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ANALYSES OF PATTERN FORMATION PROCESSES IN STOCHASTIC ACTIVATOR-INHIBITOR SYSTEMS WITH SATURATION IN GROWTH DOMAINS

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Abstract. *In the natural world, we often observe many kinds of spatio-temporal patterns such as patterns of animal coat, crystals of snow, etc. An analysis of generation processes of such patterns is very important as a basic problem in various fields of engineering including biological and material engineering. Among many spatio-temporal patterns, we study the pattern formation processes of sea shells from the viewpoint of biological engineering. As the mathematical model to generate the sea shells patterns, we consider an activator-inhibitor system, which is one of the reaction diffusion systems. Taking into consideration the fact that some kind of disturbance exists to a greater or lesser degree in the natural world, we propose stochastic activator-inhibitor systems as the mathematical model of pattern formation processes of sea shells. We analyze the influence of the random noise on the shell pattern formations and show that spatio-temporal patterns generated by the proposed model are robust for the random disturbance.*
Keywords: Activator-inhibitor systems, Stochastic reaction diffusion equations, Numerical simulations, Pattern formation

1. **Introduction.** Many studies of spatio-temporal pattern formations have been performed in the past, see for example, [1-8] and the references therein. Most of such studies are considered in the deterministic framework. Although the study in [2] treats stochastic problems of pattern formations, pattern formations of sea shells are not considered. In this paper, we study the pattern formation processes on sea shells by the stochastic activator-inhibitor systems from the viewpoint of biological engineering. The activator-inhibitor system considered here is one of the reaction diffusion systems [3,5] with two components, activator and inhibitor. The activator-inhibitor system can create complex patterns by the interaction between activator and inhibitor. One of the characteristic aspects in analysis of pattern formations on sea shells [3] is that sea shells grow with time, so that in modeling of generation of shell patterns, we must consider expansion of a spatial region in the model. Besides of expansion of the spatial region, there are several factors that determine the shell patterns such as width of stripes and figuration of patterns. As one of such factors, we can cite effect of saturation of the system state. Taking into consideration the fact that some kinds of disturbance exists to a greater or lesser degree in the natural world (see for example, [9,10]), instead of the deterministic model, we propose four types of stochastic models to create shell patterns, i.e., stochastic