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NOISE-INDUCED SYNCHRONIZATION OF UNCOUPLED NONLINEAR SYSTEMS

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Abstract. *We first derive a recursion formulae of transition probability of the noise-induced synchronization arising in a pair of identical uncoupled logistic maps linked by common noisy excitation only. It is proved that the formulae has a delta-type stationary solution which represents the perfect synchronization with probability 1. We then focus on ensemble averages of synchronization errors to investigate robustness of the solution against parameters mismatch. The numerical results shows that both the discrete- and continuous-time systems exhibit significant degree of the robustness.*
Keywords: Nonlinear dynamics, Synchronization, Markov process, Uncoupled systems

1. **Introduction.** One of the most surprising results of the last few decades in the field of the nonlinear dynamics is that a dynamical system and its copies can be synchronized with each other when they are linked by the common excitation only. For instance, the main idea of the chaotic synchronization resides in sending the output of a driving system to response systems of the same structure whose conditional Lyapunov exponents are all negative [1, 2]. Recently, stochastic counter parts of the chaotic synchronization have also been developed, showing that common excitations as elements of the original system are not necessary to produce the synchronization and can be replaced by external noisy signals. This kind of noise-induced synchronization of the dynamical system with its copies can easily be found in nonlinear systems, such as the discrete maps [3, 4], the Lorenz system [3], the Duffing oscillator [5], the single mode CO_2 laser [6], and the uncoupled neurons [7]. One of the most important results in these studies is that the perfect synchronization may arise under some suitable conditions [6, 5, 3]. Moreover, it also should be noted that the perfect synchronization exhibits significant degree of robustness against mismatches in the copies such as the parameters mismatch [5] and the independent random fluctuation of the copies [3, 8].

Physical properties of the noise-induced synchronization have mainly been characterized by Lyapunov exponents and their scaling laws [9, 4, 5, 3] in the sense of the chaotic synchronization and related fields. However, there can be another option from random dynamical systems point of view [10]. We have already reported that the noise-induced perfect synchronization of the van der Pol systems, the Duffing systems, and the nonlinear retarded systems can be characterized by point attractors of random invariant measures [11, 8], showing that if the system and its copy are subjected to a common sample path