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# PARAMETRIC APPROXIMATION OF FUZZY EXPONENT FOR COMPUTATIONALLY INTENSIVE PROBLEMS

János Botzheim[1] and Péter Földesi[2]

[1]Department of Automation
[2]Department of Logistics and Forwarding
Széchenyi István University
1 Egyetem tér, Győr 9026, Hungary
{ botzheim; foldesi }@sze.hu

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Abstract. *The paper deals with the investigation of the critical non-linear factors and NP-hard problems of real-life decision-making processes. When using non-linear utility/objective functions to represent the value of various options in the search space or when NP-hard problems arise, often soft computing techniques must be applied for optimization. Many times significant uncertainty must be handled as well, so the use of fuzzy numbers can be an efficient method to cope with ambiguity and lack of information. The fuzzy extensions for heuristic based optimizing algorithms often face the problem of an increased number of calculations required to find the solutions. Appropriate representation of the fuzzy power function for non-linear cases is to be used so that it can keep the required computation time and resources at a reasonable level.*
**Keywords:** Parametric approximation, Fuzzy exponent

## 1. Introduction.

In real life decision-making processes a very frequent question is how to maximize the output using limited resources or minimize the costs when fulfilling given requirements. In this paper three case studies are presented in order to illustrate the importance of non-linear, fuzzy objective functions.

The first case study is concerned with time-compression. The literature review gives a background to understand and handle the reasons and consequences of the growing importance of time, and the phenomenon of time inconsistency [1,2]. By using utility functions to represent the value of various delivery-times for the different participants in the supply chain, including the final customers, it is shown that the behavior and willingness of payment of time-sensitive and non time-sensitive consumers are different for varying lead times [3,4]. For optimization soft computing techniques (e.g., particle swarm optimization [5]) can be efficiently applied.

The second case study is Kano's quality model [6] that classifies the relationships between customer satisfaction and attribute-level performance and indicates that some of the attributes have a non-linear relationship to satisfaction, and the power-function must be used. For the customers' subjective evaluation these relationships are not deterministic and are uncertain. The use of fuzzy sets and fuzzy numbers can dissolve this problem as it was shown in [7]. The customers' assessment of technical attributes is very uncertain and very often it is a result of a group decision method [7,8], so in Kano's model the exponents of satisfaction functions cannot be considered as deterministic values; fuzzy numbers can give a better representation.

The third case study is the Eugenic Bacterial Memetic Algorithm for Fuzzy Road Transport Traveling Salesman Problem (FRTTSP). The aim of the classical Traveling

Salesman Problem (TSP) is to find the cheapest way of visiting all elements in a given set of cities (nodes) exactly once and returning to the starting point. In solutions presented in the literature costs of travel between nodes are based on Euclidean distances, the problem is symmetric and the costs are constant and crisp values [9]. Practical application in road transportation and supply chains are often uncertain or fuzzy. The risk attitude depends on the features of the given operation. The FRTTSP [10] handles the fuzzy, time dependent nature of the TSP and also gives a solution for the asymmetric loss aversion by embedding the risk attitude into the fitness function of the eugenic bacterial memetic algorithm [11].

Since the above mentioned cases mean a non-linear search space or NP-hard problems, the application of analytical approaches is limited; a heuristic search can be used. The fuzzy extensions for heuristic based optimizing algorithms often face the problem of an increased number of calculations required to find the solutions, since even in the case of the simplest fuzzy number (the triangular one) the necessary calculation is three times larger when considering the core value and supporting interval as minimum information. Moreover, when the fuzzy power function has to be computed the use of the extension principle [12] requires a large number of calculations that make the run of a heuristic search – such as bacterial memetic algorithm – very slow. In [13] it is shown that in some cases the extension principle is hard to calculate, and thus for practical reasons a simple parametric representation of fuzzy power function should be used, in order to keep the required computation time and resources at a reasonable level.

## 2. Case Studies.

### 2.1. Time compression.

Time has limits, consumers have become time-sensitive and choose the contents of their basket of commodities according to available time as well. The time necessary to obtain a product/service (access time) is involved in product utility to an increasing extent, the assurance of which is the task of logistics. There are more reasons for the shortening of this access time, one of the most important is the change in customer expectations, which can be related to new trends emerging in the most diverse areas with the time factor playing the main role [14]. Increasing rapidity is also encouraged by the sellers in the competition against each other based on time, because of the pressure to reduce costs and inventory and to increase the efficiency and customer satisfaction [15,16]. This paper is concerned with the time-sensitive and non time-sensitive customers behavior by using utility functions.

The time-sensitive segment of population continuously increases. The time-sensitive segment, which depends on time, expects special services with high time-quality, speed and punctuality, and because of the increasing time-preference the intertemporal decisions are present-asymmetric.

Logistics has to find delivery solutions adjusted to the consumption behavior of products, which generates many kinds of logistical needs. The importance of time is different according to production and consumption points of view but it is different due to customer segments and groups of product as well.

More research [17] has found that there is a close relationship between the entire lead time (defined as the period between a purchase order placement and its receipt by the client), the customers' demands, the willingness of payment and the customer loyalty. Karmarkar [18] pointed out that shorter delivery times are most probably inversely related to market shares or price premiums or both. Customers highly appreciate short and punctual delivery time; therefore they will not turn to competitors. Customers may be willing to pay a price premium for shorter delivery times.

Consumer-value of goods, created through production, is basically determined by the quality of the product but it can also be influenced by the time and place of its access. These two latter values are value-categories created by logistics. Time-value becomes more important as it is determined by the lead time between the appearance and the satisfaction of demand [16]. It is maximal when the search-production-obtainment of the product does not have any time-requirements; that is to say the demand can be fulfilled immediately at the moment of its appearance.

Time sensitivity is different with each consumer and product. We can speak about time sensitive consumer segments and also such kinds of products, which are very sensitive to any waiting or delay. The willingness of waiting is in relation to the importance of the product and its substitution. With the first one, the waiting-willingness is in direct proportion while with the latter one it is in the inverse ratio. Its formation determines the amount of the opportunity cost of waiting for a product for the consumer. Waiting means opportunity cost, the cost of which comes from wasted-time and wasted possibilities.

How we value time depends on several factors. First of all it depends on the customer type. We distinguish between the end user and the industrial customer. The final buyer gets more and more time-sensitive, so in his/her case the choice based on time can describe a utility function, which measures product usefulness depending on the quantity/lengths of time it takes to obtain it. The derivative function can also give information about how the marginal utility of time behaves. If we can compare it with the marginal cost function of service, we can see whether it is worth making efforts to have faster service in a certain segment.

For the buyers at higher levels of the supply chain, those who buy for further processing (producers), or for reselling (wholesalers, retailers), there is another kind of utility function to draw. The limited time-utility is due to the larger time consciousness because time costs money for companies. Like the aim to satisfy the consumer at a high level, the aim to operate efficiently as well leads to optimizing on a time basis.

There are consumers who are not sensitive to time, who do not want to or are not able to afford rapidity. There are products/services as well, where urgency is not necessary, just the opposite, quality is brought by time (e.g., process-centered services). Price is not increasing parallel to faster service (opposite direction on the lead time axis); price is constant and independent of time. The buyer does not pay more, even for a quicker service. His/her relation to time is totally inflexible [19].

Time-elasticity appears in a flexible behavior, which means a 1% relative decrease in lead time can realize a relative higher price-increment. Even a consumer surplus can arise if the reservation price (the maximum price the buyer is willing to pay for a certain time) is higher than the price fixed by the provider.

Concerning the customers' time sensitivity detailed above the following model can be set [20]. The customer satisfaction is affected by two elements:

- the actual utility of obtaining the goods
- the accuracy of the service, variance of the lead time

A simple representation of the satisfaction based on utility can be:

$$S_U(t) = u_0 - u_1 \cdot t^{\beta_U} \tag{1}$$

where $u_0$ and $u_1$ are real constants, $t$ is is the lead time, and $\beta_U > 1$ represents the time sensitivity of customers. (The value of $u_0$ shows the satisfaction of obtaining the goods with zero lead time. Negative values of $S_U$ mean dissatisfaction.)

The accuracy is considered as an attractive service element in modern logistic, just-in-time systems. When the supply chain is being extended, that is the lead time is growing, the accuracy and hit of the time-window is getting harder, thus we can write:

$$A(t) = a_0 - a_1 \cdot t, \tag{2}$$

where $A(t)$ is the measure of accuracy, $a_0$ and $a_1$ are real constants, $t$ is the time. The satisfaction measure is progressive:

$$S_A(t) = (a_0 - a_1 \cdot t)^{\beta_A}, \tag{3}$$

where $\beta_A > 1$ is the sensitivity.

The cost of the actual logistic service depends on the lead time required, the shorter lead time is the more expensive service. Since the cost reduction is not a linear function of the lead time extension we can write:

$$C(t) = c_0 + \frac{c_1}{t}, \tag{4}$$

where $C(t)$ is the cost, $t$ is the lead time and $c_0$ and $c_1$ are real constants.

The target is to maximize the total satisfaction over the costs:

$$\max \left[ \frac{S_U(t) + S_A(t)}{C(t)} \right], \quad 0 < t < \infty. \tag{5}$$

In real life decision making processes the actual sensitivity of a customer or a group of customers cannot be represented by a single, real number [8]. These values are more like intervals with some emphasis on the center of the interval, thus fuzzy numbers can be efficiently applied for representation. For optimization soft computing techniques (e.g., particle swarm optimization [5]) can be used, thus the problem of fuzzy exponent must be resolved.

2.2. **Kano's quality model.** For designing and developing products/services it is vital to know the relevancy of the performance generated by each technical attribute and how they can increase customer satisfaction. Improving the parameters of technical attributes requires financial resources, and the budgets are generally limited. Kano's quality model classifies the relationships between customer satisfaction and attribute-level performance and indicates that some of the attributes have a non-linear relationship to satisfaction; rather a power-function should be used [6]. For the customers' subjective evaluation these relationships are not deterministic and are uncertain.

In the designing process of products/services, their technical attributes must be determined so that the maximum customer satisfaction can be achieved within acceptable and reasonable financial limits. Technical attributes have different effects on the satisfaction. Kano explored [6,21] that the features and characteristics of these relationships differ; from the point of view of customers the utility functions are different as well. On the other hand customers requirements are not homogenous, they are changing in time and also differences can be detected even in the same market segment. Because of these differences in the mathematical model for Kano's quality assessment it is worth applying fuzzy numbers instead of crisp values. Results have been devoted to the relationship between technical attributes and customer requirements in correlation terms [22,23], or represented the uncertainty of budgeting by fuzzy measures [24]. Matzler et al. [25] explored the asymmetric feature of the relationship between attribute-level performance and overall customer satisfaction and indicated indirectly that linear functions are not appropriate in each case. Application of fuzzy logic for ranking technical attributes is presented in [26].

In his model [6] Kano distinguishes three types of product requirements, which influence customer satisfaction in different ways when met (see Figure 1).

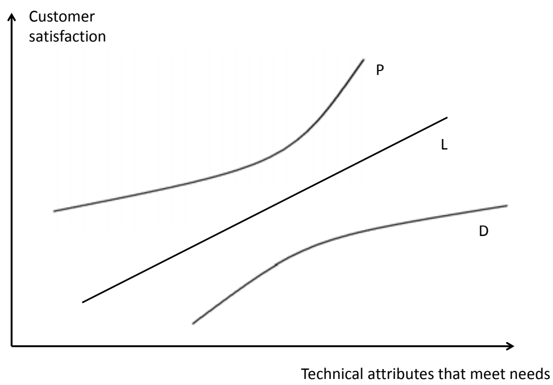


FIGURE 1. Kano's model of customer satisfaction

- Degressive or "must-be" requirements are basic criteria of a product. From a given point improving the technical attributes by unit results in minor increments of satisfaction, on the other hand not fulfilling the requirements induces dissatisfaction ("negative satisfaction").
- One-dimensional requirements. Customer satisfaction is proportional to the level of fulfillment, the higher level of fulfillment, the higher the customer satisfaction, and vice versa.
- Progressive or attractive/excitement factors: fulfilling these requirement leads to more than proportional satisfaction.

Questions of budgeting is not a key element of Kano's original model but we can reasonably assume that improving the level of technical attributes requires extra costs, so for each technical attribute a cost function can be set. The general target is to achieve the maximum economic result with the minimum use of resources, that is to maximize customer satisfaction with the minimum cost. The task can be mathematically formulated in two ways [27]:
maximize overall satisfaction $S$ not exceeding given cost limit $C_0$
or
achieve given overall satisfaction $S_0$ with minimum cost $C$.

Let

$$S_i(x_i) = b_i + a_i \cdot x_i^{\beta_i} \quad i = 1, 2, \ldots, n \tag{6}$$

be the customer satisfaction generated by technical attribute $x_i$, where

- $0 \leq x_i \ll \infty$ is a real number variable
- $a_i > 0$ is a real constant
- $\beta_i > 0$ is a real number
- $b_i$ is a constant such that $sgn(b_i) = sgn(\beta_i - 1)$
- $n$ is the number of technical attributes considered in the designing process.

Furthermore, let

$$C_i(x_i) = f_i + v_i \cdot x_i \quad i = 1, 2, \ldots, n \tag{7}$$

be the cost of manufacturing technical attribute at level $x_i$, $f_i \geq 0$, $v_i \geq 0$ are real constants. Let

$$S = \sum_{i=1}^{n} b_i + a_i \cdot x_i^{\beta_i} \tag{8}$$

be the overall satisfaction and

$$C = \sum_{i=1}^{n} f_i + v_i \cdot x_i \tag{9}$$

be the total cost, according to the "fixed costs – variable costs" methodology. Then the general formula is:

- Let $\sum_{i=1}^{n} S_i(x_i) \rightarrow \max$, subject to $\sum_{i=1}^{n} C_i(x_i) \leq C_0$, where $C_0$ is a given constant.
- Let $\sum_{i=1}^{n} C_i(x_i) \rightarrow \min$, subject to $\sum_{i=1}^{n} S_i(x_i) \geq S_0$, where $S_0$ is a given constant.

The situation is significantly different when there are several customers at the same time (which is very often the case in practice), and some customers find a given technical attribute linear, some other assess it "must-be", and again some others possibly consider it as an attractive factor. In that case the exponent $\beta$ is no longer a real number, fuzzy numbers can be applied instead of it. If $\beta$ (see Equation (6), where $\beta_i$ is the importance of the given technical attribute $x_i$) is considered as a fuzzy number then the features of each technical attribute are given by the shape of the membership function of $\widetilde{\beta}$. In that fuzzy case a very efficient approximation for fuzzy power function is needed, since the solution can only be obtained by using computational intensive heuristics, e.g., bacterial memetic algorithms [28].

2.3. **The modified TSP, the fuzzy road transport traveling salesman problem (FRTTSP).** The TSP is a very good representative of a larger class of problems known as combinatorial optimization problems [9]. The problem presented in the literature most frequently has the following features. Costs of travel between nodes (cities) are based on Euclidean distances, the problem is symmetric and the costs are constant [9,29]. Since the original formulation of the problem states: the aim is to find the "cheapest" tour, thus the cost matrix that represents the distances between each pair must be determined by calculating the actual costs of the transportation processes.

The actual costs are rarely constant and predictable, so fuzzy cost coefficients may be applied in order to represent uncertainty. In FRTTSP the sensitivity for uncertainty and risk aversion is embedded into the evaluation process, so that the priority of accuracy in relation to the actual costs can be set in advance [10].

In order to represent the uncertainty triangular fuzzy numbers are used as cost coefficients. Triangular fuzzy numbers have a membership function consisting of two linear segments joined at a peak, so they can be constructed easily on the basis of little information: the supporting interval $C = [c_L, c_R]$ as the smallest and the largest possible values, and $c_C$ which is the peak value where the membership function equals 1. In that case the triangular fuzzy number is denoted by $\widetilde{C} = (c_L, c_C, c_R)$.

When the distances between the cities are described by fuzzy numbers, it must be discussed how these fuzzy numbers are summed up in a tour in order to calculate the total distance. The arithmetic of fuzzy numbers is based on the extension principle [12]. When we calculate the total distance of a tour, then instead of adding fuzzy numbers by the extension principle, we can do an easier calculation based on the defuzzified values of the fuzzy numbers. Although this approach is straightforward, the uncertainty cannot be handled properly if only defuzzified values are considered for evaluating the quality of the tour. The FRTTSP proposes two other approaches for the evaluation of the tour based not only on the length described by the defuzzified values but the uncertainty involved in the fuzzy numbers as well. Since now we need not only the defuzzified values but the length of the tour as a fuzzy number, the fuzzy distances along the tour have to be summed up. As we have triangular fuzzy numbers described by their three breakpoints, the addition can be easily computed by summing the corresponding breakpoints. Denoting the total fuzzy tour by $\widetilde{\beta}$, its three characteristic points, $\beta_L$, $\beta_C$, $\beta_R$ can be calculated.

Let us denote by $D$ the defuzzified length of the total fuzzy tour $\widetilde{\beta}$:

$$D = defuzz(\widetilde{\beta}) = \frac{\beta_L + \beta_C + \beta_R}{3}. \tag{10}$$

Let us denote by $U$ the uncertainty belonging to the tour; i.e., the length of the support of the total fuzzy tour $\widetilde{\beta}$:

$$U = \beta_R - \beta_L. \tag{11}$$

In the first approach the tour is evaluated as:

$$f_1 = D \cdot \left(2 - e^{-U \cdot \left[\lambda_0 + \frac{\lambda_1}{D}\right]}\right), \tag{12}$$

where $\lambda_0$ and $\lambda_1$ are parameters representing the sensitivity for uncertainty. Parameter $\lambda_0$ and $\lambda_1$ are subject to the features of the problem and the priorities of the operator.

In the second approach the tour is evaluated as:

$$f_2 = D \cdot \left\{2 - \exp\left[-\widetilde{\beta} \cdot (\beta_R - \beta_L) \cdot (\beta_R - \beta_C)^w \cdot K\right]\right\}, \tag{13}$$

where $w$ and $K$ are positive parameters.

As it can be seen in Equation (13) the calculation with a fuzzy exponent is used in every fitness evaluation requiring high computational effort.

3. **Fuzzy Power Function.** According to the extension principle defined for the fuzzy exponent in [12] the $a^{\widetilde{\beta}}$ using the crisp number $a$ and fuzzy number $\widetilde{\beta}$ can be calculated as $\mu_{a^{\widetilde{\beta}}}(x) = \mu_{\widetilde{\beta}}(\log_a x)$.

3.1. **Our proposed approach.** We propose another, parametric approach for fuzzy exponent calculation, which fits better to problems mentioned in Section 2.

If $\widetilde{\beta}$ is considered as a fuzzy number then the membership function of $\widetilde{\beta}$ as shown in Figure 2 is

$$\mu_{\widetilde{\beta}}(\beta) = \begin{cases} \frac{\beta - \beta_L}{\beta_C - \beta_L} & \text{if } \beta_L \le \beta \le \beta_C, \\ \frac{\beta_R - \beta}{\beta_R - \beta_C} & \text{if } \beta_C < \beta \le \beta_R, \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

The supporting interval is $[\beta_L, \beta_R]$. Thus $\widetilde{\beta} = (\beta_L, \beta_C, \beta_R)$.

The question now is, how to represent the fuzziness of exponents. For problems investigated in this paper a promising solution is to weight the functions and sum up three weighted functions with weighted exponents (see Figure 3).

The solution for $y = a^{\widetilde{\beta}}$ in this case is:

$$y = \frac{1}{3 - 2 \cdot \alpha} a^{\beta_C} + \frac{1 - \alpha}{3 - 2 \cdot \alpha} a^{\beta_C - (1-\alpha)(\beta_C - \beta_L)} + \frac{1 - \alpha}{3 - 2 \cdot \alpha} a^{\beta_C + (1-\alpha)(\beta_R - \beta_C)}. \tag{15}$$

The general formula for any $m = 2 \cdot k + 1$ points can be given, where $k = 1, 2, 3 \ldots$ is the number of $\alpha$-cuts (see Figure 4):

$$y = \frac{1}{1 + m - \sum_{j=1}^{m} \mu_{\widetilde{\beta}}(\beta_j)} a^{\beta_{k+1}} + \sum_{i=1}^{m} \frac{1 - \mu_{\widetilde{\beta}}(\beta_i)}{1 + m - \sum_{j=1}^{m} \mu_{\widetilde{\beta}}(\beta_j)} a^{\beta_i}. \tag{16}$$

Note that $\mu_{\widetilde{\beta}}(\beta_{k+1}) = 1$, thus $1 - \mu_{\widetilde{\beta}}(\beta_{k+1}) = 0$ and $\mu_{\widetilde{\beta}}(\beta_i) = \mu_{\widetilde{\beta}}(\beta_{m+1-i})$.

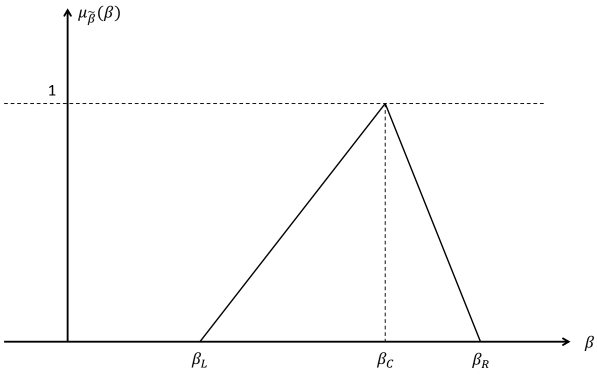


FIGURE 2. Exponent $\widetilde{\beta}$ as a fuzzy number

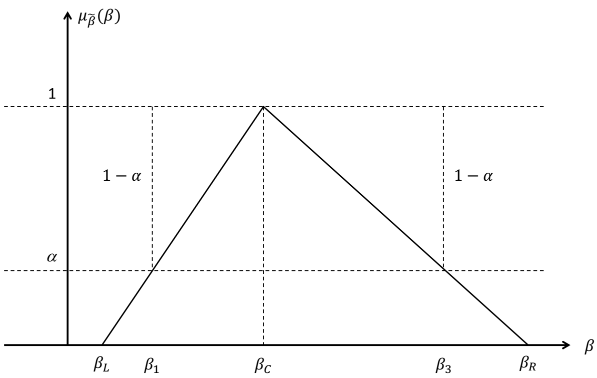


FIGURE 3. Weights and exponents based on $1 - (\alpha$-cuts)

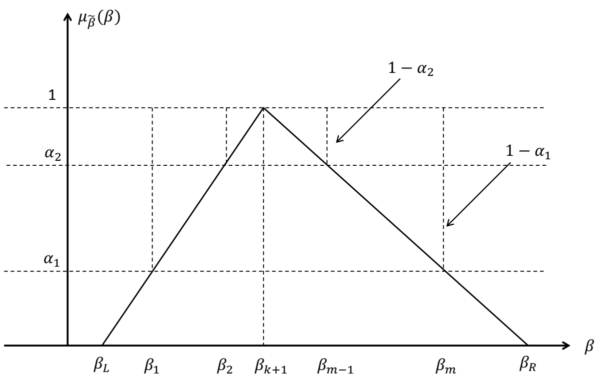


FIGURE 4. Exponents for $m$ functions based on $1 - (\alpha$-cuts)

The continuous formula is

$$y = \frac{a^{\beta_C} + \int_{\beta_L}^{\beta_R}(1 - \mu_{\widetilde{\beta}}(\beta))a^{\beta}\,\mathrm{d}\beta}{1 + \int_{\beta_L}^{\beta_R}(1 - \mu_{\widetilde{\beta}}(\beta))\,\mathrm{d}\beta}. \tag{17}$$

In the case of triangular fuzzy numbers $\int_{\beta_L}^{\beta_R}(1 - \mu_{\widetilde{\beta}}(\beta))\,\mathrm{d}\beta = \frac{\beta_R - \beta_L}{2}$ so Equation (17) can be recast as:

$$y = \frac{2}{2 + \beta_R - \beta_L}\left[a^{\beta_C} + \int_{\beta_L}^{\beta_R}(1 - \mu_{\widetilde{\beta}}(\beta))a^{\beta}\,\mathrm{d}\beta\right]. \tag{18}$$

After calculating the integral Equation (18) becomes:

$$y = \frac{2}{2 + \beta_R - \beta_L}\left[a^{\beta_C} + \frac{a^{\beta_R} - a^{\beta_L}}{\ln a} + \frac{a^{\beta_C} - a^{\beta_L}}{(\beta_C - \beta_L)\ln^2 a} + \frac{a^{\beta_R} - a^{\beta_C}}{(\beta_R - \beta_C)\ln^2 a}\right]. \tag{19}$$

In order to suppress the asymmetry caused by the right slope of the fuzzy exponent meaning that this part of the number plays a bigger role in the fuzzy power function than the left slope; a distortion has to be applied. The asymmetry can be re-adjusted by using asymmetric exponents and coefficients (see Figure 5).

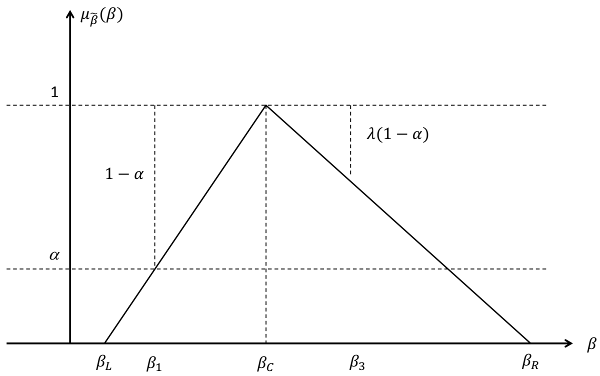


FIGURE 5. Asymmetric representation of exponents based on different $1 - (\alpha$-cuts)

Then instead of Equation (15) we obtain

$$y = \frac{1}{1 + (1 + \lambda)(1 - \alpha)}a^{\beta_C} + \frac{1 - \alpha}{1 + (1 + \lambda)(1 - \alpha)}a^{\beta_C - (1-\alpha)(\beta_C - \beta_L)} + \\ + \frac{(1 - \alpha)\lambda}{1 + (1 + \lambda)(1 - \alpha)}a^{\beta_C + (1-\alpha)\lambda(\beta_R - \beta_C)}, \tag{20}$$

where $0 \le \lambda \le \frac{1}{1-\alpha}$.

The general formula for any $m = 2 \cdot k + 1$ points can be given as follows, where $k = 1, 2, 3 \ldots$ is the number of $\alpha$-cuts:

$$y = \frac{1}{2 + k(1 + \lambda) - \sum_{j=1}^{m}\mu_{\widetilde{\beta}}(\beta_j)\lambda_j}a^{\beta_{k+1}} + \sum_{i=1}^{m}\frac{(1 - \mu_{\widetilde{\beta}}(\beta_i))\lambda_i}{2 + k(1 + \lambda) - \sum_{j=1}^{m}\mu_{\widetilde{\beta}}(\beta_j)\lambda_j}a^{\beta_i\lambda i}, \tag{21}$$

where

$$\lambda_l = \begin{cases} 1 & \text{if } l = 1, 2, \ldots, k + 1, \\ \lambda & \text{if } l = k + 2, \ldots, m. \end{cases}$$

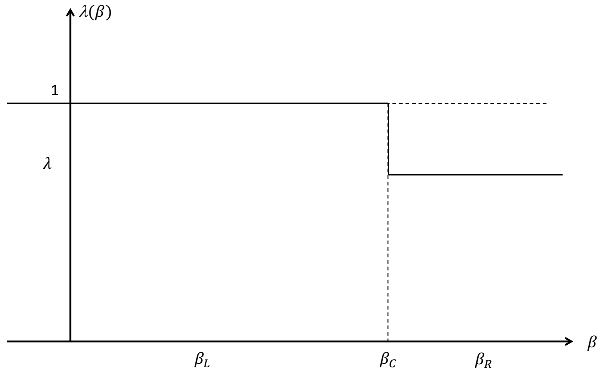


FIGURE 6. Characteristic function for $\lambda(\beta)$

The continuous formula is

$$y = \frac{a^{\beta_C} + \int_{\beta_L}^{\beta_R} (1 - \mu_{\tilde{\beta}}(\beta)) a^{\beta\lambda(\beta)} \lambda(\beta) \, \mathrm{d}\beta}{1 + \int_{\beta_L}^{\beta_R} (1 - \mu_{\tilde{\beta}}(\beta)) \lambda(\beta) \, \mathrm{d}\beta}, \tag{22}$$

where (see Figure 6)

$$\lambda(\beta) = \begin{cases} 1 & \text{if } \beta_L \le \beta \le \beta_C, \\ \lambda & \text{if } \beta_C < \beta \le \beta_R. \end{cases}$$

In the case when $\beta_L = \beta_C = \beta_R$ then our formulas lead to the crisp solution: $y = a^{\beta_C}$.

3.2. **Equivalence with the extension principle.** Using these parametric representations of fuzzy numbers a heuristic search can be easily applied to reach the maximum of Equations (5) and (8) or the minimum of Equations (9) and (13). Bacterial memetic algorithms were successfully used for similar problems [10,11,28]. The advantage of our parametric based exponent calculation compared to the extension principle is that our approach can provide a crisp solution faster than the extension principle does. If we use the extension principle for calculating the fuzzy power function then we obtain a fuzzy set as a solution. In order to get a crisp representation of this fuzzy set a defuzzification method should be used, which can be a time consuming task. In applications formulated in Section 2 lots of fuzzy power calculations are necessary. Evolutionary algorithms need lots of evaluation of individuals, which should be done quickly in order to do a more effective search for finding the optimal solution.

On the other hand, the extension principle generalizes the crisp functions for the fuzzy concept. Therefore, considering our approach, it is crucial to obtain the same result as the result given by the extension principle. The extension principle gives the fuzzy set as the power function, which needs to be defuzzified to obtain a crisp solution. In our parametric approach, the solution depends on the parameter $\lambda$. By adjusting this parameter properly the defuzzified result of the extension principle (which depends on the defuzzification technique) can be obtained by our method as well. In Equation (20), for given $\alpha$, $a$, $d$ (the defuzzified value of the exponent calculated by the extension principle) and $\beta_L$, $\beta_C$, $\beta_R$ an appropriate $\lambda$ value can be chosen in order for the result to be equal to the result calculated by the extension principle as follows:

$$\begin{aligned}
c_1 &= \alpha \cdot a^{\beta_C-(1-\alpha)(\beta_C-\beta_L)} - a^{\beta_C} - a^{\beta_C-(1-\alpha)(\beta_C-\beta_L)} + 2 \cdot d - \alpha \cdot d \\
c_2 &= (1-\alpha)a^{\beta_C} \\
c_3 &= (1-\alpha)(\beta_R - \beta_C) \\
c_4 &= \alpha \cdot d - d \\
c_5 &= \frac{c_4}{c_2} \\
c_6 &= \frac{c_1}{c_2} \\
\lambda a^{\lambda c_3} &+ \lambda c_5 = c_6. \qquad (23)
\end{aligned}$$

This equation can be approximately solved by iterative algorithms.

Considering the continuous formula in Equation (22), after calculating the integrals it becomes:

$$y = \frac{a^{\beta_C} - \frac{a^{\beta_R}\lambda - a^{\beta_L}}{\ln a} + \frac{a^{\beta_C} - a^{\beta_L}}{(\beta_C-\beta_L)\ln^2 a} - \frac{a^{\beta_C}\lambda - a^{\beta_R}\lambda}{(\beta_C-\beta_R)\lambda\ln^2 a}}{1 + \frac{\beta_C-\beta_L}{2} + \lambda\frac{\beta_R-\beta_C}{2}}. \qquad (24)$$

If $y$ is intended to be equal to any defuzzified value of the result calculated by the extension principle then this goal can be achieved in a similar way as in the discrete case above.

3.3. **Approximate estimation of time demand.** In order to see the efficiency of our approach compared with the extension principle, an approximate estimation of time demand can be useful.

The fastest version of our approach is the case where only one $\alpha$-cut is used. This is described in Equation (20). For computing the $y$ value we need 3 exponential calculations, 5 additions, 4 subtractions, 7 multiplications and 3 divisions. (If the same calculation appears in another part of the equation too, then we count this calculation only once.) In the case of the extension principle, the calculation of $\mu_{a^{\tilde\beta}}(x) = \mu_{\tilde\beta}(\log_a x)$ for a given $x$ is

$$\mu_{a^{\tilde\beta}}(x) = \mu_{\tilde\beta}(\log_a x) = \begin{cases} \dfrac{\log_a x - \beta_L}{\beta_C - \beta_L} & \text{if } \log_a x \le \beta_C, \\ \dfrac{\log_a x - \beta_R}{\beta_C - \beta_R} & \text{if } \log_a x > \beta_C. \end{cases} \qquad (25)$$

For this, we need 1 comparison, 1 logarithm calculation, 2 subtractions and 1 division. If the extension principle is applied for the exponential calculation, the function should be defuzzified. There are several methods proposed for defuzzification in the literature [30].

Center of Gravity (COG) is one of the most widely used defuzzification techniques. Applying COG for the exponential function given by the extension principle is

$$y = \frac{\int_{a^{\beta_L}}^{a^{\beta_R}} \mu_{a^{\tilde\beta}}(x)x\,\mathrm{d}x}{\int_{a^{\beta_L}}^{a^{\beta_R}} \mu_{a^{\tilde\beta}}(x)\,\mathrm{d}x}. \qquad (26)$$

If Equation (26) is discretized then we obtain:

$$y = \frac{\sum_{i=1}^{n} \mu_{a^{\tilde\beta}}(x_i)x_i}{\sum_{i=1}^{n} \mu_{a^{\tilde\beta}}(x_i)}, \qquad (27)$$

where $n$ represents the granularity of the discretization.

For this calculation we need $n$ comparisons, $n$ logarithm calculations, $(2n-2)$ additions, $2n$ subtractions, $n$ multiplications, and $(n+1)$ divisions. Comparing this time demand with our proposed technique even for a small $n$ the extension principle is worse in this sense.

If a simpler defuzzification method is considered, e.g., calculating the arithmetic mean of the three characteristic points of the exponent function, then we obtain:

$$y = \frac{a^{\beta_L} + a^{\beta_C} + a^{\beta_R}}{3}. \tag{28}$$

This result is the same as the result calculated by our method with Equation (20) in the case of $\alpha = 0$ and $\lambda = 1$. This equivalence is not surprising because if we do not apply distortion ($\lambda = 1$), and $\alpha = 0$, then the three characteristic points are obtained.

If Center of Maxima type defuzzification is applied, then the result given by the extension principle provides the core value, $a^{\beta_C}$. This result is the same as the result calculated by our method with Equation (20) in the case of $\alpha = 1$. This equivalence is also not surprising because in our approach if $\alpha = 1$ then the crisp solution must be obtained.

3.4. **Trapezoidal generalization.** Our approach can be generalized to trapezoidal shaped fuzzy numbers as well. Since the fuzzy numbers can be given using trapezoidal form, the general formulas are to be described as well. The trapezoidal fuzzy number can be interpreted very similar to the triangular one, the difference is that there are two core values ($\beta_{C1}$ and $\beta_{C2}$ in our case). In this case when only one $\alpha$-cut is used then Equation (15) becomes (see Figure 7):

$$\begin{aligned} y = {} & \frac{1-\alpha}{3-2\cdot\alpha} a^{\beta_{C1}-(1-\alpha)(\beta_{C1}-\beta_L)} + \frac{1-\alpha}{3-2\cdot\alpha} a^{\beta_{C2}+(1-\alpha)(\beta_R-\beta_{C2})} + \\ & + \frac{(\beta_{C1}-\beta_L)a^{\beta_{C1}} + (\beta_R-\beta_{C2})a^{\beta_{C2}}}{(3-2\cdot\alpha)(\beta_{C1}-\beta_L+\beta_R-\beta_{C2})}. \end{aligned} \tag{29}$$

In this formula the two extreme points of the core (the infimum and supremum of the core) are used to describe the core and they are weighted with the width of the corresponding slope of the trapezoid. If the core is singleton ($\beta_{C1} = \beta_{C2}$) then the formula for the triangular case (Equation (15)) is obtained.

Similarly, when more $\alpha$-cuts are used then Equation (16) becomes (see Figure 8):

$$\begin{aligned} y = {} & \sum_{i=1}^{k} \frac{1-\mu_{\widetilde{\beta}}(\beta_i)}{1+m-\sum_{j=1}^{m}\mu_{\widetilde{\beta}}(\beta_j)} a^{\beta_i} + \sum_{i=m-k+1}^{m} \frac{1-\mu_{\widetilde{\beta}}(\beta_i)}{1+m-\sum_{j=1}^{m}\mu_{\widetilde{\beta}}(\beta_j)} a^{\beta_i} + \\ & + \frac{(\beta_{C1}-\beta_L)a^{\beta_{C1}} + (\beta_R-\beta_{C2})a^{\beta_{C2}}}{\left[1+m-\sum_{j=1}^{m}\mu_{\widetilde{\beta}}(\beta_j)\right](\beta_R-\beta_L)} + \\ & + \sum_{i=k+2}^{m-k-1} \frac{\beta_i-\beta_{i-1}}{\left[1+m-\sum_{j=1}^{m}\mu_{\widetilde{\beta}}(\beta_j)\right](\beta_R-\beta_L)} a^{\beta_i}. \end{aligned} \tag{30}$$

In this formula $k$ is the number of $\alpha$-cuts, $m = 4\cdot k$, $k$ points are taken from the left slope, $k$ points from the right slope, and $2\cdot k$ points are taken from the core. The points taken from the core are equidistant. The weight of the infimum and supremum of the core are based on the width of the left and the right slope of the trapezoid, respectively, while the weights of the other core points are equal. The total weight is 1. The continuous formula is

$$\begin{aligned} y = {} & \frac{\int_{\beta_L}^{\beta_{C1}}(1-\mu_{\widetilde{\beta}}(\beta))a^{\beta}\,\mathrm{d}\beta + \int_{\beta_{C2}}^{\beta_R}(1-\mu_{\widetilde{\beta}}(\beta))a^{\beta}\,\mathrm{d}\beta}{1+\int_{\beta_L}^{\beta_R}(1-\mu_{\widetilde{\beta}}(\beta))\,\mathrm{d}\beta} + \\ & + \frac{(\beta_{C1}-\beta_L)a^{\beta_{C1}} + (\beta_R-\beta_{C2})a^{\beta_{C2}} + \int_{\beta_{C1}}^{\beta_{C2}} a^{\beta}\,\mathrm{d}\beta}{(\beta_R-\beta_L)\left[1+\int_{\beta_L}^{\beta_R}(1-\mu_{\widetilde{\beta}}(\beta))\,\mathrm{d}\beta\right]}. \end{aligned} \tag{31}$$

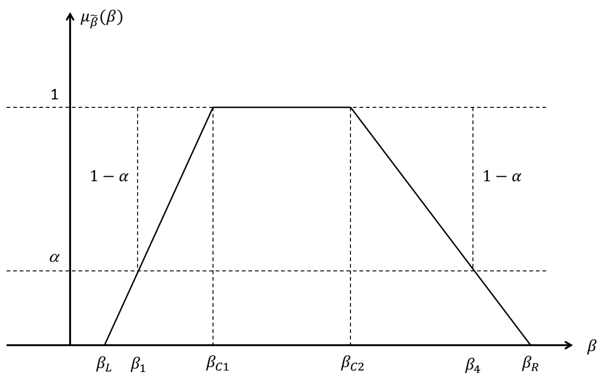


FIGURE 7. Weights and exponents based on $1-(\alpha$-cuts) in trapezoidal case

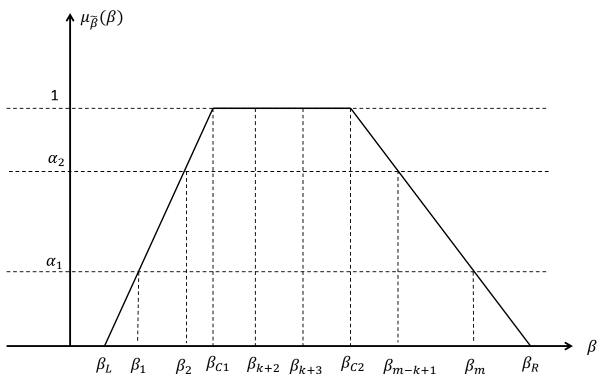


FIGURE 8. Exponents for $m$ functions based on $1-(\alpha$-cuts) in trapezoidal case

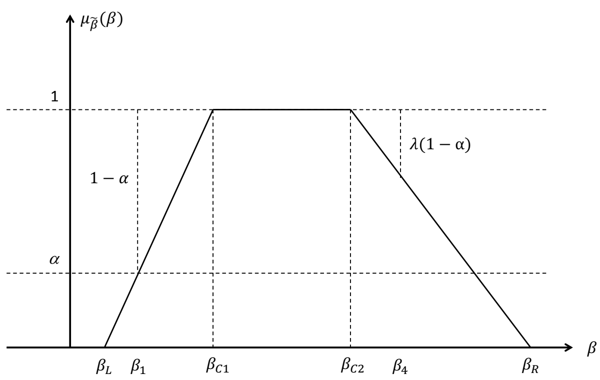


FIGURE 9. Asymmetric representation of exponents based on different $1-$ ($\alpha$-cuts) in trapezoidal case

Distortion can be applied on the right slope of the trapezoid as presented in Figure 9, similarly to the triangular based case. The formula for one $\alpha$-cut becomes:

$$y = \frac{1-\alpha}{1+(1+\lambda)(1-\alpha)} a^{\beta_{C1}-(1-\alpha)(\beta_{C1}-\beta_L)} + \frac{(1-\alpha)\lambda}{1+(1+\lambda)(1-\alpha)} a^{\beta_{C2}+(1-\alpha)\lambda(\beta_R-\beta_{C2})} + \\ + \frac{(\beta_{C1}-\beta_L)a^{\beta_{C1}} + (\beta_R-\beta_{C2})a^{\beta_{C2}}}{[1+(1+\lambda)(1-\alpha)](\beta_{C1}-\beta_L+\beta_R-\beta_{C2})}. \tag{32}$$

For more a-cuts:

$$\begin{aligned} y = {} & \sum_{i=1}^{k} \frac{1-\mu_{\widetilde{\beta}}(\beta_i)}{1+k(3+\lambda)-\sum_{j=1}^{m}\mu_{\widetilde{\beta}}(\beta_j)\lambda_j} a^{\beta_i} + \\ & + \sum_{i=m-k+1}^{m} \frac{(1-\mu_{\widetilde{\beta}}(\beta_i))\lambda}{1+k(3+\lambda)-\sum_{j=1}^{m}\mu_{\widetilde{\beta}}(\beta_j)\lambda_j} a^{\beta_i\lambda} + \\ & + \frac{(\beta_{C1}-\beta_L)a^{\beta_{C1}} + (\beta_R-\beta_{C2})a^{\beta_{C2}}}{\left[1+k(3+\lambda)-\sum_{j=1}^{m}\mu_{\widetilde{\beta}}(\beta_j)\lambda_j\right](\beta_R-\beta_L)} + \\ & + \sum_{i=k+2}^{m-k-1} \frac{\beta_i-\beta_{i-1}}{\left[1+k(3+\lambda)-\sum_{j=1}^{m}\mu_{\widetilde{\beta}}(\beta_j)\lambda_j\right](\beta_R-\beta_L)} a^{\beta_i}, \end{aligned} \tag{33}$$

where

$$\lambda_l = \begin{cases} 1 & \text{if } l = 1, 2, \ldots, m-k, \\ \lambda & \text{if } l = m-k+1, \ldots, m. \end{cases}$$

The continuous formula is

$$\begin{aligned} y = {} & \frac{\int_{\beta_L}^{\beta_{C1}}(1-\mu_{\widetilde{\beta}}(\beta))a^{\beta}\,\mathrm{d}\beta + \int_{\beta_{C2}}^{\beta_R}(1-\mu_{\widetilde{\beta}}(\beta))\lambda a^{\beta\lambda}\,\mathrm{d}\beta}{1+\int_{\beta_L}^{\beta_R}(1-\mu_{\widetilde{\beta}}(\beta))\lambda(\beta)\,\mathrm{d}\beta} + \\ & + \frac{(\beta_{C1}-\beta_L)a^{\beta_{C1}} + (\beta_R-\beta_{C2})a^{\beta_{C2}} + \int_{\beta_{C1}}^{\beta_{C2}} a^{\beta}\,\mathrm{d}\beta}{(\beta_R-\beta_L)\left[1+\int_{\beta_L}^{\beta_R}(1-\mu_{\widetilde{\beta}}(\beta))\lambda(\beta)\,\mathrm{d}\beta\right]}, \end{aligned} \tag{34}$$

where

$$\lambda(\beta) = \begin{cases} 1 & \text{if } \beta_L \le \beta \le \beta_{C2}, \\ \lambda & \text{if } \beta_{C2} < \beta \le \beta_R. \end{cases}$$

The above formulas lead to the triangular formulas if $\beta_{C1} = \beta_{C2}$.

4. **Numerical Example.** In this section the fuzzy exponent calculation is investigated by numerical examples in the case of triangular and trapezoidal fuzzy exponents. We give an example of how parameter $\lambda$ should be adjusted in order to obtain the same result for a given triangular fuzzy exponent following the extension principle. Time comparison between the extension principle and our method will be presented as well.

4.1. **Triangular example.** In the first numerical tests triangular fuzzy numbers are investigated. Considering the $2^{\widetilde{\beta}}$ expression, where 2 is a crisp number and $\widetilde{3}$ is a fuzzy number with a triangular shaped membership function with a core value 3 and $[1, 5]$ support interval, the fuzzy exponent calculated by the extension principle can be seen in Figure 10.

Using our approach with Equation (20) with $\lambda = 1$ (without distortion) the result for different $\alpha$-cuts is illustrated in Figure 11.

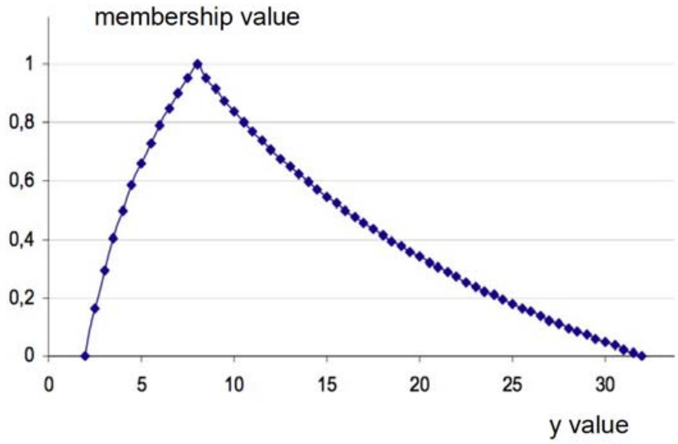


FIGURE 10. Fuzzy exponent calculated by the extension principle

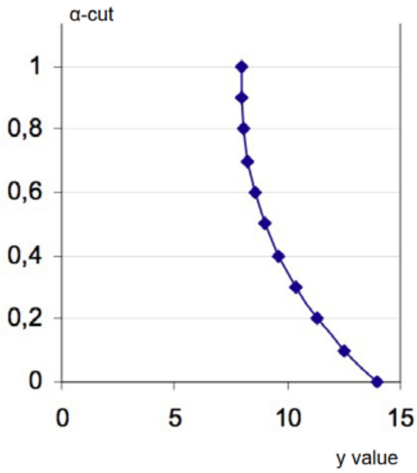


FIGURE 11. Fuzzy exponent calculated by Equation (20)

Comparing Figures 10 and 11 we can diagnose that the uncertainty is larger in the result calculated by the extension principle. The result calculated by our approach is somehow "closer" to common sense. Also if distortion is applied, meaning that $\lambda < 1$, then the uncertainty in this case is even less, e.g., for $\lambda = 0.8$ the result is shown in Figure 12.

Next, we analyze how $\lambda$ should be chosen for a given $\alpha$ value using Equation (23) in order to obtain the same result following the extension principle. The results can be seen in Figures 13-16.

From Figure 5 it can be seen that according to the definition of $\lambda$, the lower the value of $\lambda$ the more severe the distortion. The results presented in Figures 13-16 confirm this statement. For low $\alpha$ values, the $\alpha$-cut of $\widetilde{3}$ is wider, thus a more severe distortion is needed for the right slope of the triangular fuzzy set. For example, for $\alpha = 0.1$ the $\lambda$ must be around 0.6 if the defuzzified value is 8.

4.2. **Time comparisons.** We compared the time demand of the exponent calculation performed by our method and by the extension principle. We executed 100 million exponent calculations implemented in C on Intel Core 2 Duo P8600 @ 2.40 GHz Processor with 2 GB RAM using Windows Vista 32-bit operating system. Our method completed

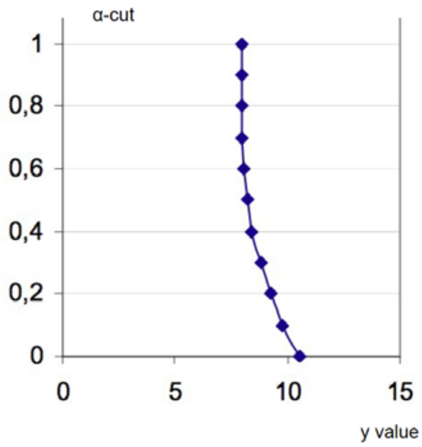


FIGURE 12. Solution for $\lambda = 0.8$

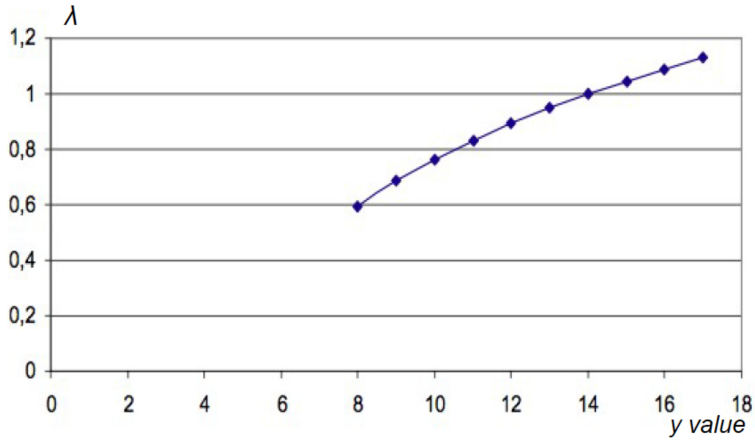


FIGURE 13. $\lambda$ values depending on the defuzzified value ($\alpha = 0$)

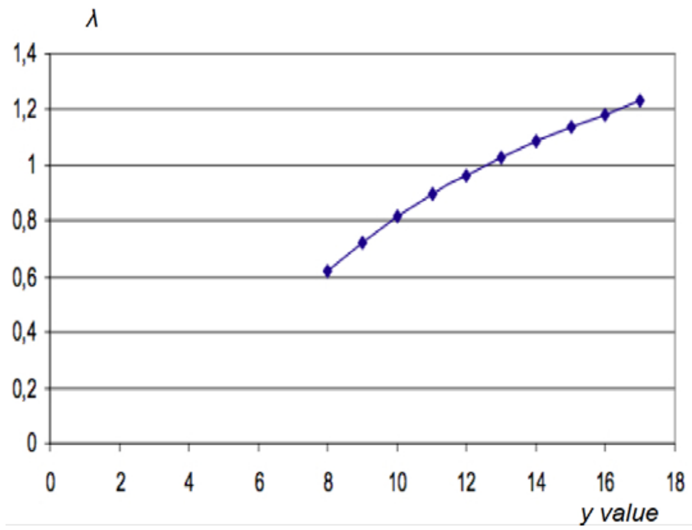


FIGURE 14. $\lambda$ values depending on the defuzzified value ($\alpha = 0.1$)

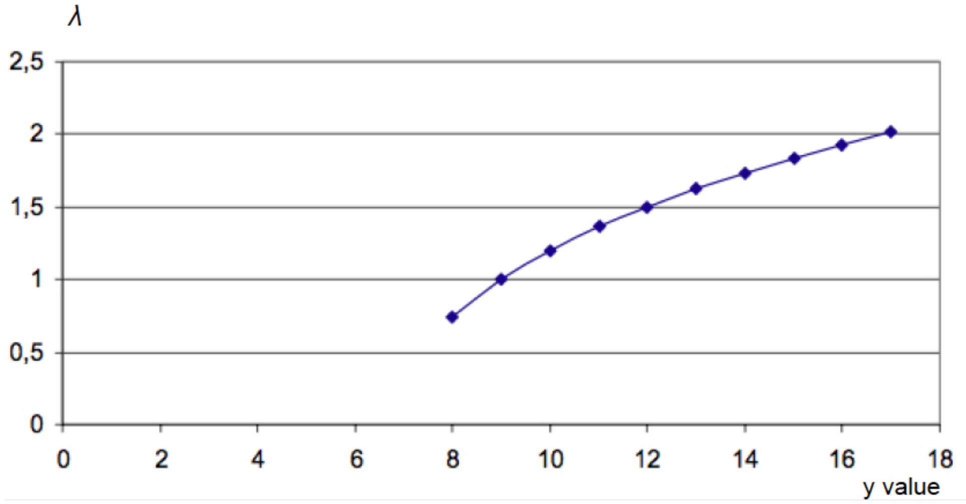


FIGURE 15. $\lambda$ values depending on the defuzzified value ($\alpha = 0.5$)

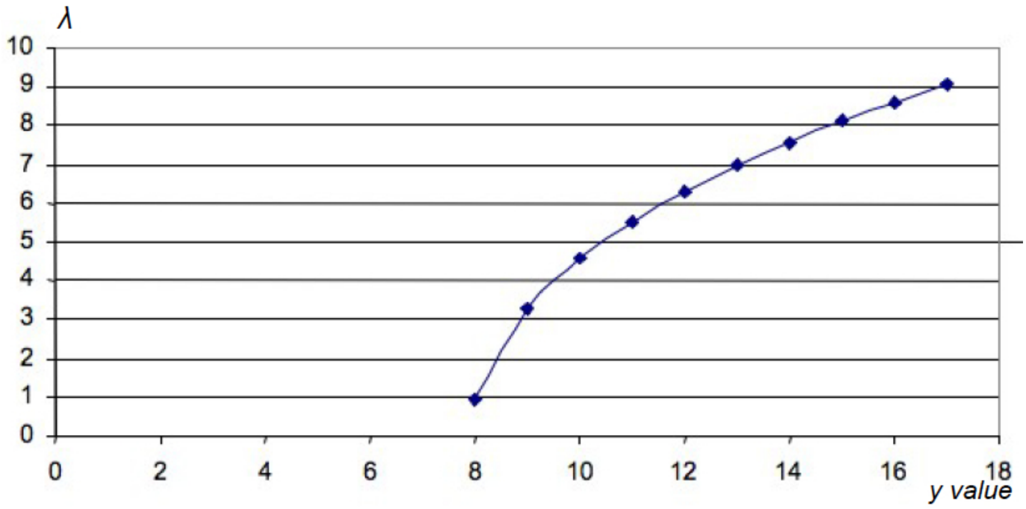


FIGURE 16. $\lambda$ values depending on the defuzzified value ($\alpha = 0.9$)

the calculations in 39 seconds. The extension principle with COG defuzzification completed the calculations in 253 seconds when the granularity of the discretization ($n$ in Equation (27)) was 10, and in 145 seconds when $n$ was 5. When $n$ is less than 5 then the discretization differs too much from the real COG value. According to this result we can state that our proposed approach was 6.49 times faster (or by applying the rougher discretization 3.72 times faster) than the classical method.

4.3. **Trapezoidal example.** In the trapezoidal fuzzy exponent calculations we compared the trapezoidal case with the triangular one. We used 2 for the crisp base and $\widetilde{3}$ as the trapezoidal fuzzy exponent with $[1, 5]$ support interval and with $\beta_{C1} = 2.5$ and $\beta_{C2} = 3.5$. So the exponent is similar to the triangular one in Section 4.1 but its core is an interval instead of a singleton. Table 1 shows the results for some $\alpha$ and $\lambda$ values in the case of the triangular fuzzy exponent used in Section 4.1 and for the trapezoidal one.

From Table 1 it can be seen that if the distortion is stronger (i.e., $\lambda$ is smaller) then the difference between the triangular and trapezoidal solutions is less. The reason for that is that a stronger distortion can reduce the role of the right slope, so in this case the

TABLE 1. Results for some $\alpha$ and $\lambda$ values

| $\alpha$ | $\lambda$ | $2^{[1,3,5]}$ | $2^{[1,2.5,3.5,5]}$ |
|---|---|---|---|
| 0 | 0 | 5 | 5.24 |
| 0 | 0.5 | 7.2 | 8.00 |
| 0 | 1 | 14 | 14.16 |
| 0.2 | 0 | 5.62 | 5.81 |
| 0.2 | 0.5 | 7.13 | 7.87 |
| 0.2 | 1 | 11.35 | 12.02 |
| 0.5 | 0 | 6.67 | 6.78 |
| 0.5 | 0.5 | 7.33 | 7.91 |
| 0.5 | 1 | 9 | 9.84 |
| 0.8 | 0 | 7.68 | 7.84 |
| 0.8 | 0.5 | 7.79 | 8.20 |
| 0.8 | 1 | 8.09 | 8.71 |
| 1 | any | 8 | 8.49 |

right part of the trapezoid has less influence in the result making it more similar to the triangular case in this way.

## 5. Conclusion.

The approximation of fuzzy arithmetic operators are proposed in the literature [31] as the application of the exact extension principle. These solutions require relatively large computational effort and resources, which is a key issue especially in evolutionary algorithms. Soft computing applications set different requirements regarding the representation of uncertain, fuzzy values. These requirements are based on the nature of uncertainty and fuzziness, and the characteristics and features of applied algorithms must be assessed as well. There are several methods that can be a theoretic foundation of parametric representation but in the case of the power function and fuzzy exponents the asymmetric features of function values must be handled as well, since for practical reasons (computation time and resources) the continuous formulas cannot be used efficiently. In this paper a simple solution for parametric representation of fuzzy power function was presented that enables the decision-maker to fit the model to any defuzzification technique by calculating and selecting the appropriate $\lambda$ parameter. Also the appropriate selection of the number and position of $\alpha$-cuts can fine-tune the model and can give a proper representation of a real life decision situation.

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