International Journal of Innovative Computing, Information and Control
Volume **1**, Number **3**, September **2005**

ICIC International ©2005 ISSN 1349-4198
pp. **289–301**

BEHAVIOR ANALYSES OF PHASE BOUNDARIES IN THE STOCHASTIC MEAN CURVATURE FLOW BY THE LEVEL SET METHOD

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Received November 2004; revised April 2005

Abstract. *An influence of a random noise on a mean curvature flow of a phase boundary is investigated. The phase boundary is defined as a curve or a surface separating different physical states such as a water-ice interface. The mean curvature flow is a motion that the phase boundary moves with a normal velocity equals the mean curvature at each point on the phase boundary. Since we consider the influence of the random noise on the mean curvature flow, we need to treat the system formulated by a stochastic level set equation, which is a nonlinear stochastic partial differential equation with a unique solution in the sense of a stochastic viscosity solution. In numerical simulations, sample behaviors of the stochastic mean curvature flows of barbell and torus shapes are studied. It is numerically shown that the sample behaviors of the phase boundary in stochastic mean curvature flows have a possibility of changing topologically by the random noise without any fattening phenomena.*

Keywords: Phase boundary, Stochastic mean curvature flow, Topology of phase boundary, Numerical simulations, Level set method, Stochastic viscosity solution

1. **Introduction.** A phase boundary is defined as a curve or a surface separating different physical states. A mean curvature flow is a motion that the phase boundary moves with a normal velocity equals the mean curvature at each point on the phase boundary. An analysis of the mean curvature flow is one of the important problems in many fields of research including material science and computer vision. For example, the grain boundary in annealing of metal moves according to the mean curvature flow [1]. The mean curvature flow plays a very important role in computer vision problems like restoration and segmentation in image processing [2]. The level set method [2-4] is a powerful tool to analyze the mean curvature flow. This method requires to solve the nonlinear partial differential equation, to which notion of a weak solution [5] based on integration by part cannot be applied because it is not a partial differential equation of divergence type. In order to overcome this difficulty, a viscosity solution [6-8], one of the weak solutions, has been developed based on the maximum principle of the partial differential equation. Since this paper is concerned with the analyses of the influence of the random noise on the mean curvature flow by numerical simulations, we need to treat stochastic mean curvature flow. The stochastic mean curvature flow is formulated by a stochastic level set equation with the unique solution in the sense of the stochastic viscosity solution [9-12]. In numerical