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SMOOTHING SPLINE CURVES AND SURFACES FOR SAMPLED DATA

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Abstract. *We consider the problem of designing optimal smoothing spline curves and surfaces for a given set of discrete data. For constructing curves and surfaces, we employ normalized uniform B-splines as the basis functions. First we derive concise expressions for the optimal solutions in the form which can be used easily for numerical computations as well as mathematical analyses. Then, assuming that a set of data in a plane is obtained by sampling some curve with or without noises, we prove that, under certain condition, optimal smoothing splines converge to some limiting curve as the number of data increases. Such a limiting curve is obtained as a functional of given curve to be sampled. The case of surfaces is treated in parallel, and it is shown that the results for the case of curves can be extended to the case of surfaces in a straightforward manner.*
Keywords: B-splines, Optimal smoothing splines, Asymptotic analysis, Statistical analysis

1. **Introduction.** The problem of optimal design of approximating or interpolating curves and surfaces for a given set of data arises in various fields of engineering and sciences. In particular, spline functions have been used frequently in such fields as computer aided design [1], numerical analysis [2], image processing [3], trajectory planning problems [4, 5], and data analysis in general [6]. Recently, the theory of smoothing splines is used to generate cursive characters based on an idea that the underlying writing motions become smooth [7, 8].

Thus splines have been studied extensively (e.g. [6]), and in particular, an approach based on optimal control theory has been employed for generating piecewise polynomial