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ON NEW SMOOTHING ALGORITHMS FOR DISCRETE-TIME LINEAR STOCHASTIC SYSTEMS WITH UNKNOWN DISTURBANCES

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Abstract. *We consider discrete-time linear stochastic systems with unknown inputs (or disturbances) and study two types of smoothing problems for these systems, i.e., the fixed-interval smoothing and the fixed-lag smoothing. We derive smoothing algorithms which are robust to the unknown inputs from the optimal filter and the fixed-point smoother proposed in our previous papers. Moreover, we show that our new algorithms reduce to the well known optimal smoothers derived from the Kalman filter when unknown inputs disappear. Thus, our new algorithms are consistent with the known smoothing algorithms for systems without unknown inputs.*
Keywords: Stochastic systems, Smoother, Optimal filter, Unknown inputs

1. Introduction.

We consider discrete-time linear stochastic systems with unknown inputs (or disturbances) and investigate recursive algorithms for estimating states of these systems. If most modelled systems made by engineers are very accurate representations of real systems, we do not need to consider systems with unknown inputs. However, in practice, most modelled systems are not free from modelling errors and should be considered as systems with unknown inputs.

The most frequently discussed problem on state estimation is the optimal filtering problem which investigates the optimal estimate of state x_t at time t or x_{t+1} at time $t+1$ with minimum variance based on the observation $\mathbf{Y}_t$ of the outputs $\{y_0, y_1, \cdots, y_t\}$, i.e., $\mathbf{Y}_t = \sigma\{y_s, s = 0, 1, \cdots, t\}$ (the smallest σ-field generated by $\{y_0, y_1, \cdots, y_t\}$ (see e.g., [19], Chapter 4)). Optimal filters for stochastic systems with unknown inputs have been investigated by many researchers in e.g., [4]-[13], [21], [22] and [24].

In this paper, we consider smoothing problems which allow us time lag for computing estimates of states. Namely, the optimal estimate $\hat{x}_{t-L/t}$ of the state x_{t-L} with $L > 0$ based on the observation $\mathbf{Y}_t$ will be discussed. We often classify smoothing problems into the following three types. For the first problem, the fixed-point smoothing, we investigate the optimal estimate $\hat{x}_{k/t}$ of the state x_k for a fixed k based on the observations $\{\mathbf{Y}_t, t = k+1, k+2, \cdots\}$. Algorithms to compute $\hat{x}_{k/t}, t = k+1, k+2, \cdots$, recursively are called fixed-point smoothers. For the second problem, the fixed-interval smoothing, we investigate the optimal estimate $\hat{x}_{t/N}$ of the state x_t at all times $t = 0, 1, \cdots, N$ based on the observation $\mathbf{Y}_N$ of all the outputs $\{y_0, y_1, \cdots, y_N\}$. Fixed-interval smoothers are algorithms computing $\hat{x}_{t/N}, t = 0, 1, \cdots, N$ recursively. The third problem, the fixed-lag smoothing, is to investigate the optimal estimate $\hat{x}_{t-L/t}$ of the state x_{t-L} based on the observation $\mathbf{Y}_t$ for a given $L \geq 1$. Fixed-lag smoothers are algorithms computing