International Journal of Innovative Computing, Information and Control
Volume **4**, Number **1**, January **2008**

ICIC International ©2008 ISSN 1349-4198
pp. 143–152

ON A ONE-DIMENSIONAL CHAOTIC DISCRETE DYNAMICAL SYSTEM WITH PIECEWISE UNIFORM INVARIANT DENSITY

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Received February 2007; revised July 2007

Abstract. *Nonlinear systems described by the simple mathematical model often exhibit extremely complicated behavior called chaos. In this paper, chaotic systems, described by the one-dimensional difference equation, are investigated. A class of nonlinear discrete system with invariant density, which is piecewise uniform, is newly introduced.*
Keywords: Chaos, Invariant measure, One-dimensional nonlinear map, Lyapunov exponent, Frobenius-Perron equation

1. **Introduction.** In scientific and engineering systems, it is well known that nonlinear systems with a simple mathematical model often exhibit extremely complicated behavior known as chaos [1,12]. In this paper, chaotic behavior, exhibited by the following deterministic system described by the one-dimensional difference equation, is discussed:

$$x_{n+1} = f\left(x_n\right), \; n = 0, 1, 2, \ldots \tag{1}$$

$$f : I \to I, I = [0, 1]$$

Concerning the nonlinear function $f(x)$, which determines the characteristic of the nonlinear system (1), if a function $p(x)$, $x \in [0, 1]$ satisfies the following Frobenius-Perron equation [6,11]

$$p\left(y\right) = \sum_{x \in f^{-1}(y)} \frac{p(x)}{|f'(x)|}, \quad f'(x) = \frac{df(x)}{dx} \tag{2}$$

and satisfies the following normalized condition

$$\int_I p(x)dx = 1$$

then the function $p(x)$ is called the invariant density of the nonlinear function $f(x)$ [9]. Important statistical information of chaotic behavior of the system (1) is included in the invariant density $p(x)$. For example, for any subset $A \subset R^1$, the equation

$$\lim_{n \to \infty} \frac{1}{n} \sum_{k=0}^{n-1} \chi_A\left(x_k\right) = \int_I \chi_A(x) p(x) dx \tag{3}$$

holds, where

$$\chi_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$