_i,p_{i+1}} \quad \text{and} \quad t_{p_k,p_{k+1}} = \frac{c_{p_k,p_{k+1}}(t_{elapsed_k})}{v}. \tag{10}$$

The total cost will be:

$$S = \sum_{i=1}^{T-1} c_{p_i,p_{i+1}}(t_{elapsed_i}) + c_{p_T,p_1}(t_{elapsed_T}), \tag{11}$$

where p_T is the city last visited, and $t_{elapsed_k}$ is the total time elapsed from the beginning of the tour till the salesman arrives in the city p_k.

In order to represent the uncertainty triangular fuzzy numbers are used as cost coefficients. Triangular fuzzy numbers have a membership function consisting of two linear segments joined at a peak, so they can be constructed easily on the basis of little information: the supporting interval $C = [c_L, c_R]$ as the smallest and the largest possible values, and c_C which is the peak value where the membership function equals 1. In that case, the triangular fuzzy number is denoted by $\widetilde{C} = (c_L, c_C, c_R)$.

When the distances between the cities are described by fuzzy numbers, it must be discussed how these fuzzy numbers are summed up in a tour in order to calculate the total distance. The arithmetic of fuzzy numbers is based on the extension principle [37]. We are using triangular shaped fuzzy numbers, which can be characterized by three values, to represent the boundaries of the support and the core value. When we calculate the total distance of a tour, then instead of adding fuzzy numbers by the extension principle, we can do an easier calculation based on the defuzzified values of the fuzzy numbers. According to [28], some defuzzification methods have invariance properties meaning that the result is invariant under linear transformations, thus there is no need to determine the whole outcome using the extension principle but only to compute the sum of the defuzzified values of each fuzzy number. If we are using triangular shaped fuzzy numbers then the Averaging Level Cuts (ALC) type of defuzzification method used in [28] gives the same result as the Center of Gravity (COG) method. So, in the first step, the fuzzy numbers are defuzzified by the COG method (which is simply the arithmetic mean of the three characteristic points of the fuzzy number) and then these crisp numbers are summed up providing the total distance of the tour. Although this approach is straightforward and it was applied in our first attempt for using fuzzy distance values in TSP like problems [8], the uncertainty cannot be handled properly if only defuzzified values are considered for evaluating the quality of the tour. We propose two other approaches for the evaluation of the tour based not only on the length described by the defuzzified values but the uncertainty involved in the fuzzy numbers as well. Since now we need not only the defuzzified values but the length of the tour as a fuzzy number, the fuzzy distances along the tour have to be summed up. As we have triangular fuzzy numbers described by their three breakpoints, the addition can be easily computed by summing the corresponding breakpoints. Denoting the total fuzzy tour by $\widetilde{\beta}$, its three characteristic points, β_L, β_C, β_R can be calculated since in case of fuzzy costs the c_{ij} constants in Equation (11) are fuzzy numbers with three characteristic points $[c_{ij_L}, c_{ij_C}, c_{ij_R}]$. The calculations need to be done three times, first for the left breakpoints, then for the center ones, and finally for the right characteristic points. For instance, the β_L is calculated by always using the left characteristic points in each fuzzy number, and in this case S in Equation (11) will be β_L. This can be done similarly for β_C and β_R.

Let us denote by D the defuzzified length of the total fuzzy tour $\widetilde{\beta}$:

$$D = defuzz(\widetilde{\beta}) = \frac{\beta_L + \beta_C + \beta_R}{3}. \tag{12}$$

Let us denote by U the uncertainty belonging to the tour, i.e., the length of the support of the total fuzzy tour $\widetilde{\beta}$:

$$U = \beta_R - \beta_L. \tag{13}$$

In the first approach, the tour is evaluated as:

$$f_1 = D \cdot \left(2 - e^{-U \cdot \left[\lambda_0 + \frac{\lambda_1}{D}\right]}\right), \tag{14}$$

where λ_0 and λ_1 are parameters representing the sensitivity for uncertainty. With growing D values the sensitivity is growing as well since the same proportion D/U is less tolerated in the case of a large D than in case of a smaller one (e.g., 20% alteration in short distance taxi fare and/or transit time can be accepted but the same dispersion in long distance haulage is not tolerated). Parameters λ_0 and λ_1 are subject to the features of the problem and the priorities of the operator [7].

In the second approach, the tour is evaluated as:

$$f_2 = D \cdot \left\{2 - \exp\left[-\widetilde{\beta} \cdot (\beta_R - \beta_L) \cdot (\beta_R - \beta_C)^w \cdot K\right]\right\}, \tag{15}$$

where w and K are positive parameters. Multiplication of a triangular fuzzy number $\widetilde{\beta}$ by scalars gives a triangular fuzzy number, and f_2 is a scalar based on the fuzzy exponent calculation detailed in [15]. Based on the calculations in [15] the evaluation function for FRTTSP in the second approach can be calculated as:

$$f_2 = D \cdot \left[2 - \frac{1}{3} \cdot \left\{ \exp\left[-\beta_L \cdot (\beta_R - \beta_L) \cdot (\beta_R - \beta_C)^w \cdot K\right] + \right.\right.$$
$$+ \exp\left[-\beta_C \cdot (\beta_R - \beta_L) \cdot (\beta_R - \beta_C)^w \cdot K\right] +$$
$$\left.\left. + \exp\left[-\beta_R \cdot (\beta_R - \beta_L) \cdot (\beta_R - \beta_C)^w \cdot K\right] \right\} \right]. \quad (16)$$

A smaller f_1 and smaller f_2 correspond to more favorable tours. The reasons for using evaluation function (15) are as follows. Related to decision making several indexes of loss aversion have been proposed in the literature [3], some of them give decomposition of the risk attitude. Distinct components have been determined: basic utility, probability weighting and loss aversion [22]. Loss aversion reflects the behavior that decision makers are more sensitive to losses than to gains. Köbberling and Wakker [22] suggested an exponential solution for constant absolute risk aversion. In our case, the basic utility is the length of the tour and an analogy between probability and fuzziness is assumed. If there is no uncertainty that is the support of the fuzzy number is zero, then the basic utility is applied, and consequently the wider support $\beta_R - \beta_L$ is the less favorable f_2 value. Furthermore, with growing total length β the sensibility for uncertainty is growing as well in logistics. For example, 20% alteration in a short distance taxi fare and/or transit time can be accepted (meaning travel between 30 minutes and 35 minutes), however, the same dispersion in long distance haulage is not tolerated, since it would mean almost a week alteration in the case of transcontinental shipments. The expression $\beta_R - \beta_C$ represents the right-hand-side dominance of the triangular fuzzy number. We assume that if two triangular fuzzy numbers have the same COG and the same support length, the one with the less right-hand side dominance is more favorable in logistics: meaning that delays in shipments are less tolerated than minor increases in tour length (time) with less of a possibility to exceed the expected value.

4.3. **Complexity analysis of the fuzzy road transport traveling salesman problem.** The problem of the modified Traveling Salesman Problem obviously contains the classical problem of searching for a Hamiltonian circle (HC) as a special case. This latter means the following: find a connected sequence of graph edges that ends in the same graph node where it starts, and the HC contains each node exactly once. The problem of deciding whether a graph contains a HC is one of the base cases of asking a question to that the answer can be given by an NP-complete search. Theorem 34.13 in [11] states exactly that the problem of deciding if a given graph contains a Hamiltonian circle is NP-complete. Thus, finding a HC is NP-hard. Consequently, the extended TSP is also NP-hard, as in some special case it results in the classical TSP. This is the lower bound for the complexity of the search. On the other hand, deciding whether an arbitrary graph of n points contains a "circle with repetition" of length $N \geq n$ can be decided in at most $(n-1)^N$ steps, by applying an exhaustive search (examining all possible continuations of the path, this latter value is true only for complete graphs, where every edge has degree $n-1$). This is the upper bound of the complexity.

If the problem is further generalized and edges are allowed to have time dependent weights, with no restriction on the time dependent behavior, the lower bound will still hold, however, the upper bound of the estimated complexity cannot be given this way: obviously there are an infinite many weight combinations that can never be exhaustively searched.

It is reasonable to assume that the time dependence of weights is restricted in some way:

1. There are finite numbers of weight possibilities for each edge. Thus, an exhaustive search can still be completed, and if k is the upper bound for the number of different weights for each edge, the upper bound for the search is $(k \cdot (n-1))^N$.
2. There are finite numbers of weight combinations for the whole graph. Then an exhaustive search is completed in at most $k \cdot (n-1)^N$ steps.

5. **EBMA for the FRTTSP.** When applying evolutionary type algorithms first of all the encoding method must be defined. The evaluation of the individuals has to be discussed, too. The operations of the algorithm have to be adapted to the given problem.

5.1. **Encoding method and evaluation of the individuals.** In the modified traveling salesman problem every city may be visited more than once. Because each city must be visited at least once, one solution of the problem does not need to be a permutation of the cities. The evident encoding of the problem into a bacterium is simply the enumeration of the cities in the order they should be visited. Therefore, the length of the bacterium may be greater than the number of cities (n), however, an upper bound for the bacterium length has to be defined, too, we allow bacteria not longer than $m \cdot n$, where m is the multiplier factor, which is a parameter of the algorithm (usually $m = 2$). In this way, we eliminate the constraints prescribed in (2) – (4) as mentioned in Subsection 4.2. The initial city is not represented in the bacteria.

The length of the bacteria can change. It may change during the evolutionary process and the individuals can have different lengths. In the initial population generation, the length of the bacteria is a random number greater than or equal to $n-1$ and less than or equal to $m \cdot n - 1$ (bearing in mind that the initial city is not presented in the individuals).

The evaluation of a bacterium is based on the time dependent distance matrix. The distances are summed up according to the calculation based on fuzzy distance values described in Subsection 4.2 considering not only the length of the tour represented by the bacterium but the uncertainty as well, as expressed by Equations (14) and (15). A smaller f_1 and smaller f_2 correspond to better bacteria.

5.2. **Bacterial mutation.** In the bacterial mutation there is an additional parameter, the length of the segment to be mutated in the clones. First, the segments of the bacterium are determined, and a random segment order is created. In the clones, the mutation of the given segment is executed. For example in Figure 1, the length of the segment is 3, and there are 4 clones. The random segment order is, e.g., {3rd segment, 1st segment, 4th segment, 2nd segment}. This means that in the first sub-cycle of bacterial mutation, the 3rd segment is mutated in the clones. After the mutation of the clones, the best one is chosen, and this clone (or the un-mutated original bacterium) transfers the mutated segment to the other individuals.

The segments of the bacterium do not need to consist of consecutive elements. The elements of the segments can come from different parts of the bacterium as it can be seen in Figure 2.

Because in the FRTTSP the number of visited cities is not predefined, bacteria with different lengths may occur in the population. Although in the initial population generation bacteria with different lengths can arise we would like to allow the changes in the length within the bacterial operations, too. Therefore, before a clone is mutated a random value is used to determine that after the mutation the length of the clone will increase, decrease or remain the same. Increasing is allowed only in the case, when the maximum bacterium length $(m \cdot n - 1)$ is not exceeded, similarly, decreasing is allowed only in the case, when the minimum bacterium length $n - 1$ is guaranteed. If the length increases, then besides changing the positions of the cities in the selected segment of the clone, new

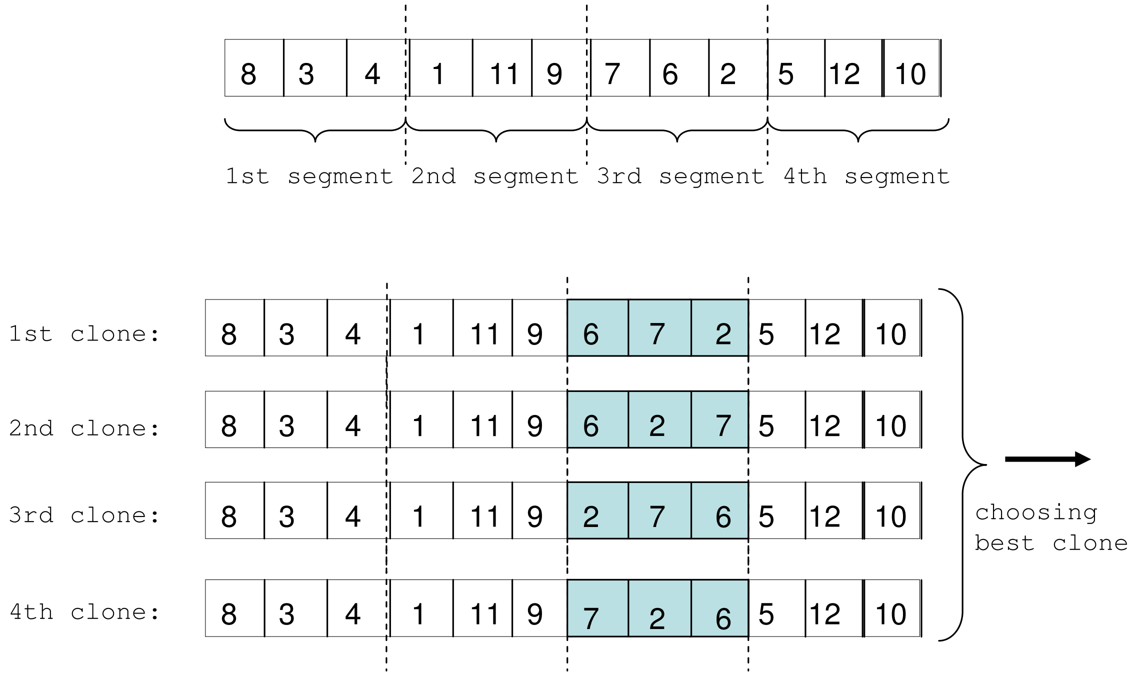

Figure 1. Bacterial mutation

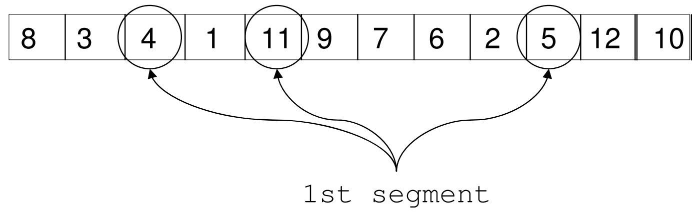

Figure 2. Segment in the bacterial mutation

cities are added to this clone randomly. If it decreases, some cities are deleted from the clone taking care that only those cities are allowed to be deleted, which have at least one other occurrence in the clone. If the length remains the same, then only the positions of the cities in the selected segment are changed.

5.3. **Local search method.** Local search techniques are crucial parts of every memetic algorithms and they can be viewed as methods that rely on vital problem specific knowledge. In the case of memetic algorithms local search algorithms are responsible for the improvement of the candidate solutions in the population. One of the most successful TSP tour improvement methods is the Lin-Kernighan algorithm [23], which is based on the k-opt algorithm. The k-opt algorithm removes k edges from the tour and reconnects the k paths optimally. We applied the 2-opt and 3-opt techniques in our EBMA algorithm.

The 2-opt technique for the classical, metric TSP is shown in Figure 3. Edge pairs $\left(\vec{AB}, \vec{CD}\right)$ are iteratively taken from the graph and the following inequality is examined: $|\vec{AB}| + |\vec{CD}| > |\vec{AC}| + |\vec{BD}|$. If the inequality is true then edge pairs are exchanged; AB

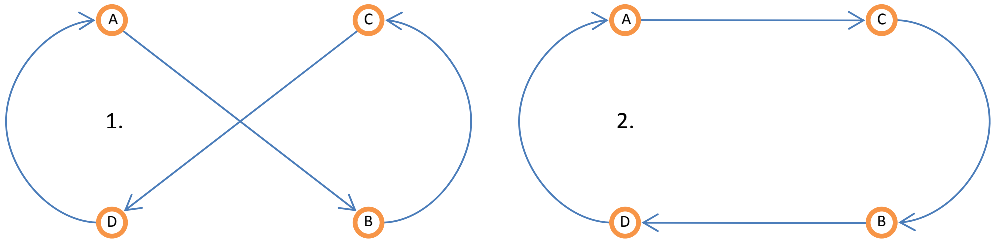

Figure 3. A graph before and after a 2-opt step

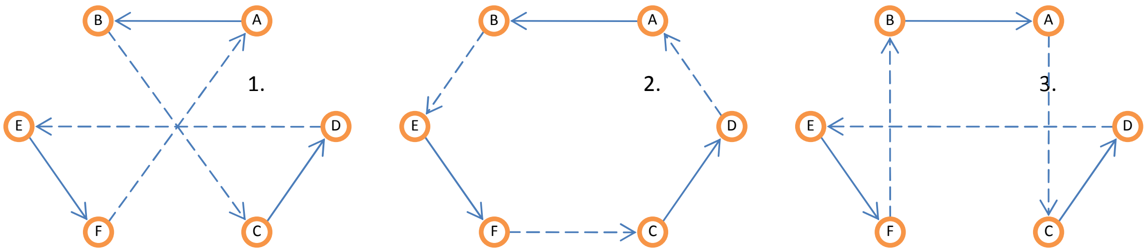

Figure 4. Possible 3-opt moves

and CD edges are deleted from the graph and AC and BD edges are inserted instead. One of the subtours between the original edges is reversed. In case of non-metric TSPs the inequality does not guarantee that the modified tour will have lower aggregated cost because cost function is not necessarily symmetric. After the edge exchange both the original and the modified tours are evaluated according to the evaluation criterion presented in Equation (14) or (15), and only the less costly tour is kept.

The 3-opt local search algorithm presented in Figure 4 works on edge triples. The selected edge triples are removed from the tour, which thus falls into 3 distinct sub-tours. Besides the original edge order there are two possible ways to reconnect these sub-tours (the other possibilities can be obtained as a result of consecutive 2-opt steps). The output of a 3-opt step is always the less costly tour.

For a higher value of k the algorithm would take more time and would provide only small improvements on the 2-opt and 3-opt techniques.

5.4. **Gene transfer.** In the gene transfer operation there is also an additional parameter, the length of the segment to be transferred from the source bacterium to the destination bacterium. In contrast with the bacterial mutation, in the gene transfer, the segment can contain only consecutive elements within the bacterium. The reason for that is the segment containing consecutive elements representing sub-tours in the bacterium, and transferring good sub-tours is the main goal of the gene transfer operation.

Figure 5 shows the gene transfer in the case of a time independent distance matrix. In the case of a time dependent distance matrix, the position where the transferred segment goes to in the destination bacterium must be the same as the position of the segment in the source bacterium.

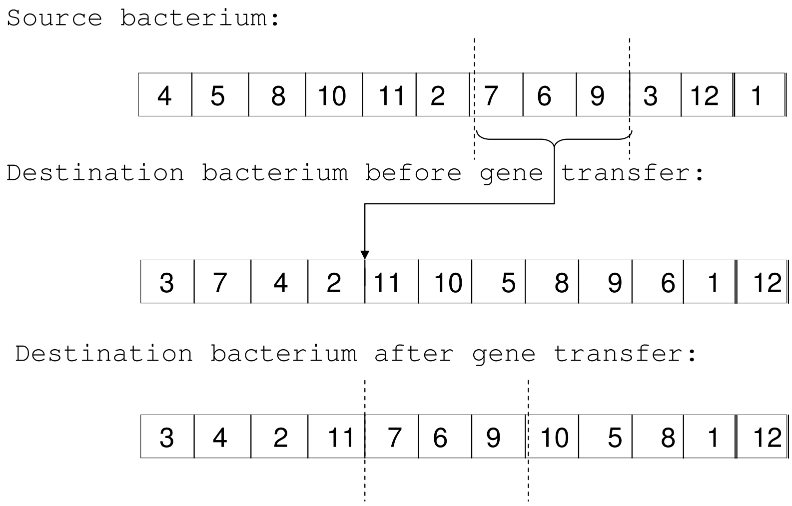

Figure 5. Gene transfer

Different individual lengths are allowed also in the gene transfer operation. After the segment was transferred to the destination bacterium, the elements that occur in the transferred segment and are already in the destination bacterium can be deleted from the destination bacterium. If the same number of elements is deleted from the destination bacterium as the length of the segment, the length of the destination bacterium remains the same. If fewer elements are deleted, its length will be increasing. If more elements are deleted (taking care that each city must have at least one occurrence in the bacterium), the length will be decreasing. We must also take care that the length of the destination bacterium must be at least $n-1$ and at most $(m \cdot n-1)$.

Gene transfer has a similar role as crossover in genetic algorithms. The advantage of gene transfer over crossover is that it fits better to the TSP like problems. Crossover works on two individuals, it combines the values of two parents creating two offspring. In TSP like problems we must be wary of the crossover operator because it can mix up the information erroneously. Several crossover operators have been proposed in the literature [29]. In the gene transfer there is no mixing up of information, only sub-tours are transferred from the source individual to the destination bacterium. Considering for instance the classical TSP, where each city must be visited exactly once, the gene transfer operator can be realized more easily than the crossover operator in the genetic algorithm, because in the crossover operation the multiple appearance of all the elements in the whole chromosome has to be checked, while in the gene transfer operation only the elements of the transferred segment need to be checked.

5.5. **Eugenic elements.** Eugenism is used in the initial population creation and in the bacterial mutation operator. This means that we put more determinism into the algorithm, which contains normally the deterministic local search and the stochastic evolutionary operators. During the initial population creation not only random individuals are generated but also some deterministic ones according to the following rule: there is an individual, which represents the tour in which always the nearest yet unvisited city is visited. There can be another initial bacterium, which represents the tour in which alternating the nearest and the second nearest city is visited. There can be a third individual, where always the second nearest city is taken. The rest of the initial population is created randomly.

TABLE 1. Parameter setting for the st70 problem

Operation	Parameter	Value
	Number of generations	300
	Number of bacteria in the population	300
Bacterial mutation	Number of clones	10
	Segment length	7
	Loose segment possibility	0.5
Gene transfer	Number of infections	50
	Segment length	15
Local search	2-opt	30%
	3-opt	10%
Eugenism	Deterministic clone	yes
	Deterministic initial individuals	3

In the bacterial mutation not only randomly mutated clones are produced, but also there will always be a deterministic clone, which performs a reverse ordering permutation on the selected segment. According to our experiences this can be effective in solving TSP like problems.

5.6. **Complexity of the algorithm.** The time complexity of the bacterial mutation in one generation is $O(N_{ind} \cdot N_{clones} \cdot (m \cdot n)^2)$, whilst for the gene transfer in one generation it is $O(N_{ind} \cdot (n + \log(N_{ind})) + N_{inf} \cdot (m \cdot n + N_{ind}))$. The time complexity of the 2-opt and 3-opt local search methods is $O((m \cdot n)^2)$ and $O((m \cdot n)^3)$, respectively.

6. **Computational Results.**

6.1. **Classical TSP.** In order to test our algorithm on classical TSP instances some benchmark problems from the publicly accessible TSPLIB were used [20]. For the relatively small st70 problem with the parameter setting presented in Table 1 ten times from ten simulations the optimal solution (675) was found (see Figure 6). The convergence of the algorithm is shown in Figure 7, where the upper, red dotted line illustrates the worst bacterium, the blue middle line represents the average tour value of the given generation, and the bottom green dotted line shows the best tour.

The local search operations (2-opt and 3-opt) were applied randomly for the given percentage of the population. Eugenic elements were also applied, meaning that one clone was deterministic in the bacterial mutation, and three deterministic individuals were created in the initial population, there was one individual, which represented the tour in which always the nearest unvisited city was visited, there was another initial bacterium, which represented the tour in which alternating the nearest and the second nearest city was visited, and there was a third individual, where always the second nearest city was taken.

For the even smaller berlin52 problem, the same parameter setting was used as for the st70 problem except the number of generations and the number of bacteria. Five simulation experiments were performed with 200 generations and 200 bacteria in the population, and another five experiments were performed with 100 generations and 100 bacteria and the optimal solution (7542) was always found. Five other simulations were also executed with 50 generations and 50 bacteria, and 20 infections (the rest of the settings were the same) and the best solution was always found as well. The best tour for the berlin52 problem is depicted in Figure 8.

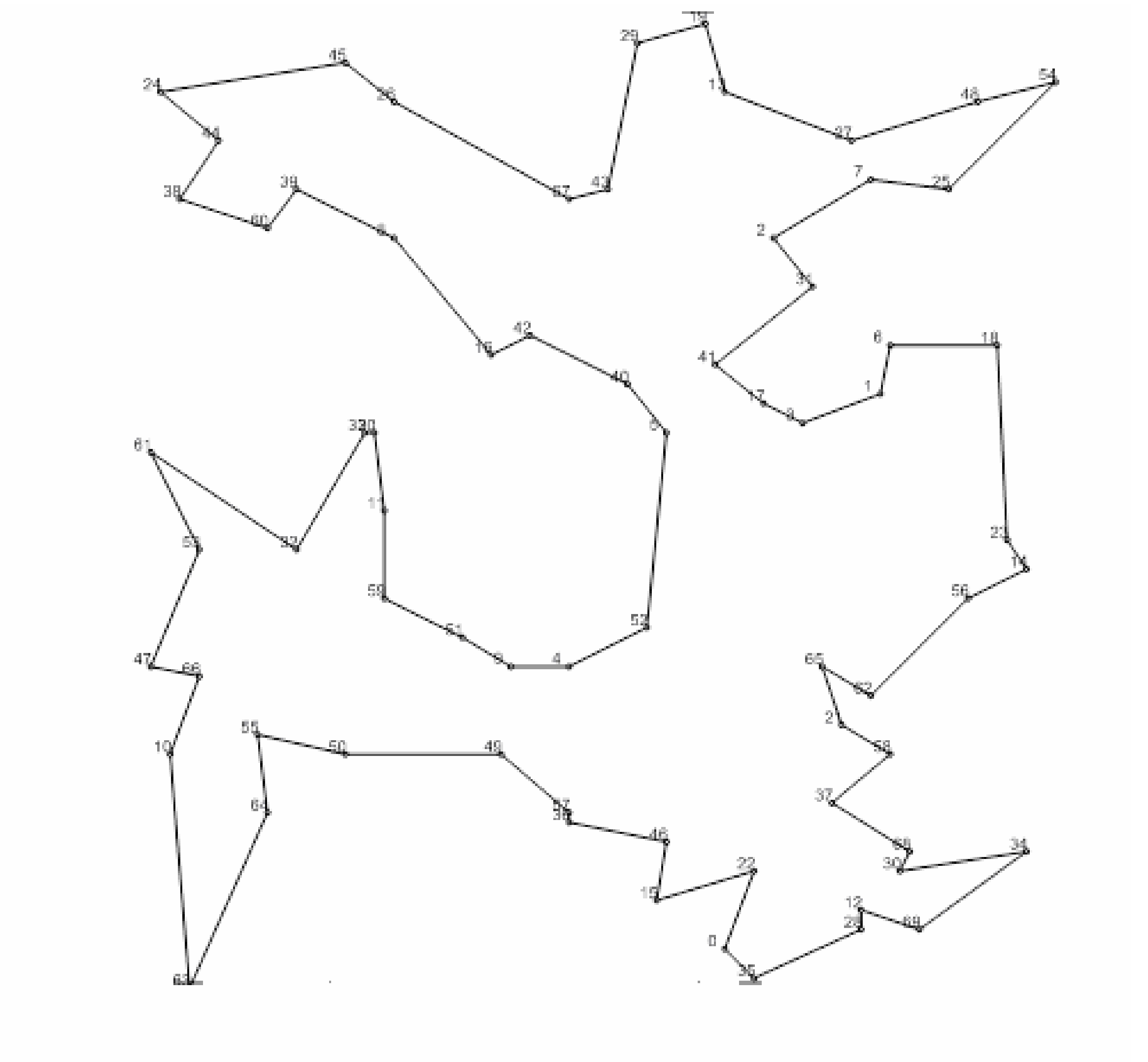

Figure 6. Representation of the best tour for the st70 problem

We performed simulation tests for other problems as well, including instances containing more cities, for example in the xqf131 problem (www.tsp.gatech.edu) containing 131 cities. The optimal solution, for this problem, found by our algorithm can be seen in Figure 9. We can diagnose that an optimal strategy for parameter settings is when the number of generations and the number of bacteria are about $2 \dots 5$ times more than the number of cities, and the rest of the settings are about similar to that one can be seen in Table 1. We can find out from this setting that the ratio of the local search is not large and the number of clones in the bacterial mutation is also not big. The reason for this is that a too intense local search can drive the population into local optima. Too many clones can also cause premature convergence. By applying about ten clones in the bacterial mutation and using the gene transfer operator, the diversity of the population can be maintained.

6.2. **FRTTSP.** For testing the FRTTSP model the st70 reference instance is fuzzified, with the following fuzzy cost matrix elements:

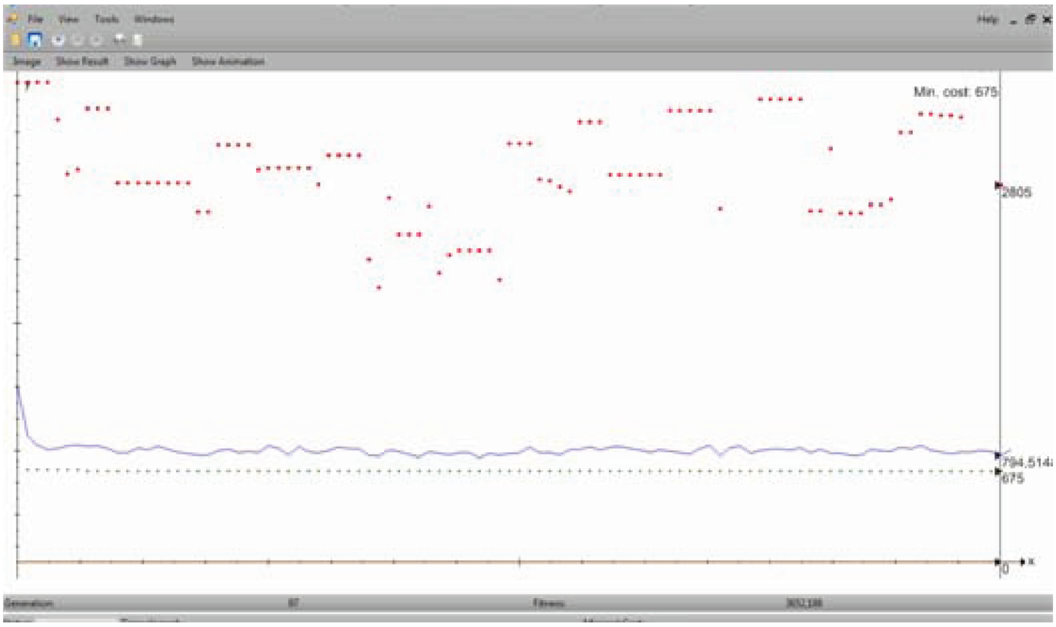

FIGURE 7. Convergence of crisp solution for the st70 problem

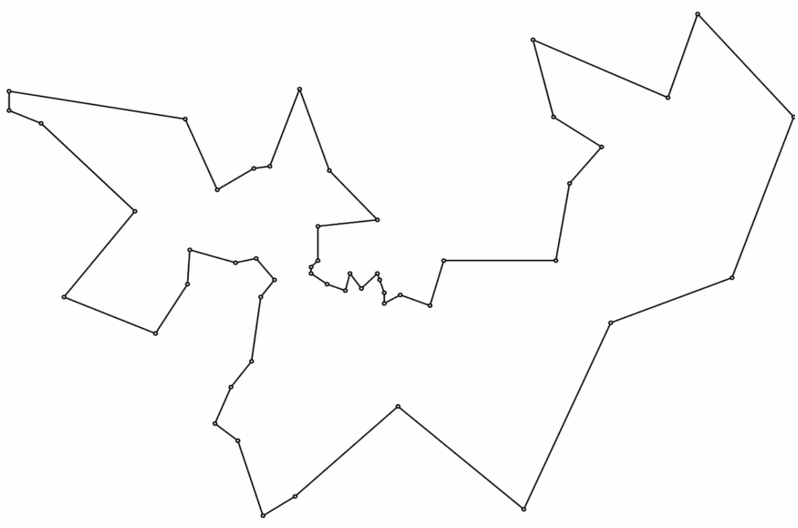

FIGURE 8. Representation of the best tour for the berlin52 problem

$\widetilde{C}_{38,44} = \widetilde{C}_{44,38} = (2, 7, 12)$ $\quad \widetilde{C}_{41,17} = \widetilde{C}_{17,41} = (1, 6, 11)$ $\quad \widetilde{C}_{49,57} = \widetilde{C}_{57,49} = (4, 9, 14)$

$\widetilde{C}_{0,22} = \widetilde{C}_{22,0} = (5, 9, 13)$ $\quad \widetilde{C}_{10,47} = \widetilde{C}_{47,10} = (4, 9, 14)$ $\quad \widetilde{C}_{28,35} = \widetilde{C}_{35,28} = (11, 12, 15)$

$\widetilde{C}_{8,16} = \widetilde{C}_{16,8} = (10, 16, 22)$ $\quad \widetilde{C}_{62,65} = \widetilde{C}_{65,62} = (5, 6, 8)$ $\quad \widetilde{C}_{40,5} = \widetilde{C}_{5,40} = (2, 6, 10)$

$\widetilde{C}_{24,45} = \widetilde{C}_{45,24} = (18, 19, 22)$ $\quad \widetilde{C}_{34,68} = \widetilde{C}_{68,34} = (11, 12, 14)$ $\quad \widetilde{C}_{47,53} = \widetilde{C}_{53,47} = (12, 13, 16)$

$\widetilde{C}_{18,23} = (15, 20, 25)$ $\quad \widetilde{C}_{57,36} = (0, 1, 2)$ $\quad \widetilde{C}_{2,7}(t) = (11, 12, 15) + 0.05t$

$\widetilde{C}_{7,2}(t) = (11, 12, 15) + 0.5t$ $\quad \widetilde{C}_{52,5}(t) = (10, 20, 30) + 0.05t$ $\quad \widetilde{C}_{5,52}(t) = (10, 20, 30) + 0.5t$

$C_{10,63}(t) = 24 + 2t$ $\quad C_{63,10}(t) = 24 + 2t$ $\quad C_{56,62}(t) = 14 - 0.01t$

$C_{62,56}(t) = 14 + 0.6t$

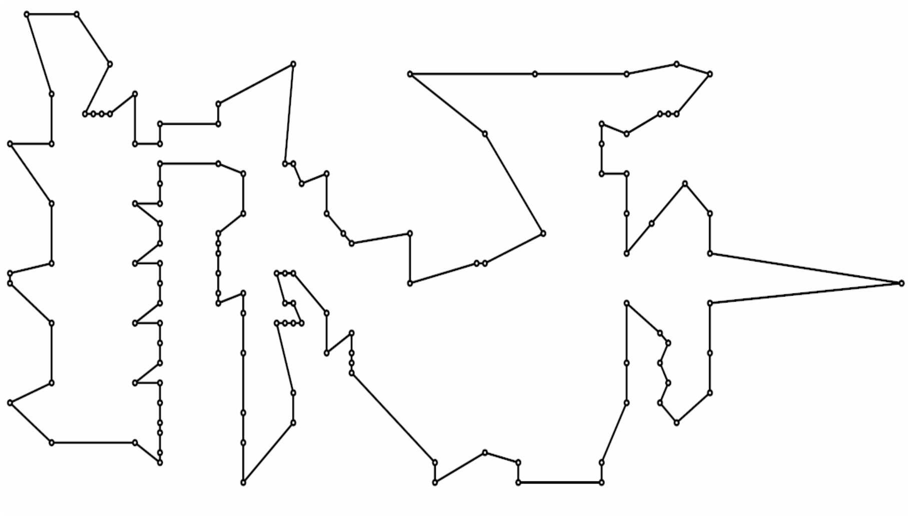

Figure 9. Representation of the best tour for the xqf131 problem

The peak values are kept from the crisp matrix. In non-time dependent cases $v = \infty$ thus $t = 0$. This means that, e.g., $C_{10,63}(t) = 24$ and $\widetilde{C}_{7,2}(t) = (11, 12, 15)$, etc.

The effect of time dependency means that a trip from node 63 to node 10 should be selected at the beginning of the tour because the cost of that edge is growing significantly in time ($C_{63,10}(t) = 24 + 2t$)). On the other hand, a trip from node 56 to node 62 should be taken as late as possible, since traffic circumstances are getting better with time.

First, the non-time dependent cases are being examined. Using the evaluation method (14) different settings of λ_0 and λ_1 were applied (see Table 2). The result for fuzzy st70 instance without any sensitivity ($\lambda_0 = 0$ and $\lambda_1 = 0$) is shown in Figure 10. Letter A refers to the difference from the crisp solution (edge 24-45 and edge 38-44), and letter X is an alternative optimum with the same sub-tour length as in the crisp version.

Table 2. Effects of λ_0 and λ_1 parameters on the evaluation function with approx. $D = 700$ and different U values

λ_0	λ_1	$U/D\%$	$\left(2 - e^{-U\cdot\left[\lambda_0 + \frac{\lambda_1}{D}\right]}\right)$
0	0.01	0.71	1.0001
0	0.01	14.29	1.0014
0	0.1	0.71	1.0007
0	0.1	14.29	1.0142
0.0001	0.5	0.71	1.0041
0.0001	0.5	14.29	1.0782
0	1	0.71	1.0071
0	1	14.29	1.1331
0.01	0.01	0.71	1.0488
0.01	0.01	14.29	1.6326
0.3	0.3	0.71	1.7773
0.3	0.3	14.29	2.0000

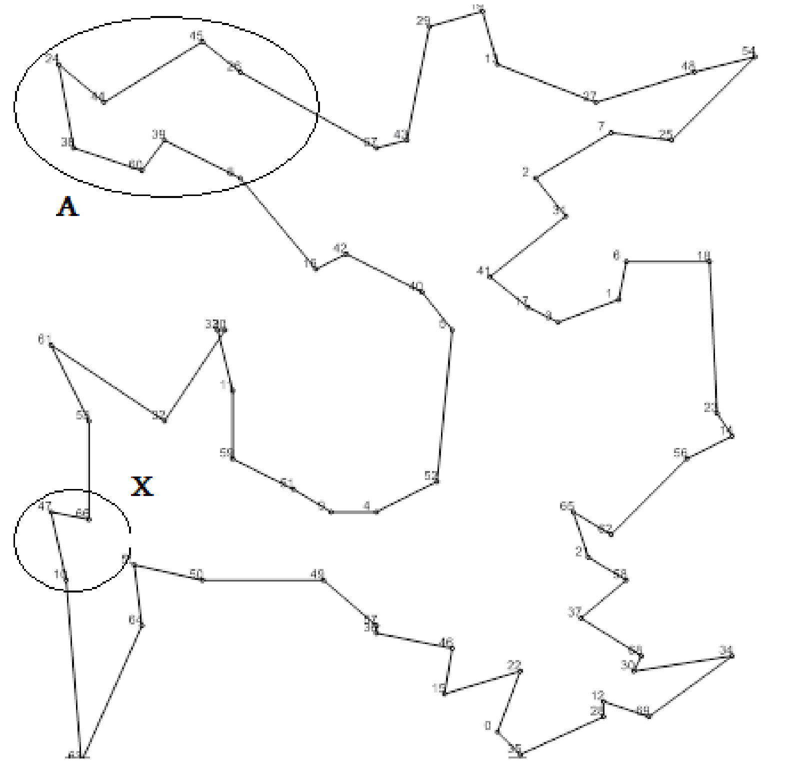

Figure 10. Fuzzy tour for st70 with $\lambda_0 = 0$ and $\lambda_1 = 0$

By increasing λ_0 and λ_1 different solutions can be obtained according to the different sensitivity for uncertainty (see Figures 11 – 13). The letters refer to the difference between the solutions:

B: edge 62-65
C: edge 0-22
D: edge 5-52
E: edge 47-10
F: edge 62-65
and X stands for an alternative optimum.

By increasing λ_0 and λ_1 the "penalty" for uncertainty is increasing as well, thus the best tour omits all fuzzy edges and gives crisp 691 value for the optimum (e.g., $\lambda_0 = 0.01$ and $\lambda_1 = 0.01$, $\lambda_0 = 0$ and $\lambda_1 = 1$, $\lambda_0 = 0.3$ and $\lambda_1 = 0.3$, see Figure 14).

In Equations (15) and (16) not only the uncertainty of the tour is punished, the model is also sensible for the right-hand dominance of triangular fuzzy numbers. Solutions with different K and w are shown in Figure 15. Parameters $w = 0.5$ and $K = 0.000001$ eliminate all fuzzy edges and again gives the crisp solution shown in Figure 14.

For demonstrating the effects of w and K parameters the following cost matrix elements were modified compared with the previous fuzzy cost matrix:

$\widetilde{C}_{38,44} = \widetilde{C}_{44,38} = (2, 7, 8)$ $\widetilde{C}_{41,17} = \widetilde{C}_{17,41} = (5, 6, 11)$ $\widetilde{C}_{49,57} = \widetilde{C}_{57,49} = (4, 9, 10)$
$\widetilde{C}_{0,22} = \widetilde{C}_{22,0} = (5, 9, 16)$ $\widetilde{C}_{10,47} = \widetilde{C}_{47,10} = (3, 9, 12)$ $\widetilde{C}_{28,35} = \widetilde{C}_{35,28} = (8, 12, 13)$
$\widetilde{C}_{8,16} = \widetilde{C}_{16,8} = (14, 16, 22)$ $\widetilde{C}_{62,65} = \widetilde{C}_{65,62} = (2, 6, 8)$ $\widetilde{C}_{40,5} = \widetilde{C}_{5,40} = (2, 6, 14)$
$\widetilde{C}_{5,52} = \widetilde{C}_{52,5} = (10, 20, 22)$ $\widetilde{C}_{2,7} = \widetilde{C}_{7,2} = (4, 12, 13)$

Figures 16 and 17 present the best tour solutions with different sensitivities; fuzzy edges are marked with circles.

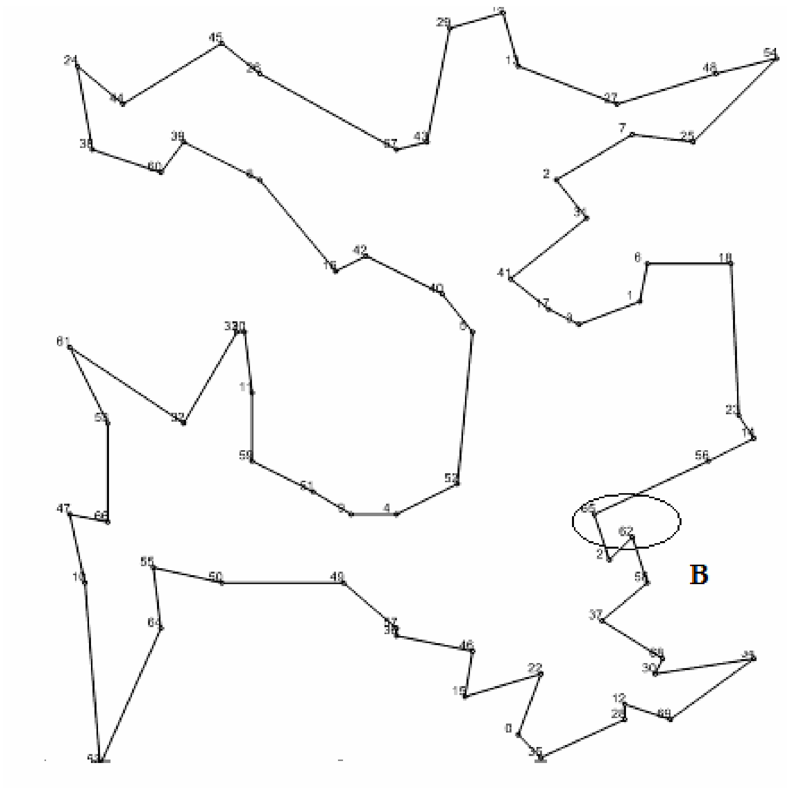

FIGURE 11. FRTTSP st70 best tour with $\lambda_0 = 0$ and $\lambda_1 = 0.01$

Finally, the time dependency is demonstrated with different velocities in Figures 18 and 19. Circle H shows that the cost of some connections of node 63 (edge 10-63 and edge 63-64) is increasing in time significantly, so that the best tour must involve edge 50-63 (see circle H) instead of them. On the other hand, edge 56-62 shows a favorable tendency in time, so when the tour with $v = 50$ reaches that region it is reasonable to select that connection (see circle G), whilst the faster tour with $v = 100$ gets there "too early", so edge 56-65 is selected.

7. **Conclusions.** The special features of road transportation and supply chains encourage the modification of the classical TSP, eliminating most of the original constraints, but making the problem more complicated in some sense. Solutions in the literature are devoted to the classical problem, so after redefining the TSP and transforming it to FRTTSP a novel approach is proposed. In the FRTTSP the costs between the nodes may depend on time and they have imprecise values involving uncertainties modeling the real life processes. These uncertainties can be represented by fuzzy numbers, which are capable of expressing the loss aversion as well, since the symmetric and asymmetric features of a fuzzy number and the peak value – support length rate can give a base for parametric quantification of the expected subjective loss.

For the solution of the FRTTSP the eugenic bacterial memetic algorithm was proposed. This approach combines the bacterial evolutionary algorithm performing a global search with local search techniques and improving the candidate solutions in order to speed up the evolutionary process. The simulation results confirm the effectiveness of the proposed technique. On the other hand, the experiments emphasize the importance of fine-tuning the model parameters responsible for converting the nature of human thinking into numerical representation.

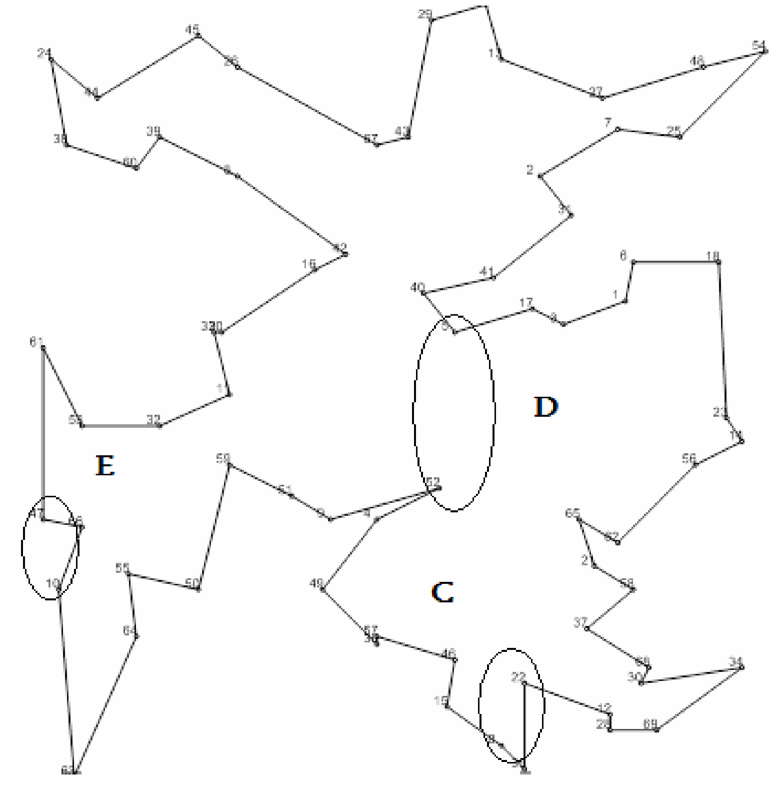

FIGURE 12. FRTTSP st70 best tour with $\lambda_0 = 0$ and $\lambda_1 = 0.1$

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REFERENCES

[1] E. E. Ammar and E. A. Youness, Study on multiobjective transportation problem with fuzzy numbers, *Applied Mathematics and Computation*, vol.166, pp.241-253, 2005.

[2] D. L. Applegate, R. E. Bixby, V. Chvátal and W. J. Cook, *The Traveling Salesman Problem, A Computational Study*, Princeton University Press, Princeton and Oxford, 2006.

[3] R. J. Aumann, Rationality and bounded rationality, *Games and Economic Behaviour*, vol.21, pp.2-14, 1997.

[4] B. Bontoux and D. Feillet, Ant colony optimization for the traveling purchaser problem, *Computers & Operations Research*, vol.35, pp.628-637, 2008.

[5] J. Botzheim, C. Cabrita, L. T. Kóczy and A. E. Ruano, Fuzzy rule extraction by bacterial memetic algorithms, *Proc. of IFSA 2005*, Beijing, China, pp.1563-1568, 2005.

[6] J. Botzheim, M. Drobics and L. T. Kóczy, Feature selection using bacterial optimization, *Proc. of the International Conference on Information Processing and Management of Uncertainty in Knowledge-based Systems*, Perugia, Italy, pp.797-804, 2004.

[7] J. Botzheim and P. Földesi, Approaching the fuzzy road transport traveling salesman problem by eugenic bacterial memetic algorithm, *Proc. of the 4th International Symposium on Computational Intelligence and Intelligent Informatics*, Egypt, pp.15-22, 2009.

[8] J. Botzheim, P. Földesi and L. T. Kóczy, Solution for fuzzy road transport traveling salesman problem using eugenic bacterial memetic algorithm, *Proc. of IFSA 2009*, Lisbon, Portugal, pp.1667-1672, 2009.

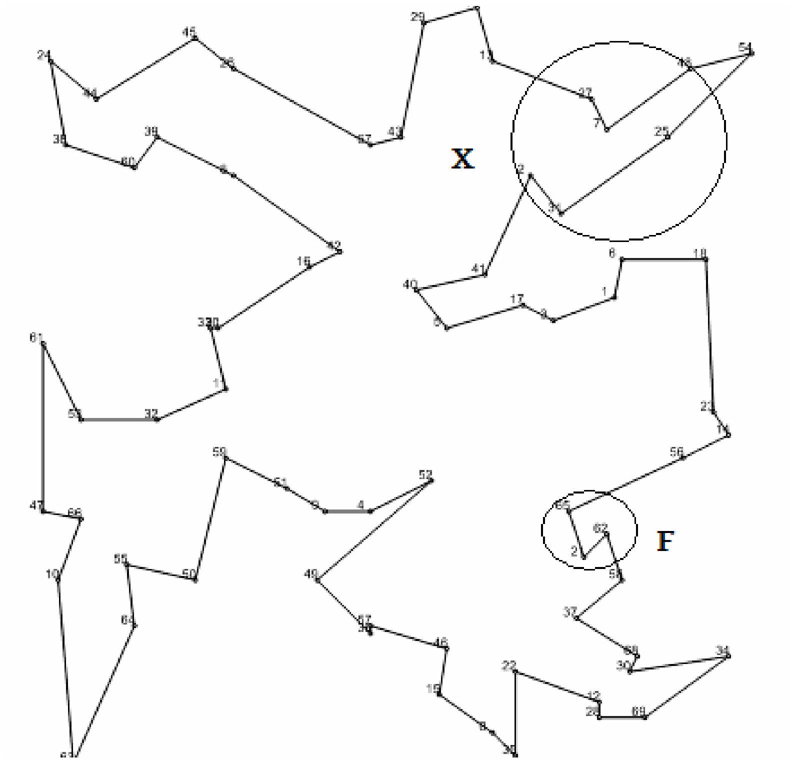

FIGURE 13. FRTTSP st70 best tour with $\lambda_0 = 0.0001$ and $\lambda_1 = 0.5$

[9] J. Botzheim, B. Hámori, L. T. Kóczy and A. E. Ruano, Bacterial algorithm applied for fuzzy rule extraction, *Proc. of the International Conference on Information Processing and Management of Uncertainty in Knowledge-based Systems*, Annecy, France, pp.1021-1026, 2002.

[10] Y.-W. Chen, Y.-Z. Lu and P. Chen, Optimization with extremal dynamics for the traveling salesman problem, *Physica A*, vol.385, pp.115-123, 2007.

[11] T. H. Cormen, C. E. Leiserson, R. L. Rivest and C. Stein, *Introduction to Algorithms*, The MIT Press/McGraw Hill, 2002.

[12] C. Ding, Y. Cheng and M. He, Two-level genetic algorithm for clustered traveling salesman problem with application in large-scale TSPs, *Tsinghua Science and Technology*, vol.12, no.4, pp.459-465, 2007.

[13] D. Favaretto, E. Moretti and P. Pellegrini, An ant colony system approach for variants of traveling salesman problem with time windows, *Journal of Information & Optimization Sciences*, vol.27, pp.35-54, 2006.

[14] P. Földesi and J. Botzheim, Solution for modified traveling salesman problem with variable cost matrix using bacterial evolutionary algorithm, *Acta Technica Jaurinensis Series Logistica*, vol.1, no.2, pp.159-171, 2008.

[15] P. Földesi, J. Botzheim and L. T. Kóczy, Fuzzy exponents for heuristic based applications, *Acta Technica Jaurinensis Series Intelligentia Computatorica*, vol.1, no.3, pp.423-435, 2008.

[16] M. R. Gholamian, S. M. T. F. Ghomi and M. Ghazanfari, A hybrid system for multiobjective problems – A case study in NP-hard problems, *Knowledge-Based Systems*, vol.20, pp.426-436, 2007.

[17] M. R. Held and R. M. Karp, A dynamic programming approaches to sequencing problems, *Journal of the Society of Industrial and Applied Mathematics*, vol.10, pp.196-210, 1962.

[18] J. H. Holland, *Adaptation in Natural and Artificial Systems: An Introductory Analysis with Applications to Biology, Control, and Artificial Intelligence*, The MIT Press, Cambridge, 1992.

[19] C.-H. Hsu, C.-S. Tsou and F.-J. Yu, Multicriteria tradeoffs in inventory control using memetic particle swarm optimization, *International Journal of Innovative Computing, Information and Control*, vol.5, no.11(A), pp.3755-3768, 2009.

[20] *http://www.iwr.uni-heidelberg.de/groups/comopt/software/TSPLIB95/.*

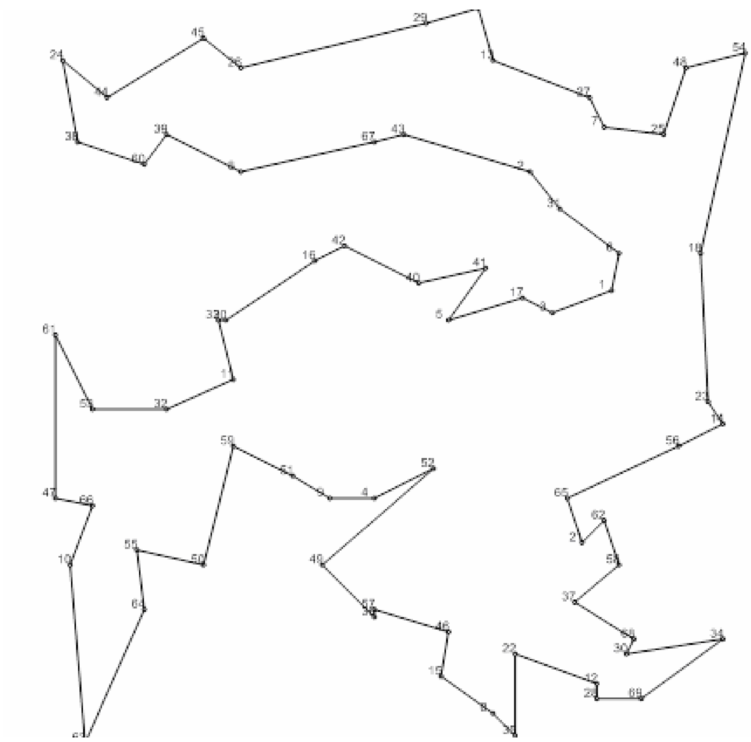

Figure 14. FRTTSP st70 best tour without fuzzy edges

[21] S. Kikuchi and P. Chakroborty, Place of possibility theory in transportation analysis, *Transportation Research – Part B*, vol.40, pp.595-615, 2006.

[22] V. Köbberling and P. P. Wakker, An index of loss aversion, *Journal of Economic Theory*, vol.122, pp.119-131, 2005.

[23] S. Lin and B. W. Kernighan, An effective heuristic algorithm for the traveling-salesman problem, *Operations Research*, vol.21, pp.498-516, 1973.

[24] Y.-H. Liu, A hybrid scatter search for the probabilistic traveling salesman problem, *Computers & Operations Research*, vol.34, pp.2949-2963, 2007.

[25] G.-C. Luh and S.-W. Lee, A bacterial evolutionary algorithm for the job shop scheduling problem, *Journal of the Chinese Institute of Industrial Engineers*, vol.23, no.3, pp.185-191, 2006.

[26] P. Moscato, On evolution, search, optimization, genetic algorithms and martial arts: Towards memetic algorithms, *Tech. Rep.*, 1989.

[27] N. E. Nawa and T. Furuhashi, Fuzzy system parameters discovery by bacterial evolutionary algorithm, *IEEE Trans. on Fuzzy Systems*, vol.7, no.5, pp.608-616, 1999.

[28] M. Oussalah, On the compability between defuzzification and fuzzy arithmetic operations, *Fuzzy Sets and Systems*, vol.128, pp.247-260, 2002.

[29] E. Ozcan and M. Erenturk, A brief review of memetic algorithms for solving Euclidean 2D traveling salesrep problem, *Proc. of Turkish Symposium on Artificial Intelligence and Neural Networks*, 2004.

[30] X. C. Shi, Y. C. Liang, H. P. Lee, C. Lu and Q. X. Wang, Particle swarm optimization-based algorithms for TSP and generalized TSP, *Information Processing Letters*, vol.103, pp.169-176, 2007.

[31] G. Sun and M. Liao, Upper bound for transport capacity of sensor networks, *ICIC Express Letters*, vol.3, no.1, pp.93-98, 2009.

[32] J. Tang, M. H. Lim, Y. S. Ong and M. J. Er, Parallel memetic algorithm with selective local search for large scale quadratic assignment problems, *International Journal of Innovative Computing, Information and Control*, vol.2, no.6, pp.1399-1416, 2006.

[33] E. J. Teoh, K. C. Tan, H. J. Tang, C. Xiang and C. K. Goh, An asynchronous recurrent linear threshold network approach to solving the traveling salesman problem, *Neurocomputing*, vol.71, pp.1359-1372, 2008.

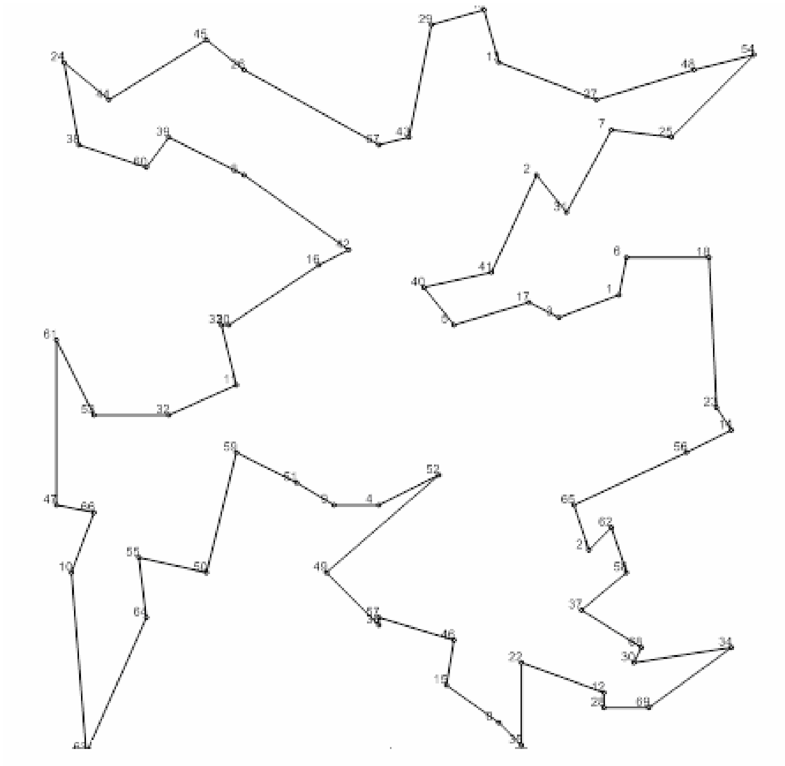

FIGURE 15. FRTTSP st70 best tour with $w = 0.3$, $K = 0.0000005$ and $w = 0.04$, $K = 0.000001$

[34] J. Ye, M. Tanaka and T. Tanino, Eugenics-based genetic algorithm, *IEICE Trans. on Information and Systems*, vol.E79-D, no.5, pp.600-607, 1996.

[35] J. Yi, G. Yang, Z. Zhang and Z. Tang, An improved elastic net method with time-dependent parameters for traveling salesman problem, *International Journal of Innovative Computing, Information and Control*, vol.5, no.4, pp.1089-1100, 2009.

[36] J. Yin and J. Branke, Evolutionary optimization in uncertain environments – A Survey, *IEEE Trans. on Evolutionary Computation*, vol.9, no.3, pp.303-317, 2005.

[37] L. A. Zadeh, The concept of linguistic variable and its application to approximate reasoning, *Inform. Sci. Part 1, Part 2, Part 3*, vol.8, pp.199-249, pp.301-357, vol.9, pp.43-80, 1975.

[38] Y. Zhu, Z. Tang, H. Dai and S. Gao, Cooperation artificial immune system with application to traveling salesman problem, *ICIC Express Letters*, vol.2, no.2, pp.143-148, 2008.

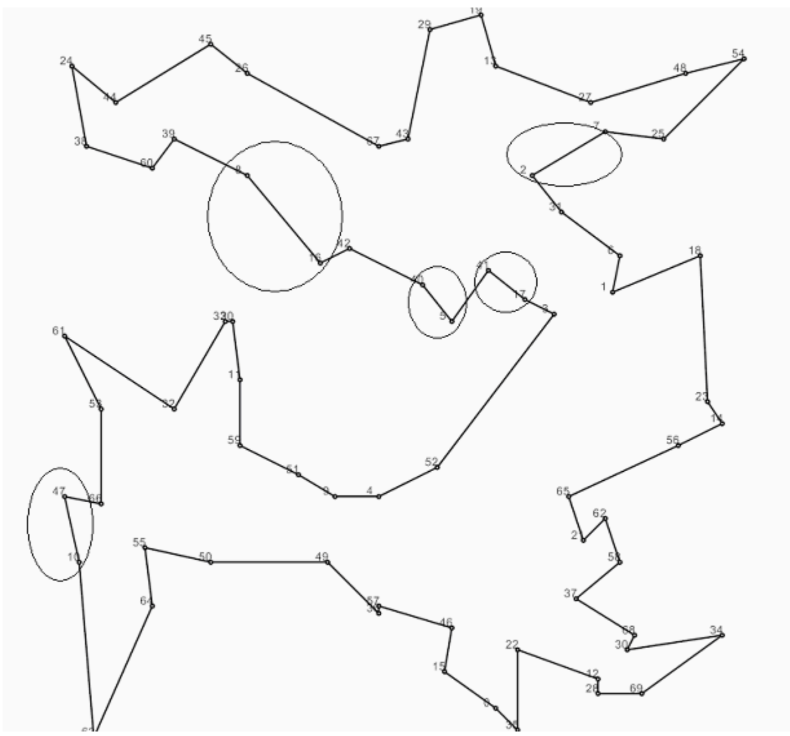

FIGURE 16. Modified FRTTSP st70 best tour with $w = 0.2$ and $K = 0.00001$

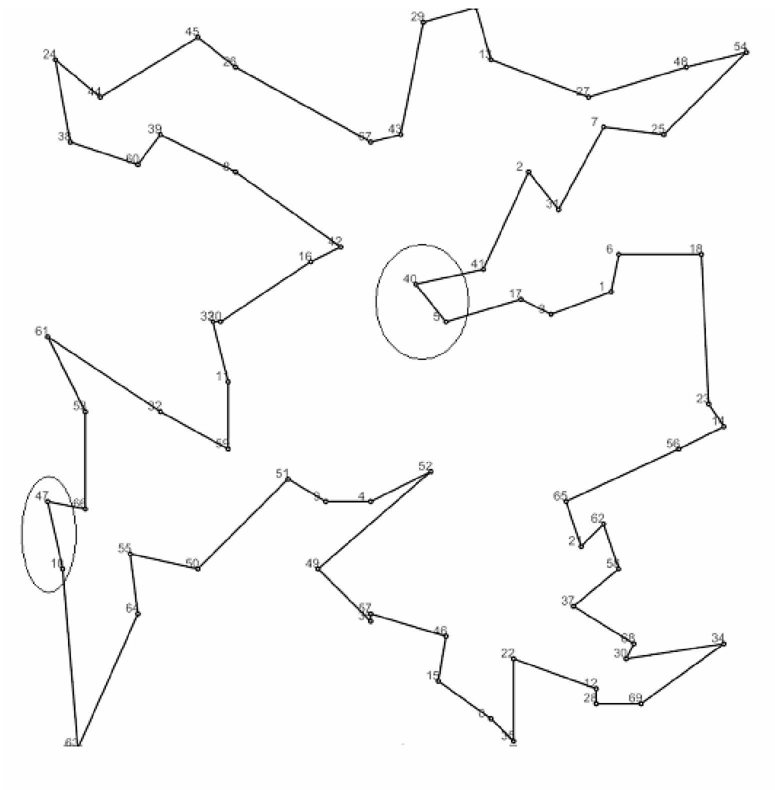

FIGURE 17. Modified FRTTSP st70 best tour with $w = 0.8$ and $K = 0.0000015$

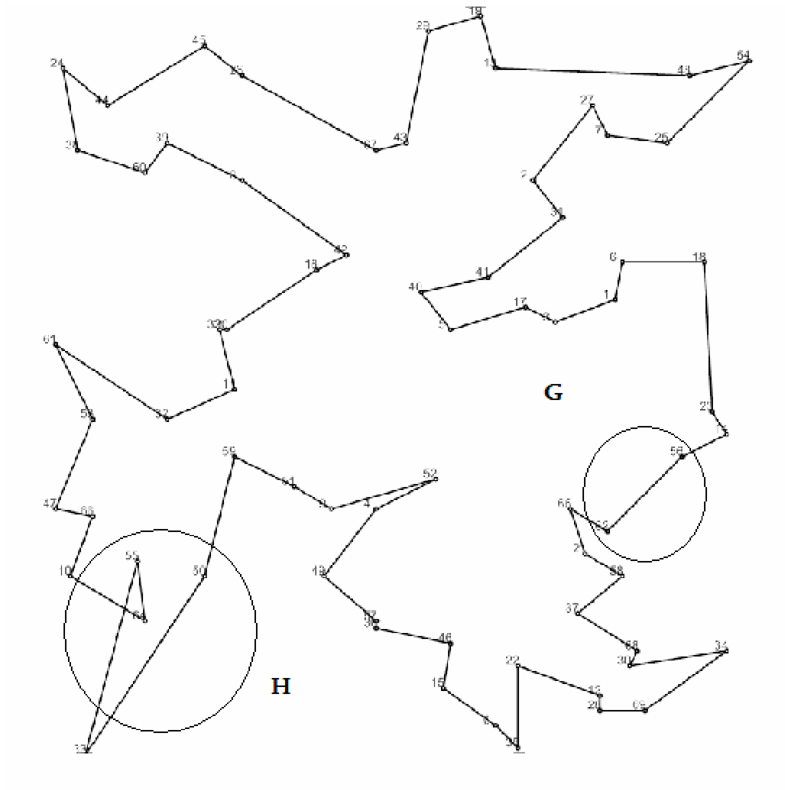

Figure 18. FRTTSP st70 best tour with $w = 0.5$, $K = 0.000001$ and $v = 50$

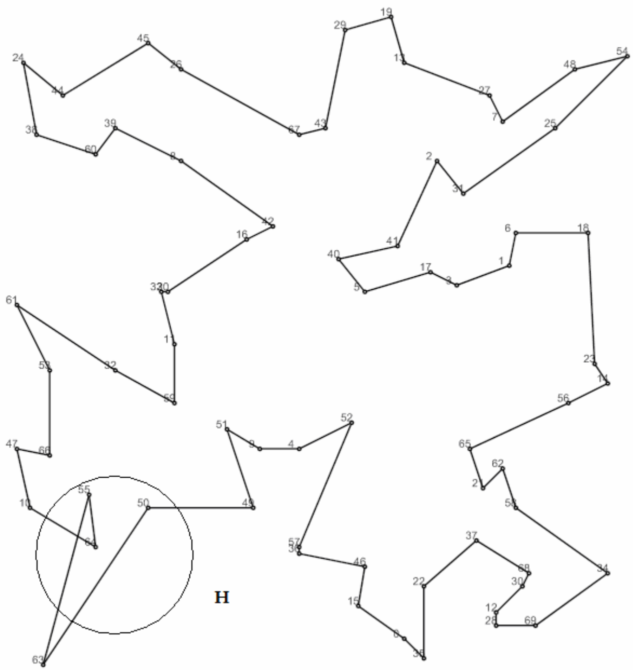

Figure 19. FRTTSP st70 best tour with $w = 0.5$, $K = 0.000001$ and $v = 100$