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AN INFORMATION THEORETIC APPROACH TO OPTIMIZATION OF LINEAR OBSERVATIONS FOR THE KALMAN-BUCY FILTER

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Abstract. *We are concerned with a problem of the optimal selection of the gain matrix of a linear observation mechanism for the Kalman-Bucy filter. By introducing an information theoretic constraint, we obtain a gain matrix which maximizes the reduction speed of an weighted estimation error. In this paper, we are especially concerned with the case where the weighting matrix is not positive definite but has positive eigenvalues as many as the dimension of the observation. By this condition, we can treat an observation with any dimension. This result is more general than the one obtained by the author using a formulation in the optimal transmission framework.*
Keywords: State estimation, Kalman filter, Gaussian processes, Optimization of observations

1. **Introduction.** The problem of optimization of observations associated with the Kalman-Bucy filter [1] has been studied in many literatures [2-5,8-9]. In most of them, the problem is formulated as a kind of optimal control problem with a quadratic performance criterion. Logothetis et. al [5], discussed a sensor scheduling problem for linear observations via information theoretic criteria. They treated a discrete-time linear system, and under the constraint that only one sensor can be used at any time, they derived equations for the optimal sensor gains which maximize the mutual information between the signal and the observation. A similar kind of problem is that of sensor allocation. Namely, for a number of sensors, which are simultaneously available at one time, we want to determine a set of gains for the sensors in such a way that the estimation error is minimized. In the view point of the Shannon's information theory, the observation is better when the mutual information between the signal and the observation takes a larger value. However, the mutual information, according to the Shannon's theory, cannot take a larger value than that of the transmission capacity. Also, the set of sensor gains which produces the same mutual information is not unique, and has a certain degree of freedom. In this paper, we consider the optimization of the sensor gains over this degree of freedom. Namely, we obtain the optimal set of gains which maximizes the reduction speed of an weighted estimation error and by which the mutual information takes a preassigned value of the capacity. We are especially concerned with the case where this weighting matrix is not positive definite but has m-positive eigenvalues, here m is the dimension of the observation. This condition makes it possible for us to treat observations with any dimension.